Competition versus Auction Design

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Abstract

We analyze the value of auction design tools relative to fundamental economic forces by estimating the impact of setting an optimal reserve price and increasing the number of potential bidders in USFS timber auctions. We allow for costly participation that selects entrants on dimensions that affect ex-post competition, since both reserve prices and potential competition can change the number and types of bidders that participate. The USFS's value of an optimal reserve price is low, both in absolute terms and relative to even marginal increases in the number of potential bidders.

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1 Introduction

A large theoretical literature provides many insights about how sellers should optimally design auctions by, for example, setting an optimal reserve price. However, there have been few attempts to evaluate the value of these design tools relative to more fundamental economic factors, such as the number of potential bidders (competition). This is especially true in realistic settings where there may be several types of bidders, auction participation is costly and entry is likely to be selective, by which we mean that potential bidders with higher values are more likely to participate, so that changes in reserve prices or increased potential competition may endogenously affect the number and types of participants. This lack of formal empirical evidence is all the more glaring since at least some respected experts, who have advised many sellers on how to design auctions, believe that fundamental factors such as competition may be much more important than many design tools (Klemperer (2002)). In this paper, we compare the relative value to a seller of using reserve prices and increasing competition in a particular empirical setting, United States Forest Service (USFS) timber auctions, where optimal reserve price policies have been the subject of considerable previous study.[1] To do this, we develop and estimate a flexible empirical entry model for independent private value (IPV) second price and ascending auctions that allows us to evaluate the degree of selection in the entry process, and which could be used to evaluate the effects of other policies or changes in market structure that would be likely to affect the entry margin.[2]

We build a model that allows for endogenous and selective entry and bidder asymmetries because these are common features of many settings in which auctions are used and, in this type of an environment, there are no unambiguous theoretical results regarding the relative value of reserve prices and increased competition to sellers. In a classic paper, Bulow and Klemperer (1996) show that a revenue-maximizing seller will always benefit more from having one more bidder than being able to set an optimal reserve price if bidders are symmetric and all bidders must participate. Even with symmetric bidders, this result is not robust to endogenous participation where assumptions about what bidders know about their values when they decide to enter can be crucial. A common assumption is that potential bidders have no information about their values when they decide to pay the entry cost (we refer to this as the LS model after Levin and Smith (1994)), so that the entry process cannot be selective. In this case, if potential bidders do not always enter, then expected revenues decline in the number of potential bidders so that a seller dislikes additional potential competition

[1]Some examples of previous studies of timber auction reserve prices include Mead, Schniepp, and Watson (1981), Mead, Schniepp, and Watson (1984), Paarsch (1997), Haile and Tamer (2003), Li and Perrigne (2003) and Aradillas-Lopez, Gandhi, and Quint (2010). All of these papers assume that entry is not endogenous.

[2]The focus on a second price model makes the exposition easier. Our empirical work allows for the fact that the auctions we analyze are actually open outcry auctions.

(the corollary to proposition 9 in Levin and Smith (1994), p. 595), and trivially prefers the ability to set an optimal reserve price (which is equal to zero for a revenue maximizing seller). An alternative extreme assumption is that potential bidders know their values prior to the entry decision (we refer to this as the S model after Samuelson (1985)). In this case, adding a potential bidder may increase or decrease expected revenues (Samuelson (1985) and Menezes and Monteiro (2000)), and it may be more valuable than the ability to set an optimal reserve price (we provide an example below). This lack of a general result also applies to models, like the one that we estimate, where potential bidders only have imperfect signals about their values when they decide to participate. When potential bidders are asymmetric there are no general results concerning the relative value of competition and reserve prices even for models with complete participation or endogenous entry but no selection.

In many real-world settings, asymmetric potential bidders are likely to have some, but imperfect, information about their values when they decide to enter the auction, and a significant part of the entry cost will be the research cost that a firm pays to identify its value. In our empirical setting (USFS timber auctions), two types of firms are potential bidders: sawmills, who have their own manufacturing facilities, and logging companies, who do not, and their observed bidding and entry behavior suggests that mills tend to have significantly higher values. A bidder's value for a particular tract will depend on the types of wood on the tract, tract characteristics such as tree diameters and density, and the bidder's own capabilities and contracts for selling either cut timber or processed wood products downstream. A firm's knowledge of the forest, its own contracts and capabilities and the information released by the USFS when announcing the sale, which can be inaccurate and lack information the bidder would like to know, should allow potential bidders to form some rough estimates of their values prior to deciding whether to participate. However, before submitting bids, bidders conduct their own surveys of each tract ("timber cruises") to determine their values precisely, and the cost of the cruise is seen as one of the most important factors that deter firms from participating in timber auctions. This environment is naturally described by a model which lies somewhere between the informational extremes of the LS and S models. This is also likely to be true in other commonly studied auction environments such as highway paving contract procurement auctions, where bidders must calculate how much the project will cost to complete, and OCS oil tract auctions, where bidders conduct expensive seismic surveys to estimate the amount of oil present.

To capture this key feature of the environment, we formulate a two-stage entry model for second price or ascending IPV auctions with asymmetric bidder types. In the first stage, potential bidders simultaneously decide whether to participate in the auction, which entails paying a sunk entry cost that enables them to learn their value exactly. In the second stage, entrants submit bids. We allow for selection by assuming that each potential entrant gets a noisy, private information signal about its value before taking an entry decision. We therefore refer to our model as the Signal model. In equilibrium, each potential bidder enters if its signal is above a type-specific threshold, and the precision of signals (which is a parameter that we estimate) determines how selective the entry process is. If signals are very uninformative, then the value distributions of entrants, marginal entrants and non-entrants will be similar (little selection) and outcomes will be similar to those of an LS model with asymmetric firms. On the other hand, when signals are very informative, these distributions will be quite different (a great deal of selection) and outcomes will approach those of the S model.

Our estimates indicate that the entry process for timber auctions is moderately selective. For a representative auction, the average mill (logger) entrant's value is 50.7% (24.3%) greater than that of the marginal mill (logger) entrant. Our estimates also imply that the value of setting an optimal reserve is typically small, both in absolute terms and relative to increasing the number of mills (the stronger type) by one. For example, for a representative auction that has four mill and four logger potential entrants, the USFS's expected value of holding the auction increases by only 0.92% when it sets an optimal reserve price rather than a non-strategic reserve price equal to its value of keeping the tract in the event of no sale. In contrast, there is a 6.66% improvement if one mill potential entrant is added, even if the USFS uses a non-strategic reserve: an increase which is 7.21 times as large. This happens even though the addition of another potential entrant makes existing firms less likely to enter because, with selection, the entrants that are lost tend to be less valuable to the seller. The gain from adding one additional logger potential entrant (1.80%) also exceeds the gain from using an optimal reserve price. Moreover, this assumes there are only incremental increases in the number of potential bidders and the result could be stronger if additional potential entrants are added. For example, in the representative auction, we find that if two potential mill entrants are added, the USFS's value of the auction rises 12.51% even if it continues to set a non-strategic reserve price.

Our paper makes two important contributions. Our first contribution is that we provide, to our knowledge, the first empirical analysis of the relative value of a commonly used auction design tool (reserve prices) and a more fundamental economic force (competition) on auction revenues. Here our empirical estimates are consistent with the view of Klemperer (2002) who suggests that a large amount of auction theory focuses on factors that do not greatly impact a seller's revenues in practical settings.[3] Understanding the relative returns of auction

[3]Klemperer provides several examples that illustrate his view. For instrance, when U.K. commercial television franchises were auctioned in the early 1990s, prices in regions where numerous firms participated sold for between 9 and 16 pounds per capita, whereas in the Midlands region only one firm participated and

design and potential competition is of obvious potential interest to any seller, and while we do not explicitly model the ways that sellers can influence potential competition, we do not view this comparison as an abstract one. Many sellers will be able to affect the number of potential competitors by, for example, engaging in advertising, timing sales appropriately or designing eligibility requirements appropriately. In fact, in many cases, it may be more straightforward for a seller to try to increase the number of potential competitors than to set the optimal reserve price, which requires the seller to know for all types of bidder, the number of potential bidders, the distribution of bidder values, the distribution of signals, and entry costs. As noted by Wilson (1987) (leading to what has come to be known as the "Wilson Doctrine"), auction design tools, including optimal reserve price policies, are not practically useful if they assume too much about the information available to the seller.

Understanding the relative effects of potential competition and reserve prices should also shape how both sellers and policy makers - such as anti-trust authorities - think about changes in the competitive environment. For example, the timber logging and processing industry has become increasingly concentrated due to both exit and mergers, and our results show that the loss of competition for particular stands of timber may substantially reduce sellers' revenues and that there will be relatively little sellers can do using auction design tools to offset these losses.[4]

Our second contribution is that we provide a structural framework, suitable for settings with asymmetric firms and unobserved market (auction) heterogeneity, that can be used to (a) estimate the degree of selection in the entry process and (b) understand how selective entry may affect outcomes in a wide range of counterfactuals in addition to those considered in this paper. We believe that there are many policy-relevant counterfactuals, in both auction and non-auction settings, that are substantively affected by selective entry. As one example, the U.S. Department of Justice and the Federal Trade Commission's *Horizontal Merger Guidelines*, and similar rules in other countries, indicate that a merger may be allowed if sufficient entry would occur within a one to two-year window to force prices back to their pre-merger levels if the merging parties were to raise prices significantly. The probability of future entry, and its ability to constrain price increases, is likely to depend on whether the entry process has already selected the most competitive firms into the market.[5] As a second example, sellers sometimes use bid subsidies, or restrict sales to particular types of buyers, in order to encourage particular types of firm, such as small or minority-owned businesses, to

it paid only one-twentieth of one penny per capita.

[4]Li and Zhang (2010) simulate mergers between bidders in Oregon timber auctions based on a model with heterogeneous bidders and a non-selective endogenous entry process.

[5]Brannman and Froeb (2000) simulate the effect of mergers in timber auctions assuming exogenous participation.

participate.[6] The cost and benefit of these programs will depend on whether the new firms that are drawn in by these programs tend to have lower values than the firms that would have participated without them, which is obviously determined by the degree of selection.[7] As a final and less obvious example, Roberts and Sweeting (2010) use the model presented here to compare the seller's expected revenue from using a simultaneous bid auction (first price or second price), to using a sequential procedure where buyers can place bids in turn, which mimics how valuable items such as companies and real estate are often sold (Bulow and Klemperer (2009)). They show that, with selective entry, the sequential procedure can generate significantly higher revenues. Some of the largest gains occur when entry is moderately selective, which our estimates suggest is an empirically relevant case, as opposed to extreme cases of the LS or S models that are the most commonly used models in the literature.

We regard this framework as an important contribution to the empirical entry literature since, while empirical studies of market entry are now common, the existing literature pays little attention to whether the entry process is selective. The key feature of the selective entry process in our model is that the firms that choose to enter are likely to be stronger competitors, which hurts rivals and benefits consumers (or sellers in auctions), than those firms that do not enter. In the non-auction literature, standard entry models (e.g. Berry (1992)), allow for there to be a shock to a firm's payoff from entering a market, which may cause some firms to enter while others do not. However, they assume that this shock does not affect the profits of other firms, so it must be interpreted as affecting sunk costs or fixed costs. Similarly, dynamic entry models (e.g. Ericson and Pakes (1995)) assume that potential entrants are symmetric apart from i.i.d. shocks to their entry costs. Instead, the natural analogue of our model in non-auction settings would be one where firms receive noisy signals about their post-entry marginal costs or qualities.

In the empirical auction literature (see Hendricks and Porter (2007) for a recent survey), most analyses that allow for endogenous entry also assume no selection, consistent with the LS model (e.g. Athey, Levin, and Seira (2011), who examine timber auctions, Bajari and Hortaçsu (2003), Palfrey and Pevnitskaya (2008), Krasnokutskaya and Seim (forthcoming), Li and Zhang (2010), Bajari, Hong, and Ryan (2010) and Ertaç, Hortaçsu, and Roberts (2011)). However, the use of the S model in theoretical work, has led to some empirical interest in whether the entry process is selective.[8] For example, Li and Zheng (2009) compare estimates

[6]Athey, Coey, and Levin (2011) study set-asides in timber auctions and Krasnokutskaya and Seim (forthcoming) study subsidies in government procurement under the assumption that the entry process is not selective.

[7]Selection's effects on cost-benefit analysis are suggested by the computational examples in Hubbard and Paarsch (2009) that show that, under perfect selection, bid subsidy programs have only small effects on auction participation.

[8]Matthews (1995), Menezes and Monteiro (2000) and Tan and Yilankaya (2006) analyze theoretical models

from both the LS and S models using data on highway lawn mowing contracts from Texas to understand how potential competition may affect procurement costs, and Li and Zheng (2010) test the LS and S models using timber auctions in Michigan. Marmer, Shneyerov, and Xu (2010) extend this literature by testing whether the Li and Zheng (2009) data is best explained by the LS, S or a more general affiliated signal model. They find support for the S and signal models, and they also estimate a simple version of their signal model.

