Price Rigidity During the Great Recession*

Camilo Morales-Jimenez       Luminita Stevens
Federal Reserve Board         University of Maryland

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Abstract
Did price rigidities become more severe during the Great Recession? If so, was the increase in distortions driven by worsening informational frictions in an uncertain environment? Using theory and data, we find that the answer to both questions is yes. First, we document that despite the relatively muted dynamics of aggregate inflation, the Great Recession featured significant cyclical changes in the distribution of price changes. The frequency, standard deviation, skewness, and kurtosis of price changes all increased, to various degrees. We also find a high and significant correlation of these pricing moments with measures of private agents’ beliefs and confidence. Next, we use the joint behavior of inflation, real economic activity, and price dispersion moments to identify the severity of the frictions that prices exhibited during the Great Recession. We use a generalized model of price rigidities that features endogenous information frictions and also allows for myopia in information choice. We estimate the model to match the dynamics of both macroeconomic aggregates and pricing moments, which turns to be crucial for identifying the degree of informational rigidities. We find that most of the rigidities in price setting come not from infrequent but from inaccurate price adjustment. Moreover, our estimation results suggest that these frictions became more severe over the course of the recession. Large shocks to the discount factor and to inflation expectations drove the worsening of informational rigidities and the increased volatility in real economic activity. Our results challenge previous conclusions that pricing flexibility rises during recessions and that monetary policy effectiveness is countercyclical.

*The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of anyone else associated with the Federal Reserve System. Contact: camilo.moralesjimenez@frb.gov and Stevens7@umd.edu.
1 Introduction

How severe were price rigidities during the Great Recession? Since these rigidities determine the ability of monetary policy to influence the real economy, the answer to this question can help shed light on the effectiveness of monetary stabilization policy during this period, and may also contain lessons for future recessions: If prices become more responsive to the state of the economy during downturns, then monetary policy tools become less potent, requiring a more aggressive response to shocks.

A recent line of research in monetary economics has linked the degree of price rigidities to moments of the distribution of price changes constructed using product-level price data. Hence, we begin by studying the dynamics of this distribution. We document that underneath the relatively muted aggregate inflation dynamics that characterized the Great Recession period and its aftermath, there were significant movements in this distribution. The frequency, standard deviation, skewness, and kurtosis of price changes all increased (to varying degrees), while the size of price changes decreased (modestly). These cyclical movements are generally consistent with evidence from past recessions (Berger & Vavra, 2018). Hence, the relatively modest volatility of the aggregate price index does not seem to be the result of more muted movements at the micro level. It seems that the Great Recession only deepened the existing micro-macro disconnect in price setting, which contrasts the sluggishness of aggregate inflation with the substantial volatility of disaggregated price data.

Based on the sufficient statistics approaches of Alvarez, Le Bihan & Lippi (2016) and Berger & Vavra (2018), and on the structural results of Vavra (2013), these movements would imply a significant decline in the effectiveness of monetary policy during the recession. Berger & Vavra (2018), for example, show that in the flexible Ss model of Caballero & Engel (2007)—which performs well in terms of matching various properties of prices at the product level—greater frequency, greater variance, and smaller kurtosis are all associated with greater price flexibility.

In this article, we cast doubt on the hypothesis of increased price flexibility during
the Great Recession. First, we emphasize that the relationship between the distribution of price changes and the degree of aggregate price flexibility depends on how pricing frictions are modeled. Moreover, the dynamics of this distribution are very useful for identifying these frictions. We find that in order to capture the dynamics of the price change distribution, we need to relax the assumption that when they update their prices, firms choose new prices based on complete information about the state of the economy. The data require that firms make imperfect decisions both in terms of the timing of price adjustment—as in the generalized Ss model of Caballero & Engel (2007), the random menu cost model of Dotsey, King & Wolman (1999), and the rationally inattentive adjustment model of Woodford (2009)—and also in terms of which price to charge conditional on adjustment—as in the generalized rational inattention model of Khaw, Stevens & Woodford (2017).

Relaxing the assumption of price resets based on complete information—common to virtually all existing models of frictional price adjustment—matters: Not only does it allow a better fit of the model with the data, but it also breaks the link that connects increased price variability with increased price flexibility. Consider interpreting the finding that the frequency of price changes increased during the downturn. Intuitively, a model featuring price resets based on complete information implies that more frequent resets will more frequently bring prices closer to the frictionless benchmark, thereby reducing pricing frictions. But a model that allows firms to make mistakes in the reset prices can feature, at least in principle, an increase in the frequency of adjustment without automatically implying a decrease in price rigidities. Next, consider the result that the standard deviation of price changes also increased, along with the frequency. As shown by Vavra (2013), a positive correlation between the frequency and the standard deviation of price changes is at odds with the predictions of the standard state-dependent pricing models, in which the dispersion of price changes collapses as the frequency of adjustment increases. Reconciling this class of models with the data requires instead an increase in the variance of fundamental shocks. In
contrast, the model with noisy price resets can generate the positive comovement between dispersion and the frequency of adjustment by having firms make larger pricing mistakes in downturns. In the second part of the paper, we numerically evaluate this mechanism in a model of price setting disciplined by data from the Great Recession.

Formally, we apply the noisy adjustment model of Khaw et al. (2017) to price-setting firms, in a general equilibrium setting. Our numerical results estimate the shocks in our model to match the dynamics of both macroeconomic aggregates and pricing moments, which turns to be crucial to identify the degree of nominal and informational rigidities. We find that most of the rigidities in price setting come not from infrequent but from inaccurate price adjustment. Moreover, our estimation results suggest that these frictions became more severe over the course of the recession. This result is at odds with previous conclusions that pricing rigidities lessen during recessions.

The best-fitting version of the model generalizes the pure rational inattention framework, by allowing for partial myopia in information choice. We allow decision-makers to have intrinsic preferences for certain actions over others. For example, firms may be reluctant to change prices even when the information they have warrants a price adjustment. Our data support the existence of these deviations from the baseline model. In particular, we find that the model with partial myopia in information choice can generate the observed price dispersion in the data without requiring as large increases in the severity of information frictions. Our takeaway from this finding is that the data warrants a worsening of decision-making in price setting, whether it is in terms of higher implied information costs or partial myopia to existing information.

