# Investor Sophistication and Capital Income Inequality[*]

Marcin Kacperczyk
Imperial College London & CEPR

Jaromir Nosal
Boston College

Luminita Stevens
University of Maryland

September 28, 2018

## Abstract

Capital income inequality is large and growing fast, accounting for a significant portion of total income inequality. We study its growth in a general equilibrium portfolio choice model with endogenous information acquisition and heterogeneity across household sophistication and asset riskiness. The model implies capital income inequality that grows with aggregate information technology. Investors differentially adjust both the size and composition of their portfolios, as unsophisticated investors retrench from trading risky securities and shift their portfolios toward safer assets. Technological progress also reduces aggregate returns and increases the volume of transactions, features that are consistent with recent U.S. data.

# 1 Introduction

The rise in income and wealth inequality has been among the most hotly discussed topics in academic and policy circles.[1] Among the possible explanations, heterogeneity in the returns on savings—due to differences in rates of return or in the composition of the risky portfolio—has been highlighted as an important driver. This factor has emerged in empirical work on the wealth distribution, such as Fagereng, Guiso, Malacrino & Pistaferri (2016a; 2016b) and in research focused on the very top of the wealth distribution (Benhabib, Bisin & Zhu, 2011).[2] However, as noted by De Nardi & Fella (2017), more work is needed to understand the determinants of such heterogeneity.

This paper studies capital income inequality growth in a portfolio choice model with information constraints. When investors differ in their capacity to process news about risky asset payoffs, both the size and the composition of the risky portfolios differ across investors. Not surprisingly, this generates inequality. More interestingly, progress in the aggregate information processing technology can exacerbate this inequality, and this effect can be economically large, as less sophisticated investors get priced out of high-return assets. This pecuniary externality arises even in a setting with a single risky asset, but is amplified in an economy with heterogeneous assets.

At the core of our model is each investor's decision of how much to invest in assets with different risk characteristics. This decision is shaped by the investors' capacity to pro-

---

[1] See Piketty & Saez (2003); Atkinson, Piketty & Saez (2011). A comprehensive discussion is also offered in the 2013 Summer issue of the Journal Economic Perspectives and in Piketty (2014).

[2] See also the review by Benhabib & Bisin (2017). Saez & Zucman (2016) emphasize the role of differential savings rates, rather than differential rates of return, in generating wealth inequality. However, their capitalization method imposes homogeneity across investors on the rates of return within asset classes, thereby ruling out one mechanism over the other.

cess information about asset payoffs, and by their choice of how to allocate this capacity across assets.[3] We model the learning choice using the theory of rational inattention of Sims (2003). While stylized, the framework captures several appealing aspects of learning. First, getting information about one's investments requires expending resources. Second, learning about more volatile assets consumes more resources. Lastly, investors can allocate their information capacity optimally across different types of assets, depending on their objective and the characteristics of the assets they invest in. Our theoretical framework generalizes existing models—Van Nieuwerburgh & Veldkamp (2010) in particular—by considering heterogeneously informed agents investing in multiple heterogeneous assets.[4]

We analytically characterize three channels of how investor heterogeneity generates capital income inequality: Investors with higher information capacity hold larger portfolios on average, tilt their average holdings towards riskier assets within the risky portfolio, and adjust their investments more aggressively in response to changes in payoffs. These patterns are consistent with the empirical literature on portfolio composition differences between wealthy and less wealthy investors, going back to Greenwood (1983), and Mankiw & Zeldes (1991), and discussed more recently by Fagereng et al. (2016b) and Bach, Calvet & Sodini (2015).

Our central result is that growth in aggregate information capacity, interpreted as a general progress in information-processing technologies, disproportionately benefits the initially more skilled investors, and leads to growing capital income inequality. As the aggregate

---

[3]In the model, we endow each investor with a particular level of information processing capacity. However, this capacity should be interpreted more broadly, as a stand-in for the individual's ability to access high quality investment advice, not limited to his or her own knowledge of or ability to invest in financial markets.

[4]In finance, rational inattention models have been used successfully to address underdiversification puzzles, price volatility and comovement puzzles, overconfidence, and the home bias, among other applications. References include Peng (2005), Peng & Xiong (2006), Van Nieuwerburgh & Veldkamp (2009; 2010), Mondria (2010). See also Maćkowiak & Wiederholt (2009; 2015), Matějka (2015), and Stevens (2018) for applications in macroeconomics. Our application to inequality is new, to our knowledge.

capacity to process information grows, all investors would like to grow their portfolios. However, in equilibrium, prices increase in response to the higher demand, and only the sophisticated investors expand their portfolios. The less sophisticated investors are priced out and retrench to lower-risk, lower-return assets, which amplifies capital income inequality. This result holds regardless of the learning technology assumed, and the specific functional form for information acquisition only affects the magnitude of the effect.

Our mechanism is amplified in a setting with heterogeneous assets, because the shifts in ownership shares occur asymmetrically across assets. Allowing investors to choose how to learn about different assets is critical here: With endogenous information choice, the sophisticated ownership share grows most for the most volatile assets, which are precisely the assets that generate the largest capital income gains. As a result, the model with multiple risky assets generates more inequality growth compared with a model with one risky asset.

To provide some guidance regarding the magnitudes of the effects identified in our model, we conduct a set of numerical experiments in a parameterized economy. We show that a 5% annual growth in aggregate information capacity[5] generates a rise in capital income inequality of 38% over 24 years. In contrast, an economy with a single risky asset generates only 20% growth. Calibrating the information capacity growth is challenging because the information that investors have when they make their investment decisions is not observable. However, for a range of plausible values of recent growth in information capacity, inequality growth ranges from 24% to 60%. The corresponding number in the *Survey of Consumer Finances* (SCF) for the 1989-2013 period is 87%.[6] General progress in information technology also generates

---

[5]This annual growth rate is chosen to generate an average market return of 7% in the model. We discuss the parameterization in detail in Section 4.

[6]We define capital income inequality as the ratio between the average capital income of the top 10% of investors by wealth and that of the bottom 90% of investors by wealth, conditioning on participation in

lower market returns, higher market turnover, and larger and more volatile portfolios. These predictions are broadly consistent with the data on turnover and ownership from CRSP and Morningstar on stocks and mutual funds over the last 25 years.

Our findings connect to the idea that generating the inequality in outcomes observed in the data requires linking rates of return to wealth–which is our indicator for access to better information on investment strategies.[7] This idea has a long history, going back to Aiyagari (1994), who discusses the wide disparities in portfolio compositions across the wealth distribution, emphasizing the fact that rich households are much more likely to hold risky assets. Subsequently, Krusell & Smith (1998) suggest that the data requires that wealthy agents have higher propensities to save, generate higher returns on savings, or both. Benhabib et al. (2011) and Gabaix, Lasry, Lions & Moll (2016) are recent theoretical treatments and Favilukis (2013), Cao & Luo (2017), and Kasa & Lei (2018) are related quantitative contributions. We complement this literature along two key dimensions. First, we study the within-period portfolio problem with multiple risky assets, rather than the dynamic savings decision with a single risky asset. Second, we study inequality in a general equilibrium context with endogenous returns, rather than with exogenous idiosyncratic investment returns. Both asset heterogeneity and the endogenous response of asset prices–and hence returns–are key sources of amplification for inequality.

Our work contributes to a broader literature on inequality in capital income, including the work on bequests (Cagetti & De Nardi (2006)), limited stock market participation (Guvenen, 2007; 2009), financial literacy (Lusardi, Michaud & Mitchell (2017)), and entrepreneurial tal-

___

financial markets. The Appendix presents all variable definitions.

[7]This connection is motivated by evidence that has linked trading strategy sophistication to asset prices, wealth and income levels, such as Calvet, Campbell & Sodini (2009), Chien, Cole & Lustig (2011), and Vissing-Jorgensen (2004).

ent (Quadrini (1999)). Our focus on differences in access to information builds on the insights of Arrow (1987). The emphasis on skill rather than risk aversion differences is supported by the portfolio-level evidence of Fagereng et al. (2016a). See Pástor & Veronesi (2016) for a one-asset model with heterogeneity in risk aversion and exogenous entrepreneurial skill differences. Also related is Peress (2004) who examines the role of wealth and decreasing absolute risk aversion in information acquisition and investment in a one-asset model.

Section 2 presents the theory. Section 3 derives analytic predictions, which is quantified in Section 4. Section 5 presents additional corroborating evidence, and Section 6 concludes.

## 2    Theoretical Framework

We set up a portfolio choice model with investors constrained in their capacity to process information about asset payoffs. Both asset characteristics and investors are heterogeneous.

**Setup**    A continuum of investors of mass one, indexed by $j$, solve a sequence of portfolio choice problems, to maximize mean-variance utility over wealth $W_j$ in each period, given risk aversion coefficient $\rho > 0$. The financial market consists of one risk-free asset, with price normalized to 1 and payoff $r$, and $n > 1$ risky assets, indexed by $i$, with prices $p_i$, and independent payoffs $z_i = \bar{z} + \varepsilon_i$, with $\varepsilon_i \sim \mathcal{N}(0, \sigma_i^2)$.[8] The risk-free asset has unlimited supply, and each risky asset has fixed supply, $\bar{x}$. For each risky asset, non-optimizing "noise traders" trade for reasons orthogonal to prices and payoffs (e.g., liquidity, hedging, or life-cycle reasons), such that the net supply available to the (optimizing) investors is $x_i = \bar{x} + \nu_i$,

---

[8]Under certain simplifying assumptions about the investors' learning technology (namely the independence of signals across assets), assuming independent payoffs is without loss of generality. See Van Nieuwerburgh & Veldkamp (2010) for a discussion of how to orthogonalize correlated assets under such assumptions.

with $\nu_i \sim \mathcal{N}(0, \sigma_x^2)$, independent of payoffs and across assets.[9] Following Admati (1985), we conjecture that prices are $p_i = a_i + b_i \varepsilon_i - c_i \nu_i$, for some coefficients $a_i, b_i, c_i \geq 0$.

Investors know the distributions of the shocks, but not the realizations $(\varepsilon_i, \nu_i)$. Prior to making their portfolio decisions, investors can obtain information about some or all of the risky asset payoffs, in the form of signals. The informativeness of these signals is constrained by each investor's capacity to process information. We consider two investor types: mass $\lambda \in (0, 1)$ of investors, labeled *sophisticated*, have high capacity to process information, $K_1$, and mass $1 - \lambda$, labeled *unsophisticated*, have low capacity, $K_2$, with $0 < K_2 < K_1 < \infty$.

Higher capacity can be interpreted as having more resources to gather and process news about different assets, and it translates into signals that track the realized payoffs with higher precision. A bound on this capacity limits investors' ability to reduce uncertainty about payoffs. Given this constraint, they choose how to allocate attention across different assets. We use the reduction in the entropy (Shannon (1948)) of the payoffs conditional on the signals as a measure of how much capacity the chosen signals consume. Starting with Sims (2003), entropy reduction has become a frequently used measure of information in a variety of contexts in economics and finance. Entropy has a number of appealing properties as a measure of uncertainty. For example, for normally distributed random variables, it is linear in variance. Moreover, the entropy of a vector independent random variables is the sum of the entropies of the individual variables. While stylized, this learning process captures the key trade-offs investors face when deciding how to allocate their limited capacity across multiple investment decisions, as a function of their objective and of the risks they face.

---

[9]For simplicity, we introduce heterogeneity only in the volatility of payoffs, although the model can easily accommodate additional heterogeneity in supply and in mean payoffs.

**Individual optimization** Optimization occurs in two stages. In the first stage, investors solve their information acquisition problem, and in the second stage, they choose portfolio holdings. We first solve the optimal portfolio choice in the second stage, for a given signal choice. We then solve for the ex-ante optimal signal choice.

Given prices and posterior beliefs, the investor chooses portfolio holdings to solve

$$U_j = \max_{\{q_{ji}\}_{i=1}^n} E_j(W_j) - \frac{\rho}{2} V_j(W_j) \tag{1}$$

$$s.t. \ W_j = r\left(W_{0j} - \sum_{i=1}^n q_{ji} p_i\right) + \sum_{i=1}^n q_{ji} z_i, \tag{2}$$

where $E_j$ and $V_j$ denote the mean and variance conditional on investor $j$'s information set, and $W_{0j}$ is initial wealth. Optimal portfolio holdings depend on the mean $\widehat{\mu}_{ji}$ and variance $\widehat{\sigma}_{ji}^2$ of investor $j$'s posterior beliefs about the payoff $z_i$, and is given by $q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i}{\rho \widehat{\sigma}_{ji}^2}$.

Given the optimal portfolio holdings as a function of beliefs, the ex-ante optimal distribution of signals maximizes ex-ante expected utility, $E_{0j}[U_j] = \frac{1}{2\rho} E_{0j}\left[\sum_{i=1}^n \frac{(\widehat{\mu}_{ji} - rp_i)^2}{\widehat{\sigma}_{ji}^2}\right]$. The choice of the vector of signals $s_j = (s_{j1}, ... s_{jn})$ about the vector of payoffs $z = (z_1, ..., z_n)$ is subject to the constraint $I(z; s_j) \leq K_j$, where $K_j$ is the investor's capacity for processing news about the assets and $I(z; s_j)$ quantifies the reduction in the entropy of the payoffs, conditional on the vector of signals (defined below).

For analytical tractability, we assume that the signals $s_{ji}$ are independent across assets and investors. Then, the total quantity of information obtained by an investor is the sum of the quantities of information obtained for each asset, $I(z_i; s_{ji})$. We can think of the information problem as a decomposition of each payoff into the signal component and a residual component that represents the information lost because of the investor's capacity constraint, $z_i = s_{ji} + \delta_{ji}$. If the signal and the residual are independent, then posterior beliefs

7

are also normally distributed random variables, with mean $\widehat{\mu}_{ji} = s_{ji}$ and variance $\widehat{\sigma}^2_{ji} = \sigma^2_{\delta ji}$. The investor chooses the precision of posterior beliefs for each asset to solve[10]

$$\max_{\{\widehat{\sigma}^2_{ji}\}^n_{i=1}} \sum_{i=1}^n G_i \frac{\sigma^2_i}{\widehat{\sigma}^2_{ji}} \quad s.t. \quad \frac{1}{2} \sum_{i=1}^n \log\left(\frac{\sigma^2_i}{\widehat{\sigma}^2_{ji}}\right) \le K_j, \tag{3}$$

$$G_i \equiv (1 - rb_i)^2 + \frac{r^2 c_i^2 \sigma_x^2}{\sigma_i^2} + \frac{(\overline{z} - ra_i)^2}{\sigma_i^2}, \tag{4}$$

where $G_i$ are the utility gains from learning about asset $i$. These gains are a function of equilibrium prices and asset characteristics only; they are common across investor types, and taken as given by each investor.

**Lemma 1.** *The solution to the capacity allocation problem (3)-(4) is a corner: Each investor allocates capacity to reducing posterior uncertainty for the asset with the largest learning gain $G_i$. If multiple assets have equal gains, the investor randomizes among them.*

The linear objective and the convex constraint imply that each investor specializes, monitoring only one asset, regardless of her level of sophistication. For all other assets, portfolio holdings are determined by prior beliefs. If there are multiple assets are tied for the highest gain, the investor randomizes among them, with probabilities that are determined in equilibrium. But she continues to allocate all capacity to a single asset. Spreading individual capacity across multiple assets–even if they have equal gains from learning–would lower utility. This result extends the specialization results of Van Nieuwerburgh & Veldkamp (2010) to the case of heterogeneous assets and investors.

**Equilibrium**   Given the solution to the individual optimization problem, equilibrium prices are linear combinations of the shocks.

---

[10]The investor's objective omits terms from the expected utility function that do not affect the optimization. See the Appendix for detailed derivations.

**Lemma 2.** *The price of asset $i$ is given by $p_i = a_i + b_i\varepsilon_i - c_i\nu_i$, with*

$$a_i = \frac{1}{r}\left[\overline{z} - \frac{\rho\sigma_i^2\overline{x}}{(1+\Phi_i)}\right], \quad b_i = \frac{\Phi_i}{r(1+\Phi_i)}, \quad c_i = \frac{\rho\sigma_i^2}{r(1+\Phi_i)}, \tag{5}$$

$$\Phi_i \equiv m_{1i}\lambda\left(e^{2K_1} - 1\right) + m_{2i}(1-\lambda)\left(e^{2K_2} - 1\right), \tag{6}$$

*where $\Phi_i$ measures the information capacity allocated to learning about asset $i$ in equilibrium, and $m_{1i}, m_{2i} \in [0,1]$ are the fractions of sophisticated and unsophisticated investors who choose to learn about asset $i$.*

Prices reflect payoff and supply shocks, with relative importance determined by amount of attention allocated to each asset, $\Phi_i$. If there is no learning, the price only reflects the supply shock $\nu_i$. As the attention allocated to an asset increases, the price co-moves more with the payoff. As $K_j \to \infty$, the price approaches the discounted realized payoff, $z_i/r$.

Given prices, we can now determine the allocation of attention across assets. Let assets be indexed so that $\sigma_i > \sigma_{i+1}$, and let $\xi_i \equiv \sigma_i^2(\sigma_x^2 + \overline{x}^2)$ summarize the properties of asset $i$.

**Lemma 3.** *Let $k$ denote the endogenous number of assets that are learned about. The allocation of information capacity across assets, $\{\Phi_i\}_{i=1}^n$, is uniquely pinned down by the conditions $G_i = \max_{h\in\{1,...,n\}} G_h$ for all $i \in \{1,...,k\}$, and $G_i < \max_{h\in\{1,...,n\}} G_h$ for all $i \in \{k+1,...,n\}$, where in equilibrium the gain from learning about each asset is $G_i = \frac{1+\rho^2\xi_i}{(1+\Phi_i)^2}$.*

The equilibrium gains from learning are asset-specific and depend only on the properties of the asset, $\xi_i$, and on the amount of attention devoted to that asset, across all investors, $\Phi_i$. The model uniquely pins down the number of assets that are learned about and the amount of attention allocated to each asset. Aggregate capacity in the economy may be high enough that in equilibrium it is spread across multiple assets. In this case, each investor continues

to allocate her entire capacity to a single asset, but is now indifferent in terms of which of these assets to learn about. The investor randomizes, with the probability of learning about each asset being determined by the equilibrium conditions in Lemma 3.

With heterogeneous investor capacity, the model does not pin down how much attention each investor class contributes: All that matters is the total capacity $\Phi_i$ allocated to each asset. In the absence of empirical evidence to guide us on how the two groups are distributed, for our analytical and numerical results we will consider a symmetric distribution in which investors of the two types contribute capacity in proportion to their size in the population, so that $m_{1i} = m_{2i}$. This assumption is motivated by our result that the gains from learning are the same for the two investor types, so that it is not obvious why they would choose different strategies. It also implies that capacity can be written as $\Phi_i = \phi_i m_i$, with $\phi_i$ an exogenous measure of the economy's information capacity, which we will vary to explore how the model responds to technological progress in information.[11]

# 3   Predictions

In this section, we present analytic results implied by our information friction. Heterogeneous information implies differences in portfolio sizes, a different composition of the risky portfolio across investors, and different responsiveness to payoff shocks. Moreover, technological progress amplifies these forces, resulting in further growth in inequality.

