Economics 601, 2nd Half – Final Exam
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Do All Questions. Point values in parentheses. First, read the whole exam. Read an entire question before starting to work on it. PLEASE WRITE CLEARLY!

TIME: 2 hours, 15 minutes

1. (35%) Consider a two-sector optimal growth problem in which the social planner maximizes the utility of the representative agent given by:

\[ W = \sum_{t=0}^{\infty} \beta^t U (c_t, l_t) \]

where \( c_t \) = consumption and \( l_t \) = leisure in period \( t \). Sector 1 produces consumption goods using capital \( k_{1t} \) and labor \( n_{1t} \) according to the production function:

\[ c_t = f_1 (k_{1t}, n_{1t}) \]

Sector 2 produces capital goods using capital \( k_{2t} \) and labor \( n_{2t} \) according to:

\[ k_{t+1} = f_2 (k_{2t}, n_{2t}) \]

Total employment is \( n_t = n_{1t} + n_{2t} \), leisure \( l_t \) is constrained by the time endowment \( \bar{t} \) so that \( n_t + l_t \leq \bar{t} \). Also, \( k_{1t} + k_{2t} \leq k_t \), where \( k_0 \) is given and capital depreciates at rate \( \delta = 1 \).

(a) Formulate this problem as a dynamic programming problem. Write down the functional equation (or equations) that the value function satisfies and clearly specify the state and control variables.

(b) Find the equations describing optimal paths.

(c) Consider another economy that is similar, except that capital is sector specific. The economy starts period \( t \) with given amounts of capital \( k_{1t} \) and \( k_{2t} \) that may be used only in their respective sectors. In the period the capital-goods sector produces capital that is specific to each sector according to the transformation function:

\[ g (k_{1t+1}, k_{2t+1}) \leq f_2 (k_{2t}, n_{2t}) \]

Write down the Bellman equation associated with this problem, clearly specifying state and control variables and characterize the optimal path. What is reasonable to assume about the transformation function \( g (k_{1t+1}, k_{2t+1}) \)? How does the growth path of this economy differ from the growth path of the previous economy?

2. (40%) Consider an economy described by an overlapping generations model with government. Individuals live for 2 periods. In the first period of her life (when young), each individual works and earns \( w_t \) (wage per unit of labor), pays taxes for \( T_t \), and saves. In the second period she does not work. Savings at \( t \) can be used to buy physical assets and government bonds \( b_{t+1} \). Both physical assets and government bonds purchased at \( t \) pay the same interest rate, \( r_{t+1} \). However, only the investment in physical assets affects the capital stock. Population grows at a constant rate \( n \). The individual's utility function is:

\[ u (c^y_t, c^o_{t+1}) = \ln c^y_t + \ln c^o_{t+1} \]
(Note the discount factor between periods is 1.) The production function is
\[ F(K_t, L_t) = K_t^\alpha (A_t L_t)^{1-\alpha} \]

(a) Write down the individual’s intertemporal budget constraint and solve for \( c_t, c_{t+1}, \) and saving in physical assets, taking \( b_{t+1} \) as given.

(b) Derive an expression for the equilibrium wage as a function of \( k_t \left( \equiv \frac{K_t}{L_t} \right) \).

(c) Derive an explicit difference equation for the evolution of the capital stock per capita (NOT per unit of effective labor).

(d) Suppose that labor productivity, per capita taxes, and per capita bond issuance emissions are constant \((A_t = A, T_t = T, \text{ and } b_{t+1} = b)\). Draw the relation in part (c) in \( k_t - k_{t+1} \) space. Be precise about the details of the graph, in particular the points at which the curves cut the axes. How many equilibria does this economy exhibit? How many are stable?

(e) Assume that initially the economy is in the highest stable equilibria. The constant level of productivity falls permanently from \( A_0 \) to \( A_1 < A_0 \). What happens with the equilibrium level of capital and output? Show the effect of the fall in productivity both algebraically and in a \( k_t - k_{t+1} \) graph.

(f) Suppose that the government wants to stimulate the economy, that is to increase the equilibrium level of output. Can the government do this by reducing taxes and increasing bond issuance in the same amount \((\Delta b = -\Delta T)\)? Show the effect of this policy in a \( k_t - k_{t+1} \) graph. Explain the reason for this result.

3. (25%) Consider an economy where output per worker is produced according to
\[ y_t = A h_t^\phi \]
where the flow of new human capital is produced in a different sector with technology
\[ \dot{h}_t = \theta h_t^\phi - \delta h_t \]

Consider three cases: (1) \( \alpha = 1, \phi < 1 \), (2) \( \alpha < 1, \phi = 1 \), (3) \( \alpha < 1, \phi > 1 \).

(a) For each case, explain whether the economy displays endogenous growth. If it does, compute the growth rate of the economy and check whether it is constant. If it doesn’t, compute the steady state level of output per capita \( y^{SS} \).

(b) In general, what are the key conditions guaranteeing that an economy will grow permanently in the long-run, even without exogenous technical progress?

(c) In this model, can government policy affect the growth rate of the economy? How?