According to our results, large shocks to risk premia and inflation expectations drove the worsening of informational rigidities and the increased volatility in real economic activity. These shocks also limited the stabilization power of monetary policy. Our results underscore the point that inference regarding the degree of price rigidities depends crucially on assumptions about how well informed firms are when making
their pricing decisions. They also emphasize that inferences regarding the degree of aggregate price flexibility remain, at least for now, very much model-dependent.

2 Patterns in the Distribution of Price Changes

In this section, we document significant changes in the distribution of price changes during the Great Recession, despite the relatively muted dynamics of aggregate inflation. The volatility of these series has important implications for the severity of pricing frictions and for the degree of monetary policy effectiveness. Various moments of the price change distribution have been identified by the monetary economics literature as being particularly informative about the degree of nominal price rigidities, and hence for the ability of monetary policy to influence real outcomes.

The canonical Calvo (1983) time dependent model implies that the frequency of price changes is a sufficient statistic for the degree of nominal rigidities. Alvarez et al. (2016) have shown that the ratio of kurtosis to the frequency of price changes provides a sufficient statistic in a class of state-dependent menu cost models of price adjustment. Berger & Vavra (2018) found a role for the frequency, standard deviation, and kurtosis of this distribution, in the context of a generalized Ss model of adjustment a la Caballero & Engel (2007), while Luo & Villar Villenas (2017) argue that the skewness of this distribution can also be informative about the nature of pricing rigidities in random menu cost models a la Dotsey et al. (1999). Hence in both time-dependent and state-dependent models of price adjustment, the frequency of price changes as well as higher moments of the price change distribution are important indicators of the degree to which monetary policy can affect real economic activity.
2.1 Data

Nielsen Data  We compute measures of price adjustment and inflation using the AC Nielsen Retail Scanner Database.\footnote{This data set is provided by the Kilts Center for Marketing at The University of Chicago Booth School of Business, http://research.chicagobooth.edu/nielsen. The conclusions drawn from the Nielsen data are our own and do not reflect the views of Nielsen. Nielsen is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein. Other papers using this database include (Beraja, Hurst & Ospina, 2016) and (Stevens, 2019).} This large data set contains weekly average selling prices and quantities for products sold at thousands of stores across the contiguous United States.\footnote{Hawaii and Alaska are not part of the sample.} Approximately 35,000 individual stores from approximately 90 retail chains are surveyed each week starting in January 2006. The data are organized in three levels of classification. There are nine departments made up of 100 groups, which in turn are divided into approximately 1,100 modules. Products are grouped in departments such as health and beauty care, non-food grocery, and general merchandise, and capture approximately 27\% of the total products measured by the Consumer Expenditure Survey of the Bureau of Labor Statistics. Each product is uniquely identify by a universal product code (UPC) and is uniquely assigned to a module. The data set also contains geographic information about the stores in which the products are sold, in the form of metropolitan statistical areas (MSAs) and state codes. As is standard in the literature, we exclude the Deli, Package Meat, and Fresh Produce departments. We use data from the 5 stores with the largest number of observations for each retail chain and state. For each series, we keep only the longest contiguous subseries with at least 52 weeks of observations. The sample tracks the behavior of the CPI and, in particular, the Food and Beverage sub-index over the sample period.

Pricing Moments  We analyze the following moments of the distribution of weekly log absolute price changes: frequency, median, mean, standard deviation, skewness, and kurtosis. We construct these moments at the module by state by week level,
and we then aggregate them to the national level by computing the expenditure-weighted median across modules and states. We aggregate the resulting weekly series to monthly frequency and seasonally adjust them using the X-13 filter.

Throughout the paper, we present pricing moments that are constructed from raw price series and from inferred adjustments in pricing policies (for products that exhibit temporary sales or more complex pricing patterns). The raw (posted) price series use as unit of observation the log change in the weekly average selling price reported by Nielsen. The inferred (filtered) policy adjustment series use the log change in average prices across pricing policies, where the changes in pricing policies are identified using the break test proposed by Stevens (2019). The filtered series capture lower-frequency movements in prices that have been shown to be the principal contributor to aggregate price rigidity, and less affected by highly transitory price volatility.\(^3\)

**Aggregate Data** We supplement the Nielsen data with monthly data from the Consumer Price Index (CPI), real gross domestic product (GDP) data from FRED, and monthly seasonally adjusted unemployment rate U-3 from the Current Population Survey (CPS).\(^4\) To measure consumer expectations and consumer confidence, we use the Index of Consumer Sentiments (ICS) and the expected inflation over the next year from the Surveys of Consumers conducted by the University of Michigan.\(^5\)

To study short-term cyclical patterns, we HP-filter all series with a smoothing

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\(^{3}\)For each individual price series, the break test estimates the location of changes in the distribution of prices charged. For each identified pricing distribution, interpreted as the realization of a pricing policy, we compute the average selling price, weighted by the quantity sold. We then compute the log change in this weighted price across policy realizations. In the Supplementary Appendix, we show that results based the regular price series identified by the rolling mode filter proposed by Kehoe & Midrigan (2015) are similar to those based on the break test.

\(^{4}\)The CPS is the main labor force survey for the United States; it is the primary source of labor force statistics such as the national unemployment rate, and it is representative at the state level.

\(^{5}\)This is a monthly survey on a nationally representative sample of consumers. The ICS is computed based on two questions about the participants’ views about the future of the country (whether they expect good or bad times) and three questions about their perceived past, current and future financial status (better or worst). Regarding prices, survey participants are asked (1) whether they expect prices to go up, down, or stay the same over the next year, and (2) to quantify the expected change.
Figure 1: Pricing Moments

Note: AC Nielsen data, 2006-2015. Solid lines depict raw price series and dashed lines depict filtered price series. Shading marks the Great Recession. The ALL index denotes the monetary non-neutrality index of Alvarez et al. (2016), which increases when non-neutrality is high.

parameter equal to 1,600, and we smooth the resulting series with a 3-quarter moving average, to eliminate high frequency variations.

