

When Should Sellers Use Auctions?

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Abstract

A bidding process can be organized so that offers are submitted simultaneously or sequentially. In the latter case, potential buyers can condition their behavior on previous entrants' decisions. The relative performance of these mechanisms is investigated when entry is costly and selective, meaning that potential buyers with higher values are more likely to participate. A simple sequential mechanism can give both buyers and sellers significantly higher payoffs than the commonly used simultaneous bid auction. The findings are illustrated with parameters estimated from simultaneous entry USFS timber auctions where our estimates predict that the sequential mechanism would increase revenue and efficiency.

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1 Introduction

The simultaneous bid auction is a standard method for sellers to solicit offers from buyers. A simple alternative is for a seller to ask buyers to make offers sequentially. If it is costly for buyers to participate, the sequential mechanism will tend to be more efficient than the simultaneous auction because later potential buyers can condition their participation decisions on earlier bids. However, the sequential mechanism's greater efficiency may not produce higher revenues because while the possibility of deterring later potential entrants can lead early bidders to bid aggressively, the fact that later firms might be deterred will tend to reduce revenues. The relative revenue performance of the mechanisms will therefore depend on whether the threat of potential future competition, which can raise bids in the sequential mechanism, is more valuable to the seller than actual competition, which will tend to be greater in the simultaneous auction.

The relative revenue performance of these alternative mechanisms has direct implications for how assets should be sold. In the case of how to structure the sale of corporations, this question has attracted attention from practitioners and commentators since the Delaware Supreme Court's 1986 *Revlon* decision charged a board overseeing the sale of a company with the duty of "getting the best price for the stockholders" (*Revlon v McAndrews & Forbes Holdings* (1986)). In practice, corporate sales occur through a mixture of simultaneous and sequential mechanisms (Denton (2008)), with sequential mechanisms sometimes taking the form of "go-shop" arrangements where a seller may reach an agreement with one firm while retaining the right to solicit other offers, to which the first firm may be able to respond.^{1,2}

Surprisingly, the only attempt to date to directly address this relative performance question is Bulow and Klemperer (2009) (BK hereafter). They compare the revenue and efficiency performances of the commonly-used simultaneous bid second-price auction with a similarly simple, sequential mechanism. In this second mechanism, buyers are approached in turn, and upon observing the history of offers, each chooses whether to enter and attempt to outbid the current high bidder. If the incumbent is outbid, the new entrant can make a jump bid that may potentially deter later firms from participating. The incumbent at the end of the game pays the standing price. As BK note (see also Subramanian (2008), Wasserstein

¹A "go-shop" clause allows a seller to come to an agreement on an initial price with a buyer and retain the right to solicit bids from other buyers for the next 30-60 days. If a new, higher offer is received, then according to the "match right", which is often included in the agreement with the initial buyer, the seller must negotiate with the first buyer (for 3-5 days, for example) to see if it can match the terms of the new, higher offer.

²There are numerous theory papers, some related directly to the field of corporate finance, that consider sequential mechanisms similar to the one considered here. Examples include Fishman (1988), Daniel and Hirshleifer (1998) and Horner and Sahuguet (2007).

(2000)), these simple mechanisms can be thought of as stylized versions of sale processes that are widely used in practice. In the comparison, BK assume that potential bidders only know the distribution from which values are drawn prior to entering, and have no additional information about their own value. After entry they find out their values for sure. These assumptions are common in the auction literature as they provide greater analytic tractability. Under this informational assumption, together with the assumption that bidders are symmetric and the seller cannot set a reserve price, BK show that “sellers will generally prefer auctions and buyers will generally prefer sequential mechanisms” (p. 1547). Applying this finding to M&A market, Denton (2008) uses BK’s results to criticize the use of go-shops as effective means for a board fulfilling its *Revlon* duties.

This result holds in BK’s model because, in the equilibrium they consider, early bidders with high enough values pool and submit a common bid that deters all future potential entry (all future potential entrants have the same beliefs about their values prior to entry), and there is too much deterrence from the seller’s perspective, so that he would prefer the greater actual competition in the auction. In particular, deterrence means that later potential entrants with high values will not enter, which decreases both the expected value of the winning bidder and the price that an incumbent has to pay. In contrast, buyers prefer the sequential mechanism as expenditures on entry costs are lower. This effect is sufficient to increase social efficiency.

In light of their result, BK interpret the fact that sequential mechanisms are actually used as evidence that buyer’s preferences can determine the choice of mechanism. This is consistent with the fact that some influential buyers, such as Warren Buffett, have explicit policies that they will not “waste time” by participating in auctions.

In this paper, we consider a similar comparison, except that we allow potential buyers to receive a noisy signal about their valuation prior to deciding whether to enter either mechanism. After entry, they find out their values for sure, as in BK’s model. This structure results in a “selective entry” model, where firms enter if they receive high enough signals, and firms with higher values are more likely to enter.^{3,4} We believe that this is a natural model to describe settings where firms are likely to have some imperfect information about their value for an asset based on publicly available information, but must conduct costly research to discover additional information that will affect their value.⁵ We also allow for potential

³The precision of the signal determines how selective the entry process is. In its limits, the model can approach the polar cases of (a) perfect selection, which we term the S model after Samuelson (1985), whereby a firm knows its value exactly when taking its entry decision, and (b) no selection, which we term the LS model after Levin and Smith (1994), whereby a firm knows nothing of its value when taking its entry decision.

⁴Selective entry contrasts with standard assumptions in the empirical entry literature (e.g., Berry (1992)) where entrants may differ from non-entering potential entrants in their fixed costs or entry costs, but not along dimensions such as marginal costs or product quality that affect competitiveness or the profits of other firms once they enter.

⁵Examples include oil and gas leases, timber sales and government procurement contracts. The model also

buyers to be asymmetric, which is another important feature of many real-world settings.

Using numerical analysis, which becomes necessary once either asymmetries or selective entry are added to the model, we show that the sequential mechanism can give the seller *higher* expected revenues than the simultaneous auction even when buyers' signals about their values are quite noisy. When the entry process is quite selective and/or entry costs are large, the absolute difference in revenues can be substantial (for example, 10% or more). The gains to using the sequential mechanism are also relatively larger than the gains to using an optimal reserve price in the simultaneous auction. As in BK's analysis, the sequential mechanism is more efficient, and the sequential mechanism generally gives higher expected payoffs to both buyers and sellers. This result can obviously lead to a different interpretation of why sequential mechanisms are sometimes used, and because the sequential mechanism increases the payoffs of buyers, it is still consistent with comments like those of Warren Buffett. Our findings are also consistent with observed differences in target shareholder returns in corporate mergers and acquisitions documented by Subramanian (2008). He compares returns when companies are sold using go-shops and a process where many firms are simultaneously asked to submit bids before a winner is selected. He finds that target shareholder returns are approximately 5% higher for go-shops⁶ and argues that, even though go-shop agreements introduce asymmetries between bidders into the sale process, they are preferable for both buyers and sellers.

Why does the sequential mechanism tend to produce higher revenues when entry is selective? The key reason is that selective entry changes the nature of the equilibrium in the sequential mechanism in a way that tends to increase both its relative efficiency and the revenues that the seller can extract. With no selection, BK show that the "pre-emptive bidding [which occurs in equilibrium] is crucial: jump-bidding allows buyers to choose partial-pooling deterrence equilibria which over-deter entry relative to the social optimum" (p. 1546). Introducing any degree of selection into the entry process causes the bidding equilibrium to change so that there is full separation, with bids perfectly revealing the value of the incumbent.⁷

describes firm takeover contests as an acquiring firm faces substantial sunk costs to learn its value for a target and prepare its offer (see, for example, Easterbrook and Fischel (1982) or Bainbridge (1990)). Recently there has been some work allowing for endogenous entry in empirical auction research. The dominant way this is done is by assuming that bidders know their value precisely prior to entry, i.e. by assuming perfect selection. For example, Li and Zheng (2009) compare estimates from both the LS and S models using data on highway lawn mowing contracts from Texas to understand how potential competition may affect procurement costs, and Li and Zheng (2011) test the LS and S models using timber auctions in Michigan. Marmer, Shneyerov, and Xu (2011) extend this literature by testing whether the Li and Zheng (2009) data is best explained by the LS, S or a more general affiliated signal model. They find support for the S and signal models, and they also estimate a very simple version of their signal model. Finally, Gentry and Li (2012) show how partial identification techniques can be used to construct bounds on the primitives of a signal model.

⁶Jeon and Lee (2012) find a similar premium (5.3%) based on an even larger sample of acquisitions.

⁷This is correct for values less than the upper limit of the value distribution minus the cost of entry. An

At the same time, a potential entrant will enter if it receives a high enough signal about its value. These changes increase the efficiency of the outcome in the sequential mechanism as higher value incumbents deter more entry and higher value potential entrants are more likely to enter. Unlike in BK's model, the expected value of the winner can be higher in the sequential mechanism, which increases the surplus available to all parties. In addition, the change to a separating equilibrium affects the equilibrium level of jump bids. For some values, this will increase the expected amount that bidders pay, benefiting the seller.

We illustrate our findings using parameters estimated from a sample of (simultaneous) open outcry US Forest Service (USFS) timber auctions. This setting provides a close match to the information structure assumed in our model as a potential bidder can form a rough-estimate of its value based on tract information published by the USFS and knowledge of its own sales contracts and capabilities, and it is also standard for interested bidders to conduct their own tract surveys ("cruises") prior to bidding. It is also a setting where various auction design tools, such as reserve price policies, have been studied by both academics and practitioners in order to try to raise revenues which have often been regarded as too low.⁸ Timber auctions are also characterized by important asymmetries between potential buyers, with sawmills tending to have systematically higher values than loggers.

Our estimates imply that the entry process into timber auctions is moderately selective, while average entry costs are 2% of the average winning bid, which is large enough to prevent all potential buyers from entering the auctions. Even though such low entry costs tend to weaken the sequential mechanism's advantage over the auction, for the (mean) representative auction in our data, our results imply that using a sequential mechanism (with no reserve) would generate a nine times larger increase in revenues than setting the optimal reserve price (the focus of the existing literature) in the simultaneous auction. We also find that the efficiency gains from using the sequential mechanism are large enough that both the USFS revenues and firm profits can increase.

Some comments about the nature of our results are appropriate. First, we do not seek to compare revenues with those from the optimal mechanism. Instead, in the same spirit as BK, we are interested in the relative performance of stylized versions of commonly used

upper limit on the value distribution is required for technical reasons but we assume that it is sufficiently high that, for practical purposes, all incumbent values are revealed.

⁸Some examples of studies of timber auction reserve prices include Mead, Schniepp, and Watson (1981), Paarsch (1997), Haile and Tamer (2003), Li and Perrigne (2003) and Aradillas-Lopez, Gandhi, and Quint (forthcoming). All of these papers assume that entry is not endogenous. Academics have also provided expert advice to government agencies about how to set reserve prices (stumpage rates) for timber (e.g. Athey, Cramton, and Ingraham (2003)). In 2006, Governor Tim Pawlenty of Minnesota commissioned a task force to investigate the performance of the state's timber sale policies, and its report indicates an openness to considering alternative sales mechanisms as well as different reserve prices (Kilgore, Brown, Coggins, and Pfender (2010)).

sales mechanisms, whereas the seller optimal mechanism, which is not known for a model with imperfectly selective entry (Milgrom (2004)), is likely to involve features, such as side payments or entry fees that are rarely observed in practice, and which might require the seller to have implausibly detailed information.⁹ The seller would also need to know this information if he wants to set the optimal reserve price in an auction, and, indeed, an attraction of the simple sequential mechanism that we consider is that the only required information concerns the set of potential entrants who should be approached.¹⁰ We show below that if the seller has enough information to set an optimal reserve in the sequential mechanism, he can do even better.

On the other hand, what is known about the optimal mechanism in models with costly entry and either no selection or perfect selection suggests that the optimal mechanism should be sequential, which helps to rationalize our results. For example, Cremer, Spiegel, and Zheng (2009) consider the case with *no* selection and McAfee and McMillan (1988) consider a model where buyers know their values but it is costly for the seller to engage additional buyers. In both cases, the optimal mechanism involves some type of sequential search procedure, which stops when a buyer with a high enough value is identified.¹¹

Second, while we characterize the unique equilibrium of each mechanism under standard refinements, our revenue comparisons are numerical in nature. This is a necessary cost of allowing for either a more general model of entry, or bidder asymmetries. Our results show that these features matter because the relative performance of the mechanisms can change even when selection is quite imperfect. The computational approach also allows us to provide a substantive empirical application of our model as selective entry and bidder asymmetries are clear features of our data.

Third, we note two differences, besides the introduction of selective entry and bidder asymmetries, between our model and the model considered by BK. First, we assume that the number N of potential entrants is fixed and common knowledge to all players, whereas BK's model allows for some probability ($0 \leq \rho_j \leq 1$) of a j^{th} potential entrant if there are $j - 1$ potential entrants. As these probabilities may equal 1 for $j < N$, and 0 for $j \geq N$ for any N ,

⁹Our approach is therefore similar to analyses of practical mechanisms in other settings, such as Chu, Leslie, and Sorensen (2011) (bundling), Rogerson (2003) (contracts), McAfee (2002) (nonlinear pricing) and Neeman (2003) (auctions).

¹⁰In this sense the sequential mechanism satisfies what has come to be known as the “Wilson doctrine” (Wilson (1987)), which suggests that, from a practical standpoint, we ought to be concerned with mechanisms that do not rely on the seller possessing unrealistically detailed information about buyers.

¹¹In this environment, Ye (2007) considers two-stage bidding structures where the seller must choose how many firms pay the entry cost ahead of the first stage. His paper clearly shows how to determine the optimal number of entrants. However, he does not consider a wider range of mechanisms that might allow, for example, the seller to set a reserve price or to decide how many firms should enter only after the first stage bids are submitted.

our model is a special case of theirs. Our choice reflects the standard practice in the empirical literature, which we want to follow when estimating our model.¹² Second, when modeling the auction mechanism, we focus on the model where potential buyers make simultaneous entry decisions as well as simultaneous bid choices, whereas BK's primary focus is on a model where firms make sequential entry decisions before bidding simultaneously. However, in BK's model "no important result is affected if potential bidders make simultaneous, instead of sequential, entry decisions into the auction" (p.1560). We also give some consideration to a sequential entry, simultaneous bid model, and show in examples that our qualitative results are unchanged. Our choice to focus on simultaneous entry into the auction reflects a desire to reduce the computational burden and, more importantly, the fact that simultaneous entry is the appropriate way to model entry into the auctions in our empirical sample (Athey, Levin, and Seira (2011) also apply a simultaneous entry model (with no selection) to USFS timber auctions).

The paper proceeds as follows. Section 2 introduces the models of each mechanism and characterizes the equilibria that we examine. Section 3 compares expected revenue and efficiency from the two mechanisms for wide ranges of parameters, and provides intuition for when the sequential mechanism outperforms the auction. Section 4 describes the empirical setting of USFS timber auctions and explains how we estimate our model. Section 5 presents the parameter estimates and counterfactual results showing that the USFS could improve its revenues by implementing a sequential mechanism. Section 6 concludes. An online Appendix contains all proofs and computational details.

2 Model

We now describe the model of firms' values and signals, before describing the mechanisms that we are going to compare.

2.1 A General Entry Model with Selection

Suppose that a seller has one unit of a good to sell and gets a payoff of zero if the good is unsold. There is a set of potential buyers who may be one of $\tau = 1, \dots, \bar{\tau}$ types, with N_τ of type τ with all types known to buyers and the seller. In practice we will consider $\bar{\tau} = 2$. The set of potential buyers and their types are common knowledge to all players. Buyers have independent private values (IPV), which can lie on $[0, \bar{V}]$, distributed according to $F_\tau^V(V)$.

