Mergers in R&D Races*

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Abstract

There is a substantial literature on the effects of mergers on product prices, but the effects of mergers on other outcomes, such as research and development (R&D) investment spending, are less studied. In this paper I develop a model for evaluating the likely effects of a merger (or joint research venture) on the R&D efforts of competing firms. The R&D process is modeled as an all-pay contest (auction) among firms, with the payoff from investment going to the firm that invests the largest amount. I provide an explicit characterization of the equilibrium in a multi-player asymmetric all-pay contest model. The equilibrium solution then is applied through simulation to calibrate the effects of mergers on firms’ R&D efforts and efficiency as well as on social welfare. I find that each firm is expected to exert more efforts after a merger, but if there are only few firms premerger, a merger reduces total R&D effort. A merger may also cause inefficiency, but the loss in efficiency is low. My results also show that net surplus increases after a merger if the number of firms is small.

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Keywords: mergers; R&D; contests; all-pay auctions

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1 Introduction

Whether proposed mergers would reduce the level of competition is the focal concern of antitrust reviews. There are a huge literature on modeling and estimating the impacts of a merger, yet price effect is their only subject. Depending on specific market characteristics, competition in developing new products through research and development (R&D) activities can also be a determinant factor for consideration. Recent merger proposals have successfully drawn the attention of antitrust agencies and researchers to this question.\footnote{For example, in a number of recent high-profile cases (AT&T/T-Mobile, Applied Materials/Tokyo Electron, and Halliburton/Baker Hughes), the Department of Justice expressed concerns about the loss of innovation competition resulting from a merger between competitors. In like manner, the proposal of John Deere/Precision Planting in high-speed precision planting market was terminated (May 2017). Proposals in the chemical and seed market also raised similar concerns, yet all were approved (ChemChina/Syngenta in May 2017, Dow/DuPont in August 2017 and Bayer/Monsanto in March 2018).}

Unlike pricing, R&D usually requires substantial investments and often takes up many years. Thus, firms in concentrated markets are more likely to invest in R&D because they can appropriate more returns from innovation. Then, extending further, if there are even fewer rivals, they may want to do more R&D since more returns can be captured. On the other hand, in concentrated industries, firms have existing products and economic profits associated with them. They invest in R&D partly to protect their existing positions, and also to give themselves a chance to leap ahead of rivals with a major innovation. Once the number of rivals is reduced, thereby reducing research competition among rivals, firms may be less incentivized to invest in research and innovation. Thus, the effects of a merger that would reduce the number of active rivals in a concentrated market is ambiguous, which is confirmed by a growing body of empirical investigations on mergers and R&D. For example, Ornaghi (2009) and Steibale and Reize (2011) find a decrease in innovative effort after mergers, while Bertrand (2009) and Steibale (2013) report a significant increase in R&D intensity after mergers.

Theoretical works are even fewer, as MacDonald has concluded in his report (2016) that there is still a very limited base on which such effects can be estimated. To my knowledge, only Davidson and Ferrett (2007), Phillips and Zhdanov (2013),
Motta and Tarantino (2017), and Federico, Langus and Valletti (2017) have provided theoretical insights on this topic. In Davidson and Ferrett (2007), firms first decide the level of process R&D, which reduces production cost, and then compete in the product market. They show that when the degree of R&D complementarity is non-trivial, a merger encourages insiders (merger participants) to invest more in R&D and benefits insiders with a lower cost on product market, regardless of the strategic variable in market competition (price vs quantity). Motta and Tarantino (2017) also study a Bertrand game with cost-reducing investment. Under a variety of cases, they show that absent efficiencies, a horizontal merger reduces innovation and suppresses price competition between them.

Though process innovation, analyzed by the previous two papers, is important in certain industries, the competition among firms does not go beyond their current products. On the other hand, my focus in this paper is on product innovation, which is related to firms’ future products, and is more aligned with the concerns of recent merger proposals. For product innovation, Phillips and Zhdanov (2013) study the incentives of merger under a setting of Bertrand competition in differentiated goods. They show that large firms may optimally decide to let small firms conduct R&D and then acquire these small innovative companies. However, they assume that R&D inputs by different firms are the same and that the probability of successful innovation is evenly distributed among firms who conduct R&D. In other words, firms in their model either maintain the R&D effort or quit R&D activity completely. The lack of flexibility in adjusting R&D effort makes their model not well suited for evaluating merger effects. Moreover, they restrict the range of capital ratios of different firms and thus their results do not carry over to mergers among firms of similar size.