2.2 Pricing Patterns

Figure 1 plots the time series for the frequency, size, standard deviation, skewness, and kurtosis of the absolute value of log price changes for both raw (posted) and filtered price series. Table 1 presents the associated descriptive statistics. All the moments fluctuate substantially, especially for the filtered series, which exhibit standard deviations that range between 5% and 11% of the mean. Moreover, clear cyclical patterns emerge for the filtered series: the frequency of adjustment rises sharply both during the Great Recession and during the period of heightened uncertainty in 2011. The skewness and kurtosis also rise during both periods, while the size of price changes (as measured by the median) slightly declines. The standard deviation rises
TABLE 1: Descriptive Statistics Pricing Moments

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Size</th>
<th>Std</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>ALL index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Prices</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.332</td>
<td>0.131</td>
<td>0.130</td>
<td>1.498</td>
<td>5.000</td>
<td>15.151</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.039</td>
<td>0.008</td>
<td>0.003</td>
<td>0.117</td>
<td>0.438</td>
<td>0.977</td>
</tr>
<tr>
<td>Standard deviation (%)</td>
<td>11.6</td>
<td>6.2</td>
<td>2.5</td>
<td>7.8</td>
<td>8.8</td>
<td>6.4</td>
</tr>
<tr>
<td>BT Prices</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.028</td>
<td>0.107</td>
<td>0.069</td>
<td>0.737</td>
<td>2.790</td>
<td>101.085</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.003</td>
<td>0.007</td>
<td>0.003</td>
<td>0.059</td>
<td>0.139</td>
<td>7.157</td>
</tr>
<tr>
<td>Standard deviation (%)</td>
<td>10.9</td>
<td>6.5</td>
<td>4.9</td>
<td>8.0</td>
<td>5.0</td>
<td>7.1</td>
</tr>
</tbody>
</table>


during the recession, although with a delay, and peaking after the official end of the recession (which differs somewhat from the evidence from prior years documented by Vavra, 2013). The observed movements in the frequency and kurtosis imply a sharp decline in the index of monetary non-neutrality index of Alvarez et al. (2016), which increases when non-neutrality is high. Viewed through the lens of state-dependent price adjustment, these movements in the price change distribution imply that monetary policy effectiveness declined significantly both during the recession and during 2011.

For a more precise identification of the timing of these cyclical patterns, Figure 2 plots the pricing series against the aggregate series for the unemployment rate, real annual GDP growth, CPI annual inflation, consumer inflation expectations, and consumer sentiment index. Table 2 presents the corresponding correlation table of our HP-filtered and MA smoothed series, with the standard deviations reported on the main diagonal.
The following facts emerge about the relationship between real variables, nominal variables and private agent beliefs: (1) Inflation is significantly correlated with all pricing moments: an increase in inflation is associated with an increase in the frequency, skewness, and kurtosis, and with a decline in the size and standard deviation of the absolute size of price changes. (2) The frequency of price changes comoves negatively with real GDP growth, consistent with evidence from prior recessions. (3) Higher moments of the price change distribution appear to have moved more strongly in this recession than in past recessions (compared, for example, with the evidence of
TABLE 2: Business Cycle Comovement of Pricing Moments

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Size</th>
<th>Std</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>ALL index</th>
<th>Inflation</th>
<th>GDP growth</th>
<th>U</th>
<th>ICS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>0.002</td>
<td>-0.22</td>
<td>-0.06</td>
<td>0.72</td>
<td>-0.93</td>
<td>0.57</td>
<td>-0.27</td>
<td>-0.41</td>
<td>-0.75</td>
<td></td>
</tr>
<tr>
<td>Size</td>
<td>0.003</td>
<td>0.78</td>
<td>-0.56</td>
<td>-0.39</td>
<td>0.06</td>
<td>-0.40</td>
<td>-0.10</td>
<td>0.71</td>
<td>-0.15</td>
<td></td>
</tr>
<tr>
<td>Std</td>
<td>0.001</td>
<td>-0.16</td>
<td>-0.06</td>
<td>0.04</td>
<td>-0.50</td>
<td>-0.37</td>
<td>0.67</td>
<td>-0.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>0.036</td>
<td>0.97</td>
<td>-0.44</td>
<td>0.54</td>
<td>-0.05</td>
<td>-0.66</td>
<td>-0.32</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.088</td>
<td>-0.54</td>
<td>0.52</td>
<td>-0.10</td>
<td>-0.61</td>
<td>-0.41</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ALL index</td>
<td>4.951</td>
<td>-0.47</td>
<td>0.35</td>
<td>0.21</td>
<td>0.82</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>0.010</td>
<td>0.55</td>
<td>-0.53</td>
<td>-0.18</td>
<td>0.57</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP growth</td>
<td>0.013</td>
<td></td>
<td>0.008</td>
<td>-0.07</td>
<td>5.034</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Notes: Calculations based on AC Nielsen, FRED, BLS, and Michigan Survey data. ALL index denotes the monetary non-neutrality index of Alvarez et al. (2016), U denotes the U-3 unemployment rate, Inflation is CPI annual inflation, and ICS denotes the Michigan Survey’s consumer sentiment index.

Berger & Vavra, 2018). (5) Our measures of agents’ beliefs and sentiments are highly and significantly correlated with our pricing moments.

Different monetary models with price rigidities can generate the correct correlation among aggregate variables (for instance, output and inflation) but systematically differ in terms of the implied comovement of pricing moments. The evidence presented in this section indicates that a successful monetary model should be able to generate significant movements in pricing moments that are systematically correlated with inflation but less so with other aggregate indicators of economic activity, possibly because of the existence of various underlying forces that tend to offset each other. Additionally, the high correlation of our measures of private agents’ expectations and beliefs with all pricing moments suggest an important role for the expectations formation process. We use the information presented in this section to discipline our theoretical framework and obtain estimates of the degree of pricing rigidities specific to the Great Recession episode.
3 Theory

Motivated by the pricing dynamics identified in the previous section, we present a model of noisy price setting in which information frictions can generate significant dispersion in prices. The model introduces mistakes in pricing decisions along two dimensions: the firm’s decisions of whether or not to adjust its price and its decision of what price to set conditional on adjustment are made using imperfect information about state of the economy. The model applies the theory developed in Khaw et al. (2017; 2019), and nests a range of pricing models, including the Calvo model, the generalized Ss model of Caballero & Engel (2007), the inattentive adjustment model of Woodford (2009), and the trembling hand model of Costain & Nakov (2019).

