\[ u(C_t) = (Y_t - C_t)^2 \]
\[ u''(C_t) = 2Y_t^2 > 0 \]

\[ a \]
\[
\begin{align*}
    r_1 &= r_2 = 1, \quad r_3 \text{ is random.} \\
    \text{Maximand, budget constraint:} \\
    \max_{c_1, c_2, c_3} & \quad E \left[ u(c_1) + \beta u(c_2) + \beta^2 u(c_3) \right] \\
    \text{s.t.} & \quad a_{11} = R(a_1 + w - C_t) \quad t = 1, 2, 3 \\
    a_4 &= 0 \quad \text{and} \quad a_1 \text{ is positive and given.}
\end{align*}
\]

Note: Because of convex utility function, the standard technique (Like Dynamic Programming) cannot be applied here. 

The lifetime budget constraint is given by,
\[
    a_1 + w + \frac{w}{1+r} + \frac{w}{(1+r)^2} = c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} \\
    \text{denote it as } H
\]

With convex utility function, the individual maximizes his utility by concentrating all consumption in one period.

When \( c_2 = c_3 = 0 \Rightarrow c_1 = a_1 + H \)
\( c_1 = c_3 = 0 \Rightarrow c_2 = (1+r)(a_1 + H) \)
\( c_1 = c_2 = 0 \Rightarrow \)
First, consider a non-stochastic case where \( Y_1 = Y_2 = Y_3 = 1 \). Then consuming total human and non-human wealth in period one, gives total utility equal to,

\[
u(C_1) = C_1^2 = (a_1 + H)^2.
\]

If consumed in period 2, \( \beta U(C_2) = \beta (1+r)^2 (a_1 + H)^2 \).

If consumed in period 3, \( \beta^2 U(C_3) = \beta^2 (1+r)^4 (a_1 + H)^2 \).

So for case \( \beta(1+r) = 1 \), individual will prefer to consume zero in first two periods and everything in the last period.

However, if \( \beta(1+r)^2 = 1 \), then he will be indifferent between all his consumption options in either of the three periods.

Now when \( Y_3 \) is random;

For convex utility function \( E U(Y_3 C_3) > U(E Y_3 C_3) = U(C_3) \), expected

Hence in presence of random \( Y_3 \), the consumer's utility in period 3 is even higher than his utility, when \( Y_3 \) is certain to be 1

Thus, the optimal consumption for the consumer will be

\[
(C_1^*, C_2^*, C_3^*) = [0, 0, (1+rc)^2(a_1+H)]
\]
For convex utility function, the risk premium is negative. Hence, in our case the consumer will trade his lottery for a certain consumption bundle only if he is paid an amount equal to \( \frac{\Delta x}{\sigma} \), so that his expected utility from the lottery \( C = A \) exactly equals his utility from the consuming a certain amount \( = D \).

However, given identical consumers, such a state contingent asset will not exist in equilibrium as all consumers will be on one side of the market.

\[ V(A_t) = \max_{C_t} u(C_t) + E_t \beta V(A_{t+1}) \]

\[ A_{t+1} = R(A_t + w - C_t) \]

where \( E_t \) is taken over future realizations of \( X_t \)'s.

Dynamic Programming cannot be applied here. It can be seen from our finite horizon solution that individual will prefer to postpone his consumption to the future and for certain