In this paper, I develop an alternative model, a model of contest, for evaluating the likely effects of a merger on R&D investment of competing firms. Such a modeling choice of R&D, which overcomes the weaknesses aforementioned, is not uncommon in the literature (see, for example, Dasgupta and Stiglitz, 1980; Fudenberg et al., 1983; Harris and Vickers, 1985; and Leininger, 1991), but is rare in merger analysis. In this sense, my model is more close to Federico, Langus, and Valletti (2017) and, similar to theirs, my model is appropriate to the situation
where all firms are effective innovators. However, my model is different from theirs in several ways. First, their model is a game of complete information and firms (except the merged firms) are symmetric and thus make the same R&D effort. My model is an incomplete information game. Although firms are symmetric ex ante, they may make different R&D effort in the interim stage, which makes my model closer to an empirical application. Second, the probability of winning in their model is a function of the firm’s own effort, while in my model, the winning probability generally depends on the efforts of other firms as well. The technique used to solve for equilibrium is different. Last but not least, in spite of some common results on firms’ expected effort, we diverge on the welfare effect of a merger. They posit that consumers are worse off after the merger. Firms in their model do not differ except the status of being merged or not, and thus, a merger always decreases the probability of price competition. But, it is also reasonable to posit that firms in their model employ similar research strategies. As argued by Dasgupta and Stiglitz (1980), if firms tend to imitate each other’s research strategy, much of R&D expenditure may be essentially duplicative, and consequently socially wasteful (p. 267). Therefore, welfare analysis shall not neglect the firms’ side because they are part of an economy too. My results focus on this aspect and show that if the number of firms is small, due to a significant reduction in total R&D effort, the net surplus (which is the innovator’s realized profit subtracted by all firms’ R&D effort) tends to increase after a merger. This also suggests an argument put forth earlier by Fullerton and McAfee (1999) and Che and Gale (2003) that the optimal number of competitors is two.

1.1 Other Related Literature

My work is directly related to those on asymmetric all-pay contest/auction, because asymmetry is always involved in merger analysis regardless of modeling choice. If firms are symmetric pre-merger, then they must be asymmetric post-merger, let alone they might be asymmetric at first. Only recently, Siegel (2009, 2010) has contributed significantly to the understanding of asymmetries
in complete information all-pay auctions.\footnote{A comprehensive survey of earlier studies on all-pay contests or all-pay auctions can be found in the book of Konrad (2009).} The incomplete information case is considerably less well-understood, mostly due to the difficulty in obtaining explicit solutions. Most research in the latter area considers only 2 players. Amann and Leininger (1996) show existence and uniqueness of equilibrium under independent private value setting with continuous signals, while Szech (2011) studies such model with discrete signals. Siegel (2014) also studies discrete signals game, where signals can be correlated and values interdependent, and makes connections between incomplete and complete information games. The only exception is Parreiras and Rubinchik (2009). They model a contest among many asymmetric players and prove the existence of a unique equilibrium. Although my theoretical results nest within theirs, their model is too abstract for easy use within application. Instead, I offer an explicit characterization of a multi-player asymmetric all-pay contest/auction model for applications, which is my second contribution in this paper.


The rest of the paper is organized as follow. Section 2 introduces the asymmetric all-pay contest model in the language of a R&D race and characterizes the unique perfect Bayesian equilibrium. Its application to merger analysis is demonstrated in Section 3 and section 4 presents the simulated merger effects. Section 5 concludes. All proof are in the appendices.
2 The Model

Consider \( n = n_1 + n_2 \) firms in a R&D race (or a technology procurement). Following the assumptions in Dasgupta and Stiglitz (1980), let firms follow the same research strategy toward some patentable characteristics so that the firm who exerts the most effort will invent first and win the R&D race\(^6\) and let the winner capture all benefits that are to be had among firms (i.e. the winner takes all). Each firm possesses a private signal concerning the potential profit\(^4\). For simplicity, let each firm’s (potential) profit be equal to their signal. Firms decide their investment levels (which I call effort later on) simultaneously based on the observed signals.

Suppose firms are of two types, where \( n_1 \) are type 1 and \( n_2 \) are type 2. The signals of type 1 firms follow a distribution function \( F \) and the signals of type 2 firms follow a (different) distribution function \( G \). Assume further that all signals are independent, that \( F \) and \( G \) are both continuously differentiable functions on a common support \([0, 1]\), and that the corresponding density functions are continuous and are bounded away from zero for all values in \([0, 1]\). Finally, assume that everything described so far is common knowledge except each firm’s private signal.