¹²Examples of this assumption in the auction literature include Athey, Levin, and Seira (2011) and Li and Zheng (2009). Examples elsewhere in empirical work on entry games include Berry (1992), Seim (2006) and Ciliberto and Tamer (2009).

F_τ^V is continuous and differentiable for all types. For technical reasons, we assume that bids must lie in the range $[0, \bar{B}]$, where $\bar{B} > \bar{V}$, although this assumption is very weak as no bidder should want to bid more than its value. When we estimate our model and numerically compare our sales mechanisms we assume that values are distributed lognormal and that \bar{V} is large, so that the density of values at \bar{V} is very small, so that the upper truncation in values should not affect the results.¹³ The numerical comparisons produce very similar results using exponential, normal, Weibull or gamma distributions.

Before participating in any mechanism, a potential buyer must pay an entry cost K_τ . Once it pays K_τ it finds out its value, so that the entry cost can be interpreted as including the cost of researching the object for sale, as well as participation and bidding costs. We assume that a firm cannot participate without paying K_τ . However, prior to deciding whether to enter, a bidder receives a private information signal about its value. We focus on the case where the signal of potential buyer i of type τ is given by $S_{i\tau} = v_{i\tau} e^{\varepsilon_{i\tau}}$, where $\varepsilon_{i\tau} \sim N(0, \sigma_{\varepsilon\tau}^2)$ and draws of ε are assumed to be i.i.d. across bidders.¹⁴ Having received his signal, a potential buyer forms a posterior belief about his valuation using Bayes Rule. The independence assumptions on signals and values imply that a potential entrant's signal provides no information about other bidders' values, and, as a result, optimal entry strategies will involve entering if and only if a signal is above a particular threshold. We discuss what may change when signals provide information on common values in the conclusion.

In this model, $\sigma_{\varepsilon\tau}^2$ controls how much potential buyers know about their values before deciding whether to enter. As $\sigma_{\varepsilon\tau}^2 \rightarrow \infty$, the model will tend towards the informational assumptions of the Levin and Smith (1994) (LS) model in which pre-entry signals contain no information about values. As $\sigma_{\varepsilon\tau}^2 \rightarrow 0$, it tends towards the informational assumptions of the Samuelson (1985) (S) model where firms know their values prior to paying an entry cost (which is therefore interpreted as a bid preparation or attendance cost). For many empirical settings, it seems plausible that buyers will have some, but imperfect, information about their values prior to conducting costly research, consistent with intermediate values of $\sigma_{\varepsilon\tau}^2$.

2.2 Mechanism 1: Simultaneous Entry Auction

The first mechanism we consider is a simultaneous entry second price or open outcry auction that we model as a two-stage game. In the first stage all potential buyers simultaneously decide whether to enter the auction (pay the entry cost) based on their signal, the number

¹³To be precise, $f^V(v|\theta) = \frac{h(v|\theta)}{\int_0^V h(x|\theta)dx}$, where $h(v|\theta)$ is the pdf of the lognormal distribution.

¹⁴The normality assumption is not important, but our equilibrium does require us to assume a signal technology which, combined with the assumption that $K > 0$, means that there is always some probability that a potential entrant receives a signal which is so low that it does not choose to enter.

of potential entrants of each type and the reserve price. In the second stage, entrants then learn their values and submit bids. We assume that bidders bid up to their value in an open outcry auction, so that, as in a second price or English button auction, the good is awarded to the firm with the highest value at a price equal to the second highest value of the entrants or the reserve price if one is used.¹⁵

Following the literature (e.g. Athey, Levin, and Seira (2011)), we assume that players use strategies that form type-symmetric Bayesian Nash equilibria, where “type-symmetric” means that every player of the same type will use the same strategy. In the auction’s second stage, entrants know their values so it is a dominant strategy for each entrant to bid its value. In the first stage, players take entry decisions based on what they believe about their value given their signal. The (posterior) conditional density $f_\tau^V(v|S_i)$ that a type- τ player’s value is v when its signal is S_i is defined via Bayes Rule.

The weights that a player places on its prior and its signal when updating its beliefs about its true value depend on the relative variances of the distribution of values and ε (signal noise), and this will also control the degree of selection. A natural measure of the relative variances is the ratio $\frac{\sigma_\varepsilon^2}{\sigma_V^2 + \sigma_\varepsilon^2}$, which we will denote α . If the value distributions were not truncated above, player i ’s (posterior) conditional value distribution would be lognormal with location parameter $\alpha\mu_\tau + (1 - \alpha)\ln(S_i)$ and squared scale parameter $\alpha\sigma_{V\tau}^2$, so a lower value of α implies a more informative signal.

The optimal entry strategy in a type-symmetric equilibrium is a pure-strategy threshold rule where the firm enters if and only if its signal is above a cutoff, S'^*_τ . S'^*_τ is implicitly defined by the zero-profit condition that the expected profit of entering the auction for a firm with the threshold signal will be equal to the entry cost:

$$\int_R^{\bar{V}} \left[\int_R^v (v - x) h_\tau(x|S'^*_\tau, S'^*_{-\tau}) dx \right] f_\tau^V(v|S'^*_\tau) dv - K_\tau = 0 \quad (1)$$

where $h_\tau(x|S'^*_\tau, S'^*_{-\tau})$ is the pdf of the highest value of other entering firms (or the reserve price R if no value is higher than the reserve) in the auction, given equilibrium strategies. A pure strategy type-symmetric Bayesian Nash equilibrium exists because optimal entry thresholds for each type are continuous and decreasing in the threshold of the other type.

With multiple types, there can be multiple equilibria in the entry game when types are similar (for example, in mean values) even when we assume that only type-symmetric equilibria are played. As explained in Roberts and Sweeting (2011), we choose to focus on an equilibrium where the type with higher mean values has a lower entry threshold (lower

¹⁵In the empirical analysis of USFS open outcry auctions, our estimation procedure does not require that other losing bidders bid up to their values.

thresholds make entry more likely). This type of equilibrium is intuitively appealing and when firms' reaction functions are S-shaped (reflecting, for example, normal or lognormal value and signal noise distributions) and types only differ in the location parameters, μ_τ , of their value distributions then there is exactly one equilibrium of this form. We therefore make this assumption about the parameters in what follows.¹⁶ Solving for this equilibrium is straightforward: we find the S'^* values that satisfy the zero profit conditions for each type and which satisfy the constraint that $S_1'^* < S_2'^*$, where a type 1 firm is the high type (larger location parameter). It is important to note that the issue of type-symmetric multiple equilibria affects only the auction, not the sequential mechanism.

2.3 Mechanism 2: Sequential Mechanism

The standard alternative to buyers submitting bids simultaneously is a sequential bid process. Here we describe the simple sequential process that we consider, which is similar to the one in BK. In Section 5 we address how this mechanism could be practically implemented in settings such as USFS timber sales.

The seller places potential buyers into an order known to him and all buyers and may depend on type, but which does not depend on signals, and approaches each potential buyer in turn. We will call what happens between the seller's approach to one potential buyer and its approach to the next potential buyer a "round". In the first round, the first potential buyer observes his signal and then decides whether to enter the mechanism and learn his value by paying K . If he enters and his value is above a reserve price (which may be zero), he can choose to place a 'jump bid' weakly greater than the reserve price. If he enters and learns his value is less than the reserve price he does not participate in the mechanism. Given entry, submitting a bid is costless.

In the second round the potential buyer observes his signal, the entry decision of the first buyer and his jump bid, and then decides whether to enter himself. If the first firm did not enter and the second firm does, then the second firm can place a jump bid above the reserve price in exactly the same way as the first firm would have been able to do had he entered. If both enter, the firms bid against each other in a knockout button auction until one firm drops out, in which case it can never return to the mechanism. If the second firm wins a knockout it can choose to submit a jump bid above the standing price at the end of the knockout. In order to show uniqueness of the equilibrium in the game with more than

¹⁶When we take the model to data, we have also estimated it using a nested pseudo-likelihood procedure which does not require us to use an equilibrium selection rule. The parameter estimates in this case indicate that the difference in mean values between our two types (sawmills and logging companies) are so large that multiple equilibria cannot be supported.

two rounds, we assume that firms are only able to submit jump bids the first time that they become incumbents.¹⁷

This within-round bidding game is repeated for each remaining potential buyer, so that in each round there is at most one incumbent bidder and one potential entrant. If a firm drops out, or chooses not enter, it is assumed to be unable to re-enter at a later date. The good is allocated to the last remaining bidder at a price equal to the current bid. The set of potential entrants and the seller's chosen order are known to all players, who also observe all entry decisions and bids in previous periods.

A player's strategy in this mechanism consists of an entry rule and a bidding rule as a function of the round, the potential buyer's signal and value (for bidding) and the observed history.

2.3.1 Equilibrium with Pre-Entry Signals

We describe the equilibrium that we consider, before detailing the assumptions that lead us to focus on it and how it compares to the equilibrium in BK's model where there are no signals. For notation, we define \hat{b}_n^{pre} as the standing bid from the previous round ($\hat{b}_1^{pre} = 0$ or the reserve), \hat{b}_n as the standing bid after the knockout in round n , and $\beta_{n,\tau(n)}(v, \hat{b}_n)$ as the new incumbent's jump bid function when he is in round n and is type τ . $\Pi_{n,\tau(n)}(v, v', b)$ are the expected continuation profits of a new incumbent in round n when he has value v , future potential entrants believe that he has value v' and he submits jump bid b , given equilibrium behavior in subsequent rounds. The equilibrium we describe involves fully separating jump bidding behavior for bidders with values on $[\hat{b}_n, \bar{V} - K]$, which will include almost all bidders when \bar{V} is high, and pooling for bidders with values on $(\bar{V} - K, \bar{V}]$.

The Perfect Bayesian equilibrium is defined for each round of the game by: (i) an entry strategy for the potential entrant as a function of its signal and beliefs about the incumbent's value (if there is one); (ii) a bidding rule during the knockout phase; (iii) a jump bidding rule for a new incumbent at the end of the knockout phase; and, (iv) the beliefs of the potential entrant about the value of the incumbent given the observed history of the game.

(i) *Entry strategy:* A potential entrant in round n whose beliefs about the incumbent's value are described by the probability distribution function (pdf) $g_n(\tilde{v})$ and whose beliefs about its own value when it receives a signal S are given by the conditional pdf $f_{\tau(n)}^V(v|S)$, will enter if and only if it receives a signal above a threshold $S'^*_{n,\tau(n)}$ where $S'^*_{n,\tau(n)}$ satisfies the

¹⁷Toxvaerd (2010) studies a finite horizon dynamic limit pricing model and argues that a reasonable equilibrium selection rule, inspired by the D1 refinement, but not identical to it, would select the equilibrium where all signaling takes place in the first period.

following zero-profit condition for $n < N$

$$\int_{\widehat{b_n^{pre}}}^{\overline{V}} \int_{\widetilde{v}}^{\overline{V}} \Pi_{n,\tau(n)}(v, v, \beta_{n,\tau(n)}(v, \widetilde{v})) f_{\tau(n)}^V(v | S'_{n,\tau(n)}^{*}) g_n(\widetilde{v}) dv d\widetilde{v} - K = 0 \quad (2)$$

If the left-hand side is less than zero for all values of S (e.g., the entrant believes that the incumbent's value is greater than $\overline{V} - K$ for sure) then the entrant never enters. If there is no incumbent then the threshold is the one that solves

$$\int_R^{\overline{V}} \Pi_{n,\tau}(v, v, \beta_{n,\tau(n)}(v, R)) f_{\tau(n)}^V(v | S'_{n,\tau(n)}^{*}) dv - K = 0 \quad (3)$$

where R is the reserve price. In the final round there will be no jump bids by a firm who wins a knockout so that $S'_{N,\tau(N)}^{*}$ will satisfy

$$\int_{\widehat{b_N^{pre}}}^{\overline{V}} \int_{\widetilde{v}}^{\overline{V}} (v - \widetilde{v}) f_{\tau(N)}^V(v | S'_{N,\tau(N)}^{*}) g_N(\widetilde{v}) dv d\widetilde{v} - K = 0 \quad (4)$$

(ii) *Knockout bidding rule*: A bidder bids up to his value during a knockout auction.

(iii) *A jump bidding rule for a new incumbent in round $n < N$* : New incumbents with values on the interval $[\widehat{b}_n, \overline{V} - K]$ submit a bid from a bid function $\beta_{n,\tau(n)}(v, \widehat{b}_n)$ that is uniquely determined by the following differential equation (the j superscript denotes the partial derivative with respect to the function's j^{th} argument):

$$\frac{d\beta_{n,\tau(n)}(v, \widehat{b}_n)}{dv} = -\frac{\Pi_{n,\tau(n)}^2(v, v, \beta_{n,\tau(n)}(v, \widehat{b}_n))}{\Pi_{n,\tau(n)}^3(v, v, \beta_{n,\tau(n)}(v, \widehat{b}_n))} \quad (5)$$

and the lower boundary condition that $\beta_{n,\tau(n)}(\widehat{b}_n, \widehat{b}_n) = \widehat{b}_n$, which means that a new incumbent with value exactly \widehat{b}_n (i.e., the price at which the knockout ends) should submit a bid of \widehat{b}_n (no jump bid). As $\Pi_{n,\tau(n)}^2(\cdot) > 0$ and $\Pi_{n,\tau(n)}^3(\cdot) < 0$ there is full separation of types on the interval $[\widehat{b}_n, \overline{V} - K]$. New incumbents with values $(\overline{V} - K, \overline{V}]$ pool, submitting bids equal to $\beta_{n,\tau(n)}(\overline{V} - K, \widehat{b}_n)$. An entrant with a value less than the standing bid, or the reserve if there is no incumbent, does not submit a jump bid. There will be no jump bidding in the final round.

(iv) *Beliefs of the potential entrant about the value of an incumbent who submitted a jump bid in an earlier round m* : For a jump bid x in the interval $[\widehat{b}_m, \beta_m(\overline{V} - K, \widehat{b}_m)]$, the potential entrant will believe that the incumbent's value is $v' = \beta_{m,\tau(m)}^{-1}(x, \widehat{b}_m)$ where β^{-1} is the inverse of the bidding function; for a jump bid $\beta_{m,\tau(m)}(\overline{V} - K, \widehat{b}_m)$, the potential entrant will believe the incumbent's value lies on the interval $[\overline{V} - K, \overline{V}]$ with the pdf given by Bayes Rule; for

bids strictly greater than $\beta_{m,\tau(m)}(\bar{V} - K, \hat{b}_m)$, which will never be observed on the equilibrium path, we will assume for completeness that future potential entrants believe that such a bid is by an incumbent with value exactly $\bar{V} - K$.

Given equilibrium behavior, $\Pi_{n,\tau}(v, v', b)$ will be:

$$\Pi_{n,\tau(n)}(v, v', b) = [v - b] [\bar{F}_{n,\tau(n)}(b|v')] + \int_b^v (v - x) \bar{f}_{n,\tau(n)}(x|v') dx \quad (6)$$

where $\bar{F}_{n,\tau(n)}(x|v')$ is the probability that the maximum value of a future entrant is less than x when potential entrants believe that the incumbent's value is v' , and $\bar{f}_{n,\tau(n)}(x|v')$ is its derivative with respect to x . Explicitly:

$$\bar{F}_{n,\tau(n)}(x|v') = \prod_{k=n+1}^N \left[\int_0^x f_{\tau(k)}^V(y) dy + \int_x^{\bar{V}} F_{S,\tau(k)}(S_{k,\tau(k)}^*(v')|y) f_{\tau(k)}^V(y) dy \right] \quad (7)$$

where $F_{S,\tau(k)}(S|y)$ is the conditional cdf of the signal for a type τ potential entrant in round k when its value is y .