**The Effects of Heterogeneity on Inequality**   How do differences in capacity translate into differences in portfolio holdings and capital income? Let $q_{1i}$ and $q_{2i}$ denote the average

---

[11]In Section 4, we investigate the sensitivity of our central results to this assumption.

per-capita holdings of asset $i$ for sophisticated and unsophisticated investors, given by

$$q_{1i} = \left( \frac{\overline{z}_i - rp_i}{\rho \sigma_i^2} \right) + m_{1i} \left( e^{2K_1} - 1 \right) \left( \frac{z_i - rp_i}{\rho \sigma_i^2} \right),$$ (7)

and $q_{2i}$ defined analogously. Equation (7) shows that per-capita holdings are the quantity that would be held under the investors' prior beliefs plus a quantity that is increasing in the realized excess return. The weight on the realized excess return is asset and investor specific. It is given by the amount of information capacity allocated to this asset by this investor group. Investors hold all assets, but invest relatively more in the asset they learn about. Hence, the model generates under-diversification of portfolios, consistent with the empirical evidence (e.g., Vissing-Jorgensen (2004) and references therein).

For actively traded assets, heterogeneity in capacities generates differences in ownership across investor types at the asset level. In a symmetric equilibrium, the average per-capita ownership difference, as a share of the supply of each asset, is

$$\frac{E\left[q_{1i} - q_{2i}\right]}{\overline{x}} = \left( e^{2K_1} - e^{2K_2} \right) \frac{m_i}{1 + \phi m_i} > 0.$$ (8)

Hence, the portfolio of the sophisticated investor is not simply a scaled up version of the unsophisticated portfolio. Rather, the portfolio weights *within* the class of risky assets also differ across the two investor types.

**Proposition 1 (Ownership).** *Let $k > 1$ be the number of assets actively traded in equilibrium. Then, for $i \in \{1, ..., k\}$,*

*(i) $E\left[q_{1i} - q_{2i}\right]/\overline{x}$ is increasing in $\sigma_i^2$ and in $E\left[z_i - rp_i\right]$;*

*(ii) $q_{1i} - q_{2i}$ is increasing in realized excess returns $z_i - rp_i$.*

Sophisticated investors hold a larger portfolio of risky assets on average, tilt their portfolio

towards more volatile assets with higher expected excess returns, and adjust ownership, state by state, towards assets with higher *realized* excess returns.

To see the effects of the portfolio scale and composition differences on capital income, let capital income be $\pi_{ji} \equiv q_{ji}(z_i - rp_i)$. Average capital income diverges with the gap in capacities, differentially across assets $i$:

$$E[\pi_{1i} - \pi_{2i}] = \frac{1}{\rho}m_i G_i \left(e^{2K_1} - e^{2K_2}\right) > 0. \tag{9}$$

**Proposition 2 (Capital Income).** *Let $k > 1$ be the number of assets actively traded in equilibrium. Then, for $i \in \{1, ..., k\}$,*

*(i) $E[\pi_{1i} - \pi_{2i}]$ is increasing in asset volatility $\sigma_i$;*

*(ii) $\pi_{1i} - \pi_{2i} \geq 0$, and is increasing in realized excess returns $z_i - rp_i$.*

The average sophisticated investor realizes larger profits in states with positive excess returns, and incurs smaller losses in states with negative excess returns. The biggest difference in profits comes from investment in the more volatile, higher expected excess return assets. It is these volatile assets that drive inequality because they generate the biggest gain from learning, and hence the biggest advantage from having relatively high capacity.

To see the effects of an increase in capacity dispersion, consider an experiment in which dispersion rises but without changing the aggregate capacity in the economy.

**Proposition 3 (Capacity Dispersion).** *Let $k > 1$ be the number of assets actively traded in equilibrium. Consider an increase in capacity dispersion, $K_1' = K_1 + \Delta_1 > K_1$, $K_2' = K_2 - \Delta_2 < K_2$, with $\Delta_1$ and $\Delta_2$ such that the total information capacity $\phi$ remains unchanged. Then, for $i \in \{1, ..., k\}$,*

*(i) Asset prices and excess returns remain unchanged.*

(*ii*) *The difference in ownership shares* $(q_{1i} - q_{2i})/\overline{x}$ *increases.*

(*iii*) *Capital income gets more polarized as* $\pi_{1i}/\pi_{2i}$ *increases state by state.*

Increasing dispersion in capacities while keeping aggregate capacity unchanged implies further polarization in holdings and capital income. As dispersion reaches its maximum level, unsophisticated investors approach zero capacity and invest based on their prior beliefs. However, dispersion in capacity has no effect on asset prices. Both the number of assets learned about and the mass of investors learning about each asset remain unchanged. Hence, the adjustment reflects a pure transfer of income from the relatively unsophisticated investors to the more sophisticated investors without any general equilibrium effects.

**The Consequences of Growth in Capacity** Our central result considers the effects of growth in aggregate capacity, interpreted as general progress in information-processing technologies. The effect of capacity growth on asset prices and inequality operate through its effects on the gains from learning and on the mass of investors learning about different assets. Figure 1 shows the evolution of masses and gains from learning as aggregate capacity grows. At low capacity, all investors learn about the most volatile asset, but as capacity grows, the gains from learning about this asset decline, and strategic substitutability in learning pushes some investors to learn about less volatile assets. The threshold that endogenizes single-asset learning as an optimal outcome is given by $\phi_1 \equiv \sqrt{\frac{1+\rho^2\xi_1}{1+\rho^2\xi_2}} - 1$. For capacity above $\phi_1$, at least two assets are learned about and for sufficiently high information capacity, all assets are learned about.[12] Nevertheless, not all assets are learned about with the same intensity: The mass of investors who learn about an asset is decreasing in its volatility. This allocation

---

[12]thus endogenizing the assumption employed in models with exogenous signals.

of attention affects the holdings across assets, and hence the investors' portfolio returns.

**Proposition 4 (Symmetric Growth).** *Let $k \leq 1$ be the number of assets actively traded in equilibrium. Consider an increase in aggregate capacity $\phi$ generated by a symmetric growth in capacities to $K_1' = (1 + \gamma) K_1$ and $K_2' = (1 + \gamma) K_2$, $\gamma \in (0, 1)$. Let $k' \geq k$ denote the new number of actively traded assets. For $i \in \{1, ..., k'\}$,*

*(i) Average asset prices increase and average excess returns decrease, approaching the risk free rate in the limit.*

*(ii) Average ownership share of sophisticated investors $E[q_{1i}]/\overline{x}$ increases and average ownership share of unsophisticated investors $E[q_{2i}]/\overline{x}$ decreases, and the gap is increasing in asset volatility.*

*(iii) As long as the return on the risky portfolio exceeds the risk-free rate, average capital income gets more unequal, as $E[\pi_{1i}]/E[\pi_{2i}]$ increases, with inequality being higher for the more volatile assets.*

Higher capacity to process information means that investors have more precise news about the realized payoffs, resulting in lower gains from learning, lower average returns, and larger and more volatile positions. However, as asset prices increase and returns decline, inequality keeps increasing. Sophisticated investors increase their ownership share at the expense of the less sophisticated investors, who retreat. This pecuniary externality arises regardless of the learning technology, since it is due to the fact that posterior variance is lower for the sophisticated investors, and hence on average they want to hold a larger quantity than the unsophisticated investors. Moreover, the increase in ownership is larger for the more volatile assets that have higher gains from learning and generate higher expected

returns. Hence, asset heterogeneity combined with endogenous information choice generates differential ownership growth that in turn amplifies the growth in inequality.

As capacity continues to grow, the decline in returns eventually becomes a mitigating factor in the growth of income inequality. Intuitively, if market returns are close to the risk free rate, then there is less scope in the economy for extracting informational rents. Capital income inequality peaks as rates of return reach the risk free rate. It subsequently starts to decline, and eventually, it stabilizes at a level implied by the differences in risk-free return income earned on on previously accumulated wealth. In the limit, all information is revealed and capital income inequality becomes flat. This process is shown in Figure 2.

# 4   Quantitative Analysis

So far, we have found that progress in information technology can qualitatively generate growing capital income inequality, through changes in both portfolio size and composition across investors. We now parameterize the model to provide some guidance for the magnitudes implied by this mechanism. We use data on household capital income from the SCF and data on the financial market from CRSP. We parameterize the model based on data from the first half of the SCF sample (1989-2000), and then we consider an experiment in which aggregate information capacity in the economy grows at a constant rate, to generate predictions for the second half of the sample (2001-2013).

## 4.1 Technological Progress and Inequality Growth

Table 1 presents parameter values and targets for the baseline results. The parameters characterizing the financial market are the risk free rate, $r = 2.5\%$, which matches the 3-month T-bill rate net of inflation over the period, the number of risky assets, $n$, which we set to 10 arbitrarily, and the means and volatilities of payoffs and noise shocks. In the absence of detailed information regarding holdings of different types of securities at the household level, we target volatility moments from the U.S. equities market. We set the dispersion in the volatilities of asset payoffs $\sigma_i$ to target a dispersion in idiosyncratic return volatilities of 3.54, as measured by the the ratio of the $90^{th}$ percentile to the median of the cross-sectional idiosyncratic volatility of stock returns.[13] We set the volatility of shocks from noise traders to $\sigma_x = 0.4$ to target an average monthly turnover (defined as the total monthly volume divided by the number of shares outstanding), equal to 9.7%. We normalize the level of prices by normalizing the mean payoff and the mean supply for each asset to $\bar{z}_i = 10$, $\bar{x}_i = 5$.[14]

The investor-level parameters we need to pin down are the risk aversion coefficient $\rho$, the information capacities of the two investor types $(K_1, K_2)$, and the fraction of sophisticated investors $(\lambda)$. We select those parameters to target the market return of 11.9% (corresponding to 1989-2000 average); the fraction of assets that investors learn about, which, in the absence of empirical guidance, we set to 50%; the equity ownership share of sophisticated investors of 69%; and the return spread between sophisticated and unsophisticated households of four percentage points. To compute the last two moments, we use data from the

---

[13]We normalize the lowest volatility to $\sigma_n = 1$, and we set $\sigma_i = \sigma_n + \alpha(n - i)/n$, which implies the volatility distribution is linear. The dispersion target generates a slope coefficient $\alpha = 0.65$.

[14]Changing the number of assets in the parameterization does not have a major impact on our results.

*Survey of Consumer Finances.* Although not as comprehensive as tax records data, the SCF provides detailed information about the balance sheets of a representative sample of U.S. households.[15] We restrict our sample to participants in financial markets, defined as households that report holding stocks, bonds, mutual funds, receiving dividends, or having a brokerage account. On average, 34% of households participate.[16] We classify as sophisticated investors the participants in the top decile of the wealth distribution, and relatively unsophisticated investors as the remaining 90% of participants.[17] Using this definition, the equity ownership share of sophisticated investors is 69%.[18]

In order to quantify the return heterogeneity, for each household, we compute capital income divided by holdings of risky securities (stocks, bonds, and mutual funds), and then use these return measures to capture the heterogeneity between the two groups of households. Specifically, we compute the ratio of the median return of the unsophisticated households relative to the median return of the sophisticated households, which is 69.2% over the first half of the sample. We use this gap to obtain targets for the levels of returns of each household type, given the market return. The weights used in computing the aggregate return are the fraction of risky securities held by each type of household in the SCF (31%

---

[15]We use the weights provided in the public use data sets of the SCF in order to make the results representative of the population of U.S. households. These weights account for both the oversampling of wealthy households and for differential patterns of nonresponse. For a discussion of weights and aggregate analysis of the quality of SCF data, see Kennickell & Woodburn (1999) and Kennickell (2000). See also Saez & Zucman (2016) for a detailed comparison of the SCF to U.S. administrative tax data. In short, they find that the SCF is representative of trends and levels of inequality in the U.S., but understates inequality inside the top 1% of the wealth distribution.

[16]We also consider a broader measure of participation that includes all households with equity in a retirement account. This raises the participation rates, but does not alter our main findings.

[17]In the Appendix, we show that in the data people with higher initial wealth use more sophisticated investment instruments, hold larger portfolios, and invest a lower proportion of their assets in money-like instruments. Additional evidence that links wealth to investment sophistication includes Calvet et al. (2009) and Vissing-Jorgensen (2004).

[18]To compute the number, we first compute the dollar value of the risky part of the financial holdings of households (stocks, bonds, non-money market funds, and other financials) for each decile of the wealth distribution. Then, we compute the share of these risky assets held by the top decile.

versus 69%). That gives us the difference between sophisticated and unsophisticated returns of four percentage points, which together with the target for market return above implies the target for sophisticated return of 13.1% and the unsophisticated return of 9.1%.[19]

Table 2 presents the model's response to aggregate capacity growth chosen to match the market return in the entire sample of 7%. It implies a 4.9% growth in capacity and additionally generates an increase in trading volume, as better informed investors adjust their holdings more aggressively. Quantitatively, a capacity growth of 4.9% over 24 years generates a decline in market returns to 2.6%, bringing the average return for the entire period to 7%, as in the data, while turnover increases from 9.7% in the first half of the sample to 16.8% in the second half, versus 16.0% in the data. This technological progress leads to higher capital income inequality, which grows by 38% over the period. This figure suggests that aggregate capacity growth is quite potent in generating capital income inequality growth. For reference, in the corresponding period capital income inequality growth in the SCF equals 87%.[20]

Inequality grows due to two main effects: (i) larger relative exposure of sophisticated investors to the asset market, marked by higher ownership shares across all assets, and (ii) a shift of sophisticated investors towards high risk, high return assets and that of unsophisticated investors towards lower risk and lower return assets. As capacity increases, less sophisticated investors are priced out of trading the more risky assets and shift their portfolio weights towards less risky, lower-return assets. As a result, the ownership share of sophisticated investors, relative to their population share, rises relatively more for the assets that

---

[19]We perform a detailed grid search over parameters until all the simulated moments are within a 10% distance from target. That gives sophisticated ownership within 0.7%, sophisticated and unsophisticated returns within 7%, ratio of volatilities within 2% and all other targets matched exactly.

[20]We compute this inequality growth as follows. For each survey year, we sort the sample of participants by the level of total wealth, and we calculate inequality as the ratio of average capital income of the top 10% to that of the bottom 90% of participants.

are above the median in terms of volatility relative to the assets that are below the median in terms of volatility. For both types of assets, sophisticated owners are over-represented relative to their size in the population (both numbers are greater than 1), reflecting their larger overall portfolios. But the difference is larger for the more volatile assets: at the end of the simulation period, sophisticated investors hold 21% more of high-risk assets relative to their population weight, compared to 14% more for low-risk assets. This gap measures the retrenchment of unsophisticated investors from the most profitable assets.

To isolate the effects due to portfolio composition and volatility dispersion, we solve and parameterize our model with just one risky asset. In a one asset economy, the rates of return on risky portfolios–which we use in the calibration of the benchmark model–are the same across the two types of investors, since there is now only one risky asset. The differences in capital income come only from the differences in holdings of the risky asset, both on average and state by state. Hence, we use ownership and turnover to discipline the one-asset numerical exercise. The resulting growth in capital income inequality is almost half of the growth generated by the benchmark model: 20% versus 38%. Hence, the different exposure to assets with different characteristics, and the asymmetric shifting of weights across assets as capacity grows play a significant role in driving capital income inequality.[21]

Our growth simulation increases the relative share of the sophisticated group in the

---

[21]In terms of the parameterization, the model with one risky asset takes away three targets from the benchmark model: heterogeneity in asset volatility, fraction of actively traded assets, and the return difference between sophisticated and unsophisticated investors. We keep the value of the risk aversion coefficient the same as in the benchmark model and set the volatility of the single asset payoff equal to the median payoff volatility of the benchmark model. That leaves three parameters: volatility of the noise trader demand $\sigma_x$, and the two capacities of sophisticated and unsophisticated investors. We choose these to match: the average market return, the average asset turnover, and the share of sophisticated ownership. That gives $(K_1, K_2, \sigma_x) = (0.0544, 0.0163, 0.37)$. In the dynamic simulation, we pick the growth rate of aggregate capacity to match the decline in the market return (just as in the benchmark simulation). That implies a 6.7% growth rate of technology.

economy's total information capacity $\phi$. To quantify the relevance of this force, we consider a simulation in which we grow capacity differentially so as to keep the shares of relative capacity of the investor types constant at the levels in the initial period. This change results in an inequality growth of 32% versus the benchmark 38%. The relatively limited effect reflects the fact that the sophisticated share in overall capacity is high to begin with.[22]

Calibrating the information capacity growth is challenging because the information that investors have when they make their investment decisions is not observable. Hence, our strategy is to set capacity growth so as to match the decline in market returns seen in the data, and to complement these results with robustness checks on this growth rate. We consider two alternative annual growth rates: 4% and 8%, based on the annual growth rate of the number of stocks actively analyzed by the financial industry, and the growth rate of the number of analysts per stock in the financial industry, respectively.[23] These rates imply 24% and 60% inequality growth. Although the results are sensitive to the growth rate of information capacity, the model generates a quantitatively significant rise in capital income inequality relative to the data.

## 4.2 Robustness

Two features of our specification have important implications for our results: the information acquisition technology and the equilibrium selection mechanism. We now discuss how changing our assumptions along these two dimensions affects inequality.

---

[22]In the Online Appendix, we also provide an exercise in which the capacity grows in proportion to the rates of return of the portfolio, capturing explicitly the idea that capacity is linked to wealth. That exacerbates the growth in inequality. Keeping the average capacity growth the same as in the benchmark economy, linking capacity growth to returns implies a 49% increase in capital income inequality.

[23]Our information friction implies that growth in information capacity translates into growth in actively analyzed stocks, and also more information capacity allocated per stock, consistent with these growth trends

**Marginal Cost Predictions**  In our benchmark model, we endow each investor with some level of capacity to process information. What happens to investor choices and inequality if we model a marginal cost of acquiring information instead? Let investors differ in their marginal cost of information, $0 < \kappa_1 < \kappa_2$. Then the investor's objective becomes $\max_{\{v_{ji}\}_{i=1}^n} \sum_{i=1}^n \left[ G_i \frac{\sigma_i^2}{\hat{\sigma}_{ji}^2} - \frac{\kappa_j}{2} \log \frac{\sigma_i^2}{\hat{\sigma}_{ji}^2} \right]$, and the information problem is independent across assets as investors decide how much information to purchase for each asset separately. Hence, instead of a corner solution for learning, each investor purchases information about all assets whose gains exceed their marginal cost, up to the point at which the gain from learning reaches the marginal cost. In equilibrium, the gains from learning decline endogenously the more information investors purchase and the sophisticated, low marginal cost investors are the marginal buyers of information, driving the gains from learning down to *their* marginal cost for all assets. The unsophisticated investors, who have a higher marginal cost, are now priced out of the information market altogether. As in the benchmark case, there is a preference for volatility, with the quantity of information purchased declining with asset volatility. The difference is that now for each asset, either the gains from learning are too small relative to the costs that *neither* investor learns about it, *or* only the sophisticated investors learn about it. For a given amount of information in the economy, the marginal cost specification results in larger inequality in both holdings and capital income relative to the endowed capacity case, in which both types of investors learn. Moreover, technological progress in information processing has no direct effect on the unsophisticated investors: As long as their marginal cost remains above that of the sophisticated investor, they purchase no information and invest in all assets according to their prior beliefs.