Each firm decides how much effort to exert in R&D activities simultaneously. Suppose the strategies are symmetric within each type. Specifically, denote \( a_i = \alpha(x_i) \), the effort a type 1 firm with signal \( x_i \) will exert, and \( b_j = \beta(y_j) \) for a type 2 firm with signal \( y_j \). Then, the expected payoffs for firms of each type can be

\(^3\)Fudenberg et al. (1983) also assume that an higher level of effort leads to an innovation sooner, though their effort levels are discrete. In this sense, it is also the firm with the most effort that is the first to invent and realizes profits.

\(^4\)As discussed by Bhattacharya (2016), firms’ different profits can be stemmed from their differences in delivering a patent to a commercial good, or can be their different assessments of a potential procurement contract.

\(^5\)This last assumption is the sufficient condition for the uniqueness of equilibrium, according to Parreiras and Rubinchik (2009).
written as

\[ \Pi_1(a_i, x_i; \alpha, \beta) = x_i F^{n_1-1}(\alpha^{-1}(a_i)) G^{n_2}(\beta^{-1}(a_i)) - a_i, \]

\[ \Pi_2(b_j, y_j; \alpha, \beta) = y_j F^{n_1}(\alpha^{-1}(b_j)) G^{n_2-1}(\beta^{-1}(b_j)) - b_j. \]

The problem is usually solved through first-order conditions, which typically forms a system of differential equations. To simplify the analysis, I follow Amann and Leininger (1996) and define \( k(x) = \beta^{-1}(\alpha(x)) \), which maps the signal of a type 1 firm into the signal of a type 2 firm who exerts the same effort. \( k(x) \) is well defined on \((0,1]\) and maps \([0,1]\) to \([0,1]\). Then, the unique equilibrium\(^7\) can be characterized as follow

**Theorem 1** (Unique Equilibrium). The following strategies form the unique perfect Bayesian equilibrium

\[ \alpha(x) = \int_x^{\max\{k^{-1}(0)\}} k(t) d[F^{n_1}(t) G^{n_2-1}(k(t))] \]

\[ \beta(x) = \alpha(k^{-1}(x)) \]

where \( k(x) \) is the solution to the following ordinary differential equation with boundary condition \( k(1) = 1 \),

\[ k(x)[F^{n_1}(x) G^{n_2-1}(k(x))]' = x[F^{n_1-1}(x) G^{n_2}(k(x))]' \]

The proof essentially transforms the system of differential equations (first-order conditions) into an ordinary differential equation using the defined mapping \( k \). The boundary condition of \( k(1) = 1 \) simply means that firms with the highest signal exert the same level of effort regardless of their types.

\(^6\)k(x) may have a mass at 0. See the proof of Theorem 1 in Appendix A for more detail.

\(^7\)This result is nested in Parreiras and Rubinchik (2009) whose model is more general yet too abstract. For application purpose, I explicitly solve the equilibrium strategies for a case of restricted asymmetry.
3 Application to Merger Analysis

In this section, I follow the methodology of Dalkir, Logan and Masson (2000). Suppose each firm has some signal draws from a common distribution function and selects the highest one. Premerger I define the number of draws by firm \( i \) as \( q_i \). The merger of two firms, \( i \) and \( j \), are modeled as a single firm with a total number of draws \( q_{\text{merger}} = q_i + q_j \). If all firms have the same number of draws prior to merger, the post merger number of draws will differ among firms. This necessitates the analysis of asymmetric model in the previous section. If firms are asymmetric premerger, they may or may not be symmetric post merger.

Let us start with the number of firms equal to the number of i.i.d signal draws, which is denoted as \( n \). Mergers are thus a “regrouping” of the signals among the rest of firms. In a typical two-firm merger, the merged firm with two signals faces \( n - 2 \) rivals each with a single signal. To simplify the analytical work (not simulation yet), assume all signals follows a cumulative distribution \( F(x) = x^a \) \((a > 0)\) on \([0,1]\).

3.1 The symmetric market

I review the symmetric case first in order to have easier comparison with the asymmetric case later. Using the language in the previous section, \( F(x) = G(x) = x^a \) and \( n_1 + n_2 = n \) fully depict the situation. Therefore, the optimal effort of a firm with signal \( x \) is

\[
\int_0^x t dt = \frac{(n - 1)a}{(n - 1)a + 1} x^{(n-1)a+1}.
\]

Accordingly, the expected effort of a firm is

\[
\int_0^1 \frac{(n - 1)a}{(n - 1)a + 1} x^{(n-1)a+1} dx = \frac{(n - 1)a^2}{[(n - 1)a + 1](na + 1)}.
\]