An important limitation of the Li and Zheng (2009), Li and Zheng (2010) and Marmer, Shneyerov, and Xu (2010) papers is that they assume that potential bidders are symmetric and, in the last one, that there is no unobserved auction heterogeneity (for example, in the good being sold). These are significant limitations as bidder asymmetries are common and often important for interesting counterfactuals, and unobserved auction heterogeneity has been shown to be important in many settings, including timber auctions (Athey, Levin, and Seira (2011) and Aradillas-Lopez, Gandhi, and Quint (2010) for timber and Krasnokutskaya (forthcoming), Roberts (2009), Hu, McAdams, and Shum (2009) in other settings), and may affect the level of optimal reserve prices (Roberts (2009) and Aradillas-Lopez, Gandhi, and Quint (2010)). The estimation method we use in this paper, which builds on the importance sampling procedure proposed by Ackerberg (2009), allows for asymmetric bidders (mills and loggers in our application) and for rich observed and unobserved heterogeneity in all of the structural parameters across auctions. We focus on second price and ascending auctions, although, at the cost of additional computation, the method can be extended to first price or low bid procurement auctions.[9]

The paper proceeds as follows. Section 2 presents our model. Section 3 introduces the empirical context and data. Section 4 describes our estimation method. Section 5 presents our structural estimates. Section 6 uses them to compare the relative benefits to the USFS of increasing potential competition and setting an optimal reserve price and Section 7 concludes. The Appendix contains Monte Carlo studies of our estimation method.

2 Model

We now present our model of entry into auctions and show that equilibria are characterized by a cutoff strategy whereby a bidder only enters an auction when its signal is sufficiently

that assume potential bidders know their values when deciding whether to enter. Ye (2007) presents a model of "indicative bidding" in which bidders can update their assessment of their value upon paying an entry cost. Examples of theoretical papers that allow for endogenous entry, but assume that it is not selective, include French and McCormick (1984), McAfee and McMillan (1987), Tan (1992) and Engelbrecht-Wiggans (1993).

[9] An incomplete working paper by Einav and Esponda (2008) proposes a method for estimating a partially selective entry model for low bid auctions with bidder asymmetries but no unobserved auction heterogeneity.

high. The model description assumes that the auction format is second price sealed bid. However, as we assume that bidders have independent private values, equilibrium strategies would be the same in an English button auction. In Section 4 we will explain how we apply our model to data from an open outcry auction.

Consider an auction a with $N_{\tau a}$ potential bidders (firms) of type τ with $\overline{\tau}$ types in total. In our setting $\overline{\tau}$ is 2 and the types are sawmills and loggers. Type τ firm values V are i.i.d. draws from a distribution $F^V_{\tau a}(V)$ (with associated pdf $f^V_{\tau a}(V)$), which is continuous on an interval $[0, \overline{V}]$. The distribution can depend on the characteristics of the auction, although the support is fixed. Both the $N_{\tau a}$s and the $f^V_{\tau a}(V)$s are common knowledge to all potential bidders. In practice, we will assume that the $f^V_{\tau a}(V)$s will be proportional to the pdfs of lognormal distributions with location parameters $\mu_{\tau a}$ and squared scale parameters σ^2_{Va} on the $[0, \overline{V}]$ interval, and, as a labeling convention, that $\mu_{1a} > \mu_{2a}$. We will also choose a value for $\overline{V}$ which is significantly above the highest price observed in our data.

Firms play a two stage game. In the first (entry) stage, each firm independently decides whether to enter the auction, which requires paying an entry cost K_a. Prior to taking this decision, each firm i receives an independent, private information signal s_i about its value, where $s_i = v_i z_i$, $z_i = e^{\varepsilon_i}$, $\varepsilon_i \sim N(0, \sigma^2_{\varepsilon a})$. We assume that part of the entry cost is the cost of researching the object being sold, so that a player who pays the entry cost finds out its true value v_i. These entrants can participate in the second (auction) stage of the game, and we assume that only firms that pay the entry cost can do so. Note that the parameters σ^2_{Va}, $\sigma^2_{\varepsilon a}$ and K_a are assumed to be common across the types. We make this assumption for reasons connected with equilibrium selection, which we explain below.

In the second (bidding) stage, entrants submit bids in a second price auction, so that if a bid is submitted above the auction's reserve price, R_a, the object is sold to the bidder with the highest bid at a price equal to the maximum of the second highest bid and the reserve price.

2.1 Equilibrium

Following the literature (e.g. Athey, Levin, and Seira (2011)), we assume that players use strategies that form type-symmetric Bayesian Nash equilibria, where "type-symmetric" means that every player of the same type will use the same strategy. In the second stage, entrants know their values so it is a dominant strategy for each entrant to bid its value. In the first stage, players take entry decisions based on what they believe about their value given their signal. By Bayes Rule, the (posterior) conditional density $g_{\tau a}(v|s_i)$ that a player of type τ's value is v when its signal is s_i is

$$g_{\tau a}(v|s_i) = \frac{f^V_{\tau a}(v) \times \frac{1}{\sigma_{\varepsilon a}}\phi\left(\frac{\ln\left(\frac{s_i}{v}\right)}{\sigma_{\varepsilon a}}\right)}{\int_0^{\overline{V}} f^V_{\tau a}(x) \times \frac{1}{\sigma_{\varepsilon a}}\phi\left(\frac{\ln\left(\frac{s_i}{x}\right)}{\sigma_{\varepsilon a}}\right) dx} \tag{1}$$

where $\phi(\cdot)$ denotes the standard normal pdf.

The weights that a player places on its prior and its signal when updating its beliefs about its true value depend on the relative variances of the distribution of values and ε (signal noise), and this will also control the degree of selection. A natural measure of the relative variances is $\frac{\sigma^2_{\varepsilon a}}{\sigma^2_{Va}+\sigma^2_{\varepsilon a}}$, which we will denote α_a. If the value distribution were not truncated above, a player i's (posterior) conditional value distribution would be lognormal with location parameter $\alpha_a\mu_{\tau a} + (1-\alpha_a)\ln(s_i)$ and squared scale parameter $\alpha_a\sigma^2_{V\tau a}$.

The optimal entry strategy in a type-symmetric equilibrium is a pure-strategy threshold rule whereby the firm enters if and only if its signal is above a cutoff, $S'^*_{\tau a}$.[10] $S'^*_{\tau a}$ is implicitly defined by the zero-profit condition that the expected profit from entering the auction of a firm with the threshold signal will be equal to the entry cost:

$$\int_{R_a}^{\overline{V}} \left[\int_{R_a}^{v} (v-x)h_{\tau a}(x|S'^*_{\tau a}, S'^*_{-\tau a})dx\right] g_{\tau a}(v|s)dv - K_a = 0 \tag{2}$$

where $g_{\tau a}(v|s)$ is defined above, and $h_{\tau a}(x|S'^*_{\tau a}, S'^*_{-\tau a})$ is the pdf of the highest value of other entering firms (or the reserve price R_a if no value is higher than the reserve) in the auction.

A pure strategy type-symmetric Bayesian Nash equilibrium exists because optimal entry thresholds for each type are continuous and decreasing in the threshold of the other type.

2.2 Marginal and Inframarginal Bidders with Selection

We now illustrate how the model produces selective entry and how the degree of selection depends on α. We do this by examining the difference between the value distributions of inframarginal entrants and those bidders who received a signal just equal to the signal threshold, what we term to be "marginal entrants". This is an important distinction when analyzing any policy that impacts entry margins since it will affect the participation decisions

[10] A firm's expected profit from entering is increasing in its value, and because values and signals are independent across bidders and a firm's beliefs about its value is increasing in its signal, a firm's expected profit from entering is increasing in its signal. Therefore, if a firm expects the profit from entering to be greater (less) than the entry cost for some signal S, it will also do so for any signal $\widetilde{S}$ where $\widetilde{S} > S$ ($\widetilde{S} < S$). As it will be optimal to enter when the firm expects the profit from entering to be greater than the entry costs, the equilibrium entry strategy must involve a threshold rule for the signal, with entry if $S > S'^*_{\tau a}$.

of marginal entrants.

These distributions must be the same in the LS model. In the Signal model, however, the bidder receiving a signal $S = S'^*$ will tend to have lower values than entrants. This is clearly shown in Figure 1 for the symmetric bidder case. The left (right) panel of Figure 1 displays the average value distributions of entrants and marginal bidders in the Signal model with a precise (imprecise) signal and in the LS model. As the signal noise increases, the Signal model approaches the LS model. Likewise, as $\sigma_\varepsilon \to 0$, the Signal model approaches the S model where entrants' value distributions are perfectly truncated at S'^*.

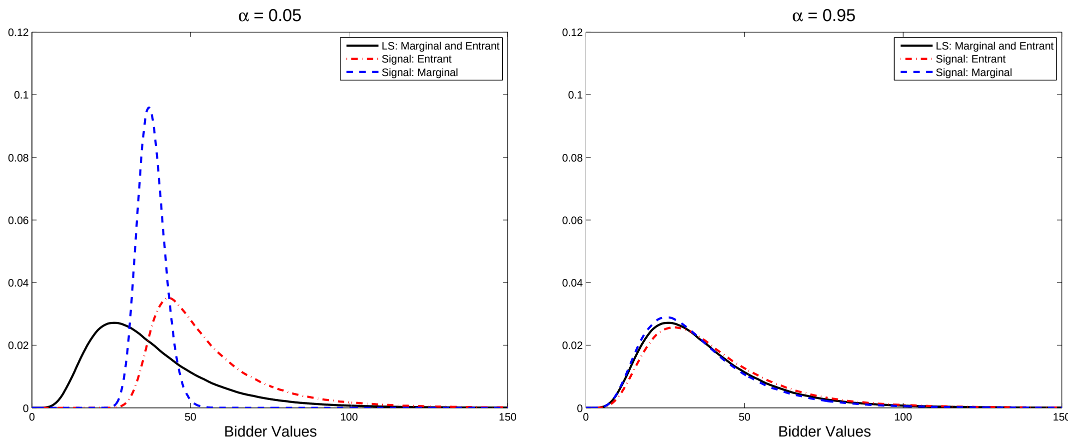

Figure 1: Comparing the value distributions for marginal bidders and entrants in the LS and Signal models. In both panels $V \sim logN(3.5, 0.5)$, $K = 5$, $N = 5$ and $R = 0$. In the Signal model $S = VZ$, $Z = e^\varepsilon$, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. The left panel pertains to a case with a high degree of selection: $\sigma_\varepsilon = 0.115$ ($\alpha = 0.05$). The right panel makes the comparison when there is less selection: $\sigma_\varepsilon = 2.179$ ($\alpha = 0.95$). In both panels the solid lines are for the LS model (in which the marginal and inframarginal bidder value distributions are the same), the dashed line is for the marginal bidder in the Signal model and the dash dot line is for a typical entrant in the Signal model.

2.3 Multiple Equilibria and Equilibrium Selection

Even when we restrict attention to type-symmetric equilibria, a game with more than one type may have multiple equilibria where different types of firm have different thresholds. For example, in our empirical setting, some parameters would support both equilibria where the mills have a lower entry threshold ($S'^*_{\text{mill}} < S'^*_{\text{logger}}$), and equilibria where loggers have a lower threshold ($S'^*_{\text{mill}} > S'^*_{\text{logger}}$).

This is illustrated in the first panel of Figure 2, which shows the reaction functions for the entry thresholds of both types of firm, when there are two firms of each type, $\sigma_V = 0.05, K = 4, \alpha = 0.1$ ($\sigma_\varepsilon = 0.0167$) and $\mu_1 = \mu_2 = 5$, so that the types are actually identical.[11] The

[11] In this diagram the reaction function represents what would be the symmetric equilibrium best response between the two firms of a particular type when both firms of the other type use a particular S'.

reserve price R is set to 20. There are three equilibria (intersections of the reaction functions), one of which has the types using identical entry thresholds (45° line is dotted), and the others involving one of the types having the lower threshold (and so being more likely to enter). The fact that there are at most three equilibria follows from the inverse-S shapes of the reaction functions.

The second panel in Figure 2 shows the reaction functions when we set $\mu_1 = 5.025$ and $\mu_2 = 5$, holding the remaining parameters fixed. This change causes the reaction function of type 1 firms to shift down (for a given S_2' they wish to enter for a lower signal) and the reaction function of the type 2 firms to shift outwards (for a given S_1', type 2 firms are less willing to enter). There are still three equilibria, but because of these changes in the reaction functions, there is only one equilibrium where the stronger type 1 firms have the lower entry threshold so that they are certainly more likely to enter. When the difference between μ_1 and μ_2 is increased, there is only one equilibrium and it has this form, as illustrated in the third panel of Figure 2.

The result that there is a unique equilibrium with $S_1'^* < S_2'^*$ when $\mu_1 \geq \mu_2$ and σ_V, σ_ε and K are the same across types holds generally if the reaction functions have only one inflection point.[12] Under these assumptions it is also generally true that the game has a unique equilibrium, in which it will be the case that $S_1'^* < S_2'^*$, if $\mu_1 - \mu_2$ is large enough.