**Asymmetric Equilibrium Predictions**    In our benchmark model, we pin down the total amount of capacity devoted to each asset, but not the contribution of each investor group to that total. When deriving our analytic and numerical results, we impose a symmetric equilibrium, assuming that the fraction of investors that learn about each asset is the same for both investor types. We base this assumption on our result that the gains from learning about different assets are the same for both sophisticated and unsophisticated investors. However, the same equilibrium allocation of attention (and hence asset prices) could be achieved with a different distribution of investors across assets. How sensitive are our results to deviations from the symmetric equilibrium? First, it is useful to note that all our results hold at the individual level: If we compare two investors who both monitor the same asset, one sophisticated and one unsophisticated, they will differ in their holdings, capital income, and response to capacity growth as expected. But when we compare the average holdings of the two groups, asset-level predictions depend on how many investors learn about the asset in each group. It is possible to conceive of an allocation of investors across assets such that for some assets, the per capita ownership of unsophisticated investors is larger than that of the sophisticated investors. But it remains the case that on average across all assets the per capita ownership–and hence capital income–of the unsophisticated investors is strictly lower than that of the sophisticated investors. Moreover, growth in aggregate capacity continues to increase capital income inequality (as long as returns remain above the risk free rate), even if we consider a reshuffling of masses most advantageous to the unsophisticated investors, namely one that assigns all the unsophisticated investors learning about an asset to the highest volatility asset. Such a reshuffling yields positive, albeit lower, inequality growth. Numerically, we find that in our parameterized economy such a reshuffling has minimal

effects on inequality growth (reducing it by less than one percentage point), because the data favor a parameterization in which the unsophisticated investors contribute minimally to the allocation of attention for each asset, so that how we reshuffle them across assets has very limited effects on the dispersion of ownership and capital income.

# 5   Empirical Evidence

We now provide auxiliary evidence supporting our mechanism and its implications.

**Skill versus Risk**   How much of the growth in inequality comes from differences in exposure to risk versus differences in skill? Our model is one in which both risk-taking differences and pure compensation for skill generate return heterogeneity. Sophisticated investors are more exposed to risk because they choose to hold a larger share of risky assets (compensation for risk); and because they have an informational advantage (compensation for skill). Quantitatively, in our model the compensation for skill accounts for approximately 75% of the return differential between the two investor groups, with the remaining 25% reflecting more risk taking.[24]

Empirically, Fagereng et al. (2016a) document that risk taking is only partially responsible for the difference in returns among Norwegian households, with approximately half of the return difference being attributed to unobservable heterogeneity. Corroborating this finding, we consider more aggregated data from the U.S. financial market. We compare returns from different types of mutual funds, using data from Morningstar, which contains information for two types of funds: those with a minimum investment of $100,000 (institutional funds)

---

[24]The Appendix presents the details of the calculation.

and those without such restrictions (retail funds). These two types of funds are suggestive of the kind of investment returns sophisticated versus unsophisticated investors can access. Since the institutional funds have a minimum investment threshold, less sophisticated, less wealthy investors do not have access to the higher returns earned by institutional funds, even for "plain vanilla" assets like equities.[25] Our fund data span the period 1989 through 2012. We compare the returns of the two groups adjusting for differences in exposure to common risk factors, a methodology that is standard in asset pricing literature. Our choice of common risk factors follows Carhart (1997) and includes market excess returns, return on the size factor, return on the value factor, and return on the momentum factor. To compute quantitative differences between the two investor groups we calculate a hedge portfolio, defined as a difference between monthly returns on the sophisticated portfolio and monthly returns on the unsophisticated portfolio. We then estimate the time-series regression of the hedge returns on the four factors. Our coefficient of interest is an intercept, which measures abnormal returns over and above premia for risk. The hedge portfolio generates a statistically significant positive return of 33 basis points per month, which is almost 4% on an annual basis. Hence, we conclude that differences in risk exposures alone are unlikely to explain the differences in returns between sophisticated and unsophisticated investors.

Nevertheless, by shutting down the risk aversion channel, we are likely minimizing the effect that risk has on inequality outcomes. The overall growth in inequality can be increased by assuming either decreasing absolute risk aversion or differences in risk attitudes that, like information capacity, are correlated with wealth. The less risk averse investors would hold

---

[25]In the Appendix, we present additional evidence that the there are both institutional and informational barriers that prevent unsophisticated households from gaining access and delegating their investment decisions to high quality investment services.

a greater share of risky assets, and hence they would have higher expected capital income.[26]
In a CRRA framework, the model solution under no capacity differences predicts the same
portfolio shares for risky assets, *independent of wealth.* Intuitively, if agents have common
information, then wealth differences affect the composition of their allocations between the
risk-free asset and the risky portfolio, but not the composition of the risky portfolio, which
is determined optimally by the (common) belief structure. As a result, differences in ca-
pacity are a necessary component for the model to generate any risky return differences
across agents. Similarly, within our mean-variance specification, a growing difference in risk
aversion produces growing *aggregate* ownership in risky assets of less risk averse investors,
and a uniform, proportional retrenchment from all risky assets of more risk averse investors.
However, heterogeneity in risk aversion alone cannot generate the empirical investor-specific
rates of return on equity, differences in portfolio weights within a class of risky assets or dif-
ferential growth in ownership by asset volatility. Hence, the information asymmetry remains
central to matching several recent trends in U.S. financial markets.[27]

**The Extensive Margin of Limited Participation**   Limited participation in U.S. finan-
cial markets has long been a source of inequality in total income and wealth (e.g., Mankiw
& Zeldes (1991)). How important is the limited participation margin for generating capi-
tal income inequality? Using data from the SCF, we find that much of the recent growth
in financial wealth inequality has occurred among household who participate in financial
markets, and that trends in capital income growth mirror trends in total financial wealth

---

[26]Such setting would also encompass situations in which investors are exposed to different levels of volatil-
ity in areas outside capital markets, like labor income.

[27]Additionally, Gomez (2016) shows that when macro asset pricing models with heterogenous risk aversion
are parameterized to match the volatility of asset prices, they require a degree of heterogeneity in preferences
that leads to counterfactual predictions about wealth inequality.

inequality. Our evidence on capital income inequality reinforces existing results using more detailed U.S. and European data, e.g. Saez & Zucman (2016), Fagereng et al. (2016b) and Bach et al. (2015).

First, participation is hump-shaped over time. Moreover, inequality in total financial wealth has grown within the group of households who participate in financial markets, but it has remained essentially unchanged along the extensive margin (defined as the ratio of average financial wealth of the bottom 10% of participating households to that of the non-participating households). Thus the dynamics of financial wealth inequality do not appear to be driven by the participation margin. These trends are shown in Figure 3 and Figure 4.[28]

Second, among participants, the increase in inequality in financial wealth tracks the accumulation of capital income from the risky assets (namely, income from dividends, interest income, and realized capital gains). To see this, we consider the counterfactual financial wealth obtained from accruing capital income only.[29] Figure 5 suggests that past capital income realizations may be sufficient to explain the evolution of financial wealth inequality, without resorting to mechanisms that involve savings rates from other income sources.[30]

Third, among participating households, capital income inequality is large and growing fast. Panel (a) of Figure 7 shows that in the cross-section, capital income is an order of magnitude more unequal than either labor or total income. For example, in 1989, the average capital income of the top 10% of participants was 21 times larger than that of the

---

[28]Financial wealth in the SCF contains holdings of risky assets (stocks, bonds, mutual funds), passive assets (life insurance, retirement accounts, royalties, annuities, trusts), and liquid assets (cash, checking and savings accounts, money market accounts).

[29]For example, the counterfactual financial wealth level in 1995 is equal to the actual financial wealth in 1989 plus 3 times the capital income reported in the prior survey years (in this case, 1989 and 1992).

[30]By construction, the two wealth levels are identical in 1989, so the figure also implies that the counterfactual levels of financial wealth for each group are very close to those in the data. Still, we treat this evidence as suggestive, since our exercise imposes a panel interpretation on a repeated cross-section.

bottom 90% of participants. This ratio increased to nearly 40 in 2013. By comparison, the corresponding ratio for wage income was 2.4 in 1989 and 3.9 in 2013. To compare the dynamics of inequality across income sources, we normalize the inequality of each income measure to 1 in 1989, and plot growth rates for capital, labor, and total income inequality in panel (b) of the figure. As is well known, labor income inequality has grown significantly during this period, and so has capital income inequality, which nearly doubled.

We complement this evidence with additional data on flows into and out of mutual funds from Morningstar by sophisticated (institutional) and unsophisticated (retail) investors. As shown in Figure 6, the cumulative flows from sophisticated investors into equity and non-equity funds increase steadily over the entire sample period. In contrast, since 2000, unsophisticated investors have been shifting their funds out of equity mutual funds and into less risky non-equity funds. To the extent that direct equity holdings are more risky than diversified equity portfolios, such as mutual funds, this implies that unsophisticated investors have been systematically reallocating their wealth from riskier to safer asset classes. This trend is consistent with our model, which predicts that as aggregate capacity grows, sophisticated investors expand their ownership of risky assets by order of volatility: starting from the highest volatility assets and then moving down.

# 6 Concluding Remarks

What contributes to the growing capital income inequality across households? We propose a theoretical information-based framework that links capital income to investor sophistication. Our model implies income inequality that rises with the total information in the

market. Predictions on asset ownership, market returns, and turnover provide additional support for the economic mechanism we propose.

The overall growth of investment resources and competition among investors with different skill levels are generally considered signs of a well-functioning financial market. Our work highlights how advances in information processing technologies also have consequences beyond the financial market, affecting the distribution of income.

# References

Admati, Anat (1985), "A noisy rational expectations equilibrium for multi-asset securities markets," *Econometrica* 53(3): 629–657.

Aiyagari, Rao S. (1994), "Uninsured idiosyncratic risk and aggregate saving," *Quarterly Journal of Economics* 109(3): 659–684.

Arrow, Kenneth J (1987), "The demand for information and the distribution of income," *Probability in the Engineering and Informational Sciences* 1(01): 3–13.

Atkinson, Anthony B, Thomas Piketty & Emmanuel Saez (2011), "Top incomes over a century or more," *Journal of Economic Literature* 49: 3–71.

Bach, Laurent, Laurent Calvet & Paolo Sodini (2015), "Rich pickings? Risk, return, and skill in the portfolios of the wealthy," Working Paper Stockholm School of Economics.

Benhabib, Jess & Alberto Bisin (2017), "Skewed wealth distributions: Theory and empirics," *Journal of Economic Literature* forthcoming.

Benhabib, Jess, Alberto Bisin & Shenghao Zhu (2011), "The distribution of wealth and fiscal policy in economies with finitely lived agents," *Econometrica* 79(1): 123–157.

Cagetti, Marco & Mariacristina De Nardi (2006), "Entrepreneurship, frictions, and wealth," *Journal of Political Economy* 114: 835–870.

Calvet, Laurent E, John Y Campbell & Paolo Sodini (2009), "Measuring the financial sophistication of households," *American Economic Review* 99(2): 393–398.

Cao, Dan & Wenlan Luo (2017), "Persistent heterogeneous returns and top end wealth inequality," *Review of Economic Dynamics* 26: 301–326.

Carhart, Mark M. (1997), "On persistence in mutual fund performance," *Journal of Finance* 52: 57–82.

Chien, YiLi, Harold Cole & Hanno Lustig (2011), "A multiplier approach to understanding the macro implications of household finance," *Review of Economic Studies* 78 (1): 199–234.

De Nardi, Mariacristina & Giulio Fella (2017), "Saving and wealth inequality," CEPR Working Paper 11746.

Fagereng, Andreas, Luigi Guiso, Davide Malacrino & Luigi Pistaferri (2016a), "Heterogeneity and persistence in returns to wealth," NBER Working Paper No 22822.

Fagereng, Andreas, Luigi Guiso, Davide Malacrino & Luigi Pistaferri (2016b), "Heterogeneity in returns to wealth and the measurement of wealth inequality," *The American Economic Review* 106(5): 651–655.

Favilukis, Jack (2013), "Inequality, stock market participation, and the equity premium," *Journal of Financial Economics* 107 (3): 740–759.

Gabaix, Xavier, Jean-Michel Lasry, Pierre-Louis Lions & Benjamin Moll (2016), "The dynamics of inequality," *Econometrica* 84(6): 2071–2111.

Gomez, Matthieu (2016), "Asset prices and wealth inequality," Working Paper Princeton University.

Greenwood, Daphne (1983), "An estimation of US family wealth and its distribution from microdata," *Review of Income and Wealth* 29 (1): 23–44.

Guvenen, Fatih (2007), "Do stockholders share risk more effectively than non-stockholders?" *Review of Economics and Statistics* 89(2): 275–288.

Guvenen, Fatih (2009), "A parsimonious macroeconomic model for asset pricing," *Econometrica* 77(6): 1711–1750.

Kasa, Kenneth & Xiaowen Lei (2018), "Risk, uncertainty, and the dynamics of inequality," *Journal of Monetary Economics* 94: 60–78.

Kennickell, Arthur B (2000), "Wealth measurement in the Survey of Consumer Finances: Methodology and directions for future research," .

Kennickell, Arthur B & R Louise Woodburn (1999), "Consistent Weight Design for the 1989, 1992 and 1995 SCFs, and the Distribution of Wealth," *Review of Income and Wealth* 45(2): 193–215.

Kessler, Dennis & Edward N. Wolff (1991), "A comparative analysis of household wealth patterns in France and the United States," *Review of Income and Wealth* 37 (1): 249–266.

Krusell, Per & Anthony A. Smith (1998), "Income and wealth heterogeneity in the macroeconomy," *Journal of Political Economy* 106(5): 867–896.

Lusardi, Annamaria, Pierre-Carl Michaud & Olivia S. Mitchell (2017), "Optimal financial knowledge and wealth inequality," *Journal of Political Economy* forthcoming.

Maćkowiak, Bartosz & Mirko Wiederholt (2009), "Optimal sticky prices under rational inattention," *American Economic Review* 99 (3): 769–803.

Maćkowiak, Bartosz & Mirko Wiederholt (2015), "Business cycle dynamics under rational inattention," *The Review of Economic Studies* 82(4): 1502–1532.

Mankiw, N Gregory & Stephen P Zeldes (1991), "The consumption of stockholders and nonstockholders," *Journal of Financial Economics* 29(1): 97–112.

Matějka, Filip (2015), "Rationally inattentive seller: Sales and discrete pricing," *The Review of Economic Studies* 83(3): 1125–1155.

Mondria, Jordi (2010), "Portfolio choice, attention allocation, and price comovement," *Journal of Economic Theory* 145(5): 1837–1864.

Pástor, Luboš & Pietro Veronesi (2016), "Income inequality and asset prices under redistributive taxation," *Journal of Monetary Economics* 81: 1–20.

Peng, Lin (2005), "Learning with information capacity constraints," *Journal of Financial and Quantitative Analysis* 40(2): 307–329.

Peng, Lin & Wei Xiong (2006), "Investor attention, overconfidence and category learning," *Journal of Financial Economics* 80(3): 563–602.

Peress, Joel (2004), "Wealth, information acquisition and portfolio choice," *The Review of Financial Studies* 17(3): 879–914.

Piketty, Thomas (2014), *Capital in the Twenty-First Century*, Harvard University Press.

Piketty, Thomas & Emmanuel Saez (2003), "Income inequality in the United States, 1913–1998," *The Quarterly Journal of Economics* 118(1): 1–41.

Quadrini, Vincenzo (1999), "The importance of entrepreneurship for wealth concentration and mobility," *Review of Income and Wealth* 45(1): 1–19.

Saez, Emmanuel & Gabriel Zucman (2016), "Wealth inequality in the United States since 1913: Evidence from capitalized income tax data," *The Quarterly Journal of Economics* 131(2): 519–578.

Shannon, Claude E (1948), "A mathematical theory of communication," *Bell System Technical Journal* 27: 379–423 and 623–656.

Sims, Christopher A (2003), "Implications of rational inattention," *Journal of Monetary Economics* 50(3): 665–690.

Stevens, Luminita (2018), "Coarse pricing policies," Working Paper, University of Maryland.

Van Nieuwerburgh, Stijn & Laura Veldkamp (2009), "Information immobility and the home bias puzzle," *Journal of Finance* 64(3): 1187–1215.

Van Nieuwerburgh, Stijn & Laura Veldkamp (2010), "Information acquisition and under-diversification," *Review of Economic Studies* 77(2): 779–805.

