2. Consider the standard Diamond OLG model with utility of an individual born at $t$ given by

$$U(c^y_t, c^y_{t+1}) = \gamma \ln c^y_t + (1 - \gamma) \ln c^y_{t+1}$$

(a) Write down the budget constraints of the individual. Derive the saving function of the young.

(b) Let the aggregate production function display constant returns to scale and assume labour-augmenting technical progress at a constant exogenous rate $g > 0$. Derive the fundamental difference equation of the model.

(c) Illustrate the dynamics in a diagram. Is the model necessarily “well-behaved”? Comment.

(d) If your answer in part (c) is “no”, what restrictions would you want to put on the production function to address this problem?

(e) Suppose the economy is in steady state. Then at time $t_1$ an unanticipated permanent positive shift in the level of total factor productivity occurs. Assuming the future rate of technical progress is not affected, illustrate the dynamics in a diagram. (Note, as in part (c) above and part (g) below, the diagram should be a discrete-time diagram.)

(f) How, if at all, is the real wage in the long run affected by the shock? How, if at all, is the real interest rate in the long run affected by the shock?

(g) Suppose that the shock at time $t_1$ is anticipated as of time $t' < t_1$. Illustrate the dynamics in a diagram if the economy was in steady state before $t'$, with special reference to the effect at $t'$.

(h) What would be the effect on the real wage and real interest rate over time in this case, that is: at $t'$; between $t'$ and $t_1$; at $t_1$; and in the long run?

(i) What can we learn from your answer in part (h), if anything, about the macroeconomic effects of the introduction of new technology? Do you find this a realistic description? Why or why not?