The empirical literature on estimating discrete choice games provides several approaches for estimating games with multiple equilibria including assuming that a particular equilibrium is played, estimating a statistical equilibrium selection rule that allows for different equilibria to be played in the data (Sweeting (2009) and Bajari, Hong, and Ryan (2010)) and partial identification tecnhiques that may only give bounds on the parameters (e.g. Ciliberto and Tamer (2009) and Beresteanu, Molchanov, and Molinari (2009)). In this paper we assume that the parameters σ_V, σ_ε and K are the same across types and that, if there are multiple equilibria, the equilibrium played will be the unique one where $S_1'^* < S_2'^*$. We view our focus on this type of equilibrium as very reasonable, given that it is clear in our data that sawmills (our type 1) tend to have significantly higher average values than loggers (our type 2), so that it is almost certain that only one equilibrium will exist (a presumption that we verify based on our parameter estimates).

In Section 5, we discuss our experimentation with other estimation approaches, such as nested pseudo-likelihood, that make weaker assumptions about the equilibrium that is played or which allow for the types to have different σ_V, σ_ε and K parameters. These approaches

[12] In general, the exact shape of the reaction functions depends on the distributional assumptions made for the distributions of values and signal noise. Under our distributional assumptions, we have verified that the reaction functions have no more than one inflection point based on more than 40,000 auctions involving different draws of the parameters and different numbers of firms of each type.

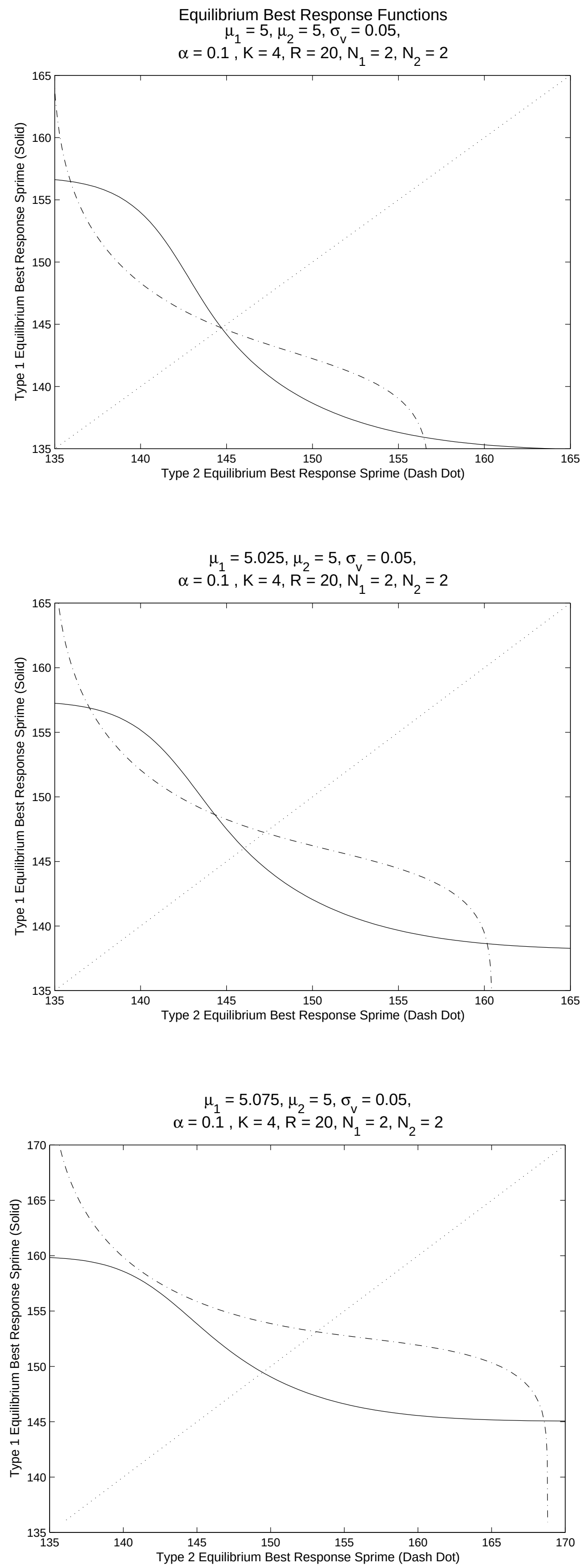

Figure 2: Reaction functions for symmetric and asymmetric bidders. In the first (top) panel, the types are identical so that $\mu_1 = \mu_2 = 5$ and there are two firms of each type, $\sigma_V = 0.05, K = 4, \alpha = 0.1$ ($\sigma_\varepsilon = 0.0167$). In the next two panels we introduce asymmetries in means only and the solid (dash dot) lines correspond to the type with the higher (lower) mean. In the second (middle) panel $\mu_1 = 5.025$ and $\mu_2 = 5$ and the remaining parameters are held fixed. In the third (bottom) panel $\mu_1 = 5.075$ and $\mu_2 = 5$ and the remaining parameters are held fixed. The 45° line is dotted.

give qualitatively similar findings for the degree of selection and for the comparison between the value of reserve prices and potential competition. On the other hand, the advantage of the particular estimation approach that we use (which for practical implementation requires us to make this selection assumption) is that it allows us to control, in a very rich way, for observed and unobserved heterogeneity across auctions, which are clear features of our data.

Given our equilibrium selection rule it is computationally straightforward to solve for the equilibrium entry thresholds. Specifically we use a standard non-linear equation solver (in MATLAB) to solve the zero profit conditions (equation 2) subject to the constraint that $S_1'^* < S_2'^*$. The integrals in equation 2 are evaluated on a 10,000 point grid, which runs from 0.01 to 500 (significantly above the maximum price that we observe in our data).[13]

2.4 Identification

Gentry and Li (2010) and Marmer, Shneyerov, and Xu (2010) discuss identification of a very similar imperfectly selective entry model with symmetric bidders, no unobserved heterogeneity and a first price auction format. The papers allow for a more general information structure that only requires a bidder's signal and value to be affiliated.

They show that, in general, the cost of participating (which is assumed to be constant across bidders) and the joint distribution of values and signals are not non-parametrically identified. The central problem is that since pre-entry signals are inherently unobservable, the distribution of values conditional on entrants' signals exceeding some threshold, which entry decisions depend on, cannot be recovered from the data. However, Gentry and Li (2010) show that when the econometrician observes the number of potential entrants and the bids submitted by entrants, exogenous variation in the number of potential entrants can be used to form bounds on these primitives, focusing primarily on the case when the lower bound of the support of bidder values exceeds the reserve price (so that the number of entrants is equal to the number of total bids submitted). Additionally, the model is fully identified if there exist covariates that exclusively affect entry costs.

If the auction format were instead second price sealed bid or English button, where bids could be interpreted as bidder values, the arguments of Gentry and Li (2010) could be used to partially identify our model's primitives. We also note that when the equilibrium entry threshold is very low, so that bidders fully participate, standard identification arguments (see Athey and Haile (2002)) can be used to non-parametrically identify the distribution of bidder values, even if the joint distribution of values and signals is not. The equilibrium

[13]For some of the counterfactuals it is important to do this to a high degree of accuracy, so we set a tolerance of 1e-13. In a first price auction it is necessary to also solve for the equilibrium bid functions given a value of S'^*, which creates additional computational costs.

entry threshold falls when there are few potential entrants or the reserve price is low.

In practice, our empirical application has several features that further complicate identification: asymmetric bidders, unobserved heterogeneity, binding reserve prices (so that we do not necessarily observe the number of entrants) and an open outcry auction format in which bids cannot be interpreted as values. For these reasons, we make parametric assumptions to estimate the model.[14] However, we can show (Section 3.1) that the data obviously reject a model with no selection.

3 Data

We analyze federal auctions of timberland in California.[15] In these auctions the USFS sells logging contracts to individual bidders who may or may not have manufacturing capabilities (mills and loggers, respectively). When the sale is announced, the USFS provides its own "cruise" estimate of the volume of timber for each species on the tract as well as estimated costs of removing and processing the timber. It also announces a reserve price and bidders must submit a bid of at least this amount to qualify for the auction. After the sale is announced, each bidder performs its own private cruise of the tract to assess its value. These cruises can be informative about the tract's volume, species make-up and timber quality.

We assume that bidders have independent private values. This assumption is also made in other work with similar timber auction data (see for example Baldwin, Marshall, and Richard (1997), Brannman and Froeb (2000), Haile (2001) or Athey, Levin, and Seira (2011)). A bidder's private information is primarily related to its own contracts to sell the harvest, inventories and private costs of harvesting and thus is mainly associated only with its own valuation. In addition, we focus on the period 1982-1989 when resale, which can introduce a common value element, was limited (see Haile (2001) for an analysis of timber auctions with resale).

We also assume non-collusive bidder behavior. While there has been some evidence of bidder collusion in open outcry timber auctions, Athey, Levin, and Seira (2011) find strong evidence of competitive bidding in these California auctions.

Our model assumes that bidders receive an imperfect signal of their value and there is a participation cost that must be paid to enter the auction. Participation in these auctions is costly for numerous reasons. In addition to the cost of attending the auction, a large fraction of a bidder's entry cost is its private cruise, and people in the industry tell us that

[14]Similar features in other empirical applications have lead previous researchers to adopt a parametric estimation approach even if they assume no selection (e.g. Athey, Levin, and Seira (2011) and Krasnokutskaya and Seim (forthcoming)).

[15]We are very grateful to Susan Athey, Jonathan Levin and Enrique Seira for sharing their data with us.

firms do not bid without doing their own cruise. There are several reasons for this. First, some information that bidders find useful, such as trunk diameters, is not provided in USFS appraisals. In addition, the government's reports are seen as useful, but noisy estimates of the tract's timber. For example, using "scaled sales", for which we have data on the amount of timber removed from the tract, we can see that the government's estimates of the distribution of species on any given tract are imperfect. The mean absolute value of the difference between the predicted and actual share of timber for the species that the USFS said was most populous is 3.42% (std. dev. 3.47%).

We use data on ascending auctions. From these, we eliminate small business set aside auctions, salvage sales and auctions with missing USFS estimated costs. To eliminate outliers, we also remove auctions with extremely low or high acreage (outside the range of [100 acres, 10000 acres]), volume (outside the range of [5 hundred mbf, 300 hundred mbf]), USFS estimated sale values (outside the range of [$184/mbf, $428/mbf]), maximum bids (outside the range of [$5/mbf, $350/mbf]) and those with more than 20 potential bidders (which we define below).[16] We keep auctions that fail to sell. We are left with 887 auctions.

Table 1 shows summary statistics for our sample. Bids are given in $/mbf (1983 dollars). The average mill bid is 20.3% higher than the average logger bid. As suggested in Athey, Levin, and Seira (2011), mills may be willing to bid more than loggers due to cost differences or the imperfect competition loggers face when selling felled timber to mills.

The median reserve price is $27.77/mbf. Reserve prices in our sample are set according to the "residual value" method, subject to the constraint that 85% of auctions should end in a sale.[17] In our sample 5% of auctions end in no sale.

We define potential entrants as the auction's bidders plus those bidders who bid within 50 km of an auction over the next month. One way of assessing the appropriateness of this definition is that 98% of the bidders in any auction also bid in another auction within 50 km of this auction over the next month. The median number of potential bidders is eight (mean of 8.93) and this is evenly divided between mills and loggers.

We define entrants as the set of bidders we observe at the auction, even if they did not submit a bid above the reserve price.[18] The median number of mill and logger entrants are three and one, respectively. Among the set of potential logger entrants, on average 21.5%

[16] For estimation we set $\overline{V}$ = $500/mbf, which is substantially above the highest price observed in our sample.

[17] Essentially, the USFS constructed an estimate of the final selling value of the wood and then subtracted logging, transportation, manufacturing and other costs required to generate a marketable product to arrive at a reserve price. The value and cost estimates are based on the government's cruise. See Baldwin, Marshall, and Richard (1997) for a detailed discussion of the method.

[18] This is the definition in Table 1 and we do estimate our model using this definition. However, in our preferred specification, we interpret the data more cautiously and allow bidders that do not submit bids to have entered (paid K), but learned that their value was less than the reserve price.

Variable	Mean	Std. Dev.	25^{th}-tile	50^{th}-tile	75^{th}-tile	N
WINNING BID ($/mbf)	86.01	62.12	38.74	69.36	119.11	847
BID ($/mbf)	74.96	57.68	30.46	58.46	105.01	3426
LOGGER	65.16	52.65	26.49	49.93	90.93	876
MILL	78.36	58.94	32.84	61.67	110.91	2550
LOGGER WINS	0.15	0.36	0	0	0	887
FAIL	0.05	0.21	0	0	0	887
ENTRANTS	3.86	2.35	2	4	5	887
LOGGERS	0.99	1.17	0	1	1	887
MILLS	2.87	1.85	1	3	4	887
POTENTIAL ENTRANTS	8.93	5.13	5	8	13	887
LOGGER	4.60	3.72	2	4	7	887
MILL	4.34	2.57	2	4	6	887
SPECIES HHI	0.54	0.22	0.35	0.50	0.71	887
DENSITY (hundred mbf/acre)	0.21	0.21	0.07	0.15	0.27	887
VOLUME (hundred mbf)	76.26	43.97	43.60	70.01	103.40	887
HOUSING STARTS	1620.80	261.75	1586	1632	1784	887
RESERVE ($/mbf)	37.47	29.51	16.81	27.77	48.98	887
SELL VALUE ($/mbf)	295.52	47.86	260.67	292.87	325.40	887
LOG COSTS ($/mbf)	118.57	29.19	99.57	113.84	133.77	887
MFCT COSTS ($/mbf)	136.88	14.02	127.33	136.14	145.73	887

Table 1: Summary statistics for sample of California ascending auctions from 1982-1989. All monetary figures in 1983 dollars. Here, ENTRANTS are the set of bidders we observe at the auction, even if they did not submit a bid above the reserve price. We count the number of potential entrants as bidders in the auction plus those bidders who bid within 50km of an auction over the next month. SPECIES HHI is the Herfindahl index for wood species concentration on the tract. SELL VALUE, LOG COSTS and MFCT COSTS are USFS estimates of the value of the tract and the logging and manufacturing costs of the tract, respectively. In addition to the USFS data, we add data on (seasonally adjusted, lagged) monthly housing starts, HOUSING STARTS, for each tract's county.

enter, whereas on average 66.1% of potential mill entrants enter.