Vissing-Jorgensen, Annette (2004), "Perspectives on behavioral finance: Does "irrationality" disappear with wealth? Evidence from expectations and actions," in *NBER Macroeconomics Annual 2003, Vol. 18*, pp. 139–208, The MIT Press, Cambridge.
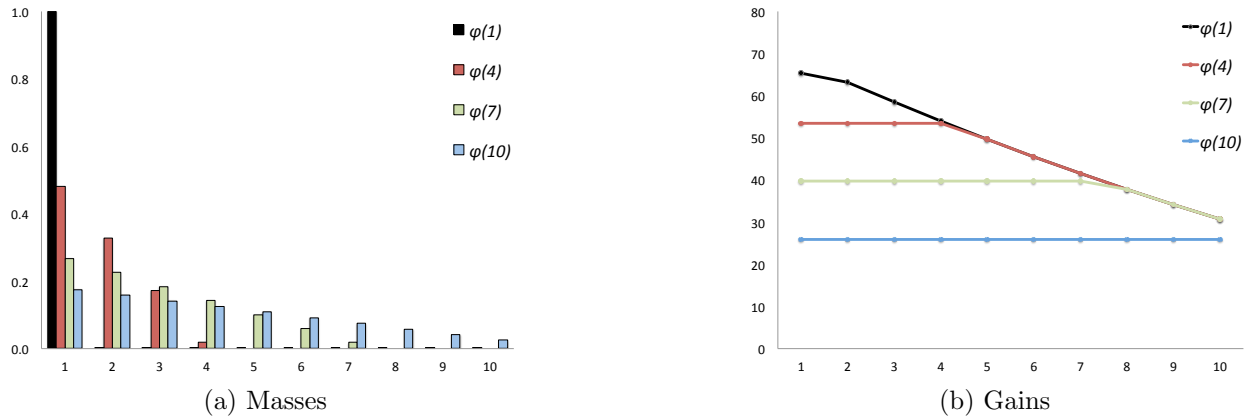
(a) Masses            (b) Gains

Figure 1: The evolution of masses and gains from learning as aggregate capacity is increased. $\phi(k)$ indicates the level of aggregate capacity for which $k$ assets are learned about in equilibrium. Gains are higher for higher volatility assets. As capacity increases, gains fall. Gains are equated for all assets that are learned about in equilibrium. On the x-axis, assets are ordered from most (1) to least (10) volatile.
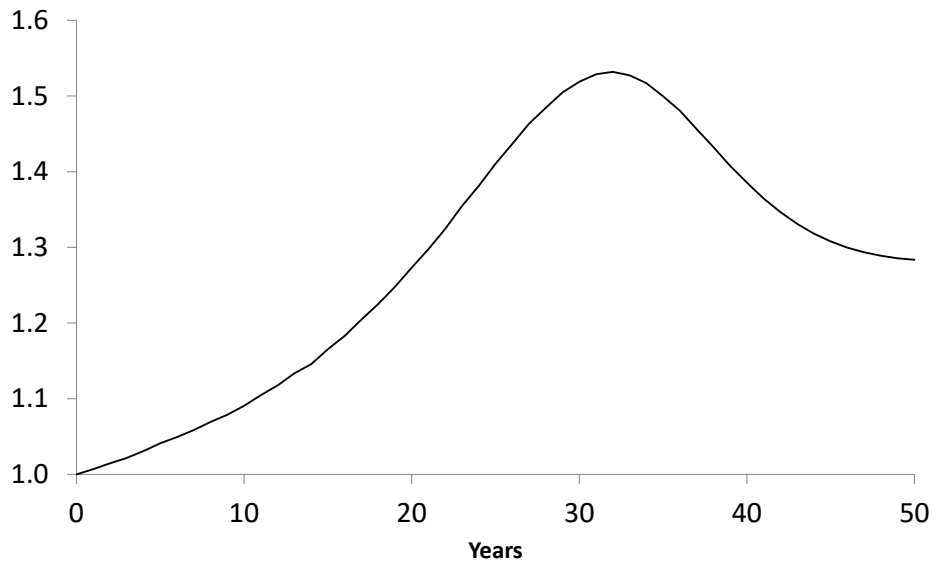


Figure 2: Model: capital income inequality in the long run.
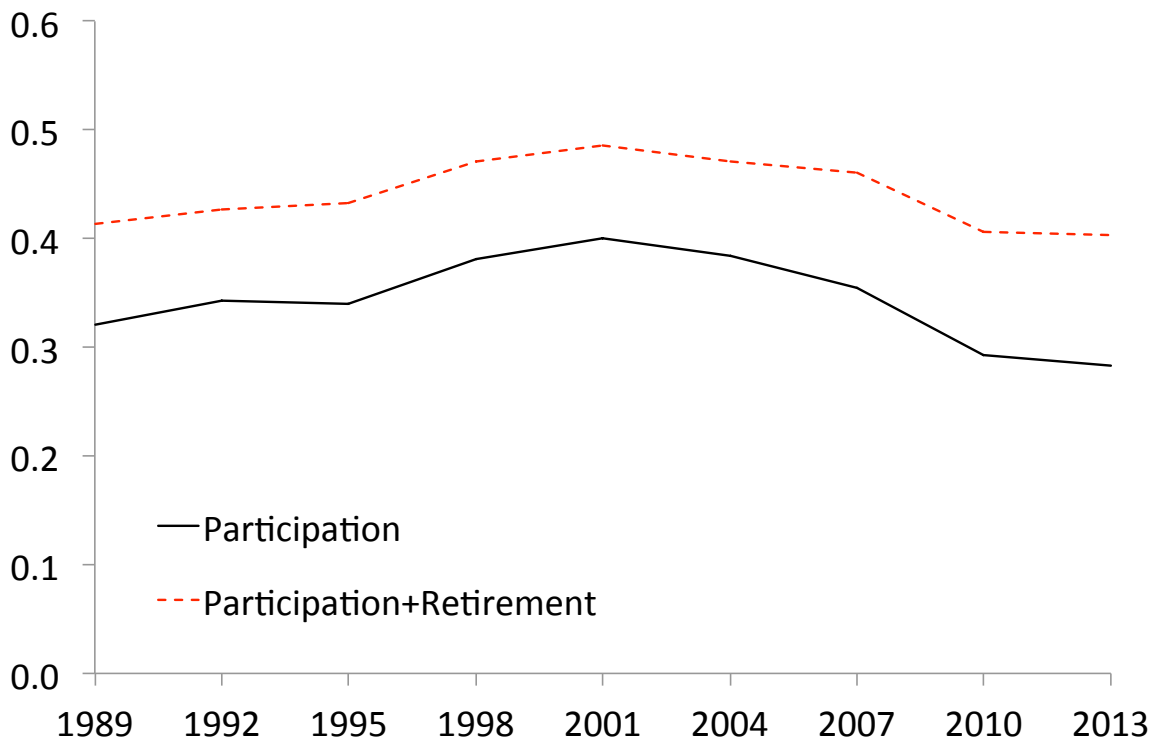
Figure 3: Financial markets participation in the SCF. Participants are individuals who have a brokerage account or who report stock holdings, bonds, money market funds, or non-money market funds. For a broader measure, we also consider households who have equity in retirement accounts.
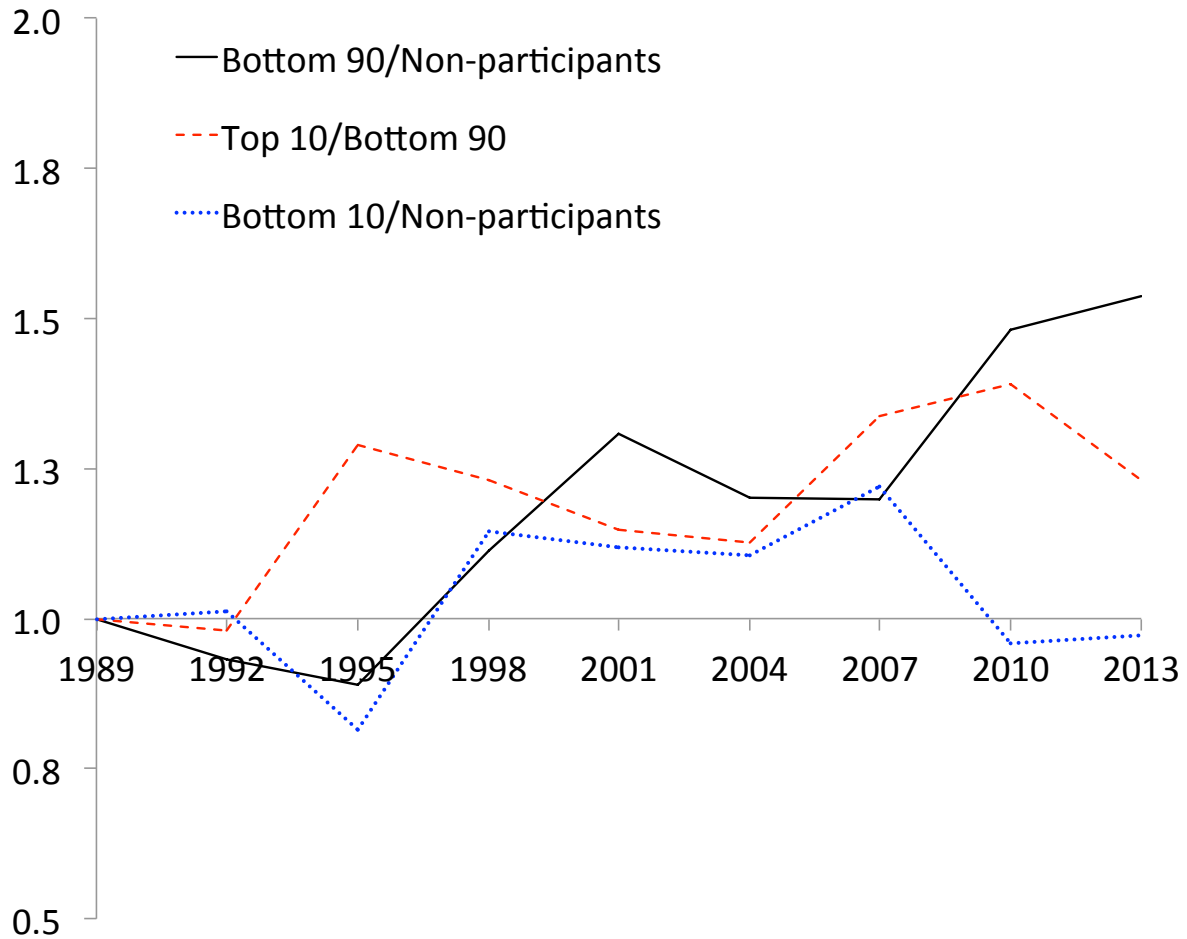
Figure 4: Extensive and intensive margins in financial wealth inequality in the SCF. 'Top 10/Bottom 90' measures inequality within the group of participants, defined as the ratio of financial wealth of the top wealth decile to that of the bottom 90% of participants. 'Bottom 10/Non-participants' measures inequality at the participation margin, measured as the ratio of financial wealth of the bottom 10% of participating households to that of all non-participating households.
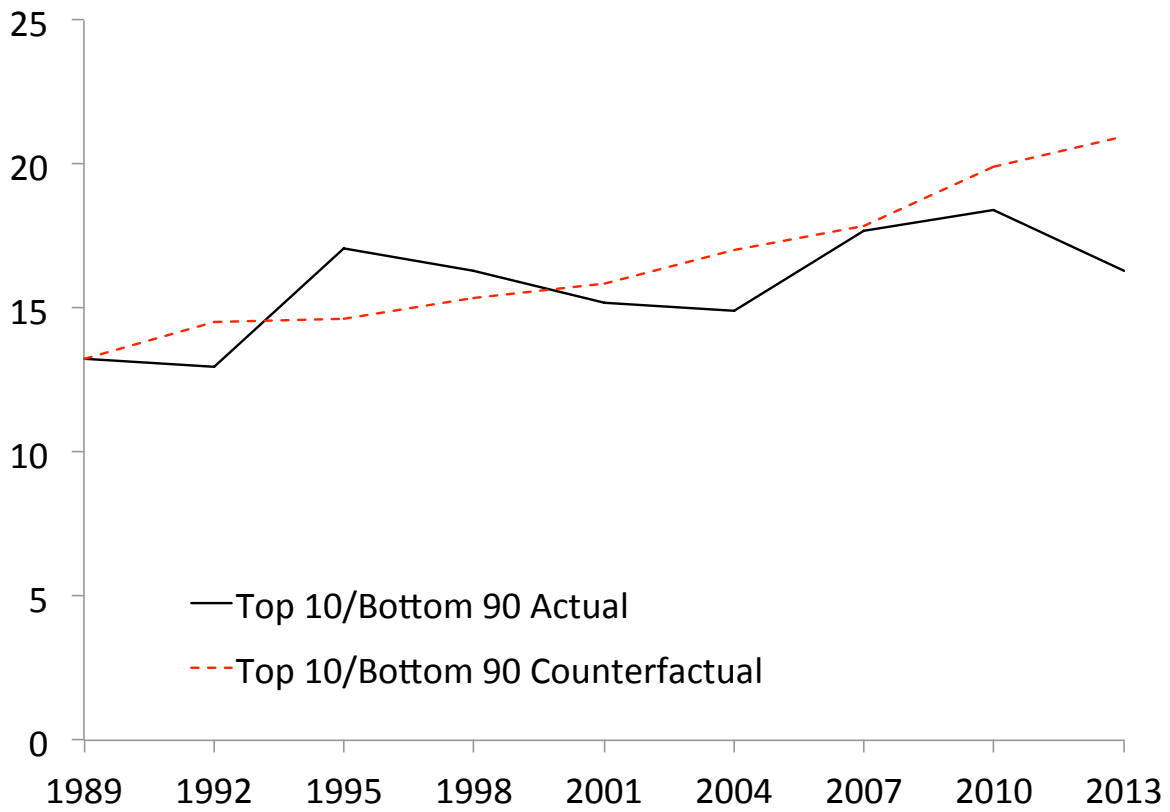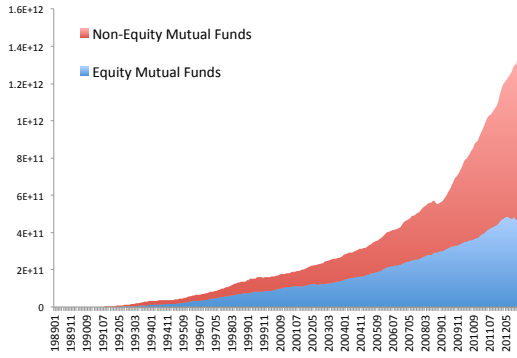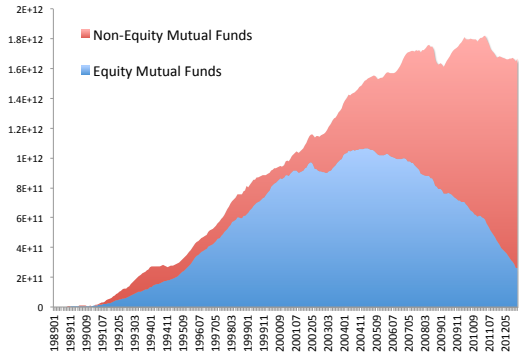
Figure 5: Financial wealth inequality and counterfactual financial wealth inequality constructed by accruing capital income.
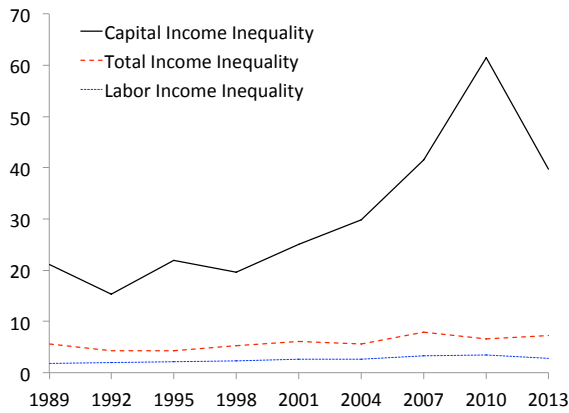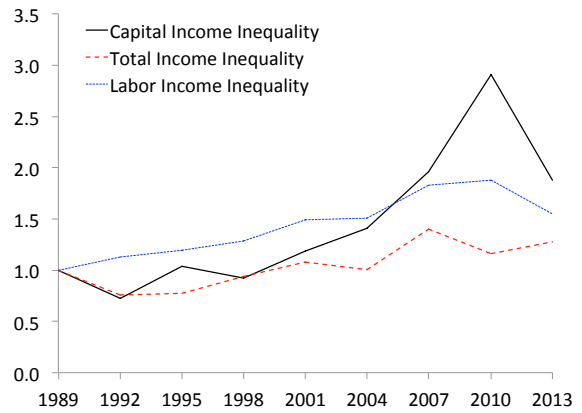
(a) Institutional

(b) Retail

Figure 6: Cumulative Flows to Mutual Funds: Institutional versus Retail Funds. Morningstar data.



(a) Raw

(b) Normalized

Figure 7: Income inequality growth in the SCF. Inequality is the ratio of the top 10% to the bottom 90% (in terms of total wealth) of participants in financial markets. (a) Inequality for capital income, labor income and other income in levels. (b) Same series, normalized to 1 in 1989.

Table 1: Parameter Values in the Baseline Model

| Parameter | Symbol | Value | Target (1989-2000 averages) |
|---|---|---|---|
| Risk-free rate | $r$ | 2.5% | 3-month T-bill $-$ inflation $= 2.5\%$ |
| Number of assets | $n$ | 10 | Normalization |
| Vol. of asset payoffs | $\sigma_i$ | $\in [1, 1.59]$ | p90/p50 of idio. return vol $= 3.54$ |
| Vol. of noise shocks | $\sigma_{xi}$ | 0.4 for all $i$ | Average turnover $= 9.7\%$ |
| Mean payoff, supply | $\bar{z}_i, \bar{x}_i$ | 10, 5 for all $i$ | Normalization |
| Risk aversion | $\rho$ | 1.032 | Average return $= 11.9\%$ |
| Information capacities | $K_1, K_2$ | 0.37, 0.0037, | Sophisticated share $= 69\%$ |
| and investor masses | $\lambda$ | 0.675 | Share actively traded $= 50\%$ |
| | | | Sophisticated return $= 13.1\%$ |

*Note:* Data are from CRSP for idiosyncratic stock return volatility, turnover, and average return and from SCF for return spread and sophisticated ownership share. Targets are for the 1989-2000 period.

Table 2: Aggregate Capacity Growth Outcomes

| Statistic | Baseline | Data | |
|---|---|---|---|
| Capacity growth (%) | 4.9 | | |
| Average market return (%) | 7 | 7 | |
| Capital income inequality growth (%) | 38 | 87 | |
| Sophis ending ownership share, top | 1.21 | | |
| Sophis ending ownership share, bottom | 1.14 | | |
| | One asset | Low growth | High growth |
| Capacity growth (%) | 6.7 | 4 | 8 |
| Average market return (%) | 7 | 8 | 5 |
| Capital income inequality growth (%) | 20 | 24 | 60 |
| Sophis ending ownership share, top | 1.1 | 1.16 | 1.43 |
| Sophis ending ownership share, bottom | | 1.08 | 1.41 |

*Note:* The average market return is the market return over the entire 1989-2013 period, and is targeted in the baseline and one-asset economy. All other numbers are not targeted. "Sophis ownership share" represents the ownership share of sophisticated investors, relative to their population share, for the assets that are above the median in terms of volatility ("top") and for assets that are below the median in terms of volatility ("bottom"), at the end of the simulation period. In the one-asset economy, it represents the sophisticated ownership share in the one risky asset.

# Investor Sophistication and Capital Income Inequality
# ONLINE APPENDIX

Marcin Kacperczyk          Jaromir Nosal          Luminita Stevens

Imperial College          Boston College          University of Maryland

**Abstract**

This file contains supplementary material for the paper 'Investor Sophistication and Capital Income Inequality', by Kacperczyk, Nosal, and Stevens.