Household The model features a representative household that derives utility from consumption, leisure, and holdings of real money balances. The household is the owner of all firms in the economy, and supplies labor in a perfectly competitive market to the goods producers in the economy. The per-period utility function is

\[ U_t = \frac{c_t^{1-\gamma}}{1-\gamma} - \chi_t \cdot n_t + \nu \cdot \log(m_t) \] (1)

where \( c_t \) is consumption and \( n_t \) labor supply, and \( m_t \) is holdings of real money balances. The household discounts future utility by a factor \( \beta \), can invest in a risk-free nominal bond \( B_t \), and faces the period budget constraint given by

\[ w_t n_t + d_t + m_t + \frac{i_{t-1}}{\pi_t} b_{t-1} = c_t + b_t + t_t + \frac{m_{t-1}}{\pi_t} \] (2)

where \( w_t \) is the real wage rate, \( d_t \) stands for real dividends from firms, \( t_t \) are real lump-sum transfers from the government, \( i_t \) is the nominal interest rate on the risk-free bond, and \( \pi_t \) is the gross inflation rate between \( t-1 \) and \( t \). We assume that there is no capital and, as consequence, bond are in zero net supply. The consumption good
is a CES aggregator with an elasticity of substitution $\varepsilon > 0$,

$$
c_t = \left[ \int c_{jt}^{\varepsilon-1} \, dj \right]^{\frac{\varepsilon}{\varepsilon-1}} 
$$

(3)

The household’s optimality conditions are standard and omitted for brevity.

**Producers** There is a continuum of firms indexed by $j$ and with mass normalized to 1 who produce using a production function that is linear in labor. Productivity has both an idiosyncratic and an aggregate component:

$$
Y_{jt} = A_{jt} n_{jt} 
$$

(4)

$$
\log A_{jt} = a_t + a_j. 
$$

(5)

The firm’s period profit is given by

$$
\Pi_t(P_{jt}, A_{jt}) = \frac{P_{jt}}{P_t} Y_{jt} - w_t n_{jt}. 
$$

(6)

In absence of any labor market frictions, the labor market clears by equating the household’s marginal rate of substitution to the real wage,

$$
\frac{\chi_t}{c_t^{-\gamma}} = \frac{W_t^*}{P_t} 
$$

(7)

$$
n_t = \int_j Y_{jt} A_{jt} \, dj. 
$$

(8)

To dampen the movement in the marginal cost, we introduce a reduced form of nominal wage rigidities in which the current nominal wage is a weighted average of the competitive labor market wage (7) and the lagged wage indexed by lagged inflation:

$$
W_t = (1 - \alpha_w) W_t^* + \alpha_w W_{t-1} \pi_{t-1}. 
$$

(9)
where \( \alpha^w \) is the degree of wage rigidity.

**Informational Friction** Firms face informational frictions that affect both the probability of price adjustment conditional on the state of the economy and the distribution from which a price is drawn conditional on adjustment. We model the information frictions as in the rational inattention literature (Sims, 2003), using relative entropy as a measure of the amount of information that the firm acquires in order to deviate from its priors when making its pricing decisions. These informational frictions imply that firms adjust prices in period \( t \) with probability \( \Lambda_{jt} \) and draw the reset price from the distribution \( f_{jt}(p) \). More severe frictions along the first dimension imply less state dependence in the timing of price changes, and in the limit imply a constant probability of adjustment \( \bar{\Lambda} \), as in the Calvo model. On the other hand, more severe frictions along the second dimension imply a noisier price conditional on adjustment, and in the limit approach the firm’s prior \( \bar{f}_j(p) \) regarding the optimal distribution from which prices should be drawn irrespective of the state of the economy.

Motivated by the experimental evidence of Khaw et al. (2017), we follow that paper and also allow for intrinsic preferences for certain actions along both dimensions, independent of the payoff associated with choosing these actions. For example, some firms may be reluctant to change prices even when the information they have warrants a price adjustment. In particular, suppose that there is a cost \( c(p) \) associated with choice of price \( p \) and likewise a cost \( c^{adj} \) associated with adjustment of the firm’s price, and a cost \( c^{non} \) of non-adjustment. These quantities can be either negative or positive, reflecting intrinsic costs of or preferences toward certain actions. The generalized objective of the rationally inattentive firm becomes

\[
E \left\{ \sum_{t=1}^{\infty} \beta^t [\pi_t - \theta_t I_t - \theta_p \Lambda_t I_p - \Lambda_t c^{adj} - (1 - \Lambda_t) c^{non} - \Lambda_t \int c(p) f_t(p) dp] \right\} 
\]

(10)

where \( \pi_t \) is profit in units of marginal utility, \( I_t \) is a measure of the amount of infor-
mation used in deciding in each period whether to adjust the price; \( I_p \) is a measure of the amount of information used in deciding what price to charge; \( \theta_l, \theta_p > 0 \) are attention cost parameters for the two types of information; and \( E\{\cdot\} \) indicates an expectation over the possible sequences of realizations of shocks and decisions.

The problem is then to choose a reference adjustment probability \( \bar{\Lambda} \), a reference measure over possible price points \( \bar{f} \) and sequences of functions \( \Lambda_t \) and distributions \( f_t(p) \), so as to maximize (10).