We can show that equilibrium bidding and entry strategies are unique under additional, and quite standard, assumptions. First, we assume that all players use the weakly dominant strategy of bidding up to their value in a knockout auction both on and off the equilibrium path. This eliminates the possibility of an equilibrium where some incumbent never drops out in a knockout auction, and later potential entrants do not enter.¹⁸ Second, we place two restrictions on the inferences that later potential entrants can make when observing an off-the-equilibrium path bid. The first restriction, consistent with a sequential equilibrium (Fudenberg and Tirole (1991)), is that all potential entrants draw the same inference about an incumbent's type when observing the jump bid. The second restriction, consistent with the D1 refinement (Cho and Sobel (1990) and Ramey (1996)), is that when they observe an off-the-equilibrium-path jump bid, potential entrants place zero posterior weight on the deviating incumbent having value v if an incumbent with value v' would strictly prefer the deviation for any inferences that the future potential entrants could make which would give the v incumbent a weak incentive to deviate (Ramey (1996), p. 516). This restriction allows us to rule out the existence of pooling equilibria.

We show that the equilibrium exists and is unique in the Appendix, but it is worth highlighting the features of the game that lead to these results. All else equal, an incumbent's expected profits are higher when future potential entrants are less likely to enter for any value. This provides an incentive for an incumbent to raise its jump bid in order to distinguish itself

¹⁸It also allows us to specify the incumbent's continuation payoff when placing a jump bid as only a function of his jump bid, not his subsequent knockout bidding strategy which is determined by our assumption.

from incumbents with lower values.

A single crossing condition on the incumbent's expected payoffs ensures that a higher value incumbent who faces the possibility of entry, will always be willing to pay more than a lower value incumbent to raise the potential entrant's belief about his value. Mailath and von Thadden (2011), Theorems 2 and 4, show how single crossing, together with other conditions, leads to the existence of a unique separating equilibrium bid schedule that satisfies differential equation (5) and the lower boundary condition. The D1 refinement on how out-of-equilibrium actions are interpreted ensures that incumbents with values less than $\bar{V} - K$ will not pool. Intuitively, if an incumbent in any hypothetical pool deviated by submitting a slightly higher bid, single crossing implies that, under the refinement, this deviation would be interpreted as being made by the highest valued incumbent in the pool, providing the highest type in a pool with a strict incentive to deviate. This logic does not apply for incumbents with values greater than $\bar{V} - K$ who do pool in equilibrium. No potential entrant will enter against these incumbents, so there is no incentive for any member of the pool to raise its bid in order to try to signal that it has a higher value.

Given the nature of this equilibrium we can solve the game recursively. Full details are given in the Appendix. For the final potential entrant, who believes that he will win if his value is greater than the incumbent's (in which case the final price will be the incumbent's value), we can solve for the equilibrium entry thresholds for a grid of values of an incumbent firm. Next we consider the previous potential entrant. Assuming that this firm becomes an incumbent, we solve for its equilibrium bid functions as a function of the standing bid using the final round thresholds, and we then use the expected profits that this bid function and the final round thresholds imply to compute its entry thresholds for the grid of values for an incumbent in the previous round. We then repeat these steps for the previous potential entrants until we reach the first round, where there is no incumbent.

2.3.2 Comparison with BK's Equilibrium

BK show that when there are no pre-entry signals and firms are *ex ante* symmetric equilibrium jump bidding and entry is quite different to our model. They find a partial pooling equilibrium where incumbent bidders separate into two groups. In particular, BK show that when an incumbent's value is less than some endogenously determined cut-off V^S , which is the same across rounds, the incumbent submits no jump bid and all later potential entrants respond by entering. Incumbents with values more than V^S post a common jump bid which deters all future entry, reflecting the fact that all future potential entrants have common information

about their values.¹⁹ In equilibrium, entry therefore happens in every round until a potential entrant has a value above V^S when it ceases forever. BK show that this outcome is more efficient than the auction, because it economizes on entry costs, but that from the seller's standpoint, there is excess deterrence, reducing revenues.

2.3.3 Illustrative Example of the Sequential Mechanism's Equilibrium

To provide additional clarity about how the mechanism works, given equilibrium strategies, Table 1 presents what may happen in a game with four potential entrants and one type of firm with values distributed proportional to $LN(4.5, 0.2)$ on $[0, 200]$, $K = 1$ and $\sigma_\varepsilon = 0.2$ ($\alpha = 0.5$).

Round	Initial		Potential Entrant			Post-Knockout Standing Bid	Post-Jump Bid Standing Bid
	Standing Bid	Value	Signal	S'^*	Entry		
1	-	80.0	90.1	75.0	Yes	-	69.3
2	69.3	75.4	50.5	69.4	No	69.3	69.3
3	69.3	116.0	114.9	61.7	Yes	80.0	87.1
4	87.1	100.0	114.0	107.0	Yes	100.0	100.0
Seller's Revenue = 100.0, social surplus (winner's value less total entry costs) = 113.0							

Table 1: A simple example of how the sequential mechanism works in a game with four potential entrants and one type of firm with values distributed proportional to $LN(4.5, 0.2)$ on $[0, 200]$, $K = 1$ and $\sigma_\varepsilon = 0.2$.

The first potential entrant enters if he receives a signal greater than 75.0, which is the case here. The signal thresholds in later rounds depend on the number of rounds remaining and the incumbent's value. So, when the incumbent is the same as in the previous round, the threshold S'^* falls since the expected profits of an entrant who beats the incumbent rise (because he will face less competition in the future). On the other hand, S'^* does not depend on the level of the standing bid given the incumbent's value, because it has no effect on the entrant's profits if he beats the incumbent in a knockout (since the standing bid must be below the incumbent's value). In round 2, the incumbent does not face entry, so there is no change in the standing bid because incumbents do not place additional jump bids.²⁰ In round 3, the standing bid rises during the knockout, and the *new* incumbent places an additional

¹⁹ V^S is independent of the round of the game and history to that point. It is determined by the condition that future potential entrants are indifferent to entering if they know that the incumbent firm's value is at least V^S . The level of the deterring bid, which may differ by round, is determined by the condition that a firm with value exactly V^S is indifferent between submitting the bid and deterring all future entry and submitting no jump bid and having entry occur.

²⁰There would also have been no change in the standing bid if the entrant had come in, because the entrant's value was below the current bid, so the standing bid would not have risen in the knockout.

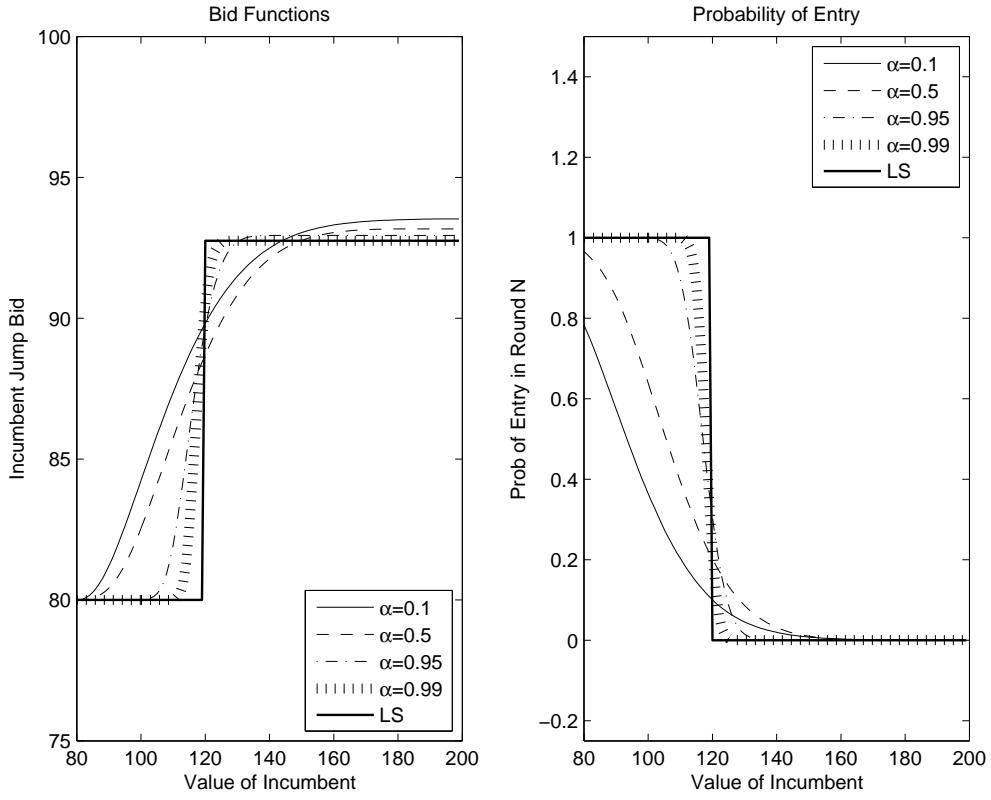


Figure 1: Penultimate round bid function for a new incumbent and probability of entry for the final round potential entrant, with symmetric firms, values $LN(4.5, 0.2)$ on $[0, 200]$, $K = 1$ and standing bid of 80.

jump bid. In round 4 the last potential entrant participates, but his value is less than the incumbent's and so revenue is the price at which this last entrant drops out.

We can also use this example to give intuition for how introducing selection affects bid functions and entry probabilities. With selection, the level of bids is determined by the fact that bids must be sufficiently high that firms with lower values will not want to copy them. In particular, if the entry decisions of later potential entrants are likely to be more sensitive to beliefs about the incumbent's value, then the equilibrium bid function must increase more quickly in v . A straightforward way to illustrate this is to focus on the last two rounds when a new incumbent in the penultimate round only needs to worry about one more potential entrant, and the final round potential entrant would face no further entry if he enters and outbids the incumbent. An example is shown in Figure 1, which compares the equilibrium bid functions in the penultimate round, and equilibrium probabilities of entry in the final round of the sequential mechanism for varying degrees of selection.

The left panel displays bid functions for a new incumbent in the penultimate round, when the previous incumbent's value was 80. The right panel gives the probability that the final round potential entrant participates as a function of this new incumbent's value. Successively lower degrees of selection change the bid function so that when $\alpha \rightarrow 1$ it approaches the bid

function in the LS (no selection) model (the bold line), which is a step function that jumps at a value of 119 (the level of the incumbent's value that deters all future entry). The slope of the bid function is more gradual for lower α s since the probability that the final round potential entrant participates declines more smoothly when α is lower.

3 Comparison of Expected Revenues and Efficiency

This section compares the revenue and efficiency performance of the sequential mechanism and simultaneous auction for a wide range of entry cost (K) and selection (α) parameters. This allows us to show that the sequential mechanism performs better than the auction quite generally, and to investigate what factors lead to the difference in performance. In Section 5 we will perform a more detailed comparison using particular parameters estimated from USFS timber auctions.

As a base case, we consider 8 symmetric potential entrants whose values are distributed $LN(4.5, 0.2)$ so that the value distribution has a mean of 91.6 and a standard deviation of 18.6.²¹ Additionally, we allow an optimal reserve price to be used in the simultaneous auction but restrict attention, for now, to a sequential mechanism with no reserve. In this way the results are biased *against* a seller preferring the sequential mechanism.

Figure 2 shows the results of comparing expected revenues from the sequential mechanism (with no reserve) and a simultaneous entry auction with an optimal reserve in (K, α) space.²² Filled squares represent outcomes where the expected revenues from the sequential mechanism are higher by more than 4% (of auction revenues), while hollow squares are outcomes where they are higher but only by between 0.1% and 4%. Diamonds represent cases where the simultaneous auction gives higher revenues. Crosses on the grid mark locations where the difference in revenues is less than 0.1%. Due to the possibility of small numerical errors in solving differential equations and simulation error in calculating expected revenues, we take the conservative approach of not signing revenue differences in these cases.

²¹The comparisons in this section are based on a lognormal distribution of values. We do this to ease comparison with our empirical framework which also models values as being distributed lognormally. However, we have verified that the qualitative conclusions we draw in this section hold for alternative value distributions such as normal, exponential, Weibull and gamma.

²²Sequential (auction) mechanism's expected revenues are calculated using 200,000 (5,000,000) simulations. The optimal reserve price is calculated by numerically solving for the revenue maximizing reserve price, where revenues are calculated via simulation using the same set of simulations. That is, for each candidate reserve price and number of bidders we solve for the equilibrium entry thresholds in the auction and the equilibrium entry thresholds and jump bidding functions in the sequential mechanism. Then we draw $N \times S$ simulated values, where N is the number of bidders per auction and S is the number of simulated mechanisms and compute the average revenue across the S simulated mechanisms based on the equilibrium for that reserve price and number of bidders. The Appendix contains more details of our calculation of optimal reserve prices.

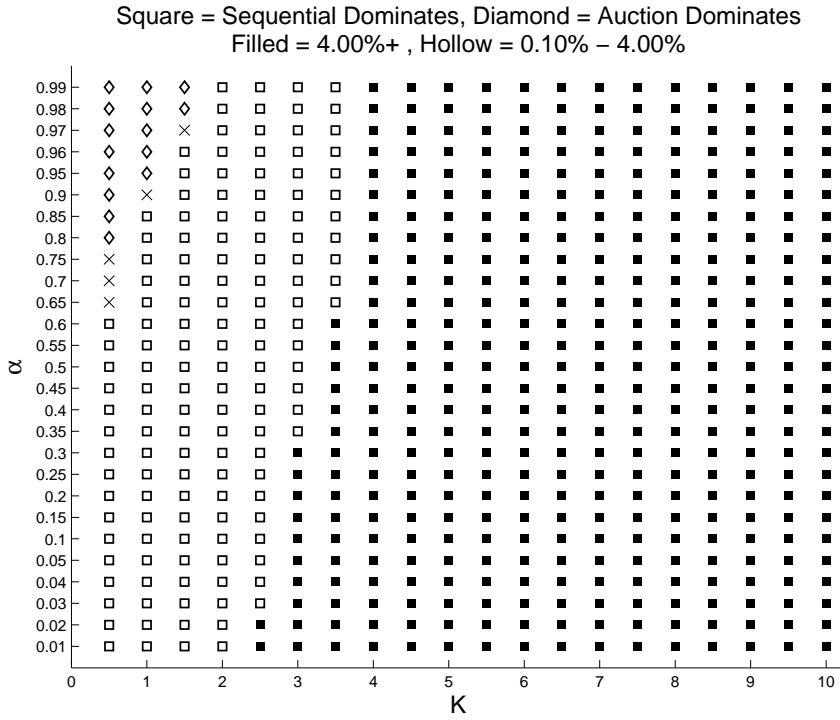


Figure 2: Expected revenue comparison for 8 symmetric firms, values $LN(4.5, 0.2)$, optimal reserve price in auction, no reserve price in sequential mechanism.

The sequential mechanism generally gives higher revenues than the auction, even though the auction has an optimal reserve. As entry costs rise, the differences in revenues can be large. For example, when $\alpha = 0.5$ and $K = 6$ (6.5% of the average value), the sequential mechanism gives expected revenues that are 7.3% higher than the auction. When $\alpha = 0.5$ and $K = 10$, expected revenues in the sequential mechanism are 11.5% higher. The only cases where the auction does better are when both K is very low and α is close to 1 (little selection). These points are consistent with BK's results as their assumptions require no selection and that at least two firms enter the auction, implying low entry costs. However, in these cases the revenue advantage of the auction is small (at most it can raise expected revenues by 1.1%).