Our model assumes that differences in values explain why mills are more likely than loggers to enter an auction. However, this pattern could also be explained by differences in entry costs, which we allow to vary across auctions, but not across mills and loggers within an auction. This is unlikely for three reasons. First, there is little reason to believe that the cost of performing cruises differs substantially across firms for any sale since all potential entrants (a) must attend the auction if they want to submit a bid and (b) are interested in similar information when performing their own cruise (even if their values are different). Second, Table 1 clearly shows that mills also bid more than loggers, suggesting that the meaningful distinction between the types are their value distributions. Third, conditional on entering, loggers are still much less likely to win than mills (15.5% vs. 27.9%). This last point argues against the possibility that loggers enter and win less because of high (relative to mills) entry costs, as this would lead them to enter only when they expect to win.

3.1 Evidence of Selection

In this subsection we argue that the data are best explained by a model that allows for potential selection. First, Athey, Levin, and Seira (2011) show that in the type-symmetric mixed strategy equilibrium of a model with endogenous, but non-selective, entry and asymmetric bidder types, whenever the weaker type enters with positive probability, the stronger type enters with probability one. Thus, for any auction with some logger entry, a model with no selection would imply that all potential mill entrants enter. In 54.5% of auctions in which loggers participate, and there are some potential mill entrants, some, but not all, mills participate. Likewise, they show that whenever the stronger type enters with probability less than one, a model with no selection implies that weaker types enter with probability zero. However, in the data we find that in 61.1% of auctions in which only some mill potential entrants participate and potential logger entrants exist, some loggers enter. A model with selective entry can rationalize partial entry of both bidder types into the same auction.

Second, a model without selection implies that bidders are a random sample of potential entrants. We can test this by estimating a Heckman selection model (Heckman (1976)), where in the first stage we estimate a probit model of the decision to enter as a function of tract characteristics and a flexible polynomial of potential other mill and logger entrants and then use the predicted probabilities to form an estimate of the inverse Mills ratio and include it in a second stage bid regression. The exclusion restriction is that potential competition affects a bidder's decision to enter an auction, but has no direct effect on values. The results appear in Table 2. Column (1) shows OLS results from regressing all bids on auction covariates and whether the bidder is a logger. Column (2) shows the second stage estimates from the selection model. The positive and significant coefficient on the inverse Mills ratio is consistent with bidders being a positively selected sample of potential entrants. In addition, comparing the coefficient on LOGGER across the columns illustrates that ignoring selection masks the difference between logger and mill values.[19] This is expected when only those loggers whose values are likely to be high participate and most mills enter ($S'^{*}_{\text{mill}} < S'^{*}_{\text{logger}}$).

The evidence presented in this section strongly suggests that the entry process is selective. However, it does not pin down the degree of selection, much less guarantee that the S model

[19] An alternative way to test for selection is to regress bids on covariates, including measures of potential entrants. If there is selection, the potential entrants should have a positive impact on submitted bids as the entry threshold will increase. We have run these regressions, using both all bids and the winning bid, and consistently find a positive impact of potential entrants (in logs or levels). The result also holds if we instrument for potential entrants using lagged participation of mills in nearby auctions.

	(1)	(2)
CONSTANT	-5.475***	-5.792***
	(0.849)	(0.852)
LOGGER	-.090***	-.203***
	(0.026)	(0.04)
SCALE SALE	0.003	-.017
	(0.054)	(0.054)
SPECIES HHI	0.025	0.064
	(0.056)	(0.057)
DENSITY	0.016	0.013
	(0.063)	(0.063)
VOLUME	0.0003	0.0002
	(0.0003)	(0.0003)
HOUSING STARTS	0.0002**	0.0002*
	(0.00008)	(0.00008)
log SALE VALUE	2.750***	2.775***
	(0.081)	(0.081)
log LOG COSTS	-1.052***	-1.093***
	(0.066)	(0.067)
log MFCT COSTS	-.262*	-.181
	(0.147)	(0.148)
$\widehat{\lambda}$		0.159***
		(0.044)
R^2	0.4297	0.4319
N	3,426	3,426

Table 2: Selection evidence. In both columns the dependent variable is log of the bid per volume and year dummies are included. Column (2) displays the second stage results of a two step selection model. The first stage probit is of entry where the exogenous shifters are potential other mill and logger entrants, incorporated as a flexible polynomial. $\widehat{\lambda}$ is the estimated inverse Mills ratio from the first stage.

is appropriate. Therefore, we now describe how we estimate our model to measure the degree of selection.

4 Estimation

To take the model to data, we need to specify how the parameters of the model may vary across auctions, as a function of observed auction characteristics and unobserved heterogeneity. Both types of heterogeneity are likely to be important as the tracts we use differ greatly in observed characteristics, such as sale value, size and wood type, and they also come from different forests so they are likely to differ in other characteristics as well. Both observed and unobserved heterogeneity may affect entry costs and the degree of selection, as well as mean values.

Our estimation approach is based on Ackerberg (2009)'s method of simulated maximum likelihood with importance sampling. This method involves solving a large number of games with different parameters once, calculating the likelihoods of the observed data for each of these games, and then re-weighting these likelihoods during the estimation of the distributions for the structural parameters. This method is attractive when it is believed that the parameters of the model are heterogeneous across auctions and it would be computationally prohibitive to re-solve the model (possibly many times in order to integrate out over the heterogeneity) each time one of the parameters changes.[20]

To apply the method, we assume that the parameters are distributed across auctions according to the following distributions, where X_a is a vector of observed auction characteristics and $TRN(\mu, \sigma^2, a, b)$ is a truncated normal distribution with parameters μ and σ^2, and upper and lower truncation points a and b.

$$\text{Location Parameter of Logger Value Distribution: } \mu_{a,\text{logger}} \sim N(X_a\beta_1, \omega^2_{\mu,\text{logger}})$$
$$\text{Difference in Mill/Logger Location Parameters: } \mu_{a,\text{mill}} - \mu_{a,\text{logger}} \sim TRN(X_a\beta_3, \omega^2_{\mu,\text{diff}}, 0, \infty)$$
$$\text{Scale Parameter of Mill and Logger Value Distributions: } \sigma_{Va} \sim TRN(X_a\beta_2, \omega^2_{\sigma_V}, 0.01, \infty)$$
$$\alpha\text{: } \alpha_a \sim TRN(\beta_4, \omega^2_{\alpha}, 0, 1)$$
$$\text{Entry Costs: } K_a \sim TRN(X_a\beta_5, \omega^2_K, 0, \infty)$$

The set of parameters to be estimated are $\Gamma = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \omega^2_{\mu,\text{logger}}, \omega^2_{\mu,\text{diff}}, \omega^2_{\sigma_V}, \omega^2_{\alpha}, \omega^2_K\}$, and a particular draw of the parameters $\{\mu_{a,\text{logger}}, \mu_{a,\text{mill}}, \sigma_{Va}, \alpha_a, K_a\}$ is denoted θ_a.

These specifications reflect our assumptions that σ_V, α and K are the same for mills and loggers within any particular auction, even though they may differ across auctions due to unobserved (to the econometrician) heterogeneity.[21] Li and Zheng (2009) take an alternative approach to handling unobserved heterogeneity when estimating a version of the S (perfect selection) model. They assume that there is an additional additive random term representing

[20]Hartmann (2006), Hartmann and Nair (forthcoming) and Wang (2010) provide applications of these methods to consumer dynamic discrete choice problems. Bajari, Hong, and Ryan (2010) use a related method to analyze entry into a complete information entry game with no selection.

[21]The lower bound on σ_{Va} is set slightly above zero simply to avoid computational problems that were sometimes encountered when there was almost no dispersion of values. The problem arises from the fact that numerically solving the game requires calculating expected player profits on a grid of possible values, and numerical problems are encountered when the numerical value calculated for value distribution's pdf is non-zero at only one or two grid points. In practice, when taking draws of the parameters for solving the games, we are also imposing upper truncation points on all of the parameters. We choose large values for these, and varying them has no significant effects on our estimates.

unobserved auction heterogeneity. Firm values and entry costs may both be a function of this unobserved term and so values and entry costs can be correlated in their framework. While the specification above assumes that the various parameters are distributed independently across auctions, this assumption could be relaxed by introducing a full covariance matrix. However, this would significantly increase the number of parameters to be estimated and, when we have tried to estimate these parameters, we have not found these coefficients to be consistently significant across specifications.

Denoting the outcome for an observed auction by y_a, the log-likelihood function for a sample of A auctions is

$$\sum_{a=1}^{A} \log \left(\int L_a(y_a|\theta)\phi(\theta|X_a,\Gamma)d\theta \right) \tag{3}$$

where $L_a(y_a|\theta)$ is the likelihood of the outcome y in auction a given structural parameters θ, $\phi(\theta|X_a,\Gamma)$ is the pdf of the parameter draw θ given Γ, our distributional assumptions, the unique equilibrium strategies implied by our equilibrium concept and auction characteristics including the number of potential entrants, the reserve price and observed characteristics X_a.

Unfortunately, the integral in (3) is multi-dimensional and cannot be calculated exactly. A natural simulation estimator would be

$$\int L_a(y_a|\theta)\phi(\theta|X_a,\Gamma)d\theta \approx \frac{1}{S}\sum_{s=1}^{S} L_a(y_a|\theta_s) \tag{4}$$

where θ_s is one of S draws from $\phi(\theta|X_a,\Gamma)$. The problem is that this would require us to make new draws of θ_s and re-solve the model S times for each auction in our data each time one of the parameters in Γ changes. Instead we follow Ackerberg by recognizing that

$$\int L_a(y_a|\theta)\phi(\theta|X_a,\Gamma)d\theta = \int L_a(y_a|\theta)\frac{\phi(\theta|X_a,\Gamma)}{g(\theta|X_a)}g(\theta|X_a)d\theta \tag{5}$$

where $g(\theta|X_a)$ is the importance sampling density whose support does not depend on Γ, which is true in our case because the truncation points are not functions of the parameters. This can be simulated using

$$\frac{1}{S}\sum_{s} L_a(y_a|\theta_s)\frac{\phi(\theta_s|X_a,\Gamma)}{g(\theta_s|X_a)} \tag{6}$$

where θ_s is a draw from $g(\theta|X_a)$. Critically, this means that we can calculate $L_a(y_a|\theta_s)$ for a given set of S draws that do not vary during estimation, and simply change the weights $\frac{\phi(\theta_s|X_a,\Gamma)}{g(\theta_s|X_a)}$, which only involves calculating a pdf when we change the value of Γ rather than re-solving the game.

This simulation estimator will only be accurate if a large number of θ_s draws are in the range where $\phi(\theta_s|X_a, \Gamma)$ is relatively high, and, as is well known, simulated maximum likelihood estimators are only consistent when the number of simulations grows fast enough relative to the sample size. We therefore proceed in two stages. First, we estimate Γ using $S = 2,500$ where $g(\cdot)$ is a multivariate uniform distribution over a large range of parameters which includes all of the parameter values that are plausible. Second, we use these estimates $\widehat{\Gamma}$ to repeat the estimation using a new importance sampling density $g(\theta|X_a) = \phi(\theta_s|X_a, \widehat{\Gamma})$ with $S = 500$ draws per auction. The Appendix provides Monte Carlo evidence that the estimation procedure works well even for smaller values of S.

To apply the estimator, we also need to define the likelihood function $L_a(y_a|\theta)$ based on the data we observe about the auction's outcome, which includes the number of potential entrants of each type, the winning bidder and the highest bids announced during the open outcry auction by the set of firms that indicated that they were willing to meet the reserve price. A problem that arises when handling data from open outcry auctions is that a bidder's highest announced bid may be below its value, and it is not obvious which mechanism leads to the bids that are announced (Haile and Tamer (2003)).

