

Macroeconomics Comprehensive Examination Question 1, January 2008

Consider an economy with one consumer and a government. The consumer has the following preferences:

$$(1) \quad E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

where $0 < \beta < 1$, where c and l refer to consumption and labor supply, and where the period utility function satisfies

$$(2) \quad U(c, l) = \log(c - (1/2)l^2)$$

Assume that the consumer is unable to borrow or save. She chooses consumption and labor supply in a static manner each period in order to maximize utility subject to the following budget constraint:

$$(3) \quad c_t = (1 - \tau_t) w_t l_t + g_t$$

where w_t is the wage rate at time t ; τ_t is the proportional tax rate levied by the government at time t ; and g_t is a lump-sum transfer given to the consumer by the government at t . Wages, taxes and government spending are all potentially stochastic from the consumer's point of view.

(a) Warm-up question: solve the consumer's problem and write down an expression for the consumer's indirect utility at time t as an analytic function of w_t , τ_t , and g_t . Re-write the consumer's objective function (1) replacing the direct utility function $U(c_t, l_t)$ with this indirect utility function. (Don't forget to use the functional form (2) in writing down this indirect utility function).

Now, turn to the problem of the government. The government has access to capital markets and can borrow or save from international capital markets at an interest rate of r . Each period it chooses a proportional tax rate τ_t , which in principle can be positive or negative (if τ is negative, then it represents a subsidy to the consumer). It also chooses a lump-sum transfer g_t to give to the consumer. To make the problem interesting, we assume that the lump sum transfer must be non-negative, so that the government cannot raise funds using lump-sum taxation. The government's constraints are thus

$$(4) \quad b_{t+1} = (1+r)(b_t + \tau_t w_t l_t - g_t)$$

$$(5) \quad g_t \geq 0$$

where b_t is the level of the government's assets at time t , where b may be positive or negative (a negative value indicating government debt).

The government is benevolent and chooses its path of (τ_t, g_t) in a dynamic manner to maximize the consumer's lifetime expected utility, subject to constraints (4) and (5). The government takes into account the impact of its tax and transfer policies on the consumer's choices of consumption and labor supply. Since we have an analytic expression for the consumer's optimal behavior in this problem, it is convenient to represent the government's objective function as the re-written version of (1), in which the direct utility function $U(c,l)$ is replaced by the indirect utility function you solved for in part (a).

(b) Identify the state and control variables for the government's problem. Write down the Bellman's equation, incorporating both constraints (4) and (5). Take first order and envelope conditions and find an Euler Equation involving wages, tax rates, and government transfers at dates t and $t+1$. The multiplier on the constraint (5) should not appear in this Euler equation.

(c) Prove that the government's optimal policy must satisfy the following:

(i) If $g_t > 0$, then $\tau_t = 0$. In words, it is never optimal to have non-zero taxes and positive transfers at the same time.

(ii) If $g_t = 0$, then $\tau_t \geq 0$, with $\tau_t > 0$ if the constraint (5) is binding at time t . In words, the government never finds it optimal to use subsidies.

(d) Explain the intuition for results (i) and (ii). Why is it never optimal to have non-zero taxes and positive transfers at the same time? Why is it never optimal to use subsidies?