Why does the sequential mechanism perform better? The natural way to analyze this question is in terms of the expected total surplus that each mechanism produces, measured as the value of the winner less total entry costs, itself an economic outcome of interest, and then the seller's ability to extract as much of this surplus as possible. BK show that with no selection the sequential mechanism is more efficient (generates more surplus). The sequential mechanism remains more efficient in our model, dominating the auction for every grid point

in Figure 2. There are two reasons for this. First, it is generally true that there is less entry into the sequential mechanism. For example, continuing with the case from above, when $\alpha = 0.5$ and $K = 6$ the average number of entrants into the sequential mechanism is 1.59, which is less than the average number of entrants into the auction, 2.66. Second, the expected winner's value can be higher in the sequential mechanism because later potential entrants will enter if the incumbent's value is low. For example, when $K = 1$ the expected winner's value is higher in the sequential mechanism if $\alpha < 0.25$, and it is always higher for larger values of K . An following with the example from above, when $\alpha = 0.5$ and $K = 6$ the expected value of the winner in the sequential mechanism is 115.68, whereas the expected value of the winner in the auction is 113.92.

With no selection, BK show that bidders capture all of the additional surplus created by the sequential mechanism, reflecting the ability of early entrants to deter entry, and reducing the seller's expected revenues below the level in the simultaneous auction. The seller's revenue in the auction is simply the second highest value among the entrants, whereas its revenue in the sequential mechanism is the maximum of the second highest value among the entrants and the jump bid submitted by the winner. Lower entry into the sequential mechanism reduces the expected second highest value, and when there is no selection, even high value incumbents may be able to submit relatively low, but fully deterring, jump bids in the partial pooling equilibrium. When there is selection, the greater ability of the sequential mechanism to select high value entrants tends to increase the second highest value among the entrants, just as it does for the highest value entrant, but for all the grid points in Figure 2 the expected value of this statistic is higher in the auction. However, by changing the nature of equilibrium jump bidding, selection can also raise the jump bid submitted by incumbents in the sequential mechanism.

An example of this effect can be seen in Figure 1, which shows jump bidding functions in the penultimate round when the previous incumbent had a value of 80 for different values of α . When the new incumbent has a value less than 119, the bid functions with selection lie above the bid function with no selection (LS). These values are the ones that are most likely to be observed in practice: for example, when $\alpha = 0.1$, the mean and 90th percentile values of a new penultimate round incumbent who would find himself submitting a jump bid are 101 and 121, respectively. With selection, these jump bids reduce the probability of final round entry (right panel), but this reduction will be smaller for potential entrants with values above the jump bid, whose entry is valuable to the seller. The figure also shows that, with selection, equilibrium jump bids are also higher for incumbents with very high values. This reflects their incentive to submit higher jump bids to reduce the probability that a high value potential entrant will enter in the final round.

In many settings, such as in the USFS timber sales studied below, buyers will be asymmetric. In a simultaneous entry auction weaker bidders need to consider the odds of competing against stronger bidders. While this is still true in the sequential mechanism, if the weaker bidders are approached last, in the separating equilibrium they know the value of the highest strong type that has entered. This permits more efficient entry of the weaker bidders and achieves a more efficient allocation of the good relative to the auction. For example suppose that $K = 5$, $\alpha = 0.4$, $N = 4$ and the first two bidders approached have values proportional to $LN(4.5, 0.2)$, while the last two bidders approached have values proportional to $LN(4.4, 0.2)$.²³ Each of the weaker firms enters the simultaneous auction with probability 0.20 and the probability that one of them wins is only 0.17. On the other hand, in the sequential mechanism the entry probabilities are similar (0.204 for the first and 0.175 for the second) but the probability that one of them wins increases to 0.283. This is much closer to the probability that one of the weaker firms will have the highest value (0.33). In this case, the sequential mechanism's expected revenues (efficiency) of 83.34 (97.96) exceed those of the auction, which are 78.40 (94.05). When bidders are asymmetric, sellers may prefer a first price auction with type-specific reserve prices to a second price auction with a uniform reserve. However, continuing with this example, even a first price auction with type-specific optimal reserve prices only generates expected revenues of 80.41, and so it is outperformed by the sequential mechanism with no reserve price.

The ability of the sequential mechanism to more efficiently allocate goods to weaker bidders while also raising revenues is particularly relevant given the commitment of many government agencies to award a certain proportion of contracts to minority-owned firms and small businesses who are likely to be weaker bidders. For example, the federal government seeks to award at least 23% of its \$400 billion of annual contracts to small businesses (Athey, Coey, and Levin (2011)). Existing methods for achieving these distributional goals include bid subsidies for preferred firms, which may allow these firms to win even when they have lower values, and set-asides, where other firms are prevented from participating. In the context of USFS timber auctions, Athey, Coey, and Levin (2011) show that the set-aside program that was used created significant revenue and efficiency losses relative to a counterfactual bid subsidy program, while in the context of highway procurement, Krasnokutskaya and Seim (2011), find that bid subsidies have only small effects on procurement costs. Our results suggest that the sequential mechanism may provide a mechanism for achieving distributional goals, while increasing efficiency and revenues, without requiring the seller to know the type of information required to compute an optimal bid subsidy program, although we leave a

²³Our simulations show that approaching all of the high value firms first, followed by all of the low value firms is better than doing the opposite.

more complete analysis of this issue to future work.

Our computations show that a simple, stylized version of real-world sequential mechanisms tends to outperform the commonly used auction, even when the optimal reserve price is set in the auction. The sequential mechanism's advantage over the auction could be increased through additional design elements, an obvious example being a reserve price. Figure 3 computes expected revenues when an optimal reserve price is added to each mechanism when there are five or eight symmetric bidders using the same value distribution parameters as before and assuming $K = 5$. For the sequential mechanism, only one reserve price is used, which is constant across all rounds in the mechanism. Generally, the seller could do better with a round-specific reserve price, but we view a constant reserve price as approximately imposing the same informational demands on the seller as does setting the optimal reserve price in the simultaneous auction.

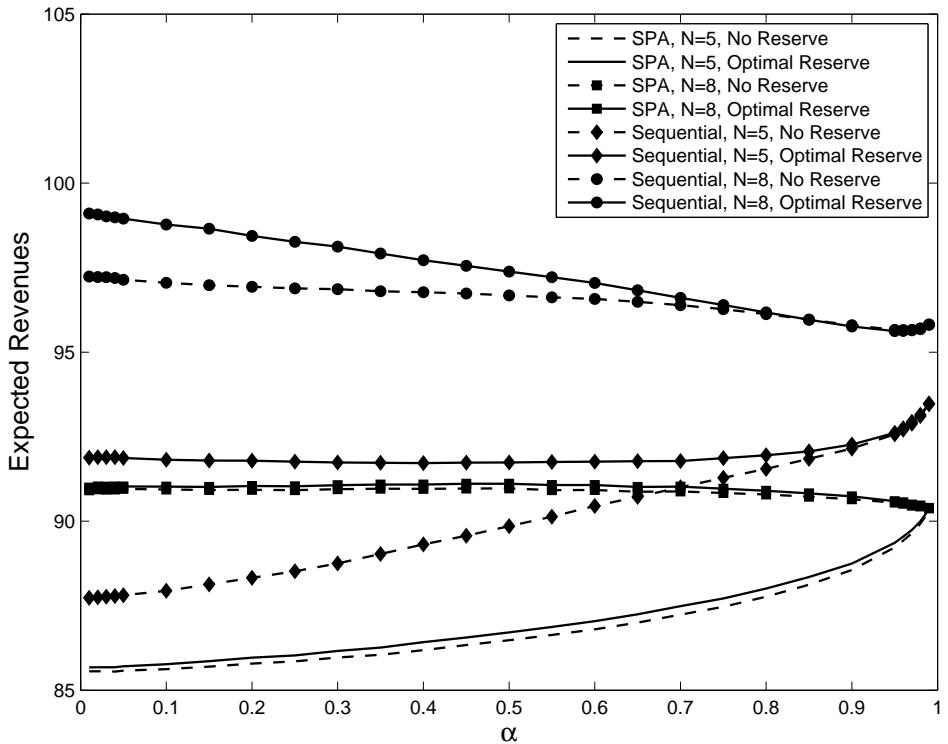


Figure 3: Expected revenue comparison for varying N , with and without reserves. Firms are symmetric with values distributed $LN(4.5, 0.2)$ and $K = 5$.

Figure 3 shows that when α is low, in contrast to the simultaneous auction, adding a constant reserve price to the sequential mechanism may substantially improve revenues. The reserve price affects sequential mechanism revenues in two ways. First, in the event that no

firm has entered through the first $N - 1$ rounds, a reserve price guards the seller against giving the good away for free to the last potential entrant. Second, a reserve price raises the first entrant's deterring bid function.

The efficacy of a reserve price varies across mechanisms and across different values of N and α . There are two main reasons for this. First, when entry is endogenous, a reserve price has a smaller impact when the level of entry is greater, as is generally the case (i) in the auction or (ii) when N is greater (unless α is very close to 1), as is clearly shown in Figure 3. Second, a reserve price excludes some bidders and if these are valuable to the seller, this reduces the value of a reserve price. This effect can be seen in Figure 3 by noticing that the impact of a reserve price in the sequential mechanism falls for higher values of α : less selection implies that marginal and inframarginal entrants are more similar, which makes the exclusion of marginal bidders more costly to the seller (it is also true that the level of entry increases in α , which also limits a reserve price's impact).

Using the parameters in Figure 3 and $N = 5$, we have also investigated the importance of our assumption that bidders make simultaneous entry decisions into the auction, as well as simultaneous bids. An alternative, even though it has not been considered in the empirical literature on auctions, is sequential entry into the auction so that later firms can condition on the participation decisions, but not values, of the firms that move earlier.²⁴ In general, sequential entry increases expected revenues and efficiency in the auction by lowering the probability that very few or very many firms enter. However, the sequential mechanism, which, in equilibrium, also allows later firms to more efficiently condition their entry decisions on the values of earlier entrants, still outperforms the auction in terms of revenues. For example, with no reserve prices, when $\alpha = \{0.1, 0.5, 0.9\}$, the sequential mechanism gives expected revenues of $\{88.0, 89.8, 92.3\}$, the sequential entry auction $\{87.3, 87.2, 89.6\}$ and the simultaneous entry auction $\{85.6, 86.4, 88.6\}$.²⁵

²⁴There is an increased computational burden in solving for equilibrium in the sequential entry, simultaneous bid auction model. This arises from the fact that later potential entrants' equilibrium entry thresholds are a function of the complete history of the game and thresholds in earlier rounds, so that it is necessary to solve for all of the thresholds simultaneously. In contrast, in the sequential mechanism we consider, a potential entrant's equilibrium threshold only depends on the value of the incumbent which (with any degree of selection) is completely revealed by its jump bid.

²⁵We have also computed expected revenues for the sequential entry auction using the parameters from USFS auctions reported in Table 4. For these parameters the sequential mechanism also gives higher expected revenues and efficiency than the sequential entry auction.

4 Empirical Application

We now turn to our empirical application that focuses on USFS timber auctions. These auctions provide a good fit to the informational assumptions of our model. Moreover, unlike other environments where this might be true, such as the M&A market, we can convincingly estimate the parameters of the model because we see many similar objects (tracts of timber) being sold. Finally, while a great deal of work has concentrated on auction design tools, such as reserve prices, as means to increasing revenues in timber auctions, we show that a shift to a sequential sales process has a much larger impact. We are brief in our discussion of the data, estimation and reduced form evidence of selection as Roberts and Sweeting (2011) discuss these topics in detail.

4.1 Data

We analyze federal auctions of timberland in California. In these auctions the USFS sells logging contracts to individual bidders who may or may not have manufacturing capabilities (mills and loggers, respectively). When the sale is announced, the USFS provides its own “cruise” estimate of the volume and value of timber for each species on the tract as well as estimated costs of removing and processing the timber. It also announces a reserve price and bidders must indicate a willingness to pay at least this amount to qualify for the auction. After the sale is announced, interested potential bidders perform their own private cruises in order to assess the tract’s value. These cruises are informative about the tract’s volume, species make-up and timber quality.

We assume that bidders have independent private values. This assumption is also made in other work with similar timber auction data (see for example Baldwin, Marshall, and Richard (1997), Haile (2001) or Athey, Levin, and Seira (2011)). A bidder’s private value is primarily related to its own contracts to sell the harvest, inventories and private costs of harvesting. In addition, we focus on the period 1982-1989 when resale, which can introduce a common value element, was limited (see Haile (2001) for an analysis of timber auctions with resale).

We also assume non-collusive bidder behavior. While there has been some evidence of bidder collusion in open outcry timber auctions, Athey, Levin, and Seira (2011) find strong evidence of competitive bidding in these California auctions.

Our model assumes that bidders receive an imperfect signal of their value and they must pay a participation cost to enter the auction.²⁶ We interpret the USFS’s publicly available

²⁶We note that we are not the first to model endogenous entry decision into these auctions (e.g. Athey, Levin, and Seira (2011)).

tract appraisal and a firm's own knowledge of its sales contracts and capabilities as generating its pre-entry signal. Participation in these auctions is costly for numerous reasons. In addition to the cost of attending the auction, a large fraction of a bidder's entry cost is its private cruise. People in the industry tell us that firms do not bid without doing their own cruise, which can provide information that bidders find useful, such as trunk diameters, but is not provided in USFS appraisals.

We use data on 887 ascending auctions.²⁷ Table 2 shows summary statistics for our sample. Bids are given in \$ per thousand board feet (mbf) in 1983 dollars. The average mill bid is 20.3% higher than the average logger bid. As suggested in Athey, Levin, and Seira (2011), mills may be willing to bid more than loggers due to cost differences or the imperfect competition loggers face when selling felled timber to mills.

Variable	Mean	Std. Dev.	25 th -tile	50 th -tile	75 th -tile	N
WINNING BID (\$/mbf)	86.01	62.12	38.74	69.36	119.11	847
BID (\$/mbf)	74.96	57.68	30.46	58.46	105.01	3426
LOGGER	65.16	52.65	26.49	49.93	90.93	876
MILL	78.36	58.94	32.84	61.67	110.91	2550
LOGGER WINS	0.15	0.36	0	0	0	887
FAIL	0.05	0.21	0	0	0	887
ENTRANTS	3.86	2.35	2	4	5	887
LOGGERS	0.99	1.17	0	1	1	887
MILLS	2.87	1.85	1	3	4	887
POTENTIAL ENTRANTS	8.93	5.13	5	8	13	887
LOGGER	4.60	3.72	2	4	7	887
MILL	4.34	2.57	2	4	6	887
SPECIES HHI	0.54	0.22	0.35	0.50	0.71	887
DENSITY (hundred mbf/acre)	0.21	0.21	0.07	0.15	0.27	887
VOLUME (hundred mbf)	76.26	43.97	43.60	70.01	103.40	887
RESERVE (\$/mbf)	37.47	29.51	16.81	27.77	48.98	887
SELL VALUE (\$/mbf)	295.52	47.86	260.67	292.87	325.40	887
LOG COSTS (\$/mbf)	118.57	29.19	99.57	113.84	133.77	887
MFCT COSTS (\$/mbf)	136.88	14.02	127.33	136.14	145.73	887

Table 2: Summary statistics for sample of California ascending auctions from 1982-1989. All monetary figures in 1983 dollars. SPECIES HHI is the Herfindahl index for wood species concentration. SELL VALUE, LOG COSTS and MFCT COSTS are USFS estimates of the value of the tract and the logging and manufacturing costs of the tract, respectively.

We define potential entrants as the auction's bidders plus those firms who bid within 50 km of an auction over the next month. One way of assessing the appropriateness of this definition is that 98% of the bidders in any auction also bid in another auction within 50

²⁷Roberts and Sweeting (2011) include a detailed description of the sample selection process.

km of this auction over the next month and so we are unlikely to be missing many actual potential entrants. The median number of potential bidders is eight (mean of 8.93) and this is evenly divided between mills and loggers.