3.2 The asymmetric market

In a two-firm merger, there are \( n_1 = n - 2 \) unmerged firm. Their number and distribution of signals remain unchanged, i.e. \( q_i = 1 \) and \( F(x) = x^a \). For the
merged firm, it has $q_{\text{merger}} = 2$ signals each following $F$, and thus the highest signal follows $G(x) = F^2(x) = x^{2a}$. Using the result of Theorem 1, $k(x)$ can be solved analytically,

$$
k(x) = \begin{cases} 
1 + \frac{1}{2} \ln x & \text{if } n = 3 \\
1 + \frac{1}{n-3} (1 - x^{\frac{n-3}{2}}) & \text{if } n \geq 3
\end{cases}
$$

which is the solution for uniform distribution ($a = 1$). Solutions for other values of $a$ and the detailed derivation is omitted for expository easiness and they can be found in Appendix B. Based on the expression of $k(x)$, I can calculate the equilibrium effort $\alpha(x)$ and $\beta(x)$.

In Figure 1, I illustrate the equilibrium effort using uniform distribution ($a = 1$) for the case of 4 premerger firms (postmerger, there are two unmerged firms and one merged firm). It is shown that the merged firm always exerts positive effort, while the unmerged firms do not exert any effort for small signals, which is 0.25 and below in this case. However, when signals are relatively large (0.39 and

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8The number of firms does not qualitatively change the shape of equilibrium strategies.
above), unmerged firms exert a higher effort than the merged firm. This kind of strategies seems strange at first glance, as compared with their counterpart in other asymmetric auction formats where the strategies of different types do not cross in the interior of bidding interval. The core intuition behind is as follows. The unmerged firms are “weaker” than the merged firm in the sense of first-order stochastic dominance. When the signal is low, weaker firms know that they are not likely to win the competition and thus avoid this irreversible effort. On the other hand, when the signal is high, it is in the interest of weak firms to exert high effort as the chance of winning from doing so is sufficiently high. The reason for this is that although the strong firm is fully aware of the equilibrium strategy of weak firms, the likelihood of her weak rivals having high signals and exert aggressive efforts is rather small, and thus, the strong firm almost “overlooks” the weak firms.

4 Effects of Merger

Before calibrating the model, I would like to discuss some features of the model. Firstly, my model implies that there will be a winner anyway. Then the effort of all firms (including the winner) is a “waste” from the viewpoint of a social planner. The ideal situation would be that the winner realizes all profits from innovation while every firm spend little R&D input. Nevertheless, governments (or procurers) may not have access to firms’ signals and thus have to encourage innovations and award patents/contracts through such costly races. Therefore, it is important to calibrate the total effort level in the industry, in addition to the effort of individual firms. It seems that merger would reduce total effort and cause less waste. Nevertheless, it shall not be taken as an implication of this model that monopoly is the best market structure. A monopolist in my model would exert ε effort, which is not desirable because the probability of innovation is also negligible. Moreover, potential entrants, who are absent in my model, shall play

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9This seems strange, as innovation is not deterministic. But my model does not impose time constraints on firms. There will be some firm who makes an innovation at last and this firm is the winner.
a more significant role in a monopoly market.\footnote{For example, Harris and Vickers (1985) and Leininger (1991) study patent races between incumbent and challenger under complete information and in their environments, both firms’ efforts are socially waste and such waste is inevitable.}

Secondly, inefficiency is present whenever there is asymmetry. By inefficiency, I mean that the firm with the highest signal in the market does not win the R&D race. As can be inferred from Figure 1, inefficiency may arise in two scenarios. One is that the merged firm does not possess the highest signal but happens to win because all signals of unmerged firms are low and fall inside a neighbourhood of the no-effort region. The other case, which is more likely, is that the merged firm does have the highest signal, yet she loses the race because some unmerged firm exerts a higher level of effort.

Finally, based on the previous two arguments, a merger appears to raise a tradeoff between a reduction in total effort and an increase in inefficiency. Thus, the overall effect, which is the realized profit (the winning signal) subtracted by the total effort, is ambiguous and is worthy of simulation. I label this overall effect as the net surplus on the firms’ side from the R&D race.

### 4.1 Calibration of merger effects

Calibration of the model is conducted through numerical calculation and simulation. Except where otherwise stated, I use uniform distribution for simulation. In all cases, I only consider a merger of 2 firms.

Table 1 shows the baseline results. The first column is the market structure, depicting the number of firms both premerger and post-merger, and column 2 is the number of signals per firm. Column 3 is the ex ante expected effort of each firm, which adds up to be the expected total effort presented in column 4. It shows that after a merger, each firm is expected to exert more effort, no matter merged or not. On an aggregate level, post-merger total effort increases faster in the number of premerger firms. A merger reduces total effort as long as the number of firms is small, and the cutoff number here is 6, which is a close description of a concentrated industry. Column 5 records the simulated total effort.