# Contents

# List of Figures

# 1 Appendix: Proofs

## 1.1 Model

**Portfolio Choice**. In the second stage, each investor chooses portfolio holdings $q_{ji}$ to solve

$$\max_{\{q_{ji}\}_{i=1}^n} U_j = E_j(W_j) - \tfrac{\rho}{2} V_j(W_j) \quad s.t. \quad W_j = r\left(W_{0j} - \sum_{i=1}^n q_{ji} p_i\right) + \sum_{i=1}^n q_{ji} z_i,$$

where $E_j$ and $V_j$ denote the mean and variance conditional on investor $j$'s information set:

$$E_j(W_j) = E_j\left[rW_{0j} + \sum_{i=1}^n q_{ji}(z_i - rp_i)\right] = rW_{0j} + \sum_{i=1}^n q_{ji}\left[E_j(z_i) - rp_i\right],$$

$$V_j(W_j) = V_j\left[rW_{0j} + \sum_{i=1}^n q_{ji}(z_i - rp_i)\right] = \sum_{i=1}^n q_{ji}^2 V_j(z_i).$$

Let $\widehat{\mu}_{ji} \equiv E_j[z_i]$ and $\widehat{\sigma}_{ji}^2 \equiv V_j[z_i]$. The investor's portfolio problem is to maximize

$$U_j = rW_{0j} + \sum_{i=1}^n q_{ji}(\widehat{\mu}_{ji} - rp_i) - \tfrac{\rho}{2} \sum_{i=1}^n q_{ji}^2 \widehat{\sigma}_{ji}^2.$$

The first order conditions with respect to $q_{ji}$ yield $q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i}{\rho \widehat{\sigma}_{ji}^2}$. Since $W_{0j}$ does not affect the optimization, we normalize it to zero. The indirect utility function becomes

$$U_j = \tfrac{1}{2\rho} \sum_{i=1}^n \frac{(\widehat{\mu}_{ji} - rp_i)^2}{\widehat{\sigma}_{ji}^2}. \qquad \square$$

**Posterior Beliefs**. The signal structure, $z_i = s_{ji} + \delta_{ji}$, implies that

$$\widehat{\mu}_{ji} = \overline{z} + \frac{Cov(s_{ji}, z_i)}{\sigma_{sji}^2}(s_{ji} - \overline{s}_{ji}) = s_{ji},$$

$$\widehat{\sigma}_{ji}^2 = \sigma_i^2\left(1 - \frac{Cov^2(s_{ji}, z_i)}{\sigma_{sji}^2 \sigma_i^2}\right) = \sigma_{\delta ji}^2. \qquad \square$$

**Information Constraint**. Let $H(z)$ denote the entropy of $z$, and let $H(z|s_j)$ denote the conditional entropy of $z$ given the vector of signals $s_j$. Then

$$I(z; s_j) \equiv H(z) - H(z|s_j) \overset{(1)}{=} \sum_{i=1}^n H(z_i) - H(z|s_j) \overset{(2)}{=} \sum_{i=1}^n H(z_i) - \sum_{i=1}^n H(z_i|z^{i-1}, s_j)$$

$$\overset{(1)}{=} \sum_{i=1}^n H(z_i) - \sum_{i=1}^n H(z_i|s_j) \overset{(3)}{=} \sum_{i=1}^n H(z_i) - \sum_{i=1}^n H(z_i|s_{ji}) = \sum_{i=1}^n I(z_i; s_{ji})$$

where (1) follows from the independence of the payoffs $z_i$; (2) follows from the chain rule for entropy, where $z^{i-1} = \{z_1, ..., z_{i-1}\}$; (3) follows from the independence of the signals $s_{ji}$.

For each asset $i$, the entropy of $z_i \sim \mathcal{N}(\overline{z}, \sigma_i^2)$ is $H(z_i) = \tfrac{1}{2}\ln(2\pi e \sigma_i^2)$.

The signal structure, $z_i = s_{ji} + \delta_{ji}$, implies that

$$I\left(z_i; s_{ji}\right) = H\left(z_i\right) + H\left(s_{ji}\right) - H\left(z_i, s_{ji}\right) = \tfrac{1}{2} \log\left(\frac{\sigma_i^2 \sigma_{sji}^2}{\left|\Sigma_{z_i s_{ji}}\right|}\right) = \tfrac{1}{2} \log\left(\frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right),$$

where $\left|\Sigma_{z_i s_{ji}}\right| = \sigma_{sji}^2 \sigma_{\delta ji}^2$ is the determinant of the variance-covariance matrix of $z_i$ and $s_{ji}$.

Hence $I\left(z_i; s_{ji}\right) = 0$ if $\sigma_{\delta ji}^2 = \sigma_i^2$.

Across assets, $I\left(z; s_j\right) = \sum_{i=1}^n I\left(z_i; s_{ji}\right) = \tfrac{1}{2} \sum_{i=1}^n \log\left(\frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right) = \tfrac{1}{2} \log\left(\prod_{i=1}^n \frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right) \leq K_j.$

$\square$

**Information Objective**. Expected utility is given by

$$E_{0j}\left[U_j\right] = \tfrac{1}{2\rho} E_{0j}\left[\sum_{i=1}^n \frac{\left(\widehat{\mu}_{ji} - rp_i\right)^2}{\widehat{\sigma}_{ji}^2}\right] = \tfrac{1}{2\rho} \sum_{i=1}^n \frac{E_{0j}\left[\left(\widehat{\mu}_{ji} - rp_i\right)^2\right]}{\widehat{\sigma}_{ji}^2} = \tfrac{1}{2\rho} \sum_{i=1}^n \left(\frac{\widehat{R}_{ji}^2 + \widehat{V}_{ji}}{\widehat{\sigma}_{ji}^2}\right),$$

where $\widehat{R}_{ji}$ and $\widehat{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns, $\widehat{\mu}_{ji} - rp_i$. Conjecture (and later verify) that prices are normally distributed, $p_i \sim \mathcal{N}\left(\overline{p}_i, \sigma_{pi}^2\right)$.

$$\widehat{R}_{ji} \equiv E_{0j}\left(\widehat{\mu}_{ji} - rp_i\right) = \overline{z} - r\overline{p}_i,$$

$$\widehat{V}_{ji} \equiv V_{0j}\left(\widehat{\mu}_{ji} - rp_i\right) = Var\left(\widehat{\mu}_{ji}\right) + r^2 \sigma_{pi}^2 - 2r Cov\left(\widehat{\mu}_{ji}, p_i\right).$$

The signal structure implies that $Var\left(\widehat{\mu}_{ji}\right) = \sigma_{sji}^2$.

Following Admati (1985), we conjecture (and later verify) that prices are $p_i = a_i + b_i \varepsilon_i - c_i \nu_i$, for some coefficients $a_i, b_i, c_i \geq 0$. We compute $Cov\left(\widehat{\mu}_{ji}, p_i\right)$ exploiting the fact that posterior beliefs and prices are conditionally independent given payoffs. We obtain

$$\widehat{V}_{ji} = \sigma_{sji}^2 + r^2 \sigma_{pi}^2 - 2r b_i \sigma_{sji}^2 = \left(1 - rb_i\right)^2 \sigma_i^2 + r^2 c_i^2 \sigma_x^2 - \left(1 - 2rb_i\right) \widehat{\sigma}_{ji}^2.$$

Hence the distribution of expected excess returns is normal with mean and variance:

$$\widehat{R}_{ji} = \overline{z} - ra_i \quad \text{and} \quad \widehat{V}_{ji} = \left(1 - rb_i\right)^2 \sigma_i^2 + r^2 c_i^2 \sigma_x^2 - \left(1 - 2rb_i\right) \widehat{\sigma}_{ji}^2.$$

Expected utility becomes

$$E_{0j}\left[U_j\right] = \tfrac{1}{2\rho} \sum_{i=1}^n G_i \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} - \tfrac{1}{2\rho} \sum_{i=1}^n \left(1 - 2rb_i\right),$$

where $G_i \equiv \left(1 - rb_i\right)^2 + \frac{r^2 c_i^2 \sigma_x^2}{\sigma_i^2} + \frac{\left(\overline{z} - ra_i\right)^2}{\sigma_i^2}$, and where the second summation is independent of the investor's choices.

$\square$

**Proof of Lemma 1 (Information Choice).** The linear objective function and the convex constraint imply that each investor allocates all capacity to learning about a single asset. Let $\widehat{\sigma}^2_{ji} = e^{-2K_{ji}}\sigma^2_i$. Then the optimization problem can be written as $\max_{\{K_{ji}\}^n_{i=1}} \sum^n_{i=1} G_i e^{2K_{ji}}$ s.t. $\sum^n_{i=1} K_{ji} \leq K_j$. Suppose the investor allocates capacity to learning about 2 assets. WLOG, let these assets be indexed by 1 and 2, and suppose $G_2 \leq G_1$. Then, $K_{j1} + K_{j2} = K_j$ and the value of the objective function is

$$\sum^n_{i=1} G_i e^{2K_{ji}} = G_1 \left(e^{2K_{j1}} - 1\right) + G_2 \left(e^{2K_{j2}} - 1\right) + \sum^n_{i=1} G_i$$

$$\overset{(1)}{\leq} G_1 \left(e^{2K_{j1}} - 1\right) + G_1 \left(e^{2K_{j2}} - 1\right) + \sum^n_{i=1} G_i$$

$$\overset{(2)}{\leq} 2G_1 \left(e^{K_j} - 1\right) + \sum^n_{i=1} G_i \overset{(3)}{<} G_1 \left(e^{2K_j} - 1\right) + \sum^n_{i=1} G_i,$$

where (1) follows from the assumption WLOG that $G_2 \leq G_1$, (2) follows from the fact that for $K_{j1} + K_{j2} = K_j$, $e^{2K_{j1}} + e^{2K_{j2}}$ is maximized at $K_{j1} = K_{j2} = K_j/2$, and (3) follows from the fact that $(e^{2K_j} - 1)^2 > 0$ for $K_j > 0$. This chain of inequalities shows that splitting capacity across two assets yields strictly lower utility than investing all capacity in a single asset, even if the gains from learning are equal across assets. Splitting capacity among more than two assets would lower utility even more.

Let $l_j$ index the asset about which investor $j$ learns. The investor's objective is $\left(e^{2K_j} - 1\right) G_{l_j} + \sum^n_{i=1} G_i$. Since $e^{2K_j} > 1$, the objective is maximized by allocating all capacity to the asset with the largest utility gain: $l_j \in \arg\max_i G_i$. The variance of the investor's posterior beliefs is

$$\widehat{\sigma}^2_{ji} = \begin{cases} e^{-2K_j}\sigma^2_i & \text{if } i = l_j, \\ \sigma^2_i & \text{if } i \neq l_j. \end{cases}$$

$\square$

**Conditional Distribution of Signals.** Conditional on the realized payoff, the signal is a normally distributed random variable, with mean and variance given by

$$E\left(s_{ji}|z_i\right) = \overline{s}_{ji} + \frac{Cov(s_{ji}, z_i)}{\sigma^2_i}\left(z_i - \overline{z}\right) = \begin{cases} \overline{z} + \left(1 - e^{-2K_j}\right)\varepsilon_i & \text{if } i = l_j \\ \overline{z} & \text{if } i \neq l_j, \end{cases}$$

$$V\left(s_{ji}|z_i\right) = \sigma^2_{sji}\left(1 - \frac{Cov^2(s_{ji}, z_i)}{\sigma^2_{sji}\sigma^2_i}\right) = \begin{cases} \left(1 - e^{-2K_j}\right)e^{-2K_j}\sigma^2_i & \text{if } i = l_j \\ 0 & \text{if } i \neq l_j. \end{cases}$$

$\square$

**Holdings.** Individual portfolio holdings are given by

$$q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i}{\rho\widehat{\sigma}^2_{ji}} = \frac{e^{2K_{ji}}\left(\widehat{\mu}_{ji} - rp_i\right)}{\rho\sigma^2_i}.$$

For the assets that investor $j$ does not learn about, $K_{ji} = 0$, and $q_{ji} = (\overline{z} - rp_i)/\rho\sigma^2_i$. This quantity is the same for all investors, regardless of their type.

For the asset that investor $j$ learns about, $K_{ji} = K_j$, and $q_{ji} = e^{2K_j} \left( \widehat{\mu} - rp_i \right) / \rho\sigma_i^2$.

Total holdings for the group of sophisticated investors learning about this asset are

$$\int_{M_{1i}} q_{ji} dj = \int_{M_{1i}} \frac{e^{2K_1}(\widehat{\mu}_{ji} - rp_i)}{\rho\sigma_i^2} dj = \frac{e^{2K_1}}{\rho\sigma_i^2} \left( \int_{M_{1i}} \widehat{\mu}_{ji} dj - \lambda m_{1i} rp_i \right),$$

where $M_{1i}$ is the set of sophisticated investors learning about asset $i$ and $m_{1i} \in [0,1]$ denotes the fraction of sophisticated investors learning about asset $i$. Using the conditional distribution of signals, $\int_{M_{1i}} \widehat{\mu}_{ji} dj = \lambda m_{1i} \left[ \overline{z} + (1 - e^{-2K_1})\varepsilon_i \right]$, and $\int_{M_{1i}} q_{ji} dj = \lambda m_{1i} \left[ \frac{\overline{z} - rp_i}{\rho\sigma_i^2} + \left( e^{2K_1} - 1 \right) \frac{z_i - rp_i}{\rho\sigma_i^2} \right]$.

Total holdings of this asset by both informed *and* uninformed sophisticated investors are:

$$Q_{1i} = \int_{M_{1i}} q_{ji} dj + \lambda(1 - m_{1i}) \frac{(\overline{z} - rp_i)}{\rho\sigma_i^2} = \lambda m_{1i} \left[ \frac{\overline{z} - rp_i}{\rho\sigma_i^2} + \left( e^{2K_1} - 1 \right) \frac{z_i - rp_i}{\rho\sigma_i^2} \right] + \lambda(1 - m_{1i}) \frac{\overline{z} - rp_i}{\rho\sigma_i^2}$$

$$= \lambda \left( \frac{\overline{z} - rp_i}{\rho\sigma_i^2} \right) + \lambda m_{1i} \left( e^{2K_1} - 1 \right) \left( \frac{z_i - rp_i}{\rho\sigma_i^2} \right).$$

Per capita holdings for the sophisticated investors are

$$q_{1i} = \frac{Q_{1i}}{\lambda} = \left( \frac{\overline{z} - rp_i}{\rho\sigma_i^2} \right) + m_{1i} \left( e^{2K_1} - 1 \right) \left( \frac{z_i - rp_i}{\rho\sigma_i^2} \right).$$

Analogous expressions hold for the unsophisticated investors, with $m_{2i}$ denoting the fraction of unsophisticated investors learning about asset $i$:

$$Q_{2i} = (1 - \lambda) \left( \frac{\overline{z} - rp_i}{\rho\sigma_i^2} \right) + (1 - \lambda)m_{2i} \left( e^{2K_2} - 1 \right) \left( \frac{z_i - rp_i}{\rho\sigma_i^2} \right),$$

$$q_{2i} = \frac{Q_{2i}}{1 - \lambda} = \left( \frac{\overline{z} - rp_i}{\rho\sigma_i^2} \right) + m_{2i} \left( e^{2K_2} - 1 \right) \left( \frac{z_i - rp_i}{\rho\sigma_i^2} \right).$$

Finally, total sophisticated and unsophisticated demand for asset $i$ is

$$Q_{1i} + Q_{2i} = \left( \frac{\overline{z} - rp_i}{\rho\sigma_i^2} \right) + \Phi_i \left( \frac{z_i - rp_i}{\rho\sigma_i^2} \right),$$

where we define $\Phi_i \equiv \lambda m_{1i} \left( e^{2K_1} - 1 \right) + (1 - \lambda)m_{2i} \left( e^{2K_2} - 1 \right).$ $\qquad \square$

**Proof of Lemma 2 (Equilibrium Prices).** The market clearing condition for each asset in state $(z_i, x_i)$ is

$$\int_{M_{1i}} \left( \frac{s_{ji} - rp_i}{e^{-2K_1}\rho\sigma_i^2} \right) dj + \int_{M_{2i}} \left( \frac{s_{ji} - rp_i}{e^{-2K_2}\rho\sigma_i^2} \right) dj + (1 - m_{1i} - m_{2i}) \left( \frac{\overline{z} - rp_i}{\rho\sigma_i^2} \right) = x_i,$$

where $M_{1i}$ denotes the set of measure $m_{1i} \in [0, \lambda]$ of sophisticated investors who choose to learn about asset $i$, and $M_{2i}$ denotes the set of measure $m_{2i} \in [0, 1 - \lambda]$, of unsophisticated investors who choose to learn about asset $i$.

Using the conditional distribution of the signals, $\int_{M_{1i}} s_{ji} dj = m_{1i} \left[ \bar{z} + \left(1 - e^{-2K_1}\right) \varepsilon_i \right]$ for the type-1 investors, and analogously for the type-2 investors. Then, the market clearing condition can be written as $\alpha_1 \bar{z} + \alpha_2 \varepsilon_i - x_i = \alpha_1 r p_i$, where

$$\alpha_1 \equiv \frac{1 + m_{1i}\left(e^{2K_1} - 1\right) + m_{2i}\left(e^{2K_2} - 1\right)}{\rho \sigma_i^2} \text{ and } \alpha_2 \equiv \frac{m_{1i}\left(e^{2K_1} - 1\right) + m_{2i}\left(e^{2K_2} - 1\right)}{\rho \sigma_i^2}.$$

We obtain identification of the coefficients in $p_i = a_i + b_i \varepsilon_i - c_i \nu_i$ as

$$a_i = \frac{1}{r}\left[\bar{z} - \frac{\bar{x}}{\alpha_1}\right], \ b_i = \frac{\alpha_2}{r\alpha_1}, \text{ and } c_i = \frac{1}{r\alpha_1}.$$

Let $\Phi_i \equiv m_{1i}\left(e^{2K_1} - 1\right) + m_{2i}\left(e^{2K_2} - 1\right)$ be a measure of the information capacity allocated to learning about asset $i$ in equilibrium. Further substitution yields

$$a_i = \frac{1}{r}\left(\bar{z} - \frac{\rho\sigma_i^2 \bar{x}}{1+\Phi_i}\right), \ \ b_i = \frac{1}{r}\left(\frac{\Phi_i}{1+\Phi_i}\right), \ \ c_i = \frac{1}{r}\left(\frac{\rho\sigma_i^2}{1+\Phi_i}\right). \hspace{2em} \square$$

**Proof of Lemma 3 (Equilibrium Learning).** Substituting $a_i$, $b_i$, and $c_i$ in $G_i \equiv (1 - rb_i)^2 + \frac{r^2 c_i^2 \sigma_x^2}{\sigma_i^2} + \frac{(\bar{z} - ra_i)^2}{\sigma_i^2}$ and defining $\xi_i \equiv \sigma_i^2 \left(\sigma_x^2 + \bar{x}^2\right)$ gives $G_i = \frac{1 + \rho^2 \xi_i}{(1+\Phi_i)^2}$.

By Lemma 1, each investor learns about a single asset among the assets with the highest gain. WLOG, assets are ordered such that $\sigma_i > \sigma_{i+1}$, for all $i \in \{1, ..., n-1\}$. First suppose that all investors learn about the same asset. Since $G_i$ is increasing in $\sigma_i$, this asset is asset 1. All investors learn about asset 1 as long as $\phi \le \phi_1 \equiv \sqrt{\frac{1 + \rho^2 \xi_1}{1 + \rho^2 \xi_2}} - 1$. At this threshold, some investors switch and learn about the second asset.

For $\phi > \phi_1$, equilibrium gains must be equated among all assets with positive learning mass. Otherwise, investors have an incentive to switch to learning about the asset with the higher gain. Moreover, the gains of all assets with zero learning mass must be strictly lower. Otherwise, an investor would once again have the incentive to deviate and learn about one of these assets.