In our baseline specification we therefore make the following assumptions that we view as conservative interpretations of the information that is in the data: (i) the second highest observed bid (assuming one is observed above the reserve price) is equal to the value of the second-highest bidder[22]; (ii) the winning bidder has a value greater than the second highest bid; (iii) both the winner and the second highest bidder entered and paid K_a; (iv) other firms that indicated that they would meet the reserve price or announced bids entered and paid K_a and had values between the reserve price and the second highest bid; and, (v) all other potential entrants may have entered (paid K_a) and found out that they had values less than the reserve, or they did not enter (did not pay K_a). If a firm wins at the reserve price we assume that the winner's value is above the reserve price. Based on these assumptions, the likelihood of an observed outcome where a type 1 (mill) bidder wins the auction, a type 2 (logger) bidder submits the second highest bid of b_{2a}, and $n_{\tau a} - 1$ other firms of type τ participate (i.e., would pay the reserve or announce bids) out of $N_{\tau a}$ potential entrants would

[22]Alternative assumptions could be made. For example, we might assume that the second highest bidder has a value equal to the winning bid, or that the second highest bidder's value is some explicit function of his bid and the winning bid. In practice, 96% of second highest bids are within 1% of the high bid, so that any of these alternative assumptions give similar results. We have computed some estimates using the winning bid as the second highest value and the coefficient estimates are indeed similar.

be proportional[23] to the following, where $S'^{*}_{\tau a}$ are the equilibrium entry thresholds:

$$\begin{aligned}
L_a(y|\theta) \propto f_2(b_{2a}|\theta) * \Pr(enter_2|v_2 = b_{2a}, S^{*\prime}_{2a}, \theta) \times \left(\int_{b_{2a}}^{\overline{V}} f_1(v|\theta) \Pr(enter_1|v_1 = v, S^{*\prime}_{1a}, \theta) dv \right) \\
\times \left(\int_{R_a}^{b_{2a}} f_1(v|\theta) \Pr(enter_1|v_1 = v, S^{*\prime}_{1a}, \theta) dv \right)^{(n_{1a}-1)} \\
\times \left(\int_{R_a}^{b_{2a}} f_2(v|\theta) \Pr(enter_2|v_2 = v, S^{*\prime}_{2a}, \theta) dv \right)^{(n_{2a}-1)} \\
\times \left(1 - \int_{R_a}^{\overline{V}} f_1(v|\theta) \Pr(enter_1|v_1 = v, S^{*\prime}_{1a}, \theta) dv \right)^{(N_{1a}-n_{1a})} \\
\times \left(1 - \int_{R_a}^{\overline{V}} f_2(v|\theta) \Pr(enter_2|v_2 = v, S^{*\prime}_{2a}, \theta) dv \right)^{(N_{2a}-n_{2a})}
\end{aligned} \tag{7}$$

reflecting the contributions to the likelihood of the second highest bidder, the winning bidder, the other firms that attend the auction and those that do not attend, respectively.[24]

5 Results

Table 3 presents the parameter estimates for our structural model. We allow the USFS estimate of sale value and its estimate of logging costs to affect mill and logger values and entry costs since these are consistently the most significant variables in regressions of reserve prices or winning bids on observables, including controls for potential entry, and in the specifications in Table 2. We also control for species concentration since our discussions with industry experts lead us to believe that this matters to firms. We allow for auction-level unobserved heterogeneity (to the econometrician) in all parameters. The rightmost columns show the mean and median values of the structural parameters when we take 10 simulated draws of the parameter for each auction. For the rest of the paper, we refer to these as the "mean" and "median" values of the parameters. All standard errors are based on a non-parametric bootstrap with 100 repetitions.

The coefficients show that tracts with greater sale values and lower costs are more valuable, as one would expect. It does appear that there is both unobserved heterogeneity in

[23] This ignores the binomial coefficients, which do not depend on parameters.

[24] If an entrant wins at the reserve price, then the likelihood is calculated assuming that winning bidder's value is above the reserve.

values across auctions (the standard deviation of μ_{logger}) and heterogeneity in the difference between mill and logger mean values across auctions (the standard deviation of $\mu_{\text{mill}} - \mu_{\text{logger}}$).

Based on the mean value of the parameters, the mean values of mill and logger potential entrants are \$61.95/mbf and \$42.45/mbf, respectively, a 46% difference. Figure 3 shows the value distributions for potential entrants of both types.

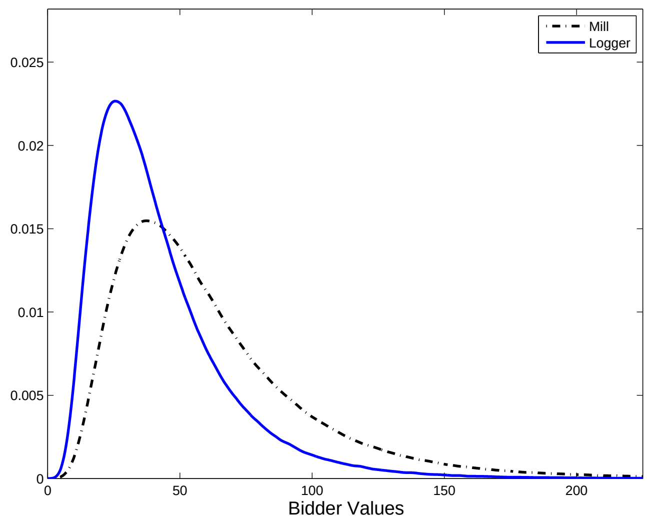

Figure 3: Comparing the value distributions for mills (dash-dot) and loggers (solid). Based on the mean value of the parameters from Table 3.

We estimate a mean entry cost of \$2.05/mbf. One forester we spoke with estimated modern day cruising costs of approximately \$6.50/mbf (in 2010 dollars). Converting our estimate to 2010 dollars yields an estimate of \$4.49/mbf. This estimate is consistent with information costs being the majority of entry costs. It is sensible that our estimate is less than the forester's estimate if firms in our data are able to use any information they learn when deciding whether to enter other auctions.

Our estimation approach assumes that, if there are multiple equilibria, the firms will play the equilibrium where mills have the lower S'^*. We checked whether our parameter estimates can support multiple equilibria by plotting type-symmetric "equilibrium best response functions" for mills and loggers for each auction (as we did in Figure 2). For each auction, our parameter estimates support only a single equilibrium. This is because our estimates imply a large difference in the mean values of loggers and mills, relatively low entry costs and a moderate amount of selection (which we discuss in more detail below).

We have also estimated our model under alternative assumptions. For example, we have assumed that there is no unobserved heterogeneity in the parameters across auctions but allowed for all of the parameters to be different across mills and loggers. Under these assumptions it is possible that there may be more than one equilibrium where the type with

		β parameters			ω parameter		
Parameter	Constant	log SELL VALUE	log LOG COSTS	SPECIES HHI		Mean	Median
$\mu_{a,\text{logger}}$	-9.6936	3.3925	-1.2904	0.2675	0.3107	3.5824	3.5375
$\sim N(X_a\beta_1, \omega^2_{\mu,\text{logger}})$	(1.3690)	(0.1911)	(0.1332)	(0.1386)	(0.0213)	(0.0423)	(0.0456)
$\mu_{a,\text{mill}} - \mu_{a,\text{logger}}$	3.6637	-0.4998	-0.0745	-0.1827	0.1255	0.3783	0.3755
$\sim TRN(X_a\beta_3, \omega^2_{\mu,\text{diff}}, 0, \infty)$	(0.8890)	(0.1339)	(0.0919)	(0.1007)	(0.0163)	(0.0242)	(0.0249)
σ_{Va}	4.0546	-0.7379	0.1393	0.0895	0.0796	0.5763	0.5770
$\sim TRN(X_a\beta_2, \omega^2_{\sigma_V}, 0.01, \infty)$	(0.7872)	(0.0994)	(0.1025)	(0.0813)	(0.0188)	(0.0273)	(0.0302)
α_a	0.7127	-	-	-	0.1837	0.6890	0.6992
$\sim TRN(\beta_4, \omega^2_\alpha, 0, 1)$	(0.0509)				(0.0446)	(0.0362)	(0.0381)
K_a	1.9622	-3.3006	3.5172	-1.1876	2.8354	2.0543	1.6750
$\sim TRN(X_a\beta_5, \omega^2_K, 0, \infty)$	(13.2526)	(2.7167)	(2.4808)	(1.5721)	(0.6865)	(0.2817)	(0.3277)

Table 3: Simulated maximum likelihood with importance sampling estimates allowing for non-entrants to have paid the entry cost. The rightmost columns show the mean and median values of the structural parameters when we take 10 simulated draws of the parameter for each auction. Standard errors based non-parametric bootstrap with 100 repetitions. $TRN(\mu, \sigma^2, a, b)$ is a truncated normal distribution with parameters μ and σ^2, and upper and lower truncation points a and b. Based on 887 auctions.

the higher location parameter for the value distribution has the lower signal threshold for entry. We estimated this model using the iterative nested pseudo-likelihood procedure of Aguirregabiria and Mira (2007). If only one equilibrium is played in the data and the researcher's initial guess of players' beliefs are close enough to those in the data then it can be hoped - although it is not guaranteed - that this iterative procedure will provide consistent estimates of the parameters even if the underlying model has multiple equilibria. All of the specifications we have tried give the following qualitatively similar conclusions: (i) mills have higher average values than loggers and (ii) the entry procedure is selective for both types, so that, for each type, inframarginal and marginal entrants have value distributions that are clearly different.

Even with the restrictive modeling assumptions we have made, both to deal with the multiple equilibria issue, as well as to take a cautious approach in interpreting the data, the model matches the data fairly well. We slightly over predict logger entry. In the data, on average 0.99 loggers attend each auction. We predict that on average 1.07 loggers will enter and have values greater than the reserve. We slightly under predict mill entry. In the data, on average 2.87 mills attend each auction. We predict that on average 2.44 mills will enter and have values greater than the reserve. We slightly under predict revenues. In the data, the mean (median) revenues are \$81.89/mbf (\$65.89/mbf) and we predict them to be \$75.58/mbf (\$58.13/mbf). We do a good job of matching prices in the event of a sale. In the data, prices average \$85.76/mbf and our model predicts an average of \$86.39/mbf.

The assumptions used to generate our preferred results in Table 3, which we use in all counterfactuals below, are based on a conservative interpretation of what we know about the entry decisions of firms that do not attend the auction. Table 4 shows estimates if we instead assume that firms that did not attend the auction did not pay K. The main differences are slightly (i) more selection and (ii) higher entry costs. This is sensible since we are now assuming that the only firms that enter are those we see and they have values greater than the reserve. A lower α and higher K are needed to rationalize this.

Our estimates of the αs across auctions indicate a moderate amount of selection in the data. Based on our estimates, we find a 46% difference in mean values for potential mill and logger entrants. This is much larger than the average difference in bids across mills and loggers (20.3%), as would be expected if entry is selective. If we consider a representative auction where the reserve and the number of potential mill and logger entrants are set to their respective medians of \$27.77/mbf, four and four, we can compare the difference in values between marginal (those who observed $S'^{*}_{\tau a}$) and inframarginal bidders. Based on the mean parameter values, the average mill entrant's value is \$68.13/mbf and the average marginal mill bidder's value is \$45.22/mbf. The fact that the average potential mill entrant's

	β parameters				ω parameter		
Parameter	Constant	log SELL VALUE	log LOG COSTS	SPECIES HHI		Mean	Median
$\mu_{a,\text{logger}}$	-8.1176	3.1520	-1.3227	0.1679	0.2809	3.5874	3.5425
$\sim N(X_a\beta_1, \omega^2_{\mu,\text{logger}})$	(1.2141)	(0.1922)	(0.1128)	(0.1428)	(0.0280	(0.0474)	(0.0502)
$\mu_{a,\text{mill}} - \mu_{a,\text{logger}}$	2.5003	-0.3126	-0.0609	-0.1337	0.1503	0.3709	0.3668
$\sim TRN(X_a\beta_3, \omega^2_{\mu,\text{diff}}, 0, \infty)$	(0.8217)	(0.1326)	(0.0823)	(0.0929)	(0.0154)	(0.0280)	(0.0290)
σ_{Va}	2.7954	-0.5725	0.2009	0.1466	0.1307	0.5807	0.5802
$\sim TRN(X_a\beta_2, \omega^2_{\sigma_V}, 0.01, \infty)$	(0.7715)	(0.1117)	(0.0942)	(0.1004)	(0.0216)	(0.0268)	(0.0277)
α_a	0.6094				0.1343	0.6086	0.6072
$\sim TRN(\beta_4, \omega^2_\alpha, 0, 1)$	(0.0321)				(0.0300)	(0.0312)	(0.0316)
K_a	1.7071	-3.0959	3.8308	-1.4711	2.2488	2.5308	2.2353
$\sim TRN(X_a\beta_5, \omega^2_K, 0, \infty)$	(7.0447)	(1.0928)	(1.1314)	(0.9802)	(0.4595)	(0.2637)	0.2688)

Table 4: Simulated maximum likelihood with importance sampling estimates assuming that non-bidders did not pay the entry cost. The rightmost columns show the mean and median values of the structural parameters when we take 10 simulated draws of the parameter for each auction. Standard errors based non-parametric bootstrap with 100 repetitions. $TRN(\mu, \sigma^2, a, b)$ is a truncated normal distribution with parameters μ and σ^2, and upper and lower truncation points a and b. Based on 887 auctions.

value is higher than the average marginal mill's value reflects the fact that most mills enter. The comparable numbers for entrant and marginal loggers are \$59.80/mbf and \$48.13/mbf, respectively. The difference between marginal and inframarginal bidders is indicative of the degree of selection in the entry process. Also note that, for these estimates, marginal loggers tend to have higher values than marginal mills. For illustration, Figure 4 compares the entrants' and marginals' value distributions for each bidder type in the representative auction based on the mean parameters. Due to selection, there is a substantial difference in the marginal and inframarginal bidders for each type.