We begin by assuming that values for \( \bar{\Lambda} \) and \( \bar{f} \) are specified, and optimizing over the other functions. We then have a problem that can be treated recursively. Let \( V_t(p) \) denote the continuation value if price \( p \) is chosen in period \( t \). The solution to this maximization problem is given by

\[
f_t(p) = \frac{\exp\{\theta_p^{-1}(V_t(p) - c(p))\}\bar{f}(p)}{\int \exp\{\theta_p^{-1}(V_t(p') - c(p'))\}\bar{f}(p')dp'} \tag{11}
\]

for all \( p \) in the support of the reference distribution \( \bar{f} \). This solution maximizes the value of adjusting,

\[
V^{adj} = \max \left\{ \int V_t(p)f_t(p)dp - \int c(p)f_t(p)dp - \theta_p I(f_t||\bar{f}) \right\}. \tag{12}
\]

The optimal adjustment hazard \( \Lambda_t \) is given by

\[
\Lambda_t = \frac{\bar{\Lambda} \exp\{\theta_l^{-1}(V^{adj}_t - c^{adj})\}}{(1 - \bar{\Lambda}) \exp\{\theta_l^{-1}(V^{adj}_t - c_{adj})\} + \bar{\Lambda} \exp\{\theta_l^{-1}(V^{adj}_t - c^{adj})\}}. \tag{13}
\]

In the pure rational inattention model, the reference probability \( \bar{\Lambda} \) and the reference distribution \( \bar{f} \) are the distributions that are consistent with the conditional probabilities integrated over the possible future states. Introducing the intrinsic costs \( c(p), c^{adj}, \) and \( c^{non} \) allows behavior to deviate from the purely rational inattention formulation. As shown in Khaw et al. (2019), we can choose a normalization for these
intrinsic costs such that

\[ \int \exp\{-\theta_p^{-1}c(p)\} \bar{f}(p) dp = 1. \] (14)

We can then define an alternative reference measure

\[ \tilde{f}(p) \equiv \exp\{-\theta_p^{-1}c(p)\} \bar{f}(p), \]

in terms of which the prediction (11) takes the more familiar form

\[ f_t(p) = \frac{\exp\{\theta_p^{-1}V_t(p)\} \tilde{f}(p)}{\int \exp\{\theta_p^{-1}V_t(p')\} \tilde{f}(p') d(p')} \] (15)

Similarly, we can define an alternative reference probability of adjustment based on a normalization of the cost of adjusting relative to that of not adjusting, given by

\[ \tilde{\Lambda} \equiv \bar{\Lambda} \exp\{-\psi^{-1}c^{adj}\}, \]

and express the hazard function for adjustment in terms of this alternative reference probability.

The firm’s price-setting behavior is summarized by equations (15) and (??), given the model parameters \( \theta_l, \theta_p \), the reference adjustment probability \( \bar{\Lambda} \), and the reference measure \( \tilde{f} \) over possible price points. In the numerical results, we will estimate the degree to which the data favors such myopic behavior over the pure RI model, by estimating the extent to which the reference probabilities differs from the pure RI-implied objects \( \bar{\Lambda} \) and \( \tilde{f} \).

**Monetary Authority and Aggregate Uncertainty** Closing the model, the growth rate of nominal money supply follows an stochastic AR(1) process:

\[ \mu_t = (1 - \rho_\mu) \mu + \rho_\mu \mu_{t-1} + e^\mu_t \] (16)
where $\mu_t = \log(M_t/M_{t-1})$. We assume a balanced government budget, which implies that the monetary authority transfers all the seigniorage revenues to the household as lump-sum transfers.

4 Quantitative Results

In this section, we present the results of calibrating and estimating the model parameters based on data for the period 2006-2015. We begin by fixing a set of parameters based on standard values in the literature and externals sources. Then, we estimate some parameters governing the steady state of the economy to match the mean of key pricing moments in the data. Finally, we estimate the matrix of variance covariance of the model to match the joint business cycle dynamics of the pricing moments, inflation and output over the sample period. We use the estimated model to understand the underlying forces that drove the worsening of price rigidities during the financial crisis.

4.1 Steady State Results

We begin with a benchmark model of myopic information acquisition in which firms use a uniform distribution for the reference probability of reset prices. This specification is equivalent to the control cost model of Costain & Nakov (2019).

Table 3 presents the parameter values of the model. We calibrate the model to a weekly frequency. We parameterize the model to match moments of the price change distribution. Preference and technology parameters $\beta, \gamma, \epsilon, \chi, \nu$ are standard. The degree of nominal wage rigidity implies that wages are reoptimized approximately once per year. The shocks all follow AR(1) processes with fixed persistence values. The remaining parameters $\theta_p, \theta_l, \bar{\lambda}$ and $\sigma_a$ are estimated to match the mean of the frequency, size, skewness, and kurtosis of the absolute size of price changes in steady state.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.999</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.000</td>
<td>Intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>$\chi$</td>
<td>6.000</td>
<td>Marginal disutility of labor</td>
</tr>
<tr>
<td>$\nu$</td>
<td>1.000</td>
<td>Marginal utility of money holdings</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>7.000</td>
<td>Elasticity of substitution</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.000</td>
<td>Money growth rate</td>
</tr>
<tr>
<td>$\alpha^w$</td>
<td>0.987</td>
<td>Degree of wage rigidity</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.950</td>
<td>Persistence of idiosyncratic shocks</td>
</tr>
<tr>
<td>$\rho_\mu$</td>
<td>0.950</td>
<td>Persistence of money growth</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.996</td>
<td>Persistence of aggregate TFP</td>
</tr>
<tr>
<td>$\rho_\beta$</td>
<td>0.996</td>
<td>Persistence of discount factor shock</td>
</tr>
<tr>
<td>$\rho_\chi$</td>
<td>0.990</td>
<td>Persistence wage markup shocks</td>
</tr>
<tr>
<td>$\rho_\epsilon$</td>
<td>0.990</td>
<td>Persistence of price markup shocks</td>
</tr>
<tr>
<td>$\rho_{\theta_l}$</td>
<td>0.990</td>
<td>Persistence of info shocks to $\theta_l$</td>
</tr>
<tr>
<td>$\rho_{\theta_\mu}$</td>
<td>0.990</td>
<td>Persistence of info shocks to $\theta_\mu$</td>
</tr>
<tr>
<td>$\rho_{\pi^e}$</td>
<td>0.990</td>
<td>Persistence of inflation expectation shocks</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_l$</td>
<td>0.011</td>
<td>Information: unit cost of pricing reviews</td>
</tr>
<tr>
<td>$\theta_\mu$</td>
<td>0.121</td>
<td>Information: unit cost of pricing decisions</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.195</td>
<td>Frequency of price adjustment</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.030</td>
<td>Standard deviation of idiosyncratic shocks</td>
</tr>
</tbody>
</table>

Note: The last four parameters are set to target the pricing moments listed in Table 4. The remaining parameters are set at conventional values.