In Table 2, entrants are defined as the set of bidders we observe at the auction, even if they did not submit a bid above the reserve price.²⁸ The median number of mill and logger entrants are three and one, respectively. Among the set of potential logger entrants, on average 21.5% enter, whereas on average 66.1% of potential mill entrants enter. The differences in bids and entry decisions are consistent with mills having significantly higher values than loggers.²⁹

4.2 Evidence of Selection

Roberts and Sweeting (2011) present reduced form evidence that the data are best explained by a model allows for selection. There are two main pieces of evidence. First, Athey, Levin, and Seira (2011) show that in the type-symmetric mixed strategy equilibrium of a model with endogenous, but non-selective, entry and asymmetric bidder types, whenever the weaker type enters with positive probability, the stronger type enters with probability one. Thus, for any auction with some logger entry, a model with no selection would imply that all potential mill entrants enter. In 54.5% of auctions in which loggers participate, and there are some potential mill entrants, some, but not all, mills participate. Likewise, they show that whenever the stronger type enters with probability less than one, a model with no selection implies that weaker types enter with probability zero. However, in the data we find that in 61.1% of auctions in which only some mill potential entrants participate and potential logger entrants exist, some loggers enter. A model with selective entry can rationalize partial entry of both bidder types into the same auction.

Second, a model without selection implies that bidders are a random sample of potential entrants. Roberts and Sweeting (2011) test this by estimating a Heckman selection model with the exclusion restriction that potential competition affects a bidder's decision to enter an auction, but has no direct effect on values. The second stage regression of all bids on auction covariates and the estimated inverse Mills ratio from a first stage probit of the decision to participate shows a positive and highly significant coefficient on the inverse Mills ratio. This is consistent with bidders being a selected sample of potential entrants.

²⁸However, in our empirical specification below, we interpret the data more cautiously and allow bidders that do not submit bids to have entered (paid K), but learned that their value was less than the reserve price.

²⁹Roberts and Sweeting (2011) present evidence that differences in values, and not entry costs, explain why mills are more likely than loggers to enter an auction.

While these tests strongly suggest a selective entry process, we need to estimate the model to recover α and perform revenue comparisons between alternative sale mechanisms.

4.3 Estimation

We estimate the model using Ackerberg (2009)'s method of simulated maximum likelihood with importance sampling, which we detail in the Appendix and Roberts and Sweeting (2011). This method requires that we allow for cross-auction heterogeneity in all of the structural parameters, which is also realistic in our setting as our sample auctions come from different forests and several different years, and also differ greatly in observed characteristics such as sale value, size and wood type. Our chosen specification uses the following parametric distributions where X_a are observed auction characteristics and $TRN(\mu, \sigma^2, a, b)$ is a truncated normal distribution with upper and lower truncation points a and b .

$$\text{Location Parameter of Logger Value Distribution: } \mu_{a,\text{logger}} \sim N(X_a\beta_1, \omega_{\mu,\text{logger}}^2)$$

$$\text{Difference in Mill/Logger Location Parameters: } \mu_{a,\text{mill}} - \mu_{a,\text{logger}} \sim TRN(X_a\beta_3, \omega_{\mu,\text{diff}}^2, 0, \infty)$$

$$\text{Scale Parameter of Mill and Logger Value Distributions: } \sigma_{V_a} \sim TRN(X_a\beta_2, \omega_{\sigma_V}^2, 0.01, \infty)$$

$$\alpha: \alpha_a \sim TRN(\beta_4, \omega_\alpha^2, 0, 1)$$

$$\text{Entry Costs: } K_a \sim TRN(X_a\beta_5, \omega_K^2, 0, \infty)$$

This specification reflects our assumption that σ_V, α and K are the same for mills and loggers within any particular auction. It also assumes that observed variables, such as wood type, do not affect the degree of selection (α) across auctions. We have estimated a number of specifications allowing for observed variables to affect α without finding economically or statistically significant effects.

To apply the estimator, we also need to define the likelihood function based on the open outcry auction data. Two problems arise when interpreting these data. First, a bidder's highest announced bid in an open outcry auction may be below its value, and it is not obvious which mechanism leads to the bids that are announced (Haile and Tamer (2003)). Second, if a firm does not know its value when taking the entry decision, it may learn (after paying the entry cost) that its value is less than the reserve price and so not submit a bid. We take a conservative approach (the details of which are provided in the Appendix) when interpreting the data by assuming that the winning bidder has a value greater than the second highest bid, the second highest observed bid is equal to the value of the second-highest bidder, all other bidders had values less than the highest observed bid and that potential entrants that we do not see bid may or may not have paid the entry cost. Standard errors are calculated

using a bootstrap where auctions, and importance sampling draws, are re-sampled 100 times.

4.3.1 Identification

While we make parametric assumptions to estimate the model, we informally discuss identification here. Gentry and Li (2012) study non-parametric identification of a class of selective entry auction models, assuming no unobserved auction heterogeneity.³⁰ They show that entry costs and the joint distribution of signals and values are exactly identified when there is sufficient exogenous variation in equilibrium entry thresholds, and that otherwise they can be bounded. Variation in entry thresholds (S'^*) can be created by variation in the number of potential entrants (possibly of different types), variables affecting entry costs (although we do not find statistically significant evidence that observed variables affect these) and reserve prices. The following argument provides intuition for identification. Suppose that in some auctions, expected competition and the reserve price are low so that all potential bidders enter and there is no selection. In this case, standard identification arguments (see Athey and Haile (2002)) imply identification of the unconditional value distributions. The degree of selection in the entry process can then be identified by how similar the value distributions of entrants are to these unconditional distributions when entry thresholds are higher and not all firms enter. For example, if entry is very selective (low α in our model) then the value distributions of entrants will be almost perfectly truncated around the threshold, whereas with no selection they will be similar to the unconditional distributions. The level of entry costs, K , will be identified by the fact that a potential entrant receiving the threshold signal must expect zero profits from entering.

Identification is more difficult in the presence of unobserved auction heterogeneity in values, which is generally viewed as an important feature of timber auction data (Athey, Levin, and Seira (2011)). In fact, even with full entry, value distributions are not exactly identified unless all bids are observed and are directly informative about values (Theorem 4 of Athey and Haile (2002)). Bids in open outcry auctions clearly cannot be interpreted in this way (e.g. Haile and Tamer (2003)). These issues explain why we, like Athey, Levin, and Seira (2011) and Krasnokutskaya and Seim (2011), use a parametric approach that also lets us allow for unobserved heterogeneity in entry costs and the degree of selection.

³⁰In their conclusion they consider identification with unobserved heterogeneity that it is only revealed to bidders after they enter. In contrast, we assume that they know factors that shift mean values prior to making entry decisions.

5 Empirical Results

In this section we present estimates of our structural model and counterfactual results measuring the benefits to the USFS of switching from the current simultaneous entry and simultaneous bid auction to our simple sequential process.

5.1 Parameter Estimates

Table 3 presents the parameter estimates for our structural model. We allow the USFS estimate of sale value and its estimate of logging costs to affect mill and logger values and entry costs since these are consistently the most significant variables in regressions of reserve prices or winning bids on observables, including controls for potential entry. We also control for species concentration since our discussions with industry experts lead us to believe that can matter to firms. The righthand columns show the mean and median values of the structural parameters when we take 10 simulated draws of the parameters for each auction. For the rest of the paper, we refer to these as the “mean” and “median” values of the parameters.

The coefficients show that tracts with greater sale values and lower costs are more valuable, as one would expect. There is unobserved heterogeneity in values across auctions (the standard deviation of μ_{logger}) and some unobserved heterogeneity in the difference between mill and logger mean values across auctions (the standard deviation of $\mu_{\text{mill}} - \mu_{\text{logger}}$).

Based on the mean value of the parameters, the mean values of mills and loggers in the population are, in 1983 dollars, \$61.95/mbf and \$42.45/mbf, respectively, a 46% difference. We estimate a mean entry cost of \$2.05/mbf, also in 1983 dollars. One forester we spoke with estimated modern day cruising costs of approximately \$6.50/mbf, or \$2.97/mbf in 1983 dollars. It is sensible that our estimate is less than the forester’s estimate if firms in our data are able to use any information they learn when deciding whether to enter other auctions.

Our estimates of the α s across auctions indicate a moderate amount of selection in the data. This is illustrated by the difference in expected values for marginal and inframarginal bidders in a representative auction where the reserve price and the number of potential mill and logger entrants are set to their respective medians of \$27.77/mbf, four and four. Based on the mean parameter values, the expected values of a marginal and inframarginal mill entrant are \$45.22/mbf and \$68.13/mbf, respectively (the former is lower than the population average because most mills enter). The comparable numbers for loggers are \$48.13/mbf and \$59.80/mbf, respectively.

Our estimation approach assumes that, if there are multiple equilibria, the firms will play the equilibrium where mills have the lower S''^* . We can check whether our parameter esti-

Parameter	β parameters					ω parameter
	Constant	log SELL VALUE	log LOG COSTS	SPECIES HHI	Mean	Median
$\mu_{a,\text{logger}}$ $\sim N(X_a\beta_1, \omega_{\mu,\text{logger}}^2)$	-9.6936 (1.3690)	3.3925 (0.1911)	-1.2904 (0.1332)	0.2675 (0.1386)	0.3107 (0.0213)	3.5824 (0.0423)
$\mu_{a,\text{mill}} - \mu_{a,\text{logger}}$ $\sim TRN(X_a\beta_3, \omega_{\mu,\text{diff}}^2, 0, \infty)$	3.6637 (0.8890)	-0.4998 (0.1339)	-0.0745 (0.0919)	-0.1827 (0.1007)	0.1255 (0.0163)	0.3783 (0.0242)
σ_{V^a} $\sim TRN(X_a\beta_2, \omega_{\sigma_V}^2, 0.01, \infty)$	4.0546 (0.7872)	-0.7379 (0.0994)	0.1393 (0.1025)	0.0895 (0.0813)	0.0796 (0.0188)	0.5763 (0.0273)
α_a $\sim TRN(\beta_4, \omega_\alpha^2, 0, 1)$	0.7127 (0.0509)	-	-	-	0.1837 (0.0446)	0.6890 (0.0362)
K_a $\sim TRN(X_a\beta_5, \omega_K^2, 0, \infty)$	1.9622 (13.2526)	-3.3006 (2.7167)	3.5172 (2.4808)	-1.1876 (1.5721)	2.8354 (0.6865)	2.0543 (0.2817)

Table 3: Simulated maximum likelihood with importance sampling estimates. The rightmost columns show the mean and median values of the structural parameters when we take 10 simulated draws of the parameter for each auction. Standard errors based non-parametric bootstrap with 100 repetitions. $TRN(\mu, \sigma^2, a, b)$ is a truncated normal distribution with parameters μ and σ^2 , and upper and lower truncation points a and b . Based on 887 auctions.

mates can support multiple equilibria by plotting type-symmetric equilibrium best response functions for mills and loggers for each auction. For every auction in our data, our parameter estimates support only a single equilibrium. This is because our estimates imply a large difference in the mean values of loggers and mills, relatively low entry costs and a moderate amount of selection, all of which tend to lead to uniqueness.

5.2 Counterfactual Results

Table 4 compares expected revenues and efficiency from the sequential mechanism and the simultaneous entry auction for a range of parameters and different numbers of firms. The simulations assume mills are approached first (in a random order) followed by loggers, although we have found some cases where a different order can strengthen the results below.³¹

The first line in Table 4 gives the results for the representative auction (four mills and four loggers) based on the mean parameter estimates from Table 3. Relative to setting no reserve price in the simultaneous entry, simultaneous bid auction, the sequential mechanism with no reserve price improves the USFS's revenues by 1.81% (s.e. of 0.17%). For a tract of average size (7,626 mbf) the expected revenue difference would be \$9,834 (s.e. of \$1,672), all \$ numbers in 1983 dollars.

The increase in revenues in this representative case of switching from the simultaneous bid auction with no reserve price to the sequential mechanism with no reserve price is 9.05 (s.e. of 1.95) times as large as the improvement from using an optimal reserve in the simultaneous bid auction, which is just 0.2% (s.e. of 0.03%). The finding that the revenue increase from using the sequential mechanism is much larger than the returns to using a reserve price in the current auction format is important since understanding optimal reserve price policies for timber auctions has been the subject of significant interest (examples include Mead, Schniepp, and Watson (1981), Paarsch (1997), Haile and Tamer (2003), Li and Perrigne (2003) and Aradillas-Lopez, Gandhi, and Quint (forthcoming)). Additionally, the sequential mechanism provides an easily implementable mechanism that does not require the USFS to possess detailed information on all of the model's primitives. Such information would be required to set an optimal reserve price. However, were the USFS to possess such information, a reserve price could also be set in the sequential mechanism. If a reserve price is used in the sequential mechanism, the increase in revenues becomes 10.43 times (s.e. of 2.27) as large as the gain to setting an optimal reserve price in the auction. This advantage would increase if we considered round-specific, or type-specific, optimal reserve prices in the sequential mechanism.

³¹ Auction results are based on 5,000,000 simulations and sequential results are based on 200,000 simulations. As the number of simulations used to compute auction and sequential mechanism revenues is so large, the standard errors reported in this section reflect uncertainty about the value of the parameters only.

Not only does the sequential mechanism have a much larger impact on revenues than does setting an optimal reserve price in the standard auction format, it also increases efficiency, as shown in the penultimate column in Table 4. In the representative case given in the first row of the table, the USFS captures the majority of the increase in surplus, but expected firm profits still increase in the sequential mechanism. As mentioned in Section 3, the sequential mechanism tends to promote more efficient entry of weaker bidders and this increases their expected profits. In the USFS auctions, switching from the current auction format to the sequential mechanism tends to increase expected logger profits without substantially harming those of mills. For example, in the representative case, expected logger profits increase 21% (s.e. of 4.4%) when the sequential mechanism is used, while mill profits only fall by 0.60% (s.e. of 0.90%).

The other rows in the table compare outcomes when we increase or decrease the number of potential entrants or structural parameters by one standard deviation from the point estimates of their means (the changing parameter is in *italics*), reflecting the fact that our estimates imply that the coefficients will differ across sales. The cases we consider indicate that using the sequential mechanism generally raises expected revenues. In case 14 the entry cost is very low and in either mechanism almost all firms participate so that revenues are essentially the same. In all cases, once a constant reserve price is used in the sequential mechanism, it earns higher revenues than the current auction format even with an optimal reserve price. We can see that setting a reserve price in the standard auction format is particularly ineffective when there are many potential entrants or when entry is less selective (α is high). In all cases the sequential mechanism increases efficiency and in only one example does total bidder surplus fall (case 10). The finding from the first row that loggers benefit from switching to the sequential mechanism holds in all rows. Additionally, when expected mill profit falls, it tends to be by a small amount, and in some cases it rises. As an example, in case 8, when μ_{diff} is low (0.169), loggers' expected profit increases by 10.18% and mills' increases by 1.40%.

The USFS also uses first price, sealed bid auctions to sell timber. We can also compare the performance of the sequential mechanism to this alternative. Across all of the cases in Table 4, with the exception of cases 6 and 14, a sequential mechanism with no reserve price earns the USFS higher revenues than a first price auction with an optimal reserve price. Introducing a reserve price to the sequential mechanism increases its advantage over the first price auction by even more so that it now dominates in all cases.

We can also compare total efficiency and revenues across all 887 auctions in our data. Our results predict that the sequential mechanism would increase our efficiency measure by \$11.3 million (s.e. of \$1.2 million) from a base of \$797 million using simultaneous auctions, with no

reserve in either mechanism. 65% of this increase in efficiency would be captured by the USFS in terms of higher revenue. In the 76% of the sample where there are at least 5 potential entrants, the sequential mechanism with no reserve also raises expected revenues by \$5.4 million (s.e. of \$1.1 million) relative to simultaneous auctions with optimal auction-specific reserve prices.