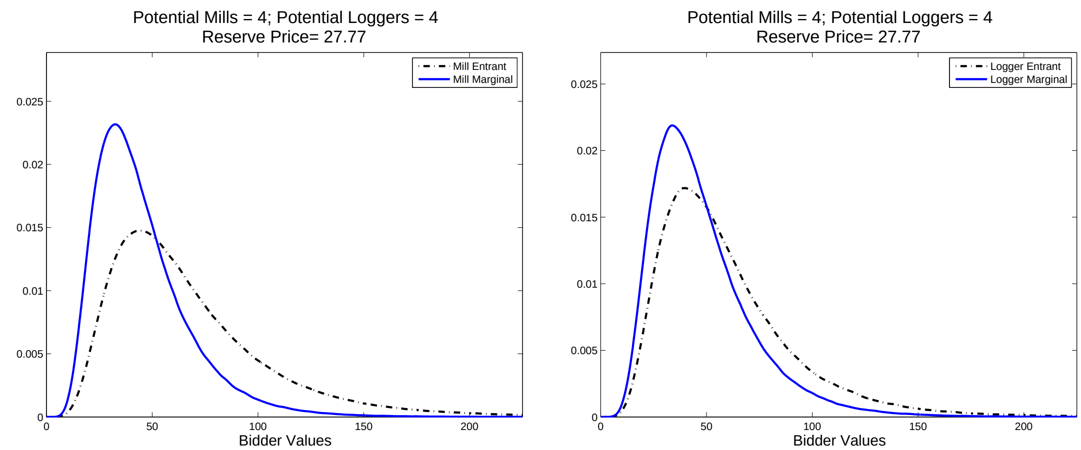

Figure 4: Comparing the value distributions for entrant and "marginal" bidders by type. Based on the mean value of the parameters from Table 3.

Selection directly impacts analysis of any counterfactual that affects the entry margin. An example is the comparison of the USFS's value of adding a marginal and inframarginal bidder to the auction but assuming that previous entrants still participate. This is the closest comparison to the experiment in Bulow and Klemperer (1996). They add an additional bidder who has the same value distribution as other bidders. We add another bidder, but consider two cases: one where the additional bidder's value distribution is the same as entrants' and a second where the additional bidder's value distribution is that of a marginal entrant. The second case is the more relevant one since, with endogenous selective entry, it is plausible that additional participants are marginal bidders.

For this, and all counterfactuals below, we define the *seller's value of an auction* as the expected winning bid when the object sells and the seller's value of holding onto the object, v_0, when it does not sell. When considering the representative auction, we assume v_0 is the median reserve price in the data, \$27.77/mbf.[25] When we need to assume a value for v_0 in any particular auction, we use that auction's reserve price. Throughout the paper, a *non-strategic* reserve price means the seller sets a reserve equal to v_0.

Consider the representative auction with four potential mill and four potential logger entrants. On average 3.18 mills and 1.16 loggers enter and the USFS's expected value of the auction is \$71.48/mbf. If we add a mill bidder whose value is assumed to be distributed as are those of typical mill entrants in this auction, the USFS's value increases by 11.3%. However, it is more likely that if another bidder were added, it would be a "marginal-like" bidder whose signal equaled $S^{*\prime}_{\text{mill}}$. The value of adding this bidder will be much less than adding an "incumbent-like" bidder. If a marginal mill is added, the USFS's value only increases by 4.2%. The respective increases from adding an inframarginal and marginal logger are 8.8% and 5.0%. This comparison is only meant to be illustrative since it assumes that other entrants do not adjust their entry decisions when the additional bidder (marginal or inframarginal) is added. In the next section we analyze the effect of changing potential competition and allow other bidders to adjust their entry decisions accordingly.

6 The Relative Value of Reserve Prices and Competition

In this section we use our structural estimates to show that the USFS's value of setting an optimal reserve price is small relative to increasing competition for its tracts. The reserve price is a natural example of auction design to consider for numerous reasons. First, it is well known that the optimal mechanism for a seller facing a fixed number of symmetric bidders is a second price auction with a correctly chosen reserve price (Myerson (1981) and Riley and Samuelson (1981)). Moreover, analyzing timber auction reserve prices has been an active area of research. Examples of such studies, all of which assume exogenous entry, include Mead, Schniepp, and Watson (1981), Mead, Schniepp, and Watson (1984), Paarsch (1997), Haile and Tamer (2003), Li and Perrigne (2003) and Aradillas-Lopez, Gandhi, and Quint (2010). In addition, government agencies appear to be quite interested in how to set reserve prices in timber auctions and have actively sought the opinions of expert economists (see for example Athey, Cramton, and Ingraham (2003)).

While reserve prices are key to optimal auction design with a given number of bidders, increasing competition may be more valuable to a seller. For example, Bulow and Klemperer

[25] Li and Zheng (2010), and Paarsch (1997), in one of his specifications, also make this assumption. Moreover, in practice this is very similar to assuming that v_0 is the median appraisal value (the USFS's estimated sale value less costs), of \$24/mbf, which is the approach taken by Aradillas-Lopez, Gandhi, and Quint (2010).

(1996) show that sellers prefer auctions with non-strategic reserve prices to those in which they can set an optimal reserve but have one less bidder if (1) bidders are symmetric, (2) all bidders must participate, (3) the marginal revenue curves associated with each bidder are downward sloping and (4) every bidder is willing to make an opening offer of at least v_0.

However, this result does not generally hold in settings with endogenous entry, where the most common way to think about increasing competition is by increasing the number of potential bidders (e.g. Samuelson (1985), Menezes and Monteiro (2000) or Li and Zheng (2009)). This can be trivially seen in the LS model. In this case, expected revenues decline in the number of potential bidders if bidders participate with probability less than 1, so that a seller dislikes additional potential competition, and prefers the ability to set an optimal reserve price (which is equal to zero for a revenue maximizing seller). Conversely, the seller may prefer to increase the number of potential bidders by one and forgo the opportunity to set a reserve when there is selective entry.[26] In general, the degree of selection affects the seller's returns to both increasing the reserve price and increasing competition because marginal entrants, who will be deterred from entering due to either change, will tend to have lower values than inframarginal entrants.

Table 5 presents results comparing the relative impact of increasing the number of potential entrants and setting an optimal reserve with the original set of potential entrants for the representative auction with $v_0 = \$27.77$/mbf. With four potential mill and four potential logger bidders, the optimal reserve $R^* = \$55.90$/mbf and this increases, relative to setting a non-strategic reserve price, the probability that the auction fails to sell. However, this benefits the seller since he sells at low prices less often and his expected value increases from \$71.48/mbf to \$72.14/mbf, or 0.92%. Given the informational demands on the seller for setting an optimal reserve (he must know the number of potential entrants, their distribution of values, entry costs, signal distributions and his own value of retaining the object), this improvement is small.

The improvement is also low relative to that based on simply increasing the number of potential entrants by one, regardless of the new firm's type. Even if the seller continues to set a non-strategic reserve price, his value rises by 6.66% (1.80%) when the number of potential mills (loggers) increases by one: 7.21 (1.96) times the improvement from setting an optimal reserve with the original set of bidders.[27] The benefit of increased competition over optimal reserve pricing is even more pronounced when more than one potential entrant is added.

[26]For example, in the symmetric equilibrium of the symmetric bidder-type S model, if values are distributed $logN(5, 0.4)$, $K = 20$ and $N = 4$, with no reserve the seller's expected revenues are 128.9. If instead $N = 3$ but the seller can set the optimal reserve price of 73.9, the seller's expected revenues are 118.17.

[27]This is despite the fact that increasing the number of potential bidders of one type lowers entry probabilities for all other bidders of both types.

N_{Mill}	N_{Logger}	R	USFS's E[Value of Auction]	Pr[No Sale]	E[n_{Mill}]	E[n_{Logger}]	USFS's Gain Relative to 1st Row
4	4	v_0	71.48	0.0016	3.18	1.16	-
4	4	R^*	72.14	0.0706	3.00	1.05	0.92%
4	5	v_0	72.77	0.0015	3.12	1.39	1.80%
4	6	v_0	73.95	0.0014	3.07	1.59	3.46%
5	4	v_0	76.27	0.0011	3.69	0.96	6.66%
6	4	v_0	80.43	0.0009	4.12	0.81	12.51%

Table 5: The relative value of competition for the representative auction based on mean parameters. For any combination of potential bidders and reserve price, the table displays the USFS's expected value of the auction (in $/mbf), probability of no sale, the expected number of entering mills and loggers and the % increase in the USFS's value relative to having four potential mill and logger entrants and setting a non-strategic reserve price, i.e. the first row in the table. This assumes $v_0 = \$27.77$/mbf. In the second row, $R^* = \$55.90$/mbf. Results are based on the mean parameter estimates in Table 3 and 5,000,000 simulated auctions.

We now consider how our comparison varies with the degree of selection, as our estimates imply that this varies significantly across the auctions in our data (for example, the accuracy of the USFS cruises may vary over time or across different forests in CA). The degree of selection is also a parameter that the seller could potentially affect by improving the quality of information he releases.[28]

To illustrate, we consider this comparison when we vary α by one standard deviation from its mean value in Table 3, holding constant the other parameters at their mean values. When α is raised by one standard deviation to $\alpha = 0.872$, then adding one more potential mill entrant, but continuing to set a non-strategic reserve price, increases the seller's value of the auction 9.35 times as much as if an optimal reserve price were set with the original group of potential entrants. If instead α is lowered by one standard deviation to $\alpha = 0.505$, then adding one more potential mill entrant, but continuing to set a non-strategic reserve price, increases the seller's value of the auction 5.53 times as much as if an optimal reserve price were set with the original group of potential entrants.[29] For these values, the rise in the relative value of potential competition as α increases is due to the fact that the return to setting an optimal reserve price falls, while the benefit of adding a potential mill entrant is fairly constant.

These findings also highlight the value of the framework that allows us to consider both intermediate values of α and heterogeneous bidders, as considering only the traditional polar

[28]For example, some state agencies selling timber (e.g., Oregon) provide more detailed pre-sale cruise reports that contain more information for bidders such as tree diameters.

[29]When $\alpha = 0.872$ ($\alpha = 0.505$), then adding one more potential logger entrant, but continuing to set a non-strategic reserve price, increases the seller's value of the auction 2.04 (1.66) times as much as if an optimal reserve price were set with the original group of potential entrants.

cases might lead one to different conclusions. Specifically, the LS (no selection) model with homogenous potential bidders suggests that increasing potential competition would reduce revenues once entry is incomplete. Instead, we find that, even though there is only moderate selection, reducing selection (increasing α) actually increases the relative value of adding more potential bidders.

The returns to both setting an optimal reserve and increasing competition fall in the number of potential bidders. Figure 5 illustrates this effect for the representative auction using the mean parameter estimates. In the figure, the top (bottom) panel shows how changing the number of potential mill (logger) entrants affects the value of adding one potential mill (logger) entrant and setting an optimal reserve price. To understand the figure, consider the top panel (the explanation is analogous for the bottom panel). The dashed line with triangles gives the percentage increase in value to the USFS from setting the optimal reserve for the number of potential mill bidders on the horizontal axis and four potential logger bidders, relative to setting a non-strategic reserve when there are that number of mill potential bidders and four potential logger bidders. The dotted line with points gives the percentage increase in value to the USFS from adding one potential mill bidder to the number on the horizontal axis when there are four potential logger bidders, but setting a non-strategic reserve, relative to setting a non-strategic reserve with the original set of potential bidders. The gains are measured on the left axis. The solid line with squares measures the change in value from adding a potential bidder and setting a non-strategic reserve, relative to setting an optimal reserve price with the original set. These relative gains are measured on the right axis. Although the returns to both strategies fall in the number of potential bidders, for this representative auction, the gains from increasing potential competition continue to outweigh those from setting an optimal reserve, and in fact the return of the former, relative to the latter, increases in N.

Since our estimation method recovers structural parameters for each auction, we can evaluate the relative impacts of increasing competition and setting an optimal reserve price for each auction in our data. To do this, we take one draw of the parameters based on the observed characteristics of each auction and assume for each auction that v_0 is the tract's observed reserve price. We then compute the % gain to the USFS from (a) setting an optimal reserve price with that auction's number of potential entrants, (b) setting a non-strategic reserve price but adding one potential mill entrant and (c) setting a non-strategic reserve price but adding one potential logger entrant, all relative to setting a non-strategic reserve price with that auction's number of potential entrants.