Since we have a parsimonious model of the economy, we augment it with a broad variety of shocks: In addition to aggregate and idiosyncratic productivity shocks and standard monetary policy shocks, we also consider similarly defined exogenous processes for labor disutility/wage markups ($\chi_t$), the discount factor ($\beta_t$), price markups ($\mu_t$), where $\mu = \text{epsilon}/\epsilon - 1$, as well as shocks to expectations of inflation.
TABLE 4: Pricing Moments in the Steady State

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Absolute size of price changes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency of price changes</td>
<td>0.028</td>
<td>0.028</td>
</tr>
<tr>
<td>Mean</td>
<td>0.107</td>
<td>0.107</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.069</td>
<td>0.075</td>
</tr>
<tr>
<td>Skweness</td>
<td>0.737</td>
<td>0.716</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.790</td>
<td>2.898</td>
</tr>
<tr>
<td>ALL index</td>
<td>101.085</td>
<td>103.080</td>
</tr>
<tr>
<td><strong>Size of price changes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.010</td>
<td>0.014</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.106</td>
<td>0.130</td>
</tr>
<tr>
<td>Skweness</td>
<td>-0.017</td>
<td>-0.152</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.736</td>
<td>2.581</td>
</tr>
<tr>
<td>ALL index</td>
<td>99.275</td>
<td>91.807</td>
</tr>
</tbody>
</table>

Note: The first four moments of the absolute size of price changes are targeted. The data moments are averages over the 2006-2015 sample period.

These shocks are a reduced form way of capturing the unmodeled dynamics of the Great Recession. We also allow for shocks to the severity of information frictions \( \theta_{lt} \) and \( \theta_{pt} \). For all these variables we assume processes of the form

\[
\log(x_t) = \rho_x \log(x_{t-1}) + \varepsilon_t^x. \tag{17}
\]

The shocks to the information costs are a reduced-form way to allow for informational rigidities to change over the cycle. An increase in these unit costs would be interpreted as a worsening of the quality of information firms use to make pricing
decisions. Our goal is not to argue that such shocks exist as exogenous sources of volatility; rather, we seek to test if the pricing dynamics observed during the recession suggest any change in the degree of informational rigidities beyond what would be captured by variations in the quantity of information alone.

Table 4 presents the resulting pricing moments. The model matches the data in terms of the steady state values for the moments characterizing the distribution of price changes. This is not particularly surprising, since prior work has shown that various models of imperfect pricing flexibility can match these moments, but the model provides a particularly good fit in terms of the shape of the entire distribution, as shown in Figure 3.

Moreover, the data warrant a breakdown of the information frictions that is particularly severe along the pricing dimension, rather than along the dimension of whether to adjust the price. First, consider Figure 4, which plots the estimated probability of a price review as a function of the firm’s price and productivity. The probability of a review is very low for much of the state space, reflecting the firm’s desire to avoid making pricing mistakes (the estimated $\theta_p$ is relatively high compared to $\theta_l$). But once the price gets far enough out of line, the hazard function steepens quickly, such
that, if this were a model in which adjustment where to occur to the optimal full information price, the estimated hazard function alone would yield only modest price rigidities.

Figure 5 shows the estimated probability of charging each price as a function of firm’s productivity. Even though firms tend to set low prices when they are highly productive, there is significant price setting dispersion: for a given productivity there is wide range of prices that the firm could charge with a significant probability reflecting a high degree of information rigidity in setting the optimal price (high $\theta_p$). The models that assume reset prices based on perfect information would yield a degenerate distribution. Here, instead, there is significant dispersion, and this dispersion is
the key determinant of both the pricing moments and the degree of aggregate price rigidity.

4.2 Model dynamics

Since we allow firms to reoptimize their information acquisition choice, we can see both the interdependence between the two decisions and how information acquisition responds to other shocks. Figure 6 shows how the acquisition of information for the adjustment decision and for the price setting decision, $I_l$ and $I_p$ respectively, respond to changes in the information costs and in other shocks. When the cost of making accurate adjustment decisions $\theta_l$ increases, the firm chooses to acquire less information for both decisions: this suggests that its adjustment decision has to be accurate enough in order to warrant allocating resources to figuring out what price to charge, rather than keeping prices unchanged. Conversely, when the cost of acquiring information to choose its price increases, the firm chooses to make its timing of price adjustment more accurate. Intuitively, the firm trades off pricing mistakes for improved timing of price reviews.

In response to other shocks, $I_l$ and $I_p$ tend to move in opposite directions, high-
lighting that there is some degree of substitutability between them. Interestingly, TFP, wage and price markup shocks all induce the firm to improve the accuracy of its pricing decision, while demand shocks like the money growth shock, the discount factor (or risk premium) shock and the shock to inflation expectations all reduce the firm’s incentive to acquire more precise pricing information.

How do our pricing moments respond to shocks in the model? We find that innovations in money growth, the discount factor, inflation expectations, and markup shocks affect the dynamics of aggregate inflation mostly through their effect on the frequency of adjustment. Conversely, the worsening of information frictions, modeled as an increase in the information unit costs $\theta_l$ and $\theta_p$, has a large effect on all pricing moments, as shown in Figure 7.

4.3 Understanding price rigidities in the Great Recession

Based on our estimated model, we now aim to understand the severity of price rigidities during the Great Recession. We do this in two steps: first, we estimate the matrix of variance covariance of the model to match business cycle properties of
our pricing moments, inflation and GDP growth. Next, we use our fully estimated model (steady state parameters and shocks covariance matrix) to filter out the shocks during our sample period by using the Kalman filter.