Of course, these gains from using a sequential mechanism would have to be weighed against sunk costs that might need to be incurred in switching from simultaneous auctions to a sequential procedure. For example, it would be necessary to develop appropriate software, while training USFS staff and educating potential bidders about the system. However, in this regard it is important to note that in the USFS context these costs could be spread across a very large number of auctions, while the gains that we estimate above come from only 18% of the USFS auctions held in one state between 1982 and 1989.³²

It is also worth considering other potential practical impediments. First, the USFS must be able to identify potential buyers. We have been told by USFS officials that they believe that they can accurately identify potential entrants for a given sale. Even if at times they are unsure, it would be straightforward to allow potential participants to costlessly identify themselves before the full details of a sale are announced. Second, were the USFS to use the sequential mechanism, there may be concern that approaching firms in an order places some of them at an advantage over others and may lead firms to try to affect the order in which they are approached. However, for all of the examples that we have considered, expected firm profits are fairly constant across the order of moves within bidder type, and there is no systematic pattern suggesting that a particular spot in the order is best. Intuitively, while the first potential entrant will be more likely to participate, he also must pay more to win. For example, in the representative auction, where the four mills are approached first followed by the four loggers, the expected profits (in \$/mbf) by order are $\{6.07, 6.09, 6.14, 6.18, 1.08, 1.05, 1.09, 1.04\}$. The maximum amount by which expected mill (logger) profits differ in this case is 0.016 (0.042). Third, there may be some concern about whether the USFS can commit to an order. However, repeated use of the mechanism likely would incentivize the USFS to maintain its credibility through consistent commitment to stated orders. Additionally, the lack of variation of profits across spots in the order could mean that firm lobbying efforts, which might dissuade a seller from sticking to a stated order, are likely to be small. Fourth, collusion may be a concern given the existing evidence from other USFS regions consistent with noncompetitive bidding (Athey, Levin, and Seira (2011)). However, as Bulow and Klemperer (2009) note (their footnote 40), the “simple auction is

³²The federal Bureau of Land Management also auctions timber and many state agencies also conduct hundreds of timber auctions each year.

Case	N_{mill}	N_{logger}	Parameters				Expected Revenues (\$/mbf)			Expected Efficiency	
			μ_{logger}	μ_{diff}	σ_V	α	K	$R = 0$	$R = R^*$	Total Δ (\$/mbf)	% of Δ to Firms
								71.25	72.54	71.39	1.88
1	4	4	3.582	0.378	0.576	0.689	2.05	50.96 83.91	52.32 85.21	51.63 83.98	1.45 2.16
2	1	4	3.582	0.378	0.576	0.689	2.05	64.42 75.87	64.46 77.60	64.69 75.97	0.91 2.31
3	γ	4	3.582	0.378	0.576	0.689	2.05				25.11
4	4	0	3.582	0.378	0.576	0.689	2.05	34.10 142.75	35.50 143.75	34.34 142.81	1.58 2.25
5	4	8	3.582	0.378	0.576	0.689	2.05				55.56
6	4	4	2.921	0.378	0.576	0.689	2.05	61.90 83.91	62.93 85.32	62.02 84.07	1.88 1.66
7	4	4	4.243	0.378	0.576	0.689	2.05				11.39
8	4	4	3.582	0.169	0.576	0.689	2.05	57.13 91.08	58.79 91.64	57.47 91.16	1.51 1.98
9	4	4	3.582	0.587	0.576	0.689	2.05				45.21 15.06
10	4	4	3.582	0.378	0.349	0.689	2.05	70.64 71.89	72.36 73.08	71.02 71.91	1.95 2.03
11	4	4	3.582	0.378	0.804	0.689	2.05				11.79 71.72
12	4	4	3.582	0.378	0.576	0.505	2.05				-9.93
13	4	4	3.582	0.378	0.576	0.872	2.05				41.38
14	4	4	3.582	0.378	0.576	0.689	2.05	75.74 66.69	75.71 69.32	75.75 67.12	0.54 2.93
15	4	4	3.582	0.378	0.576	0.689	2.05				105.56 10.24

Table 4: Comparing the impact of the sequential mechanism on expected revenues and efficiency. The first line shows the results for the representative auction with four mills and four loggers and the mean parameter estimates from Table 3, column 7. This is what we refer to as the “representative case”. Italics indicate changes from this representative auction and these changes are ± 1 standard deviation changes based on our estimates of the distributions of the structural parameters across auctions. SEQ refers to the sequential mechanism, SPA refers to the auction, $R = 0$ and $R = R^*$ indicates that either no reserve price or the optimal reserve price is used, respectively. The first three columns following the parameters give the expected revenues in the three mechanisms. The final two columns compare the expected efficiency of a sequential mechanism and an auction with no reserve. The first of these is the total change in efficiency. The second gives the percent of the change which accrues to firms.

perhaps more easily undermined, than a sequential process, by collusion.”

There may also be some concern that switching to the sequential mechanism would increase the time required to sell any stand of timber since cruises would have to be done sequentially and the mechanism allows for knockout auctions in between jump bids. While the length of the bidding process would necessarily increase, we note that there is already a sizable gap (over a month) between when a sale is announced and when it is completed.³³ Since cruising takes between a day and seven days, depending on the size of the sale, even in the extreme (assuming a large sale in which 8 potential bidders all decide to participate), a sequential mechanism could be run in under two months. Often the sequential process could happen much faster, but even an extra month may be a small price to pay to realize the sequential mechanism’s advantages.

6 Conclusion

This paper compares the performance of a sequential and a simultaneous bidding mechanism in an environment where it is costly for potential buyers to participate and they receive imperfectly-informative signals about their values prior to deciding whether to enter, so that the entry process is selective. In contrast to results when there is no selection, a very simple sequential mechanism can generate higher expected revenues for the seller than the commonly used auction, and it also has an efficiency advantage so that buyers may prefer it as well. The revenue result holds even though there is less entry (actual competition) into the sequential mechanism. Instead, with selection, the sequential mechanism can do a better job of allocating the good to the firm with the highest value and this fact, combined with the feature that firms with high values have to bid aggressively in an attempt to deter future entry, provides its revenue advantage.

We view our results as relevant and important for at least three reasons. First, our results highlight the important role that selective entry can play in the performance of different sales mechanisms. We believe that a model where potential bidders have some information about their values, but perform costly assessments prior to bidding, is a plausible description for many real-world sale or procurement settings. One example is corporate takeovers where there is an on-going debate about whether corporate boards should be able to use sequential sale procedures, such as go-shops, to fulfill their *Revlon* duty to maximize shareholder value. Our results indicate that there are circumstances in which a sequential bidding process will achieve this aim more effectively than a simultaneous auction, and they identify two factors

³³As USFS officials informed us, the gap is usually much longer since the USFS must file documents to comply with the National Environmental Policy Act.

(the level of entry costs and the degree of selection) that will determine how well these alternatives perform. Our results also address some of the concerns that some commentators have raised about sequential sale procedures treating potential bidders asymmetrically, giving the initial bidder an unfair advantage (Bloch (2010)). In our model, the initial bidder does try to deter later firms from entering, but in equilibrium this does not raise its profits significantly because it pays entry costs more often and tends to pay more when it wins.³⁴

Second, our results are consistent with recent empirical evidence on corporate takeovers, where both Subramanian (2008) and Jeon and Lee (2012), find that target shareholders earn significantly higher returns, on the order of 5%, when sequential go-shop procedures are used, as well as Subramanian's observations that both buyers and sellers often prefer go-shops. Our model can also explain other well-documented features of takeover data. For example, Betton and Eckbo (2000) and Betton, Eckbo, and Thorburn (2008) show that jump bids do not perfectly deter all future entry, and that multiple jump bids, sometimes by different firms, are observed. These features are not consistent with a sequential bidding model with no selection.

Third, the revenue differences that we identify are not trivial. When entry costs are large or there is strong selection, we show in Section 3 that the sequential mechanism can have a large absolute revenue advantage over the simultaneous auction. When entry costs are relatively low, the gains to using a sequential mechanism over a simultaneous auction are relatively much larger than the gains to use auction design tools, such as reserve prices, that are the focus of much of the auction literature. We illustrate this point in our empirical application. In contrast, we have been unable to find parameters where the simultaneous auction provides a significant advantage over the sequential mechanism.

There are, of course, some limitations of the model considered here. One example is that we assume that firms act competitively in both mechanisms. Another example is that we use an IPV framework, which would not be satisfied in a setting where potential buyers have to form imperfect opinions about an asset's innate future potential. A common value component could change jump bidding strategies in a sequential mechanism where a bidder finds out more after entering, as an incumbent bidder could now want to signal that the value of the common component is low in order to deter other firms from entering.³⁵ Understanding how

³⁴Sautter (2008) notes that “Proponents of go-shops argue that the provisions may aid the target in achieving maximum stockholder value. They reason that ... the initial acquirer is incentivized to offer the highest possible price in order to avoid a post-signing bidding war and the possibility that the deal may be successfully ‘jumped’.”

³⁵Denton (2008) notes that sequential go-shop procedures are believed to work best in takeover settings where there are potential strategic (industry) buyers. As he notes, an IPV framework, reflecting the different synergies that each firm can realize with the company being sold, is likely to be more appropriate for these settings.

common values or the threat of collusion affect the performance of sequential mechanisms appear to be profitable directions for future research.

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A Propositions and Proofs: FOR ONLINE PUBLICATION

This Appendix shows that the entry thresholds, beliefs of potential entrants and jump bidding functions defined in the text form an equilibrium and is the only equilibrium consistent with our refinement assumptions. For clearer exposition, we begin with a two period game and show there exists a unique equilibrium under the D1 refinement. We extend the result to games with more than two rounds by showing how a recursive application of the same arguments leads to the uniqueness of bidding and entry rules in earlier rounds.

A.1 Two Round Game

In a two round game, the equilibrium consists of strategies for potential entrants in both rounds, a jump bidding rule for a first round entrant and the beliefs of the potential entrant about the value of the first-round potential entrant given a jump bid. As explained in the text, we assume that both firms would bid up to their values in a second-round knockout auction.

The main proposition that we prove below is that there exists a unique equilibrium to this game under the D1 refinement. To show this, we establish the following three lemmas which immediately yield the proposition and characterize the nature of the unique equilibrium.

Lemma 1. *The expected post-entry profits of the potential entrant in round 2 are strictly increasing in its signal, S_2 .*

Proof. Given the knockout bidding assumption, the expected profit of the potential entrant who enters with signal S_2 will be

$$\int_{\widehat{b}_2^{\text{pre}}}^V \int_{\widetilde{v}}^V (v - \widetilde{v}) f_{\tau(2)}^V(v|S_2) g_2(\widetilde{v}) dv d\widetilde{v} \quad (8)$$

when his belief is that the value of the incumbent has pdf $g_2(\widetilde{v})$. If there is no incumbent, then the reserve price R can be viewed as an incumbent with known value R . The expression in (8) is weakly increasing in v and since $F_{\tau(2)}^V(v|S)$ is strictly decreasing in S , the entire expression is strictly increasing in S_2 . \square

Since the expected post-entry profits are monotonic in the second potential entrant's signal we get the following corollary.

Corollary 1. *If the expected post-entry profits are less than K for all S , then the second round potential entrant does not enter. Otherwise he enters if and only if his signal exceeds a*

threshold $S'_{2,\tau(2)}^*$ uniquely given by the solution to $\int_{b_2^{pre}}^{\bar{V}} \int_{\tilde{v}}^{\bar{V}} (v - \tilde{v}) f_{\tau(2)}^V(v|S'_{2,\tau(2)}^*) g_2(\tilde{v}) d\tilde{v} d\bar{v} - K = 0$.

Lemma 2. *There exists a unique equilibrium jump bidding function for a new entrant in period one under the D1 refinement which can be described as:*

- (i) strictly increasing for incumbent values on $[R, \bar{V} - K]$ and characterized by the differential equation in (5) and the lower boundary condition $\beta_{1,\tau(1)}(R, R) = R$;
- (ii) equal to $\beta_{1,\tau(1)}(\bar{V} - K, R)$ for incumbent values greater than $\bar{V} - K$;
- (iii) and submitting no bid for incumbent values $< R$.

Proof. We begin by showing (i). Theorems 2 and 4 of Mailath and von Thadden (2011), generalizing Mailath (1987), provide sufficient conditions under which there is a unique separating equilibrium signal function $\beta_{1,\tau(1)}(v, R)$, determined by the differential equation (5) and the initial condition $\beta_{1,\tau(1)}(R, R) = R$. We now list these conditions (a)-(f) in our setting and show that each holds.

(a) *The possible value of the incumbent and its action space are compact intervals.* This is true in our model given our assumptions that values lie on $[0, \bar{V}]$ and possible bids lie on $[0, \bar{B}]$, $\bar{V} < \bar{B}$.

(b) *If the final round potential entrant observed the value of the incumbent, the jump bidding problem of the incumbent would have a unique solution.* The optimal bid would be equal to R . This is because it cannot be optimal for the incumbent to submit a bid above its value. Further, no bid below its value affects the potential entrant's entry decision but will reduce the incumbent's profit, relative to submitting a bid equal to R , if the potential entrant stays out.

(c) $\Pi_{1,\tau(1)}(v, v', b)$ is continuous and differentiable in each argument. This is true in the model since the exact form of $\Pi_{1,\tau(1)}(v, v', b)$ is

$$\Pi_{1,\tau(1)}(v, v', b) = [v - b] \bar{F}_1(b|v') + \int_b^v (v - x) \bar{f}_1(x|v') dx, \quad (9)$$

$$\text{where } \bar{F}_1(x|v') = \left[\int_0^x f_{\tau(2)}^V(y) dy + \int_x^{\bar{V}} F_{S,\tau(2)}(S'_{2,\tau(2)}^*(v')|y) f_{\tau(2)}^V(y) dy \right] \quad (10)$$

$$\text{and } \bar{f}_1(x|v') = \frac{\partial \bar{F}_1(x|v')}{\partial x}. \quad (11)$$

These profits will be continuous and differentiable in each argument as all of the pdfs and cdfs in these functions are continuous and differentiable and $S'_{2,\tau(2)}^*(v')$, determined by the threshold rule described above, will be continuous and differentiable in v' . Below we will

make use of the fact that:

$$\frac{\partial \bar{F}_1(w|z)}{\partial z} = - \int_w^{\bar{V}} \frac{\partial \bar{f}_1(y|z)}{\partial z} dy$$

and that

$$\frac{\partial \bar{f}_1(y|v')}{\partial v'} = - \frac{\partial F_{S,\tau(2)}(S'_{2,\tau(2)}(v')|y)}{\partial S'_{2,\tau(2)}(v')} \frac{\partial S'_{2,\tau(2)}(v')}{\partial v'} f_{\tau(2)}^V(y) < 0$$

since $S'_{2,\tau(2)}$ is increasing in the potential entrant's perception of the incumbent's value because the incumbent will have a higher dropout point in a knockout auction.

(d) $\frac{\partial \Pi_{1,\tau(1)}(v,v',b)}{\partial v'} > 0$ for all (v, v') . After some algebra we have:

$$\begin{aligned} \frac{\partial \Pi_{1,\tau(1)}(v, v', b)}{\partial v'} &= -[v - b] \int_b^{\bar{V}} \frac{\partial \bar{f}_1(y|v')}{\partial v'} dy + \int_b^v (v - x) \frac{\partial \bar{f}_1(y|v')}{\partial v'} dy \\ &= - \left[[v - b] \int_v^{\bar{V}} \frac{\partial \bar{f}_1(y|v')}{\partial v'} dy + [v - b] \int_b^v \frac{\partial \bar{f}_1(y|v')}{\partial v'} dy - \int_b^v (v - x) \frac{\partial \bar{f}_1(y|v')}{\partial v'} dy \right] \\ &> - \left[[v - b] \int_v^{\bar{V}} \frac{\partial \bar{f}_1(y|v')}{\partial v'} dy \right] > 0 \end{aligned}$$

(e) $\frac{\partial \Pi_{1,\tau(1)}(v,v',b)}{\partial b} \neq 0$ for all b . This is immediate since $\frac{\partial \Pi_{1,\tau(1)}(v,v',b)}{\partial b} = -\bar{F}_1(b|v') < 0$.

