UNIVERSITY OF MARYLAND
Department of Economics

Economics 602
Macroeconomic Analysis

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EXAMINATION -- Second Half

Read the entire exam before starting to work, both to allocate your time and to think about a whole question before starting to work. Relax. Then do both questions, which have equal weight.

1. Consider the standard non-altruistic overlapping generations model in which individuals live two periods and are endowed with one unit of labor in the first period and zero units of labor in the second period of their lives. Individuals differ in their first-period wage, individual $i$ earning an exogenous wage $w^i$, where wages in the population are distributed according to a cumulative distribution function $H(w)$. The interest factor $R = 1+r$, the interest rate, is also exogenous. All individuals have the same utility function defined over first- and second-period consumption:

$$u(c_1^i, c_2^i) = (1 - \beta) \ln c_1^i + \beta \ln c_2^i,$$  \hspace{1cm} (1.1)

(for individual $i$), where $\ln x$ is the logarithm of $x$ and $1 > \beta > 0$. Initially suppose that the rate of population growth is $n > 0$.

a) Consider a steady state in which a lump-sum tax on the young finances a per capita transfer $v$ to the old in the same period, that is, a “pay-as-you-go” system. Set up the budget constraints of individuals and derive their consumption and saving functions. Derive the preferred tax for each individual as a function of $r - n$ and $w^i$. Explain your results, specifically the nature of preferences over the tax-transfer program across individuals.

b) Now let $n = 0$ and suppose that a tax is put on saving of the old to finance a lump-sum transfer, so that second-period income of individual $i$ is $(1 - \tau)R x^i + v$. There are no taxes on or transfers to the young. Derive first- and second-period consumption and saving for individual $i$ as a function of $w^i$, $\tau$, $R$, and $v$. Write down the government’s budget constraint relating tax collections to the per capita transfer $v$ in terms of average saving. Derive an explicit expression for average saving in equilibrium as a function of the average wage $\bar{w}$. Show that $c_1^i$ is unambiguously higher and $s^i$ unambiguously lower in the presence of a tax transfer system than if there were no system, but that the relation between equilibrium $c_2^i$ with and without a tax system depends on the relation between $w^i$ and $\bar{w}$.

c) Demonstrate that an individual whose income is equal to the mean (i.e., $w^i = \bar{w}$) prefers zero tax. (Hint: use your results on consumption with and without a tax system, think carefully about what they imply.) Briefly explain your result.

2. Consider a two-period of inflation choice in which a policymaker has a one-time private cost $\zeta$,
of reneging on a commitment to zero inflation, announced at the beginning of the first period. Policymakers differ in their $\zeta$, where there are two possible types of policymakers, $\zeta^w$ and $\zeta^t$, with $0 < \zeta^w < 1 < \zeta^t < \infty$. The public does not observe the type, but has an initial prior belief $p_1$ in the first period that the policymaker is of type $\zeta^t$. In each period the public uses Bayes' rule to update its beliefs.

The policymaker maximizes discounted social welfare ($\beta < 1$ is the discount factor), where his single-period loss function is

$$L(\pi, \pi^e, \zeta) = -(\pi_t - \pi^e_t) + \frac{\pi_t^2}{2} + \frac{C(\zeta)}{2}, \quad (2.1)$$

where $C = \zeta$ the first period in which the policymaker inflates, $C = 0$ otherwise.

a) Explain briefly why $\zeta$ is modeled as a private rather than a social cost and what a private cost to the monetary authority might mean.

b) Derive what the rate of inflation for each type of policymaker in a one-period model with a loss function such as (2.1) and use your result to briefly explain what is meant by the time consistency problem in monetary policy.

c) In the two-period case, derive the algebraic solution for the evolution of government actions and the public’s beliefs in the model, including: the government’s choice of inflation in each period as a function of its past actions and type; showing the government’s equilibrium actions are optimal; the evolution of the public’s beliefs about type as a function of observed past inflation rates, and, of course, equilibrium evolution of inflation. Include a discussion of the types of Perfect Bayesian equilibria, including whether separating, pooling, and mixed strategy equilibria are possible.