Denoting by $\Sigma_t = [\epsilon^1_t, ...]$ the vector of potentially correlated innovations, the matrix of variance covariance of this model is given by:

$$E[\Sigma'\Sigma] = \chi E[\varepsilon'\varepsilon] \chi'$$

(18)

where $\varepsilon$ is the vector of orthogonal shocks, $E[\varepsilon'\varepsilon]$ is the (diagonal) matrix of variance covariance of these shocks, and $\chi$ is a matrix of weights. We estimate matrix $\chi$ in order to match the weighted covariance matrix of the six pricing moments, annual inflation, and GDP growth over the sample period, where we give more weight to matching the diagonal entries of the matrix.\(^6\) In order to identify matrix $\chi$, we assume that it is lower triangular with the following ordering of innovations: productivity, wage

\(^6\)Specifically, we target 100 times the standard deviation for the diagonal entries and the correlation for the off diagonal elements.
<table>
<thead>
<tr>
<th></th>
<th>$\theta_p$</th>
<th>$\theta_l$</th>
<th>Money</th>
<th>Risk Premium</th>
<th>$\pi^e$</th>
<th>Price Markup</th>
<th>Wage Markup</th>
<th>TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_p$</td>
<td>0.140</td>
<td>-0.113</td>
<td>0.104</td>
<td>0.082</td>
<td>0.163</td>
<td>0.068</td>
<td>0.075</td>
<td>1.054</td>
</tr>
<tr>
<td>$\theta_l$</td>
<td>-0.051</td>
<td>0.067</td>
<td>-0.306</td>
<td>-0.921</td>
<td>-0.068</td>
<td>1.146</td>
<td>1.171</td>
<td></td>
</tr>
<tr>
<td>Money</td>
<td>0.000</td>
<td>0.001</td>
<td>0.004</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.005</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Risk Premium</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>$\pi^e$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>Price Markup</td>
<td>0.000</td>
<td></td>
<td>0.000</td>
<td>-0.008</td>
<td>-0.008</td>
<td>-0.036</td>
<td>0.055</td>
<td></td>
</tr>
<tr>
<td>Wage Markup</td>
<td></td>
<td></td>
<td>0.023</td>
<td>0.055</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TFP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.019</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This matrix represents the estimated Cholesky decomposition of the matrix of variance covariance of the shocks. In other words, denoting this matrix by $C$, the matrix of variance covariance of the model is given by: $C' C$.

markup, price markup, expected inflation, risk premium, money growth, unit cost of reviews, unit cost of pricing policy. The estimated matrix $\chi$ is presented in Table 5.

To infer the shocks during our sample period, we make use of the same series used for the model estimation: frequency, size, skewness and kurtosis of the absolute value of price changes along with the annual inflation rate and the annual GDP growth.\textsuperscript{7} Figure 8 plots the average value of the weekly orthogonal shocks for each quarter, and Figure 9 shows the filtered correlated shocks.

Based on the results of our filtering, the Great Recession was characterized by large increase in risk premium and large declines in inflation expectations. Figure 8 shows large risk premium and inflation expectations shocks during the great recession of up to 0.7 and 0.5 standard deviations, respectively. Figure 8 also reveals substantial monetary policy accommodation with shocks to the growth rate of money as high as 0.3 standard deviations.

\textsuperscript{7}Given that our model is calibrated to a weekly frequency and that we use quarterly data, we show in the Supplementary Appendix how to implement the Kalman filter in this case.
Figure 8: Exogenous Shocks

Figure 9: Endogenous Shocks
5 Conclusion

A growing literature has argued that monetary policy may be less effective during downturns. This argument rests on evidence that price dispersion rises during downturns, which existing models predict implies more price flexibility. Our results suggest that this chain of reasoning did not apply during the Great Recession. In fact, we would argue the opposite: price rigidities rose significantly during the downturn.

One interpretation of our results is that they show that moments of the price distribution are not sufficient to pin down the effectiveness of monetary policy. In a way this is an expansion of the point originally made by the price setting literature of a decade ago, which showed that—contrary to popular belief—the frequency of price changes is not a sufficient statistic for the degree of nominal rigidity. We push this point further by showing that different models that match a range of moments of the price distribution but have different assumptions regarding the information that is available to decision makers when making price setting decisions also have different implied degrees of monetary non-neutrality over the business cycle. Our conclusion is that more work is needed to measure the severity of information frictions under which economic agents make decisions.
References


A Appendix

A.1 Data

**Nielsen Inflation** We compute a monthly price index for each module \((m)\) and state \((s)\) as follows:

\[
P_{mst} = P_{mst-1} \left( \frac{\sum_{i \in \{m,s\}} p_{i,t} \bar{q}_{i,y(t)-1}}{\sum_{i \in \{m,s\}} p_{i,t-1} \bar{q}_{i,y(t)-1}} \right) \quad (A.1)
\]

where \(p_{it}\) is the average price of a UPC \(\times\) store observation \(i\) during month \(t\), \(\bar{q}_{i,y(t)-1}\) is the average quantity of that good sold over the previous year, where \(p_{i,0} = 1\). Then this price index is aggregated across modules and states to generate the aggregate price index

\[
P_t = P_{t-1} \Pi_{m,s} \left( \frac{P_{mst}}{P_{mst-1}} \right)^{\frac{S_{mst} + S_{mst-1}}{2}} \quad (A.2)
\]

where \(S_{mst}\) is the expenditure share of module \(m\) in state \(s\) for month \(t\).