(f) $\frac{\partial \Pi_{1,\tau(1)}(v,v',b)}{\partial b} / \frac{\partial \Pi_{1,\tau(1)}(v,v',b)}{\partial v'}$ is monotonic in v for all (v', b) . We can prove this directly.

Alternatively we can define the profit function in terms of entry thresholds instead of beliefs about the incumbent's value: $\pi_{1,\tau(1)}(v, S'_{2,\tau(2)}, b)$ and show single crossing in terms of signal threshold: $\frac{\partial \pi_{1,\tau(1)}(v, S'_{2,\tau(2)}, b)}{\partial b} / \frac{\partial \pi_{1,\tau(1)}(v, S'_{2,\tau(2)}, b)}{\partial S'_{2,\tau(2)}}$ is monotonic in v for all $(S'_{2,\tau(2)}, b)$. Roddie (2011) shows (his fact 2) that when $S'_{2,\tau(2)}(v')$ is monotonically increasing in v' , which was shown above, that this signal-threshold version of single crossing implies $\frac{\partial \pi_{1,\tau(1)}(v, v', b)}{\partial b} / \frac{\partial \pi_{1,\tau(1)}(v, v', b)}{\partial v'}$ is monotonic in v for all (v', b) . As it will be useful to have a single crossing condition written in terms of the potential entrant's signal threshold for proving that no pooling equilibria exist below, we take this second route by establishing $\frac{\partial \pi_{1,\tau(1)}(v, S'_{2,\tau(2)}, b)}{\partial b} / \frac{\partial \pi_{1,\tau(1)}(v, S'_{2,\tau(2)}, b)}{\partial S'_{2,\tau(2)}}$ is monotonic in v for all $(S'_{2,\tau(2)}, b)$.

We prove this by showing that the derivative of this expression with respect to v is always positive. Differentiating this expression with respect to v yields (using superscripts to denote partial derivatives) $\pi_{1,\tau(1)}^{13} \left[\pi_{1,\tau(1)}^2 \right]^{-1} - \pi_{1,\tau(1)}^3 \pi_{1,\tau(1)}^{12} \left[\pi_{1,\tau(1)}^2 \right]^{-2}$, which is equal to:

$$\bar{F}_1(b|S'_{2,\tau(2)}) \left[- \int_v^{\bar{V}} \frac{\partial \bar{f}_1(y|S'_{2,\tau(2)})}{\partial S'_{2,\tau(2)}} dy \right] \left[-[v - b] \int_b^{\bar{V}} \frac{\partial \bar{f}_1(y|S'_{2,\tau(2)})}{\partial S'_{2,\tau(2)}} dy + \int_b^v (v - x) \frac{\partial \bar{f}_1(y|S'_{2,\tau(2)})}{\partial S'_{2,\tau(2)}} dy \right]^{-2}$$

This expression is always positive since all three terms being multiplied are positive.

This establishes the form and the uniqueness of the separating equilibrium bid function on the interval $[R, \bar{V} - K]$. We now show that no pooling equilibria exist over this interval. Theorem 3 of Ramey (1996) shows that if the incumbent does not want to submit the maximum possible bid and $\frac{\partial \pi_{1,\tau(1)}(v, S'_{2,\tau(2)}, b)}{\partial b} / \frac{\partial \pi_{1,\tau(1)}(v, S'_{2,\tau(2)}, b)}{\partial S'_{2,\tau(2)}}$ is monotonic in v for all $(S'_{2,\tau(2)}, b)$, then no pooling equilibria can exist under D1. We just established the second condition and we know that the first condition holds since our assumption that $\bar{B} > \bar{V}$ implies that even the highest incumbent type will not submit the maximum possible bid.

We now show part (ii) of the lemma. A potential entrant who believes that the incumbent's value is $\bar{V} - K$ will not enter whatever his signal as the signal technology implies that there is some probability that the entrant's value will be less than \bar{V} . Given this, the expected benefit of entering the mechanism is less than the entry cost K . Therefore, considering only bids greater than or equal to $\beta_{1,\tau(1)}(\bar{V} - K, R)$, the strictly dominant strategy will be to bid $\beta_{1,\tau(1)}(\bar{V} - K, R)$. The single crossing condition implies that if $\beta_{1,\tau(1)}(\bar{V} - K, R)$ is preferred to a lower bid by the incumbent with value $\bar{V} - K$ then it is also preferred by an incumbent with a value greater than $\bar{V} - K$.

Part (iii) of the lemma is immediate since an incumbent should not bid more than his value as he may have to pay this bid if the potential entrant stays out or comes in with a value less than the incumbent. \square

Lemma 3. *The expected post-entry profits of the potential entrant in round 1 are strictly increasing in S_1 .*

Proof. In the first round, the expected post-entry profit of a potential entrant if it enters with signal S_1 is

$$\int_R^{\bar{V}} \Pi_{1,\tau(1)}(v, v, \beta_{1,\tau(1)}(v, R)) f_{\tau(1)}^V(v|S_1) dv \quad (12)$$

where $\beta_{1,\tau(1)}(v, R)$ is the equilibrium jump bidding strategy, characterized above, for the firm if it enters and has a value above the reserve (if it has a value less than the reserve it does not submit a bid after entering). As long as the expression in (12) is weakly increasing in v , it will be strictly increasing in S_1 since $F_{\tau(1)}^V(v|S)$ is strictly decreasing in S . We now show that the expression in (12) is weakly increasing in v .

To do this we must establish that $\Pi_{1,\tau(1)}(v, v, \beta_{1,\tau(1)}(v, R))$ is increasing in v for $v > R$. Consider any v on $[R, \bar{V} - K]$, where we know from above that the jump bidding schedule is separating. Incentive compatibility of the jump bidding strategy implies that

$$\Pi_{1,\tau(1)}(v, v, \beta_{1,\tau(1)}(v, R)) \geq \Pi_{1,\tau(1)}(v, \hat{v}, \beta_{1,\tau(1)}(\hat{v}, R)) \text{ for any } \hat{v} < v$$

and, as the payoff of a v incumbent will be higher than a \hat{v} incumbent if he wins without

having to compete in a knockout auction when both use a bid of $\beta_{1,\tau(1)}(\widehat{v}, R)$, we also know that

$$\Pi_{1,\tau(1)}(v, \widehat{v}, \beta_{1,\tau(1)}(\widehat{v}, R)) > \Pi_{1,\tau(1)}(\widehat{v}, \widehat{v}, \beta_{1,\tau(1)}(\widehat{v}, R)) \text{ for any } \widehat{v} < v$$

and thus $\Pi_{1,\tau(1)}(v, v, \beta_{1,\tau(1)}(v, R)) > \Pi_{1,\tau(1)}(\widehat{v}, \widehat{v}, \beta_{1,\tau(1)}(\widehat{v}, R))$ for any $\widehat{v} < v$, as required. For any v greater than $\bar{V} - K$, equilibrium payoffs will also be increasing in v as $\Pi_{1,\tau(1)}(v, v, \beta_{1,\tau(1)}(v, R)) = v - \beta_{1,\tau(1)}(\bar{V} - K, R)$. \square

Since the expected post-entry profits are monotonic in the first round potential entrant's signal we get the following corollary.

Corollary 2. *If the expected post-entry profits are less than K for all S , then the first round potential entrant does not enter. Otherwise he enters if and only if his signal exceeds a threshold $S'^*_{1,\tau(1)}$ uniquely given by the solution to $\int_R^{\bar{V}} \Pi_{1,\tau(1)}(v, v, \beta_{1,\tau(1)}(v, R)) f_{\tau(1)}^V(v | S'^*_{1,\tau(1)}) dv - K = 0$.*

The above lemmas immediately imply that the following:

Proposition 1. *There exists a unique equilibrium bid function and entry thresholds in the two round sequential mechanism with pre-entry signals under the D1 refinement.*

A.2 Three or More Round Games

We now explain how the above proposition's existence and uniqueness results can be extended to sequential mechanisms with three or more rounds. To do so, we use the same recursive arguments that were used in the two round game. Consider a three round game. The proofs for the equilibrium strategies in the penultimate and final rounds are exactly the same as above, except that the incumbent in the penultimate round may be bidding from a standing bid determined by the value of a previous incumbent rather than the reserve price, and the penultimate round entry threshold will depend on the agent's beliefs about the value of the incumbent if there is one. Following the arguments above, this threshold, $S'^*_{2,\tau(2)}$, is uniquely determined by the zero profit condition $\int_{b_2^{pre}}^{\bar{V}} \int_{\widetilde{v}}^{\bar{V}} \Pi_{2,\tau(2)}(v, v, \beta_{2,\tau(2)}(v, \widetilde{v})) f_{\tau(2)}^V(v | S'^*_{2,\tau(2)}) dv d\widetilde{v} - K = 0$.

We need to characterize the jump bidding function for an incumbent in the first round. After this, extending the arguments to four or more round games is straightforward as again the proofs for the equilibrium strategies in the last three rounds of a four round game would be exactly the same as in a three round game (except that the incumbent in the second round may be bidding from a standing bid determined by the value of a previous incumbent rather than the reserve price, and the second round entry threshold will depend on the agent's beliefs about the value of the incumbent if there is one).

To characterize the first round jump bidding function requires establishing the three-plus-round versions of properties (a)-(f) listed in the proof of part (i) of Lemma 2. Properties (a)-(c) and (e) are immediate. For property (d) to hold, so that the incumbent in the first round is better off being perceived as having a higher value, we must show that the entry thresholds of the subsequent potential entrants are increasing in their beliefs about his value since this implies that they are less likely to enter for any potential entrant value. We know from above that this will be the case for the final round potential entrant. As the following lemma illustrates, it is also true for the second round potential entrant and so property (d) holds.

Lemma 4. *The second round potential entrant's entry threshold is increasing in its beliefs about a round one incumbent's value v' .*

Proof. This requires showing that $\Pi_{2,\tau(2)}(v, v, \beta_{2,\tau(2)}(v, v'))$ decreases in v' , the standing bid at the end of a knockout that the potential entrant wins. This will be the case because $\beta_{2,\tau(2)}(v, v')$ increases in v' (since, by standard arguments, two bid functions defined by the same differential equation, but with different initial conditions, cannot cross) and since the final round potential entrant's entry decision depends only the second round entrant's value if he wins the knockout (since the bid function is fully revealing), then this jump bid will only serve to increase the price paid by the second round entrant in the event the final round entrant stays out. \square

The final property needed to show the existence and uniqueness of a separating equilibrium bid function, and that there are no pooling equilibria, in the first round is the three-plus-round version of single crossing, property (f) above. With three rounds, this can be more compactly proved by using the non-derivative form of single crossing.

Lemma 5. *Consider any two possible bid and entry threshold combinations $(S_A^{2'}, S_A^{3'}, b_A)$ and $(S_B^{2'}, S_B^{3'}, b_B)$ where $b_B > b_A$. For $v^H > v^L$, if $\Pi_{1,\tau(1)}(v^L, S_B^{2'}, S_B^{3'}, b_B) \geq \Pi_{1,\tau(1)}(v^L, S_A^{2'}, S_A^{3'}, b_A)$, then $\Pi_{1,\tau(1)}(v^H, S_B^{2'}, S_B^{3'}, b_B) > \Pi_{1,\tau(1)}(v^H, S_A^{2'}, S_A^{3'}, b_A)$.*

Proof. Consider all possible combinations of values and signals of the second and third round potential entrants. The required implication will hold if the profit gain to $(S_B^{2'}, S_B^{3'}, b_B)$ is not lower for the incumbent with value v^H than the incumbent with type v^L for any combination, and it is strictly greater for some combination (all combinations are possible). In the following we will use $v_{2:3,A}^{\max}$ as the maximum value of an entrant under $(S_A^{2'}, S_A^{3'}, b_A)$ conditional on the incumbent still being the incumbent after round 2.

If the switch to $(S_B^{2'}, S_B^{3'}, b_B)$ has no effect on the entry of this entrant, then the payoffs of either incumbent are only affected if $b_B \geq v_{2:3,A}^{\max}$, in which case there is a cost to both incumbents of $b_B - \max\{v_{2:3,A}^{\max}, b_A\}$, which is independent of v .

If the switch to $(S_B^{2'}, S_B^{3'}, b_B)$ causes this entrant not to enter, which will happen with positive probability for any $v_{2:3,A}^{\max}$, then label the maximum value of the highest entrant $v_{2:3,B}^{\max}$, which could be equal to zero and will be less than $v_{2:3,A}^{\max}$. If $b_A \geq v_{2:3,A}^{\max}$ then the cost to both incumbents is $b_B - b_A$ and so is independent of v . If $b_B \geq v_{2:3,A}^{\max} \geq b_A$, the cost to both incumbents is $b_B - v_{2:3,A}^{\max}$, which is independent of v . If $v^H > v^L \geq v_{2:3,A}^{\max} \geq b_B$ there is a gain to both incumbents of $v_{2:3,A}^{\max} - \max\{b_B, v_{2:3,B}^{\max}\}$ and so is independent of v . If $v_{2:3,B}^{\max} \geq v^H$ there is no impact on either incumbent's profits. In the remaining cases the H incumbent will gain strictly more than the L incumbent. This can happen when $v^H > v_{2:3,A}^{\max} > v^L > v_{2:3,B}^{\max} > b_B$ in which case the gain to the H incumbent is $v_{2:3,A}^{\max} - v_{2:3,B}^{\max}$ which exceeds the gain of $v^L - v_{2:3,B}^{\max}$ for the L incumbent. It can happen when $v^H > v_{2:3,A}^{\max} > v_{2:3,B}^{\max} > v^L > b_B$ in which case gain to the H incumbent is $v_{2:3,A}^{\max} - v_{2:3,B}^{\max}$ which exceeds no gain for the L incumbent. It can happen when $v^H > v_{2:3,A}^{\max} > v^L > b_B > v_{2:3,B}^{\max}$ in which case gain to the H incumbent is $v_{2:3,A}^{\max} - b_B$ which exceeds the gain of $v^L - b_B$ for the L incumbent. It can happen when $v_{2:3,A}^{\max} > v^H > v^L > b_B > v_{2:3,B}^{\max}$ in which case gain to the H incumbent is $v^H - b_B$ which exceeds the gain of $v^L - b_B$ for the L incumbent. It can also happen when $v_{2:3,A}^{\max} > v^H > v_{2:3,B}^{\max} > v^L > b_B$ in which case gain to the H incumbent is $v^H - v_{2:3,B}^{\max}$ which exceeds no gain for the L incumbent. Finally it can happen when $v_{2:3,A}^{\max} > v^H > v^L > v_{2:3,B}^{\max} > b_B$ in which case gain to the H incumbent is $v^H - v_{2:3,B}^{\max}$ which exceeds the gain of $v^L - v_{2:3,B}^{\max}$ for the L incumbent. \square

The arguments easily extend to more than three rounds leading to the following proposition.

Proposition 2. *There exists a unique equilibrium for entry and bidding behavior in the sequential mechanism with pre-entry signals in which:*

1. *A type $\tau(n)$ potential entrant in round n will enter if and only if it receives a signal above a threshold $S_{n,\tau(n)}^*$ defined by the zero profit condition given by equation (2) for $n < N$ and by equation (4) for $n = N$;*
2. *Any entrant participating in a knockout auction bids up to its value;*
3. *Any incumbent placing a jump bid in round n when either the reserve or the standing bid at the end of the previous knockout is \hat{b}_n bids according to a bid function $\beta_{n,\tau(n)}(v, \hat{b}_n)$ that is unique and:*
 - (a) *when $v \in [\hat{b}_n, \bar{V} - K]$ is determined by the solution to the differential equation:*

$$\frac{d\beta_{n,\tau(n)}(v, \hat{b}_n)}{dv} = -\frac{\Pi_{n,\tau(n)}^2(v, v, \beta_{n,\tau(n)}(v, \hat{b}_n))}{\Pi_{n,\tau(n)}^3(v, v, \beta_{n,\tau(n)}(v, \hat{b}_n))}$$

- with lower boundary condition: $\beta_{n,\tau(n)}(\hat{b}_n, \hat{b}_n) = \hat{b}_n$; and
- (b) when $v \in (\bar{V} - K, \bar{V}]$ is $\beta_{n,\tau(n)}(\bar{V} - K, \hat{b}_n)$.