Table 6 displays the distributions of the relative returns for (a), (b) and (c). As in the representative auction, the distribution of gains associated with adding one potential mill

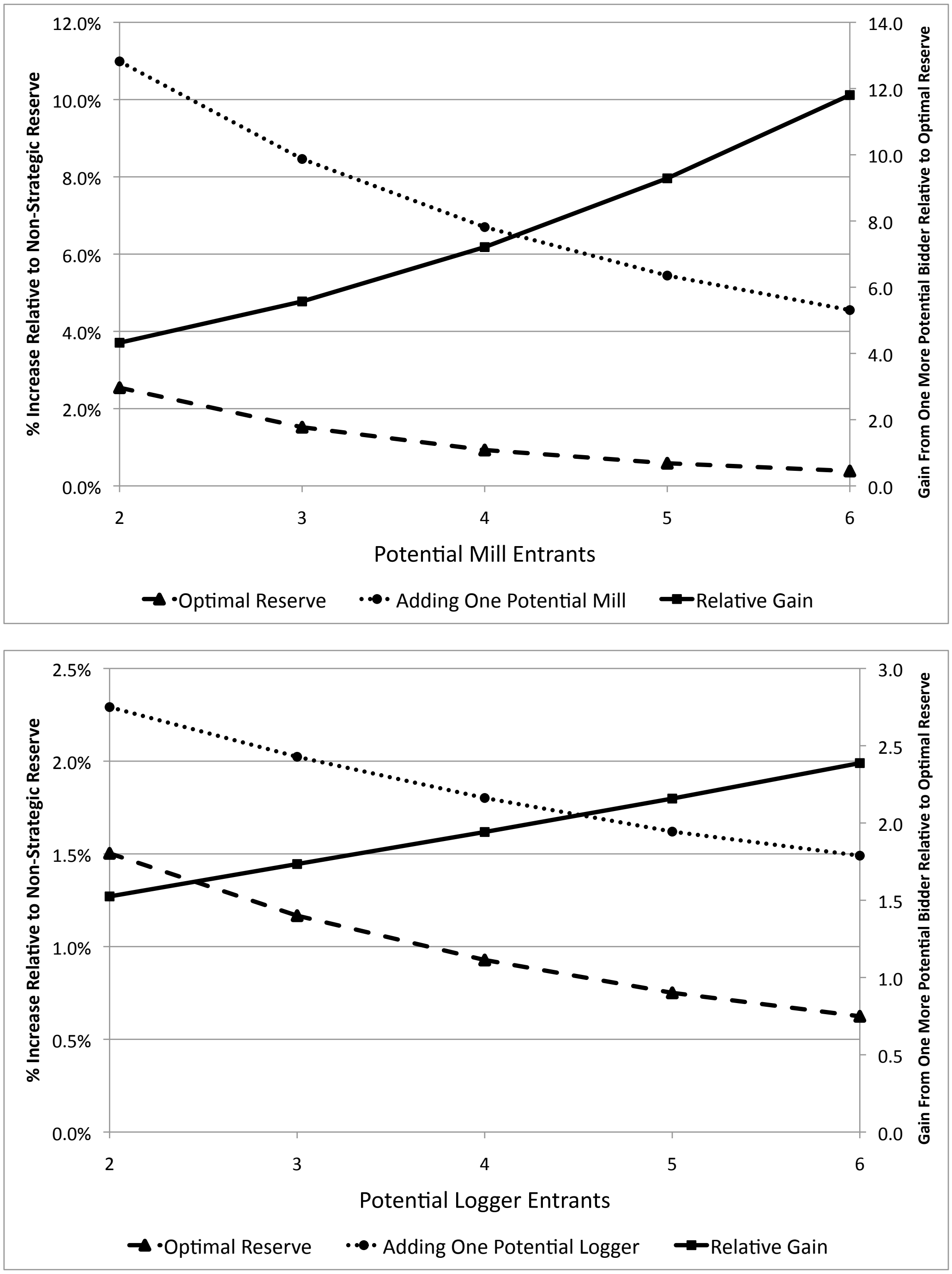

Figure 5: Comparing the effects of adding one extra potential entrant and setting an optimal reserve for different numbers of potential bidders. The top (bottom) panel shows how changing the number of potential mill (logger) entrants affects the value of adding one potential mill (logger) entrant and setting an optimal reserve price. Consider the top panel (the explanation is analogous for the bottom panel). The dashed line with triangles gives the percentage increase in value to the USFS from setting the optimal reserve for the number of potential mill bidders on the horizontal axis and four potential logger bidders relative to setting a non-strategic reserve when there are that number of mill potential bidders and four potential logger bidders. The dotted line with points gives the percentage increase in value to the USFS from adding one potential mill bidder to the number on the horizontal axis when there are four potential logger bidders, but setting a non-strategic reserve, relative to setting a non-strategic reserve with the original set of potential bidders. The gains are measured on the left axis. The solid line with squares measures the change in value from increasing the potential bidders, but setting a non-strategic reserve price, relative to setting an optimal reserve price with the original set of potential bidders.

bidder generally exceeds that for setting the optimal reserve price. For 85.5% of auctions, the USFS prefers adding one potential mill entrant and setting a non-strategic reserve price to setting an optimal reserve price with the original set of potential entrants. The results for an additional logger potential entrant are less clear cut, as the USFS prefers an additional logger for 52.0% of auctions. One reason that an additional logger potential entrant may not always be as valuable to the seller as setting an optimal reserve price is that loggers often have values below our proxy for v_0.[30] As mentioned above, this assumption is necessary for Bulow and Klemper's result that if all bidders must participate, a seller would always prefer an additional bidder to the ability to set an optimal reserve. We can also compare, for each auction, the relative return to adding a potential mill entrant to setting an optimal reserve price (i.e., the return to adding a mill divided by the return to optimal reserve). When we do this, the median value across auctions is 4.94.

There are 38 auctions with one potential bidder, and for these auctions the returns to both setting an optimal reserve price and increasing potential competition tend to be high. In the bottom panel of Table 6 we exclude these cases, and the general pattern that adding competitors is more valuable than setting the optimal reserve price is maintained.

	Percentiles of % Gains Relative to $R = v_0$ with N								
	1	5	10	25	50	75	90	95	99
All Auctions									
R^*	0.00	0.00	0.04	0.21	1.04	4.34	11.31	15.40	59.90
+1 Mill PE	0.09	1.47	2.07	3.20	5.25	9.30	17.40	24.39	83.05
+1 Logger PE	0.00	0.03	0.14	0.54	1.30	3.02	7.17	12.32	59.40
Auctions with > 1 PE									
R^*	0.00	0.01	0.03	0.18	0.93	3.90	9.03	13.09	25.04
+1 Mill PE	0.09	1.51	2.11	3.21	5.13	8.86	15.66	20.80	40.95
+1 Logger PE	0.00	0.03	0.13	0.52	1.26	2.82	6.04	9.75	22.41

Table 6: The distribution of the relative value of competition across sample of auctions. For each auction we compute the % gain to the USFS from (a) setting an optimal reserve price with that auction's number of potential entrants, (b) setting a non-strategic reserve price but adding one potential mill entrant and (c) setting a non-strategic reserve price but adding one potential logger entrant, all relative to setting a non-strategic reserve price with that auction's number of potential entrants. Each row gives percentiles of these gains across auctions and each number is a %. The top panel uses all auctions and the bottom only uses those with more than one potential bidder. Results are based on parameter estimates in Table 3. We assume for each auction that v_0 is the tract's reserve price.

A seller's return from setting an optimal reserve will generally rise in v_0 as this decreases the cost of not selling. For example, if $v_0 = 0$, which would be the case for a revenue maximizing seller, our conclusions about the relative importance of competition are strengthened.

[30] For example, our structural estimates imply that across auctions, 38.9% of loggers have values below each auction's observed reserve price (our proxy for v_0).

When recalculating the results in Table 6 with $v_0 = 0$ for all auctions, we find that the USFS now prefers an additional mill to setting an optimal reserve price for 99.5% of auctions. It prefers an additional logger for 88.4% of auctions.

On the other hand, it is possible that for high enough v_0, the return from setting an optimal reserve price is greater than that from increasing the number of potential entrants by one. However, the optimal reserve price policy in this case may result in a probability of a failed sale that is too high for some sellers. For example, for the representative auction in our data, the optimal reserve price is more valuable than one additional mill potential entrant only if v_0 is greater than \$57.50/mbf. In this case the probability that there is no sale when the optimal reserve price (\$84.64/mbf) is used is 40%, which greatly exceeds the 15% level that the USFS is allowed.

7 Conclusion

The widespread use of auctions has been cited as an example of the practical relevance of economic theory (Milgrom (2004)), but relatively little is known about the value of the design tools that are the subject of most of the theoretical and empirical literatures, relative to more fundamental economic factors, such as competition. This is especially true in settings, which are likely to be encountered in the real-world, where there may be multiple bidder types and auction participation may be costly and selective. The degree of selection in the entry process will affect the seller's returns to both increasing the reserve price (the most common design tool in the literature) and increasing competition because marginal entrants, who will be deterred from entering, will tend to have lower values than inframarginal entrants.

We develop and estimate, using data from USFS timber auctions, a flexible entry model for IPV second price and ascending auctions that allows us to evaluate the degree of selection in the entry process. Our empirical model allows for asymmetries between different types of potential bidder and unobserved (to the researcher) auction heterogeneity, which are important characteristics of our data. Our estimates indicate a moderately selective entry process, where, for a representative auction, the average value of a mill (logger) that enters the auction is 50.7% (24.3%) greater than the average value of a marginal mill (logger) entrant. Our estimates also imply the USFS's returns from increased competition, particularly from additional mill (higher value) potential entrants, are much greater than the returns from setting an optimal reserve price. For example, in the representative auction, the USFS's expected value from holding the auction increases by 6.66% when it uses a non-strategic reserve price but there is one additional mill potential entrant. In contrast, its value increases by only 0.92% when it uses an optimal reserve price without additional competition. This is true even though there has been considerable interest in the literature on optimal reserve prices in timber auctions, based on models with no endogenous entry margin.

Our approach can also be used to evaluate the benefit of selection to the seller. For example, given a set of potential entrants, a seller might like to improve the accuracy of bidders' signals, say through more accurate appraisals, if a greater amount of selection improves its value of the auction. In fact, some state forest agencies provide more detailed information about tracts than is provided by the USFS, and the USFS has, subsequent to our data, made more use of outside consultants to improve the quality of its cruises. Looking at the value of this type of policy to sellers, both as a counterfactual and empirically, is one possible direction for future work.

Our results strongly support the view that economic fundamentals can be much more valuable than design tools, as suggested by Klemperer (2002), but we recognize that we rely on some quite common maintained assumptions. First, we assume independent private values. This is a common assumption in the literature on timber auctions (Baldwin, Marshall, and Richard (1997), Brannman and Froeb (2000), Haile (2001) or Athey, Levin, and Seira (2011)) and is made more reasonable by our focus on a sample of auctions where resale possibilities were limited. However, as is well known, common value components could reduce the value of increased potential competition by exacerbating the problem of the "winner's curse" (see Bulow and Klemperer (2002) or Pinkse and Tan (2005)). Second, based on the finding in Athey, Levin, and Seira (2011), we assume that potential bidders do not collude either in their entry decisions or bids. If firms are able to collude, then both design tools and increased competition might help to encourage competitive bidding, and it would also be interesting to understand the relative value of these mechanisms in this case. Third, in this paper we consider only second price or ascending auctions, whereas it has been suggested in the literature that first-price sealed bid auctions may be effective at increasing the participation of weaker bidders and increasing the seller's revenue when potential bidders are asymmetric (Klemperer (2004) or Athey, Levin, and Seira (2011)). In a related paper (Roberts and Sweeting (2010)), we compare revenues from typical simultaneous-bid first and second price auctions with a more sequential bidding structure, and find that, in the presence of selective entry, the gains to using an optimal reserve price in first price auctions are also small.

Our selective entry model can be used to examine many counterfactuals aside from those considered here. With appropriate changes to the second (post-entry) stage of the game, our empirical approach can be used to understand the degree of selection in non-auction settings as well. In the general entry literature, the possibility of selective entry is assumed away. Instead, the different entry decisions by similar firms are explained by i.i.d. differences in fixed costs or entry costs, which do not reflect their subsequent competitiveness or post-entry market outcomes. However, selective entry, which will lead to differences in the competitiveness of marginal and inframarginal entrants, seems intuitively plausible and may affect the conclusions of *any* counterfactual that impacts the set of participating firms.

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A Monte Carlos

This appendix describes a set of Monte Carlo exercises where we investigate the performance of our Simulated Maximum Likelihood (SML) estimator, which uses Importance Sampling, to approximate the likelihood of the observed outcome for a particular auction (Ackerberg (2009)). This evidence is important because SML estimators may perform poorly when the number of simulation draws is too small. We also study the performance of our estimator under alternative definitions of the likelihood, which make different assumptions about the data available to the researcher.