**Regional Inflation** Using the regional variation of our data, we compute a monthly price index for each module \((m)\) and state \((s)\) as follows:

\[
P_{mst} = P_{mst-1} \left( \frac{\sum_{i \in \{m,s\}} p_{i,t} \bar{q}_{i,y(t)-1}}{\sum_{i \in \{m,s\}} p_{i,t-1} \bar{q}_{i,y(t)-1}} \right) \quad (A.3)
\]

where \(p_{it}\) is the average price of a UPC \(\times\) store observation \(i\) during month \(t\), \(\bar{q}_{i,y(t)-1}\) is the average quantity of that good sold over the previous year, where \(p_{i,0} = 1\). Then this price index is aggregated across modules for each state to compute the price index for state \(s\):

\[
P_{st} = P_{st-1} \Pi_{m} \left( \frac{P_{mst}}{P_{mst-1}} \right)^{\frac{S_{mst} + S_{mst-1}}{2}} \quad (A.4)
\]

where \(S_{mst}\) is the expenditure share of module \(m\) in state \(s\) for month \(t\).

A.2 Filtering

Assume that the vector of weekly state variables \((E_t)\) of size \(ne\) evolves according to:

\[
E_{t+1} = FE_t + e_t^w \quad (A.5)
\]

\[
e_t^w = \begin{bmatrix} \Omega \cdot u_t \\ \emptyset \end{bmatrix} \quad (A.6)
\]

\[
E \left[ e_t^w e_t^{w'} \right] = Q = \begin{bmatrix} \Omega \Omega' & \emptyset \\ \emptyset & \emptyset \end{bmatrix} \quad (A.7)
\]
where $\Omega$ is the impact matrix, which we assume to be lower triangular, and $v_t$ is the vector of orthogonal shocks of size $ns$ and associated matrix of variance covariance equal to the identity matrix. Then, iterating forward for $q$ periods, where $q$ is equal to the number of week in a quarter:

\[ E_{t+q} = \tilde{F}E_t + \epsilon_{t+q}^q \]  
\[ \tilde{F} = F^q \]  
\[ \epsilon_{t+q}^q = \tilde{x} \cdot [e_{t+q}^w e_{t+q-1}^w \cdots e_{t}^w]' \]  
\[ \tilde{x} = [I_{n_e} F F^2 \cdots F^{q-1}] \]  
\[ E[\epsilon_t^q \epsilon_t^q]' = \tilde{Q} \]  
\[ \tilde{Q} = \tilde{x} (I_q \otimes Q) \tilde{x}' \]  

Each quarter, we observe vector $y_{t+q}$, which collects the average weekly pricing moments over the quarter (frequency, size, skewness and kurtosis) along with the average annual inflation rate and the annual GDP growth (4 quarter change of the quarterly log-GDP). These jumping variables can be expressed as:

\[ y_{t+q} = H \cdot E_{t+q} \]

The unconditional variance for $E$, at a quarterly frequency, is given by matrix $P_{1|0}$ that satisfies:

\[ P_{1|0} = \tilde{F} P_{1|0} \tilde{F}' + \tilde{Q} \]

To filter the value of the state variables, denoted by $\hat{E}_{t|t}$, we proceed to iterate on:

\[ \hat{E}_{t|t} = \hat{E}_{t|t-q} + P_{t|t-q} H (H' P_{t|t-q} H)^{-1} (y_t - H' \hat{E}_{t|t-1}) \]  
\[ P_{t|t} = P_{t|t-q} - P_{t|t-q} H (H' P_{t|t-q} H)^{-1} H' P_{t|t-q} \]  
\[ \hat{E}_{t+q|t} = \tilde{F} \hat{E}_{t|t} \]  
\[ P_{t+q|t} = \tilde{F} P_{t|t} \tilde{F}' + \tilde{Q} \]

**Smoothing**

After collecting the filtered series, the sequence of smoothed estimates is computed as follows:

\[ \hat{E}_{t|T} = \hat{E}_{t|t} + J_t \left( \hat{E}_{t+q|T} - \hat{E}_{t+q|t} \right) \]  
\[ J_t = P_{t|t} \tilde{F}' P_{t+1|t}^{-1} \]
Filtering weekly shocks

Given a sequence of filtered (or smoothed) state variables, we inferred the sequence of aggregate shocks $e^q_t$ as follows:

$$
\hat{e}^q_t = \hat{E}_{t|T} - \tilde{F} \tilde{E}_{t-q|T}
$$

(A.22)

Given that $e^q_t$ is a linear combination of weekly shocks, filtered the weekly shocks using the Kalman filter. To this end, notice that the sequence of quarterly shocks is serially uncorrelated at a quarterly frequency. In other words, $E[e^q_t e'^q_{t-q}] = \emptyset$. Hence, we can filter weekly shocks for quarter $q$ independently of filtering process for quarter $\tilde{q}$. Then, notice that we can express this problem in state-space form as follows:

$$
\Xi_t = f\Xi_{t-1} + \tilde{e}_t
$$

(A.23)

$$
e^q_t = \tilde{x}\Xi_t
$$

(A.24)

where

$$
\Xi_t = \begin{bmatrix} e^w_t & e^w_{t-1} & \cdots & e^w_{t-q} \\
\end{bmatrix}
$$

(A.25)

$$
f = \begin{bmatrix} \emptyset_{ne \times (ne-q)}; & [I_{ne(q-1)} \ \emptyset_{ne(q-1) \times ne-q}] \\
\end{bmatrix}
$$

(A.26)

$$
\tilde{e}_t = \begin{bmatrix} e^w_t & e^w_{t-1} & \cdots & e^w_{t-q} \\
\end{bmatrix}
$$

(A.27)

$$
E[\tilde{e}\tilde{e}'] = \tilde{Q}^w = I_q \otimes Q
$$

(A.28)

Hence,

$$
\hat{\Xi}_{t|t} = \tilde{P}_{1|0} \tilde{x}' \left( \tilde{x} \tilde{P}_{1|0} \tilde{x}' \right)^{-1} e^q_t
$$

(A.29)

where $\tilde{P}_{1|0}$ solves:

$$
\tilde{P}_{1|0} = f \tilde{P}_{1|0} f' + \tilde{Q}^w
$$

(A.30)

Now, given that $\tilde{Q}^w$ may not have full rank. We should re-write this problem such that vectors $e^w_t$ and $e^q_t$ only contain those variables with an associated shock. Hence, those vectors effective size becomes $ns$ instead of $ne$. 

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