Do All Questions. Each question has equal weight. First, read the whole exam. Read an entire question before starting to work on it. PLEASE WRITE CLEARLY! Note that each question gets harder as you go along. If you are time constrained, concentrate on earlier parts of a question.

Exam Time: 2 hours and 15 minutes

1. Consider an economy described by an overlapping generations model with government. Individuals live for 2 periods. In the first period of her life (when young), each individual works and earns $w_t$ (wage per unit of labor), pays taxes for $T_t$, and saves. In the second period she does not work. Savings at $t$ can be used to buy physical assets and government bonds $b_{t+1}$. Both physical assets and government bonds purchased at $t$ pay the same interest rate, $r_{t+1}$. However, only the investment in physical assets affects the capital stock. Population grows at a constant rate $n$. The individual’s utility function is:

$$u(c_y^t, c_{o_{t+1}}) = \ln c_y^t + \ln c_{o_{t+1}}$$

(Note the discount factor between periods is 1.) The production function is

$$F(K_t, L_t) = K_t^\alpha (A_t L_t)^{1-\alpha}$$

(a) Write down the individual’s intertemporal budget constraint and solve for $c_y^t, c_{o_{t+1}}$, and saving in physical assets, taking $b_{t+1}$ as given.

(b) Derive an expression for the equilibrium wage as a function of $k_t \equiv \frac{K_t}{L_t}$?

(c) Derive an explicit difference equation for the evolution of the capital stock per capita (NOT per unit of effective labor).

(d) Suppose that labor productivity, per capita taxes, and per capita bond issuance emissions are constant ($A_t = A$, $T_t = T$, and $b_{t+1} = b$). Draw the relation in part (c) in $k_t - k_{t+1}$ space. Be precise about the details of the graph, in particular the points at which the curves cut the axes. How many equilibria does this economy exhibit? How many are stable?

(e) Assume that initially the economy is in the highest stable equilibria. The constant level of productivity falls permanently from $A_0$ to $A_1 < A_0$. What happens with the equilibrium level of capital and output? Show the effect of the fall in productivity both algebraically and in a $k_t - k_{t+1}$ graph.

(f) Suppose that the government wants to stimulate the economy, that is to increase the equilibrium level of output. Can the government do this by reducing taxes and increasing bond issuance in the same amount ($\Delta b = -\Delta T$)? Show the effect of this policy in a $k_t - k_{t+1}$ graph. Explain the reason for this result.

2. An economy produces two goods, consumption and investment, using only capital. Population is constant and equal to 1 and capital does not depreciate. The production functions for the two goods
are

\[ C(t) = K_C(t)^\alpha \]
\[ \dot{K}(t) = BK_K(t) \]

where \( K_C(t) \) and \( K_K(t) \) are the amounts of capital used in the two sectors at time \( t \) (so that \( K_C(t) + K_K(t) = K(t) \)), \( 0 < \alpha < 1 \), \( B > 0 \), and \( \dot{K}(t) \) is the time derivative of \( K(t) \). Input markets are competitive and capital is perfectly mobile between sectors.

(a) Show that on a balanced growth path (that is, a steady state) the proportion of \( K_C \) to \( K_K \) remains constant. What does this imply about the growth rate of capital in the two sectors? What about the growth rate of output in the two sectors?

(b) Let \( P_K(t) \) be the price of capital goods in terms of consumption goods. Find an expression for \( P_K(t) \) in terms of \( \alpha \), \( B \), and \( K_C(t) \) that must hold in a competitive equilibrium.

(c) Show that the interest rate in terms of consumption goods (the numeraire) must equal \( B + gP_K(t) \) where \( gP_K(t) \) is the growth rate of \( P_K(t) \).

3. Consider a two-period economy inhabited by a continuum of heterogeneous agents. The government provides a public good \( g \) that is non-rival and non-excludable and is financed by capital taxation. It also provides a public investment good \( I \) financed by lump-sum taxation. The government’s budget is balanced every period.

Everyone has the same quasi-linear preferences over private consumption in periods 1 and 2 and over government (per capita) consumption \( g \) in period 2. Consumer \( i \)'s utility is

\[ U^i = u(c^i_1) + c^i_2 + v(g) \]

where \( u(\cdot) \) and \( v(\cdot) \) are increasing, concave and satisfy the Inada conditions. Consumer \( i \)'s budget constraints are

\[ c^i_1 = e^i - \bar{\tau} - k^i \]
\[ c^i_2 = (1 - \tau_k) A(I) k^i \]

where \( e^i \) is the endowment of agent \( i \) and \( k^i \) is his private investment. \( e^i \) is distributed in the population according to a CDF \( H(e^i) \) with mean \( \mu \). Assume that the median \( e_{med} \) of the distribution is below the mean, i.e., \( e_{med}, \mu, \bar{\tau} \) and \( \tau_k \) are lump sum and capital taxes, respectively, and the gross return to private capital is given by

\[ A(I) = \begin{cases} \alpha & \text{if } I \geq \dot{I} \\ 0 & \text{otherwise} \end{cases} \]

where \( \alpha > 1 \) and \( \dot{I} \) is given. Assume the government can commit to policy instruments \( \bar{\tau} \) and \( \tau_k \) before individuals choose capital accumulation.

(a) Solve for \( c^i_1, c^i_2 \) and \( k^i \) for individual \( i \) for given fiscal policy.

(b) Write down the government’s budget constraints and solve for the equilibrium of the political-economic equilibrium when policies are chosen by majority voting.

(c) Show that compared to a symmetric distribution of endowments (that is, where \( e_{med} = \mu \)), the equilibrium in part (b) involves higher taxes and lower capital accumulation.