Off-the-equilibrium-path beliefs of potential entrants are not unique. While a potential entrant in round n that observes a jump bid x in an earlier round m between $[\hat{b}_m, \beta_{m,\tau(m)}(\bar{V} - K, \hat{b}_m)]$ will believe that the value of this incumbent is $\beta_{m,\tau(m)}^{-1}(x, \hat{b}_m)$, the density of potential entrants' beliefs of the incumbent's type over the interval $(\bar{V} - K, \bar{V}]$ upon observing a bid greater than $\beta_{m,\tau(m)}(\bar{V} - K, \hat{b}_m)$ is not pinned down. However, for all such beliefs, the equilibria have the common feature that entry will cease once this bid is placed.

B Details of Estimation Method: FOR ONLINE PUBLICATION

This appendix describes our estimation procedure based on Ackerberg (2009)'s method of simulated maximum likelihood with importance sampling.

This method involves solving a large number of games with different parameters once, calculating the likelihoods of the observed data for each of these games, and then re-weighting these likelihoods during the estimation of the distributions for the structural parameters. This method is attractive when it is believed that the parameters of the model are heterogeneous across auctions and it would be computationally prohibitive to re-solve the model many times (in order to integrate out the heterogeneity) each time one of the parameters changes.³⁶

To apply the method, we assume that the parameters are distributed across auctions according to the specification given in Section 4.3. These specifications reflect our assumptions that σ_V , α and K are the same for mills and loggers within any particular auction, even though they may differ across auctions. The lower bound on σ_{Va} is set slightly above zero simply to avoid computational problems that were sometimes encountered when there was almost no dispersion of values. Our estimated specifications also assume that the various parameters are distributed independently across auctions. This assumption could be relaxed, although introducing a full covariance matrix would significantly increase the number of parameters to be estimated and, when we have tried to estimate these parameters, we have not found these coefficients to be consistently significant across specifications. The set of parameters to be estimated are $\Gamma = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \omega_{\mu,\text{logger}}^2, \omega_{\mu,\text{diff}}^2, \omega_{\sigma_V}^2, \omega_\alpha^2, \omega_K^2\}$, and a particular draw of the parameters $\{\mu_{a,\text{logger}}, \mu_{a,\text{mill}}, \sigma_{Va}, \alpha_a, K_a\}$ is denoted θ .

³⁶Bajari, Hong, and Ryan (2010) use a related method to analyze entry into a complete information entry game with no selection.

Denoting the outcome for an observed auction by y_a , the log-likelihood function for a sample of A auctions is

$$\sum_{a=1}^A \log \left(\int L_a(y_a|\theta) \phi(\theta|X_a, \Gamma) d\theta \right) \quad (13)$$

where $L_a(y_a|\theta)$ is the likelihood of the outcome y in auction a given structural parameters θ , $\phi(\theta|X_a, \Gamma)$ is the pdf of the parameter draw θ given Γ , our distributional assumptions, the unique equilibrium strategies implied by our equilibrium concept and auction characteristics including the number of potential entrants, the reserve price and observed characteristics X_a .

Unfortunately, the integral in (13) is multi-dimensional and cannot be calculated exactly. We follow Ackerberg by recognizing that

$$\int L_a(y_a|\theta) \phi(\theta|X_a, \Gamma) d\theta = \int L_a(y_a|\theta) \frac{\phi(\theta|X_a, \Gamma)}{g(\theta|X_a)} g(\theta|X_a) d\theta \quad (14)$$

where $g(\theta|X_a)$ is the importance sampling density whose support does not depend on Γ , which is true in our case because the truncation points are not functions of the parameters to be estimated. This can be approximated by simulation using

$$\frac{1}{S} \sum_s L_a(y_a|\theta_s) \frac{\phi(\theta_s|X_a, \Gamma)}{g(\theta_s|X_a)} \quad (15)$$

where θ_s is one of S draws from $g(\theta|X_a)$. Critically, this means that we can calculate $L_a(y_a|\theta_s)$ for a given set of S draws that do not vary during estimation, and simply change the weights $\frac{\phi(\theta_s|X_a, \Gamma)}{g(\theta_s|X_a)}$, which only involves calculating a pdf when we change the value of Γ rather than re-solving the game.

This simulation estimator will only be accurate if a large number of θ_s draws are in the range where $\phi(\theta_s|X_a, \Gamma)$ is relatively high, and, as is well known, simulated maximum likelihood estimators are only consistent when the number of simulations grows fast enough relative to the sample size. We therefore proceed in two stages. First, we estimate an initial guess of Γ using $S = 2,500$ draws, where $g(\cdot)$ is a multivariate uniform distribution over a large range of parameters which includes all of the parameter values that are plausible. Second, we use these estimates $\hat{\Gamma}$ to repeat the estimation using a new importance sampling density $g(\theta|X_a) = \phi(\theta_s|X_a, \hat{\Gamma})$ with $S = 500$ per auction. Roberts and Sweeting (2011) provide Monte Carlo evidence that the estimation procedure works well even for smaller values of S .

To apply the estimator, we also need to define the likelihood function $L_a(y_a|\theta)$ based on the data we observe about the auction's outcome, which includes the number of potential entrants of each type, the winning bidder and the highest bids announced during the open

outcry auction by the set of firms that indicated that they were willing to meet the reserve price. Two problems arise when interpreting these data. First, a bidder's highest announced bid in an open outcry auction may be below its value, and it is not obvious which mechanism leads to the bids that are announced (Haile and Tamer (2003)). Second, if a firm does not know its value when taking the entry decision, it may learn (after paying the entry cost) that its value is less than the reserve price and so not submit a bid.

We therefore make the following assumptions (Roberts and Sweeting (2011) present estimates based on alternative assumptions about the data generating process that deliver similar results) that are intended to be conservative interpretations of the information that is in the data: (i) the second highest observed bid (assuming one is observed above the reserve price) is equal to the value of the second-highest bidder;³⁷ (ii) the winning bidder has a value greater than the second highest bid; (iii) both the winner and the second highest bidder entered and paid K_a ; (iv) other firms that indicated that they would meet the reserve price or announced bids entered and paid K_a and had values between the reserve price and the second highest bid; and, (v) all other potential entrants may have entered (paid K_a) and found out that they had values less than the reserve, or they did not enter (did not pay K_a). If a firm wins at the reserve price we assume that the winner's value is above the reserve price.

C Details and Robustness of Numerical Procedure for Solving Sequential Mechanism with Pre-Entry Signals: FOR ONLINE PUBLICATION

This appendix details the recursive numerical procedure used to solve for equilibrium in the sequential mechanism.

We start with the final potential entrant, who believes that he will win if his value is greater than the incumbent's. For every possible value v' of the incumbent that this final potential entrant faces, we solve for the equilibrium entry threshold $S'_{N,\tau(N)}^*(v')$ on a fine grid of evenly spaced possible values $[0, \bar{V}]$. For example, the comparisons of mechanisms in Figure 2 are based on a grid with unit spacing, but we have experimented with 1/10th unit spacing with little effect on our results but substantial increases in the time needed to solve

³⁷Alternative assumptions could be made. For example, we might assume that the second highest bidder has a value equal to the winning bid, or that the second highest bidder's value is some explicit function of his bid and the winning bid. In practice, 96% of second highest bids are within 1% of the high bid, so that any of these alternative assumptions give similar results. We have computed some estimates using the winning bid as the second highest value and the coefficient estimates are indeed similar.

the game. Since the final price, if the final potential entrant wins, will be the value of the incumbent, the entry threshold of the final potential entrant is given by:

$$K = \int_{v'}^{\bar{V}} (x - v') f_{\tau(N)}^V(x | S'_{N,\tau(N)}^*) dx \quad (16)$$

The integral in Equation (16) is approximated using the trapezoidal rule. Since the right hand side of Equation (16) is monotonic in $S'_{N,\tau(N)}$, we use the method of bisection to calculate $S'_{N,\tau(N)}^*(v')$ at every v' on $[0, \bar{V}]$. Our default tolerance for solving for signal thresholds is 10^{-6} .

Next we solve for the jump bid functions of the previous potential entrant were he to enter and win any knockout auction. The differential equation that defines the bid function (the definition of the individual terms appears in the body of the text) is given:

$$\frac{d\beta(\cdot)}{dv} = \frac{[v - \beta(\cdot)] \overbrace{\left[\frac{d\Pi_{k=n+1}^N F_{S,\tau(k)}(S'_{k,\tau(k)}^*(v))}{dv} + \frac{\partial \bar{F}_{n,\tau(n)}(\beta(\cdot)|v)}{\partial v} \right]}^{(b)} + \int_{\beta(\cdot)}^v (v - \hat{v}) \frac{\partial \bar{f}_{n,\tau(n)}(\hat{v}|v)}{\partial v} d\hat{v}}{\underbrace{\Pi_{k=n+1}^N F_{S,\tau(k)}(S'_{k,\tau(k)}^*(v))}_{(a)} + \underbrace{\bar{F}_{n,\tau(n)}(\beta(\cdot)|v)}_{(c)}} \quad (17)$$

Term (a) can be calculated directly given our parametric assumptions. The derivatives that appear in (b) are solved using numerical differentiation as we do not have analytical expressions for these terms. The integrals that appear in term (b) and (c) are approximated using the trapezoidal rule, although other methods, like Simpson's Rule, did not meaningfully change the results. All of terms (a), (b) and (c) are stored as arrays on the grid of values $[\underline{v}, \bar{v}]$ and our solver uses MATLAB's `interp1` and `interp2` to read data from them and linearly interpolating functions across the grid. Using cubic interpolation does not materially affect our results.

We solve Equation 17 using MATLAB's `ode113` solver but alternative solvers, such as MATLAB's `ode45` and `ode23`, do not materially affect our results.³⁸ To give an example, Table 5 displays summary statistics for the absolute differences in equilibrium bid functions when different differential equation solvers are used. The baseline bid function is based on `ode113` (the solver used in the paper). Each row of the table represents differences from this baseline when alternative differential equation solvers are used. These summary statistics

³⁸The `ode113` is a variable order Adams-Bashforth-Moulton PECE solver. The `ode45` and `ode23` solvers are based on explicit Runge-Kutta methods using the Dormand-Prince and Bogacki-Shampine pairs, respectively. It has been shown that solvers such as MATLAB's `ode113` can be more efficient than basic Runge-Kutta methods when the function is expensive to compute (Shampine and Reichelt (1997)).

pertain to the bid function for a potential entrant in the penultimate round when the current incumbent has a value of 90 and firms are symmetric with values distributed $\text{LN}(4.5,0.2)$ and $K = 1$ and $\alpha = 0.5$.

ODE Solver	Absolute Difference in Solved Bid Function from <code>ode113</code>					
	Mean	Min	25th-tile	Median	75th-tile	Max
<code>ode23</code>	1.0698e-05	0	0	1.6532e-06	2.1766e-05	3.1375e-05
<code>ode45</code>	1.0731e-05	0	0	1.6879e-06	2.1872e-05	3.1420e-05

Table 5: Example of robustness of equilibrium bid function to different differential equation solvers. Details for the table’s construction are found in the accompanying text.

We have also tested our bid functions using a “best-response-like” check. This involves numerically simulating the expected benefit to a bidder, say with value v_{true} , from deviating and pretending as if his value is v_{fake} by submitting a bid $b(v_{fake})$. This check is analogous to that used in Gayle and Richard (2008) to check numerical solutions to equilibrium bid functions in an asymmetric first price auction when there is no entry margin.

Take as an example the case of a potential entrant in the penultimate round who faces an incumbent with a value of 90 when firms are symmetric with values distributed $\text{LN}(4.5,0.2)$ and $K = 1$ and $\alpha = 0.5$ (this is the same as in the example above). In this case we can compute the optimal best bid deviation as just described using 100,000 simulations and compare it to the bid function that we solved for. The average absolute difference in the two bid functions is 0.09. The 25th percentile of the absolute differences is 0, the 75th percentile is 0.07 and the maximum absolute difference is 1.09. Moreover, the change in expected profits from deviating from the equilibrium bid function for this potential entrant is a negligible 0.0014.

Finally, the entry threshold $S'_{n,\tau(n)}(v')$ for $n < N$ is set so that the expected profit from entering, conditional on the threshold, is zero. Using the notation from the paper, we have that $S'_{n,\tau(n)}(v')$ must satisfy

$$\int_{v'}^{\bar{v}} \left\{ [v - \beta(v, v', n)] \left[\underbrace{\prod_{k=n+1}^N F_{S,\tau(k)}(S'_{k,\tau(k)}(v'))}_{(a)} + \underbrace{\bar{F}_{n,\tau(n)}(\beta(v, v', n)|v')}_{(b)} \right] + \int_{\beta(v, v', n)}^v (v - x) \underbrace{\bar{f}_{n,\tau(n)}(x|v)}_{(c)} dx \right\} f_{\tau(n)}^V(x|S'_{n,\tau(n)}(v')) dv = K. \quad (18)$$

As before, term (a) is easy to compute for a given distribution of signals. Term (b) is calculated via numeric integration via the trapezoidal rule, and term (c) is calculated via numeric differentiation of term (b). As in Equation (16), the left hand size is monotonic in $S'_{n,\tau(n)}(v')$, and the method of bisection can be used to determine a solution.

We also perform a check on these entry thresholds as well as the entry thresholds in the last round. We do this by numerically simulating the value of expected profits from entry at S'_n . We always find that the value of the simulated profits is very close to zero. For example, continuing with the example from above used to illustrate the bid check, the penultimate round potential entrant's equilibrium entry threshold is 71.344. The expected profit from entering with a signal equal to this threshold is 0.009.

At times in the paper (e.g. Figure 2, Figure 3 and Table 4) we calculate optimal reserve prices for the sequential mechanism and the auction. We briefly describe how this is done in Footnote 22. Here we give greater detail.

When bidders are asymmetric, or entry is endogenous and/or selective, expected revenues and optimal reserve prices must be calculated numerically. To calculate expected revenues given a particular reserve price in the simultaneous auction, we first solve the model and then calculate expected revenues using 5,000,000 sets of simulation draws of the values and signals of each potential entrant. Holding these simulation draws fixed, we can calculate expected revenues for different reserve prices, re-solving the game each time. With this number of simulation draws, expected revenues are essentially smooth in the reserve price and we are able to perform a one-dimensional maximization to find the optimal reserve price. However, we note that we find almost identical optimal reserves using a grid search.

For the sequential mechanism it is more expensive to solve the game, especially when the number of players is large. One reason for this is that the calculation of expected revenues in the sequential mechanism is based on interpolation using our solution to the differential equation, though we have checked that expected revenues are almost identical using 100,000 and 400,000 simulations. So we do not want to re-solve the game for many different reserves. Instead we exploit the fact that the expected revenue in an N player game with a reserve price of R is equal to the expected revenues from the last N players in an $N+1$ player game, where the first entrant enters and has a value of R . We therefore solve an $N+1$ player game once, which gives us later strategies for all possible values of the first round entrant. Then we simulate forward from the second round of this game to compute expected revenues. In this case we use 200,000 revenues and consider a grid (with unit spacing) of possible reserve prices. In this way we may slightly under predict expected revenues with an optimal reserve in the sequential mechanism.