To derive expressions for the mass of investors learning about each asset, we assume that the participation of sophisticated and unsophisticated investors in learning about a particular asset is proportional to their mass in the population: $m_{1i} = \lambda m_i$ and $m_{2i} = (1 - \lambda) m_i$, where $m_i$ is the total mass of investors learning about asset $i$. The necessary and sufficient conditions for determining $\{m_i\}_{i=1}^n$ are $\sum_{i=1}^k m_i = 1$; $\frac{1 + \phi m_i}{1 + \phi m_1} = c_{i1}$, for any $i \in \{2, ..., k\}$, where $c_{i1} \equiv \sqrt{\frac{1 + \rho^2 \xi_i}{1 + \rho^2 \xi_1}} \le 1$, with equality *iff* $i = 1$; and $m_i = 0$ for any $i \in \{k+1, ..., n\}$. Recursively, $m_i = c_{i1} m_1 - \frac{1}{\phi}(1 - c_{i1})$, $\forall i \in \{2, ..., k\}$. Using $\sum_{i=1}^k m_i = 1$, and defining $C_k \equiv \sum_{i=1}^k c_{i1}$, we obtain the solution for $m_1$ given by $m_1 = \frac{1}{C_k} + \frac{1}{\phi}\left(\frac{k}{C_k} - 1\right)$. Using this expression, we obtain the solution for all $m_i$, $i \in \{1, ..., k\}$, $m_i = \frac{c_{i1}}{C_k} + \frac{1}{\phi}\left(\frac{kc_{i1}}{C_k} - 1\right)$. \hspace{1em} $\square$

## 1.2 Analytic Results

**Proof of Proposition 1**. Results follow from equations that define ownership. □

**Proof of Proposition 2**. **(i)** Follows from the definition of capital income per capita. **(ii)** Since for all $i \in \{1, ..., k\}$, the gains $G_i$ are equated in equilibrium, then $E[\pi_{1i} - \pi_{2i}]$ is increasing in $m_i$, which in turn is increasing in $\sigma_i^2$. □

**Proof of Proposition 3**. **(i)** The increase in dispersion keeps $\phi$ unchanged. Therefore, the masses $m_i$ are unchanged. With both $\phi$ and $m_i$ unchanged, prices are unchanged. **(ii)** The result follows from equation (10): masses and prices do not change, and dispersion, $\left(e^{2K_1} - e^{2K_2}\right)$ increases. **(iii)** Relative capital income is

$$\frac{\pi_{1i}}{\pi_{2i}} = \frac{(\bar{z}_i - rp_i)(z_i - rp_i) + \left(e^{2K_1} - 1\right) m_i (z_i - rp_i)^2}{(\bar{z}_i - rp_i)(z_i - rp_i) + \left(e^{2K_2} - 1\right) m_i (z_i - rp_i)^2} > 1.$$

Since prices are unchanged, $(\bar{z}_i - rp_i)(z_i - rp_i)$ and $m_i (z_i - rp_i)^2$ are unchanged. Since $K_1' > K_1$ and $K_2' < K_2$, the second term in $\pi_{1i}$ increases and the second term in $\pi_{2i}$ decreases. □

**Proof of Proposition 4**. **(i)** Using equilibrium prices, $\bar{p}_i = \frac{1}{r}\left(\bar{z} - \frac{\rho\sigma_i^2\bar{x}}{1+\phi m_i}\right)$. The quantity $\phi m_i$ is increasing in $\phi$. Hence, for $i \in \{1, ..., k\}$, $\bar{p}_i$ is increasing in $\phi$. The result for equilibrium expected excess returns $E[z_i - r\bar{p}_i]$ follows.

**(ii)** Since $\lambda E[q_{1i}] + (1-\lambda) E[q_{2i}] = \bar{x}$, it is sufficient to show that for $i \in \{1, ..., k'\}$, $E[q_{1i}]$ increases in response to symmetric capacity growth. Let $K \equiv K_1$, and $K_2 = \delta K$, with $\delta \in (0, 1)$. Since

$$E[q_{1i}] = \frac{1 + m_i\left(e^{2K} - 1\right)}{(1+\phi m_i)}\bar{x}, \text{ then } \frac{dE[q_{1i}]}{dK} = \frac{\bar{x}}{(1+\phi m_i)^2}\left[\frac{d\left[m_i\left(e^{2K}-1\right)\right]}{dK}(1+\phi m_i) - \frac{d(\phi m_i)}{d\phi}\frac{d\phi}{dK}m_i\left(e^{2K}-1\right)\right].$$

Hence $sign\left(\frac{dE[q_{1i}]}{dK}\right) = sign\left(\frac{d\left[m_i\left(e^{2K}-1\right)\right]}{dK} - \frac{d(\phi m_i)}{d\phi}\frac{d\phi}{dK}\frac{m_i\left(e^{2K}-1\right)}{1+\phi m_i}\right)$.

The quantity $\frac{d\left[m_i\left(e^{2K}-1\right)\right]}{dK} > 2e^{2K}\frac{d(\phi m_i)}{d\phi} > 0$. Hence,

$sign\left(\frac{dE[q_{1i}]}{dK}\right) = sign\left(2e^{2K} - \frac{d\phi}{dK}\frac{m_i\left(e^{2K}-1\right)}{1+\phi m_i}\right)$

$$= sign\left(2e^{2K} - \frac{2m_i\left[\lambda e^{2K} + (1-\lambda)\delta e^{2K\delta}\right]\left(e^{2K}-1\right)}{1 + m_i\left[\lambda(e^{2K}-1) + (1-\lambda)\left(e^{2K\delta}-1\right)\right]}\right)$$

$$= sign\left(e^{2K} - \left(e^{2K} - 1\right)\frac{m_i\left[\lambda e^{2K} + (1-\lambda)\delta e^{2K\delta}\right]}{1 + m_i\left[\lambda e^{2K} + (1-\lambda)e^{2K\delta}\right] - m_i}\right)$$

$$\overset{(1)}{=} sign\left(e^{2K} - \left(e^{2K} - 1\right)\left[\frac{m_i\left[\lambda e^{2K} + (1-\lambda)e^{2K\delta}\right]}{1 + m_i\left[\lambda e^{2K} + (1-\lambda)e^{2K\delta}\right] - m_i}\right]\right)$$

6

$$\overset{(2)}{=} sign\left(e^{2K} - \left(e^{2K} - 1\right)\right) > 0$$

where (1) follows from $\delta \in (0,1)$, and (2) follows from the fact that the term in square brackets is less than 1.

**(iii)** Let the per capita capital income be decomposed into a component $C_i$ that is common across investor groups, and a component that is group-specific:
$\pi_{1i} = c_i + \frac{1}{\rho\sigma_i^2} m_i \left(e^{2K} - 1\right) (z_i - rp_i)^2$, where $c_i \equiv \frac{1}{\rho\sigma_i^2} (\bar{z} - rp_i)(z_i - rp_i)$, with expected value $C_i$. Then $E[\pi_{1i}] = C_i + \frac{1}{\rho\sigma_i^2} m_i \left(e^{2K} - 1\right) E\left[(z_i - rp_i)^2\right] = C_i + \frac{1}{\rho} m_i \left(e^{2K} - 1\right) G_i$, where $G_i$ is the gain from learning about asset $i$, equated across all $i \in \{1, ..., k\}$.
We then obtain that $\frac{E[\pi_{1i}]}{E[\pi_{2i}]} = \frac{C_i + \frac{1}{\rho} m_i \left(e^{2K} - 1\right) G_i}{C_i + \frac{1}{\rho} m_i \left(e^{2K\delta} - 1\right) G_i}$.
In response to an increase in $K$, $C_i$ and $G_i$ decrease, but they affect both sophisticated and unsophisticated profits in the same way. The quantity $m_i \left(e^{2K} - 1\right)$ increases by more than $m_i \left(e^{2K\delta} - 1\right)$ in response to a change in $K$. Hence overall, $\frac{E[\pi_{1i}]}{E[\pi_{2i}]}$ increases.

$\square$

## 1.3    Additional Results

### 1.3.1    Asymmetric Capacity Growth

How much would investor-specific capacity growth amplify inequality? In bringing the model to the data, we have linked investor sophistication to initial wealth. We now investigate the impact of this link by supposing that the growth in capacity for each investor type is proportional to that group's own returns, rather than to the market average return. Since the sophisticated investors earn higher returns, their capacity growth is also higher. This results in further dispersion in capacities. As shown in Section 3, larger dispersion amplifies inequality, but it does not affect asset prices or the average market return. In the parameterized economy, such a feedback from returns to capacity growth generates a 49% growth in inequality, nearly 30% more than the benchmark, as shown in Table 1. In this asymmetric specification, we keep the average capacity growth rate at 4.9% annually, as in the benchmark.
This exercise considers a reduced-form feedback loop, while maintaining the relationship between initial wealth and initial capacity as exogenous. Below we study how such a relation could arise endogenously, and what it implies for capacity differences between types.

### 1.3.2    Endogenous Capacity Choice

We provide a numerical example of an endogenous capacity choice outcome in a model in which wealth heterogeneity matters for endogenous capacity choice. In particular, we assume that investors have identical CRRA preferences with IES coefficient $\gamma$, and differ in terms of their beginning of period wealth. Then, for each investor $j$, the absolute risk aversion

coefficient is a function of wealth $W_j$, given by

$$A(W_j) = \gamma / W_j.$$

Locally, we map this into absolute risk aversion differences in a mean-variance optimization model by setting the coefficient $\rho_j$ for investor $j$ equal to $A(W_j)$. These differences in absolute risk aversion in the model imply differences in the size of the risky portfolio, and hence different gains from investing wealth in purchases of information capacity.

In particular, for a given cost of capacity given by the function $f(K)$, each investor type is going to choose the amount of capacity to maximize the ex-ante expectation of utility:

$$\frac{1}{2\rho_j} \sum_{i=1}^{n} \frac{\sigma_i^2}{\widehat{\sigma}_{ij}^2} G_i - f(K_j),$$

where, in equilibrium, $G_i$ is a function of the distribution of individual capacity choices of investors, but not of individual capacity choices, and $\widehat{\sigma}_{ij}^2 = \sigma_i^2 e^{-2K_j}$ if the investor learns about asset $i$.

The gain from increasing wealth is given by the benefit of increasing the precision of information for the asset that the investor is learning about. Since all actively traded assets have the same gain in equilibrium, we can express the marginal benefit of increasing capacity in terms of the gain of the highest volatility asset (asset 1), $\frac{1}{2\rho_j} e^{2K_j} G_1$, and then the optimization problem for capacity choice can be expressed as

$$\max_K \left\{ \frac{1}{2\rho_j} e^{2K} G_1 - f(K) \right\}. \tag{1}$$

Assumption 1 below ensures an interior solution to (1) exists.

**Assumption 1.** The following statements hold:
($i$) For all $j$, $\frac{G_1}{\rho_j} - f'(0) > 0$, where $G_1$ is evaluated at $K_j = 0$ for all $j$,
($ii$) There exists $\underline{K} > 0$, such that for all $j$ and for all $K > \underline{K}$, $2\frac{G_1}{\rho_j} e^{2K} - f''(K) < 0$,
($iii$) There exists $\bar{K} > 0$ such that for all $j$ and for all $K > \bar{K}$, $\frac{G_1}{\rho_j} e^{2K} - f'(K) < 0$.

**Numerical example**  Assume that the cost function is of the form: $f(K) = e^{aK}$. Under Assumption 1, the optimal choice of $K$ for agent $j$ is implicitly defined by:

$$\frac{G_1(\{\bar{K}_j\})}{\rho_j} = ae^{(a-2)K},$$

where we make the dependence of $G_1$ on the distribution of capacities explicit. Clearly, for any $a > 2$, the higher wealth investors (implying lower $\rho_j$) will choose higher capacity levels. However, because of the dependence of $G$ on equilibrium capacity choices, to quantify the differences we need to solve the equilibrium fixed point of the model.

Figure 1 presents the ratio of capacities as a function of the cost parameter of capacity, $a$, for

8

different values of the absolute risk aversion coefficient of the wealthy $\rho_1$ (which maps into different common relative risk aversion coefficients $\gamma$). The inequality in capacity exhibits a U-shape. First, if the cost of capacity is small, then the equilibrium inequality in capacity grows without bound, as the wealthier accumulate infinite capacity (faster than the less wealthy). For higher values of the cost of capacity, inequality exhibits a growing trend as the cost increases, very quickly approaching values in excess of 38, our benchmark value. It should be noted that even for the high values of the cost parameter, the overall cost relative to gain, $f(K_j)/\frac{1}{2\rho_j}e^{2K_j}G_1$, is relatively small, less than 1% for the wealthy and less than 6% for the less wealthy.
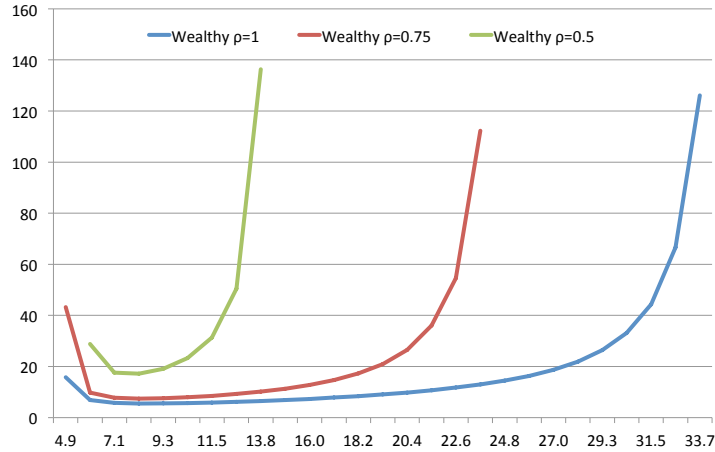


Figure 1: Inequality in information capacity $(K_1/K_2)$ as a function of $a$ and absolute risk aversion coefficient of the wealthy.

Intuitively, if investors endogenously choose different portfolio sizes, then their net benefit from investing in information increases with portfolio size, which further increases dispersion in capacity choice and hence portfolios.

### 1.3.3 CRRA Utility Specification

We also solve the main investment problem of maximizing the expected utility of wealth, where the utility function is CRRA with respect to end of period wealth:

$$\max E \frac{W^{1-\rho}}{1-\rho} \tag{2}$$

where $\rho \neq 1$. Generally, for our specification of the payoff process, i.e. $z \sim \mathcal{N}(\bar{z}, \sigma_i^2)$, wealth next period is

$$W_{t+1} = r(W_t - \sum_i p_i q_i) + \sum_i q_i z_i$$

which has a Normal distribution if $z_i$'s are Normal. In order to analytically express the expectation in (2), we start by expressing wealth as $W' = We^{\log\{[r(1-\sum p\frac{q}{W})+\sum \frac{q}{W}z]\}}$, and then use an approximation of the log return.

9

**Approximation**   To approximate $\log\{[r(1 - \sum p\frac{q}{W}) + \sum \frac{q}{W}z]\}$, define

$$f(z - rp) \equiv \log[r + \frac{1}{W}\sum pq\frac{z - rp}{p}].$$

In the above equation, the term $z$ is the only unknown stochastic term. Its Taylor approximation is

$$f(z - rp) = f(\bar{z} - rp) + f'(\bar{z} - rp)(z - \bar{z}) + \frac{1}{2}f''(\bar{z} - rp)(z - \bar{z})^2 + o(z - rp)$$

where in the above,

$$f' = \frac{1}{r + \frac{1}{W}\sum q(\bar{z} - rp)}\frac{q}{W},$$

$$f'' = -\frac{1}{(r + \frac{1}{W}\sum q(\bar{z} - rp))^2}\frac{q^2}{W^2},$$

$$f''' = 2\frac{1}{(r + \frac{1}{W}\sum q(\bar{z} - rp))^3}\frac{q^3}{W^3}.$$

With these formulas in hand, the approximation is

$$f(z - rp) = \log[r + \frac{1}{W}\sum q(\bar{z} - rp)] + \frac{1}{r + \frac{1}{W}\sum q(\bar{z} - rp)}\frac{q}{W}(z - \bar{z})$$

$$-\frac{1}{2}\frac{1}{(r + \frac{1}{W}\sum q(\bar{z} - rp))^2}\frac{q^2}{W^2}(z - \bar{z})^2$$

Denote

$$r + \frac{1}{W}\sum q(\bar{z} - rp) \equiv R(q)$$

Then we can write

$$f(z - rp) = \log[R(q)] + \frac{1}{R(q)}\frac{q}{W}(z - \bar{z}) - \frac{1}{2}\frac{1}{R(q)^2}\frac{q^2}{W^2}(z - \bar{z})^2,$$

and

$$(e^{\log(f(z - rp))})^{1 - \rho} = e^{(1 - \rho)(\log[R(q)] + \frac{1}{R(q)}\frac{q}{W}(z - \bar{z}) - \frac{1}{2}\frac{1}{(R(q))^2}\frac{q^2}{W^2}(z - \bar{z})^2)}$$

$$= (R(q))^{1 - \rho}e^{(1 - \rho)\frac{1}{R(q)}\frac{q}{W}(z - \bar{z}) - \frac{1}{2}(1 - \rho)\frac{1}{(R(q))^2}\frac{q^2}{W^2}(z - \bar{z})^2}$$

We are interested in the object $e^{(1 - \rho)\frac{1}{R(q)}\frac{q}{W}(z - \bar{z}) - \frac{1}{2}(1 - \rho)\frac{1}{(R(q))^2}\frac{q^2}{W^2}(z - \bar{z})^2}$ from the above expression. First, we approximate the term $(z - \bar{z})^2$ by its expected volatility, $\sigma_{\delta i}^2$, to get

$$e^{(1 - \rho)\frac{1}{R(q)}\frac{q}{W}(z - \bar{z}) - \frac{1}{2}(1 - \rho)\frac{1}{(R(q))^2}\frac{q^2}{W^2}\sigma_{\delta i}^2}$$

As an approximation point, we pick $\bar{z}$, which gives a constant $R(q)$, and then

$$\log EW^{1-\rho} = const. \times \log E e^{(1-\rho)\frac{1}{R(q)}\frac{q}{W}(z-\bar{z}) - \frac{1}{2}(1-\rho)\frac{1}{(R(q))^2}\frac{q^2}{W^2}\sigma_{\delta i}^2} \tag{3}$$

where the variable in the exponent is Normal, with mean (ignoring constants) $\sum q_i(\hat{\mu}_i - \bar{z}_i)$ and variance equal to $\sum q_i^2 \sigma_{\delta i}^2$. Then,

$$\log EW^{1-\rho} = const. \times (1-\rho)\left\{ \frac{1}{R}\sum \frac{q}{W}(\hat{\mu}_i - \bar{z}_i) + (1-\rho)\frac{1}{W^2 R^2}\frac{1}{2}\sum q_i^2 \sigma_{\delta i}^2 \right.$$

$$\left. -\frac{1}{2}\frac{1}{W^2 R^2}\sum q_i^2 \sigma_{\delta i}^2 \right\}$$

which gives

$$\log EW^{1-\rho} = const. \times (1-\rho)\left\{ \frac{1}{R}\sum \frac{q}{W}(\hat{\mu}_i - \bar{z}_i) - \rho\frac{1}{W^2 R^2}\frac{1}{2}\sum q_i^2 \sigma_{\delta i}^2 \right\}$$

Interior minimum (which maximizes $EW^{1-\rho}/(1-\rho)$) is

$$q_i = \frac{1}{\rho}\frac{\hat{\mu}_i - rp}{\sigma_{\delta i}^2}(Wr).$$

Plugging in gives:

$$U = \frac{1}{1-\rho}W^{1-\rho}r^{1-\rho}e^{\frac{1-\rho}{\rho}\frac{1}{2}\sum \frac{(\hat{\mu}_i - rp)^2}{\sigma_{\delta i}^2}}$$

where $\hat{\mu}_i$ and $\sigma_{\delta i}$ are the expected mean and standard deviation of the payoff process $z$, given the investor's prior, private signal, and the price signal.