Simulated Data

To generate data for the Monte Carlos, we allow the number of {mill, logger} potential entrants to take on values {3,3}, {5,5}, {8,8}, {6,2} and {2,6} with equal probability. For each auction a, there is one observed auction covariate x_a, which is drawn from a Uniform [0,1] distribution, and the vector X_a is equal to $[1\ x_a]$. We assume

$$\text{Location Parameter of Logger Value Distribution: } \mu_{a,\text{logger}} \sim N(X_a\beta_1, \omega^2_{\mu,\text{logger}})$$
$$\text{Difference in Mill/Logger Location Parameters: } \mu_{a,\text{mill}} - \mu_{a,\text{logger}} \sim TRN(X_a\beta_3, \omega^2_{\mu,\text{diff}}, 0, \infty)$$
$$\text{Scale Parameter of Mill and Logger Value Distributions: } \sigma_{Va} \sim TRN(X_a\beta_2, \omega^2_{\sigma_V}, 0.01, \infty)$$
$$\alpha\text{: } \alpha_a \sim TRN(X_a\beta_4, \omega^2_{\alpha}, 0, 1)$$
$$\text{Entry Costs: } K_a \sim TRN(X_a\beta_5, \omega^2_{K}, 0, \infty)$$

where $TRN(\mu, \sigma^2, a, b)$ is a truncated normal distribution with parameters μ and σ^2, and upper and lower truncation points a and b. The true values of the parameters are $\beta_1 = [2.8; 1.5]$, $\beta_2 = [0.3; 0.2]$, $\beta_3 = [0.5; -0.1]$, $\beta_4 = [0.5; 0]$, $\beta_5 = [4; 4]$, $\omega_{\mu,\text{logger}} = 0.2$, $\omega_{\sigma_V} = 0.3$, $\omega_{\mu,\text{diff}} = 0.2$, $\omega_\alpha = 0.2$ and $\omega_K = 2$. The reserve price can take on values of 10, 30 or 50. We allow for R to be correlated with x, as one would expect if the seller sets a higher reserve price when he believes the tract has higher value. Specifically, for each auction, we a draw u_a from a uniform $[0, 1]$ distribution and set

$$R_a = 10 \text{ if } \frac{x_a + u_a}{2} < 0.33$$
$$R_a = 30 \text{ if } 0.33 \leq \frac{x_a + u_a}{2} \leq 0.66$$
$$R_a = 50 \text{ otherwise.}$$

For each auction we find the unique equilibrium that satisifies the constraint that $S'^{*}_{\text{mill}} < S'^{*}_{\text{logger}}$, and generate data using the equilibrium strategies assuming that the auction operates as a second price sealed bid auction, or, equivalently, an English button auction. The exercises described below all use the same 100 data sets of 1,000 auctions each.

Having constructed the data we estimate the parameters in three different Monte Carlo exercises, which differ in the importance sampling density used to draw the simulated parameters.

A.1 Monte Carlo Exercise 1: Importance Sampling Density is the True Distribution of the Parameters

In the first exercise we make the (generally infeasible) assumption that the researcher knows the true distribution of each of the parameters, which depends on the value of x_a for a particular auction. The number of simulation draws per auction is set equal to 250, and different draws are used for each auction. We compute the results for four different definitions of the likelihood (the same simulation draws are used in each case) that make different assumptions about the information available to the researcher, which will vary with the exact format of the auction (open outcry vs. sealed bid) and with the information that the seller collects about entry decisions. The alternative assumptions are:

1. the researcher observes the values (as bids) and identities of all firms that pay the entry cost and have values above the reserve, and he observes the entry decision of each potential entrant;

2. the researcher observes the values (as bids) and identities of all firms that pay the entry cost and have values above the reserve, and he knows that these firms entered, but for other firms he does not know whether they paid the entry cost and found that their values were less than R, or they did not pay the entry cost;

3. the researcher observes the value and identity of the firm with the second highest value as the final price, the identity of the winning bidder (e.g. whether it is a mill or logger), the identity of all entering firms with values above the reserve price and he observes the entry decision of each potential entrant;

4. the researcher observes the value and identity of the firm with the second highest value as the final price, the identity of the winning bidder (e.g. whether it is a mill or logger), the identity of all entering firms with values above the reserve price, but for other firms he does not know whether they paid the entry cost and found that their values were less than R, or they did not pay the entry cost. This informational assumption forms the basis of the likelihood function shown in equation 7.

			Likelihood Definition			
Parameter	Variable	True Value	1	2	3	4
Logger	Constant	2.8	2.7793	2.7830	2.7918	2.7873
Location Parameter			(0.1094)	(0.1224)	(0.0893)	(0.0776)
	x_a	1.5	1.4954	1.4962	1.4945	1.4950
			(0.0999)	(0.1070)	(0.1211)	(0.1247)
	Std. Dev.	0.2	0.1904	0.1930	0.1849	0.1848
			(0.0182)	(0.0204)	(0.0320)	(0.0312)
Difference in Mill and Logger	Constant	0.3	0.3128	0.3025	0.3195	0.3139
Location Parameters			(0.0633)	(0.1477)	(0.1037)	(0.0551)
	x_a	0.2	0.1815	0.1897	0.1981	0.1925
			(0.0909)	(0.1045)	(0.1042)	(0.0942)
	Std. Dev.	0.2	0.1873	0.1888	0.1872	0.1820
			(0.0190)	(0.0278)	(0.0286)	(0.0277)
Value Distribution	Constant	0.5	0.5153	0.5190	0.5205	0.5307
Scale Parameter			(0.1311)	(0.1221)	(0.1003)	(0.0534)
	x_a	-0.1	-0.1007	-0.1022	-0.0895	-0.0795
			(0.1099)	(0.1065)	(0.0892)	(0.0759)
	Std. Dev.	0.3	0.2797	0.2749	0.2804	0.2771
			(0.0267)	(0.0268)	(0.0321)	(0.0280)
α (Degree of selection)	Constant	0.5	0.4979	0.4561	0.4809	0.5100
			(0.1358)	(0.2151)	(0.1716)	(0.0815)
	x_a	0.0	-0.0145	-0.0403	-0.0167	0.0068
			(0.1266)	(0.2003)	(0.1649)	(0.1337)
	Std. Dev.	0.2	0.1886	0.1853	0.1869	0.1891
			(0.0199)	(0.0300)	(0.0299)	(0.0407)
Entry Cost	Constant	4.0	4.0269	4.0115	4.0604	4.0743
			(0.5182)	(0.5386)	(0.7512)	(0.8108)
	K	4.0	4.2420	4.2858	4.4381	4.5171
			(0.8683)	(0.8868)	(1.3409)	(1.4110)
	Std. Dev.	2.0	1.9076	1.9194	1.9117	1.8934
			(0.2855)	(0.3024)	(0.3733)	(0.3971)

Table 7: True Importance Sampling Density Monte Carlo. The table shows the mean and standard deviation (in parentheses) for each of the parameters estimates across the 100 repetitions based on the four different definitions of the likelihood when we use the true joint distribution of the parameters as the importance sampling density, with $S = 250$ draws. See paper for descriptions of the different likelihood definitions.

Table 7 shows the mean value of each parameter and its standard deviation across the simulated datasets for each definition of the likelihood. With the true distribution as the importance sampling density and $S = 250$, all of the parameters are recovered accurately, including the standard deviation parameters. Several of the parameters appear to be recovered less precisely when less information is available to the researcher (likelihood definition 4), but the differences are never large.

A.2 Monte Carlo Exercise 2: Importance Sampling Density is a Uniform Distribution

When the true distributions are unknown, it is necessary to choose importance sampling densities that provide good coverage of the possible parameter space. In this exercise we draw parameters from independent uniform distributions where $\mu_{a,\text{logger}} \sim U[2,6]$, $\sigma_{Va} \sim U[0.01, 2.01]$, $\mu_{a,\text{mill}} - \mu_{a,\text{logger}} \sim U[0, 1.5]$, $\alpha_a \sim U[0,1]$, $K_a \sim U[0,20]$. In this case we set the number of simulation draws per auction equal to 1,000 to try to compensate for the fact that a relatively small proportion of the simulated draws are likely to be close to the parameters that really generate the data (in our empirical work we use 2,500 simulated draws per auction so that we get even better coverage). We use the four alternative definitions of the likelihood that we used for the first exercise.

Table 8 shows the mean value of each parameter and its standard deviation across the simulated datasets for each definition of the likelihood. The parameters which determine the means of each distribution are recovered accurately, but four out of the five standard deviation parameters are biased upwards. As in the first exercise, the alternative likelihood definitions appear to have only small effects on the precision of the estimates.

A.3 Monte Carlo Exercise 3: Two Step Estimation

As some of the parameter estimates appear to be biased using a uniform importance sampling density, the estimator we use in the paper uses the estimates based on a uniform importance sampling density to form new importance sampling densities that are used in a repetition of the estimation procedure. As long as the first step estimates are not too biased, this two step procedure should give accurate results, provided that the number of simulation draws is large enough.

To confirm that this is the case, we apply this two step procedure using likelihood definition 4 estimates from exercise 2 for each of the 100 datasets to form an importance sampling density from which we take $S = 250$ simulation draws for each auction (when we apply our

			Likelihood Definition			
Parameter	Variable	True Value	1	2	3	4
Logger	Constant	2.8	2.7051	2.7102	2.6895	2.7031
Location Parameter			(0.0969)	(0.0998)	(0.1324)	(0.1333)
	x_a	1.5	1.3921	1.3807	1.3331	1.2946
			(0.1766)	(0.1810)	(0.2090)	(0.2245)
	Std. Dev.	0.2	0.2536	0.2478	0.2379	0.2312
			(0.0163)	(0.0178)	(0.0223)	(0.0238)
Difference in Mill and Logger	Constant	0.3	0.3286	0.3255	0.3532	0.3407
Location Parameters			(0.0806)	(0.0848)	(0.0970)	(0.1007)
	x_a	0.2	0.3073	0.3148	0.3536	0.3819
			(0.1465)	(0.1505)	(0.1595)	(0.1713)
	Std. Dev.	0.2	0.2487	0.2913	0.2452	0.2418
			(0.0171)	(0.0232)	(0.0204)	(0.0194)
Value Distribution	Constant	0.5	0.5727	0.5694	0.5969	0.5921
Scale Parameter			(0.0812)	(0.0423)	(0.0719)	(0.0753)
	x_a	-0.1	-0.0729	-0.0607	-0.0476	-0.0176
			(0.0762)	(0.0760)	(0.1227)	(0.1289)
	Std. Dev.	0.3	0.2895	0.2913	0.3163	0.3174
			(0.0201)	(0.0232)	(0.0326)	(0.0351)
α (Degree of selection)	Constant	0.5	0.4678.	0.4811	0.4671	0.5034
			(0.0842)	(0.1112)	(0.1052)	(0.1380)
	x_a	0.0	-0.1070	-0.1164	-0.1394	-0.1849
			(0.1526)	(0.1878)	(0.1595)	(0.2112)
	Std. Dev.	0.2	0.2590	0.3088	0.2537	0.3077
			(0.0234)	(0.0385)	(0.0311)	(0.0600)
Entry Cost	Constant	4.0	4.3744	4.0931	4.7719	4.6044
			(0.7186)	(0.7834)	(1.0344)	(1.0318)
	K	4.0	3.5605	3.8494	3.0215	3.1088
			(1.5964)	(1.5916)	(1.9387)	(1.9433)
	Std. Dev.	2.0	3.5409	3.6859	3.3704	3.5099
			(0.3358)	(0.3446)	(0.3623)	(0.4164)

Table 8: Uniform Importance Sampling Density Monte Carlo. The table shows the mean and standard deviation (in parentheses) for each of the parameters estimates across the 100 repetitions based on the four different definitions of the likelihood when we use a uniform importance sampling density, with $S = 1,000$ draws. See paper for descriptions of the different likelihood definitions.

estimator to the real data we use $S = 500$). We focus on likelihood definition 4 as it is the basis of our preferred estimates in the paper.

Table 9 shows the mean and standard deviation of the estimates for each of the parameters. We see that both the mean and standard deviation parameters are recovered accurately, although the estimated standard deviation of entry costs is recovered slightly less accurately than when we used the infeasible estimator in exercise 1. Overall, we regard these Monte Carlo results as providing strong support for our estimation procedure, especially as we use more than twice as many simulation draws when we apply our estimator to the actual data.

Parameter	Variable	True Value	Definition 4
Logger	Constant	2.8	2.7313
Location Parameter			(0.1389)
	x_a	1.5	1.3720
			(0.2138)
	Std. Dev.	0.2	0.1722
			(0.0349)
Difference in Mill and Logger	Constant	0.3	0.3308
Location Parameters			(0.0976)
	x_a	0.2	0.3138
			(0.1490)
	Std. Dev.	0.2	0.2039
			(0.0257)
Value Distribution	Constant	0.5	0.5741
Scale Parameter			(0.0639)
	x_a	-0.1	-0.0380
			(0.1078)
	Std. Dev.	0.3	0.2706
			(0.0292)
α (Degree of selection)	Constant	0.5	0.4725
			(0.1321)
	x_a	0.0	-0.0902
			(0.2193)
	Std. Dev.	0.2	0.2064
			(0.0590)
Entry Cost	Constant	4.0	4.2557
			(0.9945)
	K	4.0	3.5161
			(2.0808)
	Std. Dev.	2.0	2.5403
			(0.4681)

Table 9: Two Step Estimator Monte Carlo. The table shows the mean and standard deviation (in parentheses) for each of the parameters estimates across the 100 repetitions based on the fourth different definitions of the likelihood when we use the true joint distribution of the parameters as the importance sampling density, with $S = 250$ draws. See paper for the likelihood definition.