We compute the expectation E(U) as in **?**. Some new notation is needed for that. First, denote the excess return as

$$R_i \equiv \hat{\mu}_i - rp_i$$

with mean $\hat{R}_i$. Denote the period zero volatility of $R_i - \hat{R}_i$ as $\hat{V}_i$ (which is just the volatility of $R_i$). Then, we can write (in a matrix form):

$$U = \frac{1}{1-\rho}W^{1-\rho}r^{1-\rho}e^{\frac{1-\rho}{\rho}\frac{1}{2}[(R-\hat{R})\Sigma_\delta^{-1}(R-\hat{R}) + 2\hat{R}\Sigma_\delta^{-1}(R-\hat{R}) + \hat{R}\Sigma_\delta^{-1}\hat{R}]}$$

Which gives

$$EU = \frac{1}{1-\rho}W^{1-\rho}r^{1-\rho}|I - 2\hat{V}\frac{1-\rho}{2\rho}\Sigma_\delta^{-1}|^{-1/2}\times$$

$$\exp(\frac{(1-\rho)^2}{2\rho^2}\hat{R}\Sigma_\delta^{-1}(I - 2\hat{V}\frac{1-\rho}{2\rho}\Sigma_\delta^{-1})^{-1}\hat{V}\hat{R}\Sigma_\delta^{-1} + \frac{1-\rho}{2\rho}\hat{R}\Sigma_\delta^{-1}\hat{R})$$

and

11

$$EU = \frac{1}{1-\rho}W^{1-\rho}r^{1-\rho}(\Pi_i(1-\hat{V}_i\frac{1-\rho}{\rho}\sigma_{\delta i}^{-1}))^{-1/2} \times \exp\left(\frac{1-\rho}{2\rho}\sum\frac{\hat{R}_i^2}{\sigma_{\delta i}}\left[(1+\frac{\hat{V}_i}{\sigma_{\delta i}}\frac{\rho-1}{\rho})^{-1}\right]\right).$$

Logging the negative of that and simplifying gives

$$-\log(-EU) = const. + \frac{1}{2}\sum_i\log(1+\frac{\hat{V}_i}{\sigma_{\delta i}}\frac{\rho-1}{\rho}) + \frac{\rho-1}{2\rho}\sum_i\frac{\hat{R}_i^2}{\sigma_{\delta i}+\hat{V}_i\frac{\rho-1}{\rho}}$$

This objective function is strictly decreasing in $\sigma_{\delta i}$ and convex, which means that agents are going to invest all capacity into learning about one asset. For that asset, $\sigma_{\delta i} = e^{-2K}\sigma_{yi}$, and $\sigma_{\delta i} = \sigma_{yi}$ otherwise.

### 1.3.4 Expansion of Asset Space

The last several decades have been marked by changes in idiosyncratic risk in the U.S. economy. To explore the role that such changes might play in the dynamics of income inequality, we consider an expansion of the assets available for investment. For illustrative purposes, we introduce new assets at the high end of the volatility spectrum, with each new asset being 1% more volatile than the previous highest-volatility asset. The emergence of these new assets actually *reduces* the growth in capital income inequality, as shown in the Table 1. High-volatility assets make the information processing more difficult, making effective capacity lower. In response, the ownership shares of sophisticated investors grow less rapidly and the price impact is reduced, resulting in higher market returns. This general equilibrium effect amplifies the direct effect of lower effective capacity, leading to more moderate capital income inequality growth. Because the volatility of the asset market is growing in this exercise, excess returns are higher for a given rate of capacity growth. We consider a reparameterization of the model that increases the rate of aggregate capacity growth to 7.4%, to match the decline in the market return seen in the data. In that case, the growth in capital income inequality is 85%—much higher than in the benchmark model—as sophisticated investors take advantage of the now more volatile asset set. The sophisticated investors hold 34% more of high-volatility assets and 30% more of low-volatility assets, relative to their population weights, at the end of the simulation. This result illustrates the strong link between structural change in the economy and inequality in capital income.

### 1.3.5 Skill versus Risk

How much of the growth in inequality comes from differences in exposure to risk versus differences in skill? Fagereng et al. (2016b) document that risk taking is only partially responsible for the difference in returns among Norwegian households, with approximately half of the return difference being attributed to unobservable heterogeneity. Our model is one in which both risk-taking differences and pure compensation for skill generate return heterogeneity. Sophisticated investors are more exposed to risk because they hold a larger

Table 1: Aggregate Capacity Growth Outcomes

|  | Baseline | New assets | | Asym. growth |
|---|---|---|---|---|
| Capacity growth | 4.9 | 4.9 | 7.4 | 5.1 |
| Average market return | 7 | 10 | 7 | 5.6 |
| Capital income ineq. growth | 38 | 28 | 85 | 54 |
| Sophis end own share of top | 1.21 | 1.17 | 1.34 | 1.24 |
| Sophis end own share of bottom | 1.14 | 1.08 | 1.30 | 1.17 |

share of risky assets (compensation for risk); and they have informational advantage (compensation for skill). To shed more light on the relative importance of these two effects, we decompose the returns of each investor type by computing the unconditional expectation of the return on the portfolio held by investor type $j \in \{S, U\}$:

$$R_j = E \sum_i \omega_{jit}(r_{it} - r) = \sum_i Cov(\omega_{jit}, r_{it}) + \sum_i E\omega_{jit}E[r_{it} - r], \qquad (4)$$

where $r_{it} = z_{it}/p_{it}$ is the time $t$ return on asset $i$ and $\omega_{jit}$ is the portfolio weight of asset $i$ for investor $j$ at time $t$, defined as $\omega_{jit} = q_{jit}p_{it}/\sum_l q_{jlt}p_{lt}$. The first term of the decomposition captures the covariance conditional on investor $j$ information set, that is, the investor's reaction to information flow via portfolio weight adjustment (*skill effect*); the second term captures the *average effect*, unrelated to active trading.

Quantitatively, the skill effect accounts for the majority of the return differential in the model. To show that, we compute the counterfactual return of sophisticated investors if their skill matched that of unsophisticated (plus noise) investors, but their average holdings stayed the same

$$\hat{R}_I = \sum_i Cov(\omega_{Rit}, r_{it}) + \sum_i E\omega_{Iit}E[r_{it} - r]. \qquad (5)$$

Such a portfolio would have generated an annualized return of 10.3%, which implies that the compensation for skill accounts for more than 75% of the return differential between the sophisticated and unsophisticated investors.

# 2 Appendix: Additional Data Discussion and Analysis

Our model parametrization is based on the data from the Survey of Consumer Finances (SCF) from 1989 to 2013. For all computed statistics, we weigh all observations by the weights provided by the SCF (variable `42001`). Consistent with previous studies we drop farm owners.

## 2.1 Data Constructs

**Participation** Our measure of participation in financial markets includes individuals who satisfy at least one of the following criteria: (i) have a brokerage account (coded in variable 3923), (ii) report a positive amount of stock holdings (variable 3915), (iii) report holding non money market funds (coded as a positive balance in at least one of the variables: 3822, 3824, 3826, 3828, and 3830; and also 7787 starting in survey year 2004), (iv) report positive holdings of bonds (coded as the sum of: 3906, 3908, 3910, and additionally 7633, 7634 starting in survey year 1992), (v) report dividends from their stock holdings (variable 5710), (vi) report holding money funds (coded in variables: 3507, 3511, 3515, 3519, 3523, 3527). As a robustness check, we also consider a measure of broad market participation that includes the above six, plus the condition that a household has equity in a retirement account. Specifically, we consider the criterion that (vii) a household reports that either the head or spouse or other family members have money in retirement accounts invested in equity. For survey years 1989 and 1992 it is coded in variable 3631 with values of 2 (stocks, mutual funds), 4 (combination of stocks, CDs and money market accounts, and bonds), 5 (combination of stocks and bonds), 6 (combination of CDs and money market accounts, and stocks). For survey years 1995, 1998, and 2001 it is coded in variable 3631 with values of 2, 4, 5, 6, or 16 (brokerage account/cash management account). For surveys starting in 2004, the coding shifts to variables 6555, 6563, or 6571 (head, spouse, other family members). For the 2004, 2007, and 2013 surveys, this means values 1 (all in stocks), 3 (split), or 5 (hedge fund) for at least one of the variables. For survey year 2010, this means answering 1, 3, 5, or 30 (mutual fund). Adding the category of -7 (other) to the above list does not change the results.

**Capital Income** To construct a measure of capital income, we sum up income from four sources: (i) dividend income (5710), (ii) income from non-taxable investments such as municipal bonds (5706), (iii) net gains or losses from mutual funds, sale of stocks, bonds, or real estate (5712), and (iv) other interest income (5708).

**Wealth Measures** Total wealth is a sum of financial and non-financial wealth as per **?**. Financial wealth is a sum of: (1) holdings in non money funds (sum of balance in variables: 3822, 3824, 3826, 3828, 3830, and also 7787 starting in survey year 2004), (2) bond holdings balance (the sum of: 3906, 3908, 3910, and also 7633, 7634 starting in survey year 1992), (3) balance of directly held stocks (variable 3915), (4) cash value of life insurance (4006), (5) other financial assets (future royalties, money owed to households, etc. in variable 4018), (6) balances in individual retirement accounts of all family members (variables 6551–6554, 6559–6562, 6567–6570, 6756, 6757, 6758), (7) value of certificates of deposit (3721), (8) cash value of annuities, trusts, or managed accounts (6577, 6587), (9) value of savings bonds (3902), (10) value of liquid assets (checking accounts 3506, 3510, 3514, 3518, 3522, 3526, 3529, cash or call money accounts 3930, savings and money market accounts 3730, 3736, 3742, 3748, 3754, 3760).
Non-financial wealth is a sum of: (1) value of vehicles, including motor homes, RVs, motorcycles, boats, and airplanes less the amount still owed on the financing loans for these ve-

hicles (8166+8167+8168+8188–2218–2318–2418–7169+2506+2606–2519–2619+2623–2625), (2) value of business in which a household has either active or nonactive interest (value of active business is calculated as net equity if business was sold today plus loans from the household to the business minus loans from the business to the household, plus value of personal assets used as collateral for business loans; value of non-active business is the market value; the formula used (for the 2004 SCF) is `3129+3229+3329+3335+8452+8453+3408+3412+3416+3420 +3424+3428+3124+3224+3324-(3126+3226+3326) plus 3121+3221+3321` (variables have different numbers pre-1995; some variables are not reported in 2010 and 2013 anymore), (3) value of houses and mobile homes/sites owned (`604+614+623+716`), (4) value of other real estate owned: vacation homes (`2002`) and owned share of other property (`1706*1705+1806*1805+1906*1905` divided by 10000), (5) the value of other non-residential real estate net of mortgages and other loans taken out for investment in real estate (2012–2016), (6) other non-financial assets, such as artwork, precious metals, antiques, oil and gas leases, futures contracts, future proceeds from a lawsuit or estate that is being settled, royalties, or something else (`4022+4026+4030`).

**Wage Income and Total Income**  For labor income and total income, we use the SCF responses to questions `5702` (income from wages and salaries) and `5729` (income from all sources). The difference between the two, apart from capital income, consists of social security and other pension income, income from professional practice, business or limited partnerships, income from net rent, royalties, trusts and investment in business, unemployment benefits, child support, alimony and income from welfare assistance programs.

## 2.2   Participation

In Figure 1, we present the time series of our two measures of participation. The series *Participation* follows our benchmark definition above, while *Participation + Retirement* is a broader measure that also includes individuals who participate in equity through retirement accounts.
Our participation measure changes from 32% in 1989 to a high of 40% in 2001, and down to 28% in 2013. When we additionally include participation through retirement accounts, the dynamics are very similar, except that the levels get shifted upwards. The participation level is around 35% in 1989, peaks at 44% in 2001, and goes down to 37% in 2013.
Even though both measures of participation exhibit considerable variation over time (although without any particular trend), as we point out in the paper, financial wealth inequality in the SCF data set is entirely concentrated within our participating group. Figure 2 in the paper, reproduced in Figure 3 below, presents financial wealth inequality between (i) top decile versus the rest of our participating group ('Sophisticated/Unsophisticated'), (ii) bottom decile of participants and non-participants, and (iii) bottom decile of participants and non-participants. Financial wealth inequality between the bottom participants and non-participants, exhibits no trend and the ratios are stable around 1. Additionally, also in Figure 2 in the paper, we show that all of the growth in financial wealth inequal-
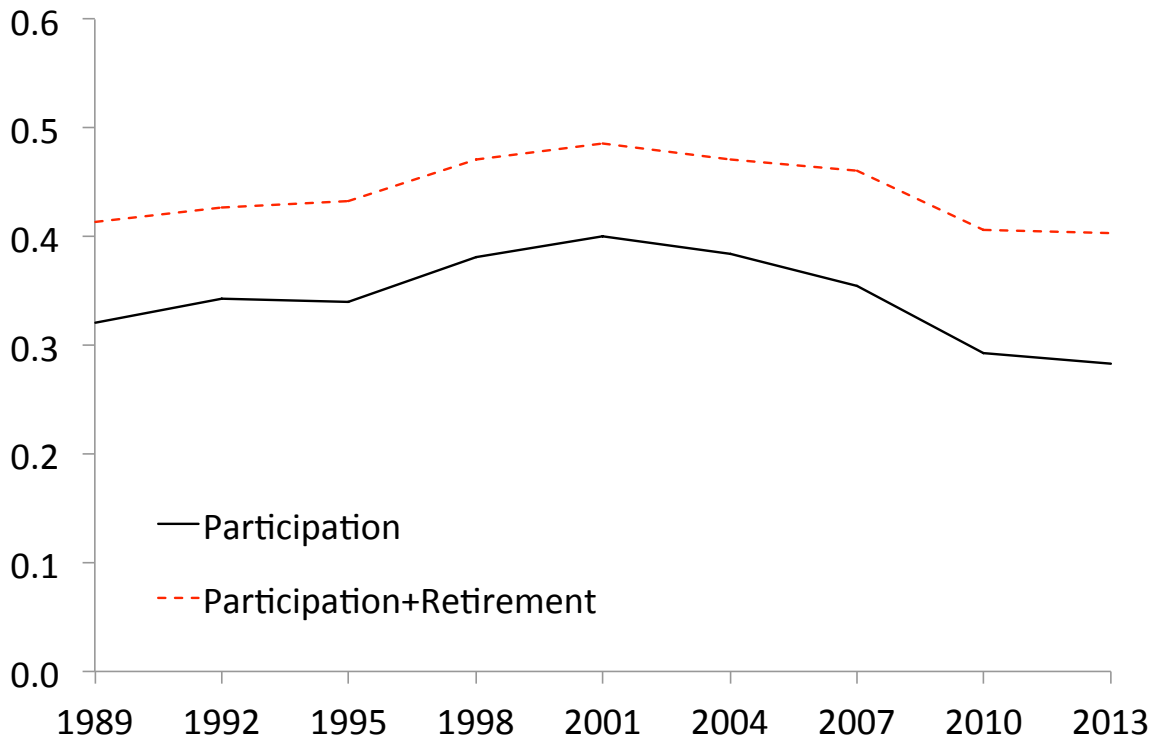
Figure 2: Financial markets participation in the SCF.

ity in our participating group can be accounted for by retained capital income. These two points suggest that the participating group is the relevant subsample to study capital income inequality.
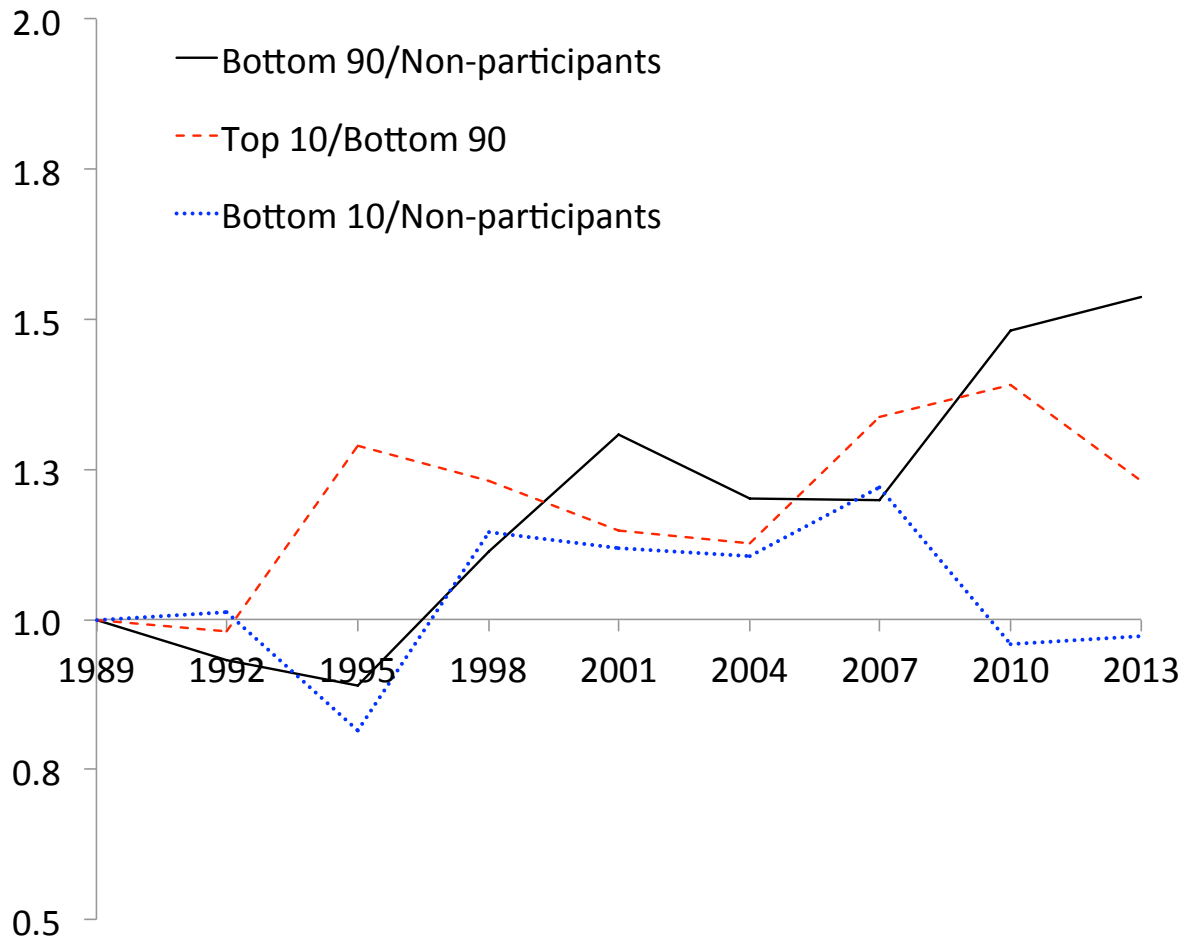
Figure 3: Extensive and intensive margins in capital income inequality.

## 2.3   Capital Income

**Inequality**   Our measure of inequality is the mean income in the top decile of the wealth distribution relative to the mean income in the rest (*of participants*). Figure 4 presents the evolution of capital income inequality in the SCF. Figure 5 presents the evolution of capital income inequality using the benchmark definition of participation as well as Participation+Retirement.
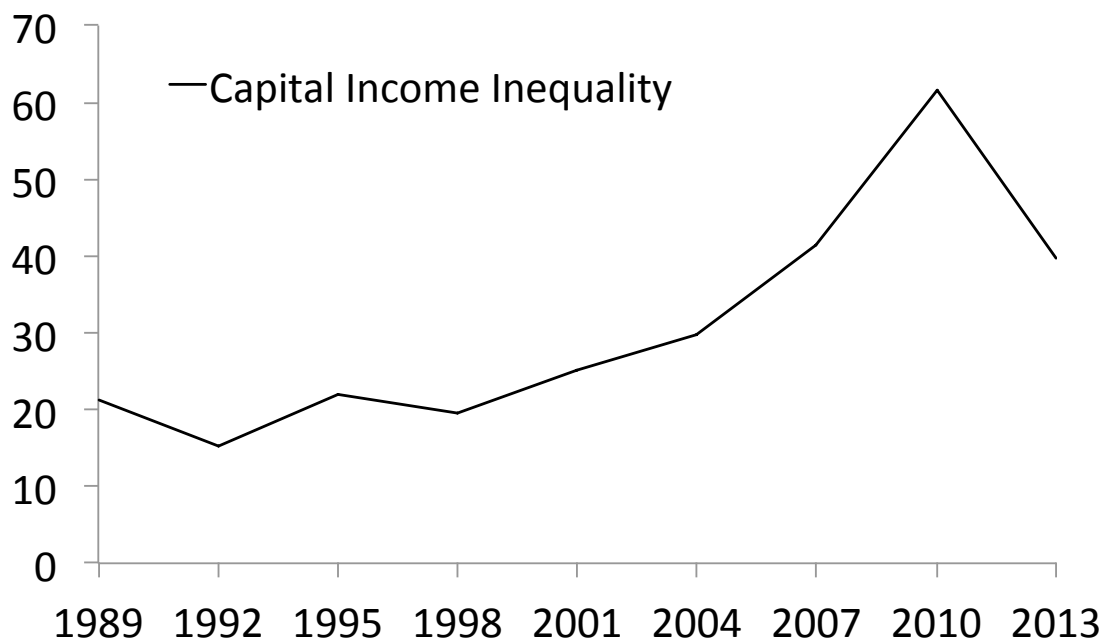


Figure 4: Capital income inequality.

**Passive Investment Policies**   We also study whether capital income differences are an outcome of time-varying *market returns* combined with *passive* buy-and-hold household strategies. It is possible that some households (the wealthy) hold a larger share of their wealth in stocks, which gives them higher returns by the mere fact that stocks outperform bonds. In Figure 6 we plot, for each year, the past 15-year cumulative return on holding the aggregate index of the U.S. stock market.[1] We contrast this return with that of a household exclusively holding bonds (with a gross return of 1).

The cumulative return on the passive strategy exhibits a declining trend, which implies that if investors used the passive strategy and the only difference was how much money they hold in the stock market versus bonds, then we should observe a declining trend in capital income inequality, as the gross return on the market converges to the gross return on bonds. This exercise highlights the importance of active decisions of when to enter and exit the stock market.

---

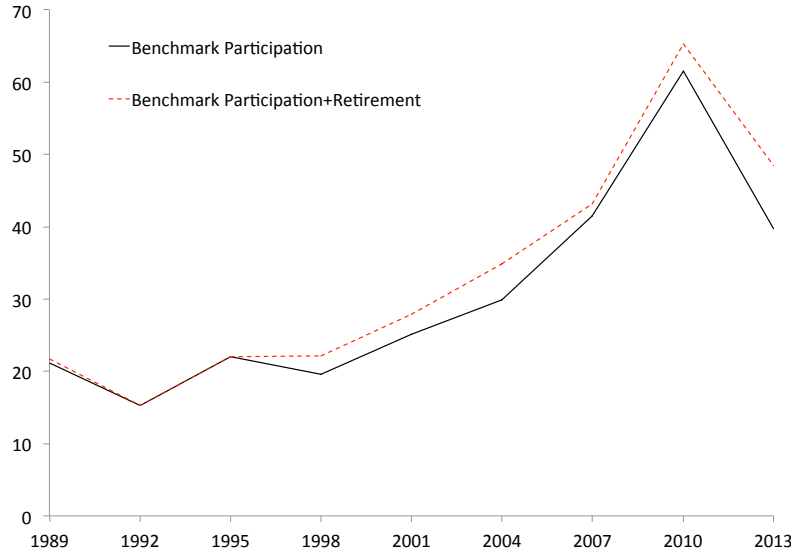[1]The patterns we document are essentially the same for other choices of the horizon: 5, 10, or 20 years.

Figure 5: Capital income inequality for different measures of participation.

## 2.4 Survey of Consumer Finances: Descriptive Statistics

To complete the characterization of the participating and non-participating groups in the SCF, Table 2 presents summary statistics for the 1989 and 2013 surveys. As expected, participants in financial markets tend to be wealthier, older and more educated. Within the participating group, the top 10% of participants also have higher financial wealth, are older, and more educated. However, Panel I of the table shows that the growth in financial wealth inequality is concentrated almost exclusively within the participating group, consistent with the trends in Figure 3. First, in the cross-section, the financial wealth of the bottom 50% of participants is only twice that of the non-participants; conversely, in 1989 the top 10% of participants has financial wealth that is 38 times larger than that of the bottom 50%. Second, between 1989 and 2013, financial wealth inequality within the participants group grew by 67% (top 10% versus bottom 50%), while inequality across groups (bottom 50% versus Non-participants) grew by mere 12%. Panels II through IV of Table 2 summarize the inequality in capital, labor, and total income for participants and non-participants. The same pattern that emerged with respect to financial wealth inequality also applies to labor and total income inequality: both the level and the growth of inequality have been concentrated within the group of participants.

Panels V through VIII of the table explore potential drivers of the growth in inequality between the top 10% and the bottom 90% or 50% of participants. First, top participants hold a much smaller fraction of their financial wealth in liquid assets (Panel V). In turn, bottom participants start out with a higher share (28% or 33% versus 21%) and also grow the fraction of financial wealth held in liquid assets significantly (from 28% and 33% in 1989 to 37% and 46% in 2013). This type of portfolio composition shift towards lower risk liquid assets for the bottom participants is consistent with our information-based mechanism. Third, top participants also have higher educational attainment and are much more likely to

Table 2: Investor Characteristics in the SCF

|  | 1989 | 2013 |
|---|---|---|
| **I. Financial Wealth** | | |
| Top 10%/Bottom 90% of Participants | 13 | 16 |
| Bottom 90%/Non-participants | 5.8 | 8.8 |
| Bottom 50%/Non-participants | 2 | 2.2 |
| **II. Capital Income** | | |
| Top 10%/Bottom 90% of Participants | 21 | 39.7 |
| Bottom 90%/Non-participants | - | - |
| **III. Wages and Salaries Income** | | |
| Top 10%/Bottom 90% of Participants | 2.4 | 3.9 |
| Bottom 90%/Non-participants | 1.8 | 2. |
| Bottom 50%/Non-participants | 1.3 | 1.4 |
| **IV. Total Income** | | |
| Top 10%/Bottom 90% of Participants | 5.6 | 7.2 |
| Bottom 90%/Non-participants | 1.9 | 2 |
| Bottom 50%/Non-participants | 1.25 | 1.27 |
| **V. Liquid Assets/Financial Wealth** | | |
| Top 10% of Participants | 21% | 19% |
| Bottom 90% of Participants | 28% | 37% |
| Bottom 50% of Participants | 33% | 46% |
| Non-participants | 52% | 75% |
| **VI. Has brokerage account** | | |
| Top 10% of Participants | 64% | 83% |
| Bottom 90% of Participants | 25% | 46% |
| Bottom 50% of Participants | 16% | 36% |
| **VII. % with college** | | |
| Top 10% of Participants | 67% | 87% |
| Bottom 90% of Participants | 40% | 56% |
| Bottom 50% of Participants | 31% | 46% |
| Non-participants | 15% | 23% |
| **VIII. Age (years)** | | |
| Top 10% of Participants | 57 | 60 |
| Bottom 90% of Participants | 51 | 54 |
| Bottom 50% of Participants | 49 | 51 |
| Non-participants | 46 | 50 |

*Source*: SCF. Capital income/Financial wealth is the ratio of average capital income to the average financial wealth in each group. Percent with college is the fraction of individuals with 16 or more years of schooling. See the Online Appendix for complete definitions.

Figure 6: Cumulative market return on a 15-year passive investment in the U.S. stock market.

have brokerage accounts (Panels VI and VII), consistent with their having a higher degree of financial sophistication. The data, however, also show a significant increase in access to brokerage accounts for the bottom participants (from 25% and 16% in 1989 to 46% and 35% in 2013). This fact, along with evidence that transaction costs on brokerage accounts have been trending down (**?**), suggests that the costs of accessing and transacting in financial markets are an unlikely explanation for the observed rise in capital income inequality. If anything, the improved access to financial markets should generate lower inequality, in the absence of informational heterogeneity. Finally, while top participants are on average older, there are no time-series dynamics to the age difference that could explain the observed capital income dynamics (Panel VIII).

## 2.5   Mutual Funds and Delegation

**Barriers to High-Return Institutional Funds**   We compare returns from different types of mutual funds, using data from Morningstar, which contains information for two types of funds: those with a minimum investment of $100,000 (institutional funds) and those without such restrictions (retail funds). Our fund data span the period 1989 through 2012.

Figure 7 plots the cumulative return series for institutional versus retail mutual funds. To construct the figure, we compute the value of one dollar invested in each fund type in January of 1989 and assume that the monthly after-fee return is subsequently reinvested until December 2012. A cumulated value of the dollar in 1989 grows to $22 for institutional funds and to $16 for retail funds. This difference amounts to about 3% return difference per year between the two types of funds. Since the institutional funds have a minimum investment threshold, less sophisticated, less wealthy investors do not have access to the higher returns earned by institutional funds, even for "plain vanilla" assets like equities.
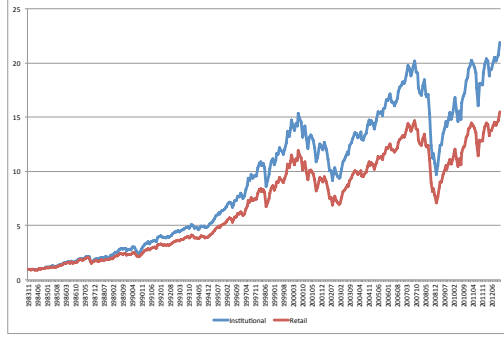
21

Figure 7: Cumulative investment returns in equity mutual funds by investor type.

**Dispersion in the Quality of Asset Management Companies**   We document a large heterogeneity in mutual fund returns in the data depending on the investment size (typically related to wealth). Additionally, below we show that the average fund does not outperform the passive benchmark and that the performance of a typical mutual fund is not persistent over time. Taken together, these findings suggest that selecting a mutual fund in any particular period is an informationally intensive task, similar to trading individual stocks.[2]

*Average Mutual Fund Does Not Outperform Passive Benchmark.* We construct a sample of risk-adjusted after-fee fund returns by regressing monthly excess fund returns, net of the risk-free rate, on four risk factors: market, size, value, and momentum as in Carhart (1997). The abnormal return from this regression is our definition of a risk-adjusted return. We present in Figure 8 a histogram of monthly returns pooled across all funds and all months in our sample. The mean and median value of the distribution are not statistically different from zero.
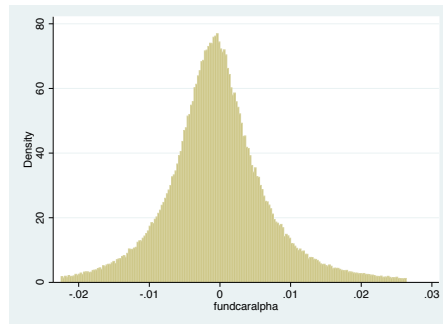


Figure 8: Distribution of equity funds' returns.

*Mutual Fund Performance Is Not Persistent Over Time.* While an average fund does not beat a passive benchmark, we observe a large cross-sectional dispersion in returns with both small and large values of alpha. It is thus possible that investors could focus their attention only on funds with positive returns thus beating the market portfolio. The issue with such

---

[2]We are not the first ones to point out these regularities. Extant literature in finance, such as **?** or **?** finds that while the average abnormal gross returns of mutual funds are positive, the distribution of returns is highly dispersed and the returns are not predictable.

approach is whether funds with positive returns tend to outperform the benchmark on a consistent basis. If not, the strategy of focusing on current winners may not be profitable. To test for such predictability, each month we sort funds into five equal-sized portfolios according to their current risk-adjusted returns and test whether the ranking of funds into such portfolios is preserved one month and one year into the future. We show the result using a transition matrix of being in a particular quintile portfolio conditional on starting in a given portfolio at time $t$. Each of the 25 cells of the transition matrix illustrates the probability of being in quintile $j = 1 - 5$ at time $t + k$ conditional on being in quintile $i = 1 - 5$ at time $t$. We set $k$ to be equal to 1 and to 12 months. The results are in Table 3.

Table 3: Transition Probabilities of Fund Performance

| | Performance quintiles | | | | |
|---|---|---|---|---|---|
| | at $t + k$ | | | | |
| at $t$ | 1 | 2 | 3 | 4 | 5 |
| | k=1 month | | | | |
| 1 | 77.8 | 16.6 | 3.4 | 1.3 | 0.7 |
| 2 | 16.5 | 56.2 | 20.7 | 5.2 | 1.4 |
| 3 | 3.6 | 20.5 | 52.1 | 20.3 | 3.4 |
| 4 | 1.2 | 5.4 | 20.3 | 56.6 | 16.4 |
| 5 | 0.8 | 1.5 | 3.6 | 16.6 | 77.4 |
| | k=12 months | | | | |
| 1 | 29.3 | 19.6 | 16.6 | 16.6 | 17.8 |
| 2 | 20.0 | 22.2 | 21.6 | 20.3 | 15.9 |
| 3 | 16.6 | 22.0 | 23.7 | 21.9 | 15.7 |
| 4 | 16.0 | 21.0 | 21.9 | 22.1 | 19.1 |
| 5 | 18.5 | 16.7 | 17.0 | 19.8 | 27.9 |

We observe that fund performance is not very persistent over time. For example, a fund that starts in the top-performing quintile at time $t$ has a 77% chance of ending up in the same quintile one month later. The same probability for one-year ahead transition drops to 28%. Similar patterns emerge for other quintiles in the matrix. We conclude that an uninformed household would face a difficult task to invest in a successful fund by simply following past winners.

## 2.6  Expansion of Ownership

As aggregate capacity grows, sophisticated investors expand their ownership of risky assets by order of volatility: starting from the highest volatility assets and then moving down.

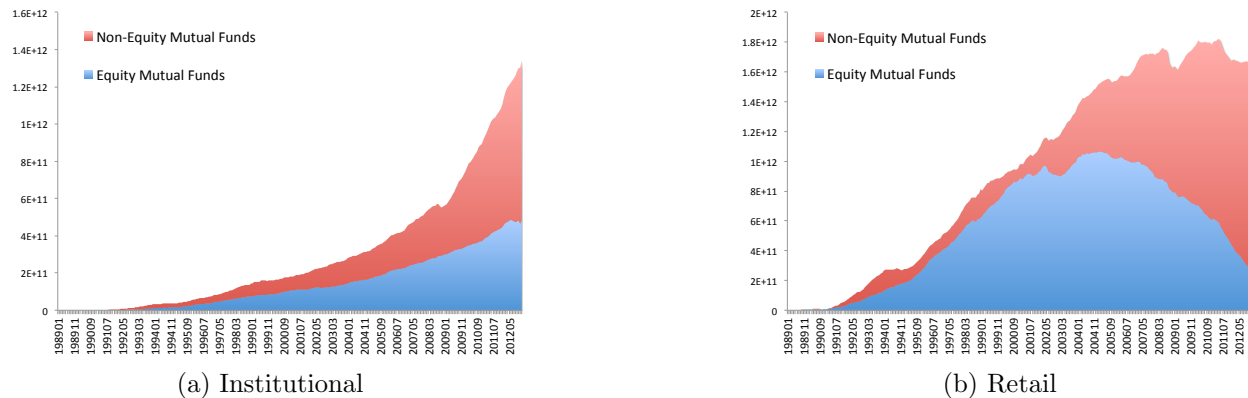(a) Institutional                            (b) Retail

Figure 9: Cumulative Flows to Mutual Funds: Institutional vs. Retail

To test this prediction, we consider flows into mutual funds by investor type. Given that equity funds are generally more risky than non-equity funds one would expect unsophisticated investors be less likely to invest in the equity funds, especially if aggregate information capacity grows.

We use data on flows into equity and non-equity mutual funds from Morningstar by sophisticated (institutional) and unsophisticated (retail) investors. As shown in Figure 9, the cumulative flows from sophisticated investors into equity and non-equity funds increase steadily over the entire sample period. In contrast, the flows from unsophisticated investors display a markedly different pattern. The flows into equity funds grow until 2000 but subsequently decrease at a significant rate to drop by a factor of 3 by 2012. Moreover, this decrease coincides with a significant increase in cumulative flows to non-equity funds. Notably, the increase in equity fund flows by unsophisticated investors observed in the early sample period is consistent with the steady decrease in holdings of individual equity in the U.S. data. To the extent that direct equity holdings are more risky than diversified equity portfolios, such as mutual funds, this implies that unsophisticated investors have been systematically reallocating their wealth from riskier to safer asset classes.

Overall, these findings qualitatively support our model's predictions: Sophisticated households have a large exposure to risky assets and subsequently add exposure to less risky assets, and as unsophisticated households face greater information disadvantage they increasingly move their money into safer assets.