The results in the last 3 columns are derived through simulation for 1000
Table 1: Effects of Merger

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-merger</td>
<td>n = 3</td>
<td>1</td>
<td>0.167</td>
<td>0.5</td>
<td>0.513</td>
<td>0.755</td>
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<tr>
<td></td>
<td>Post-merger</td>
<td>n1 = 1</td>
<td>1</td>
<td>0.190</td>
<td>0.453</td>
<td>0.420</td>
<td>0.750</td>
<td>0.106</td>
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<tr>
<td></td>
<td>Post-merger</td>
<td>n2 = 1</td>
<td>2</td>
<td>0.263</td>
<td>(0.249)</td>
<td>(0.198)</td>
<td>(0.137)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Post-merger</td>
<td>t-stat</td>
<td>-0.093</td>
<td>-0.005</td>
<td>0.088</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>-17.08</td>
<td>-2.387</td>
<td>47.40</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Pre-merger</td>
<td>n = 4</td>
<td>1</td>
<td>0.150</td>
<td>0.6</td>
<td>0.661</td>
<td>0.860</td>
<td>0</td>
<td>0.199</td>
</tr>
<tr>
<td></td>
<td>Post-merger</td>
<td>n1 = 2</td>
<td>1</td>
<td>0.171</td>
<td>0.590</td>
<td>0.549</td>
<td>0.795</td>
<td>0.106</td>
</tr>
<tr>
<td></td>
<td>Post-merger</td>
<td>n2 = 1</td>
<td>2</td>
<td>0.248</td>
<td>(0.347)</td>
<td>(0.168)</td>
<td>(0.237)</td>
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<td></td>
<td>Post-merger</td>
<td>t-stat</td>
<td>-0.052</td>
<td>-0.005</td>
<td>0.047</td>
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<tr>
<td>t-stat</td>
<td>-5.95</td>
<td>-3.035</td>
<td>11.08</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Pre-merger</td>
<td>n = 5</td>
<td>1</td>
<td>0.133</td>
<td>0.667</td>
<td>0.646</td>
<td>0.828</td>
<td>0</td>
<td>0.182</td>
</tr>
<tr>
<td></td>
<td>Post-merger</td>
<td>n1 = 3</td>
<td>1</td>
<td>0.148</td>
<td>0.666</td>
<td>0.613</td>
<td>0.825</td>
<td>0.077</td>
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<tr>
<td></td>
<td>Post-merger</td>
<td>n2 = 1</td>
<td>2</td>
<td>0.221</td>
<td>(0.400)</td>
<td>(0.148)</td>
<td>(0.207)</td>
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<td></td>
<td>Post-merger</td>
<td>t-stat</td>
<td>-0.033</td>
<td>-0.003</td>
<td>0.03</td>
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<tr>
<td>t-stat</td>
<td>-2.976</td>
<td>-2.036</td>
<td>4.831</td>
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<tr>
<td>Pre-merger</td>
<td>n = 6</td>
<td>1</td>
<td>0.119</td>
<td>0.714</td>
<td>0.726</td>
<td>0.859</td>
<td>0</td>
<td>0.134</td>
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<tr>
<td></td>
<td>Post-merger</td>
<td>n1 = 4</td>
<td>1</td>
<td>0.130</td>
<td>0.716</td>
<td>0.705</td>
<td>0.856</td>
<td>0.092</td>
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<tr>
<td></td>
<td>Post-merger</td>
<td>n2 = 1</td>
<td>2</td>
<td>0.198</td>
<td>(0.459)</td>
<td>(0.127)</td>
<td>(0.371)</td>
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<tr>
<td></td>
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<td>t-stat</td>
<td>-0.019</td>
<td>-0.003</td>
<td>0.016</td>
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</tr>
<tr>
<td>t-stat</td>
<td>-1.467</td>
<td>-3.887</td>
<td>1.812</td>
<td></td>
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<td></td>
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<tr>
<td>Pre-merger</td>
<td>n = 7</td>
<td>1</td>
<td>0.107</td>
<td>0.750</td>
<td>0.752</td>
<td>0.878</td>
<td>0</td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td>Post-merger</td>
<td>n1 = 5</td>
<td>1</td>
<td>0.115</td>
<td>0.753</td>
<td>0.732</td>
<td>0.876</td>
<td>0.064</td>
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<tr>
<td></td>
<td>Post-merger</td>
<td>n2 = 1</td>
<td>2</td>
<td>0.178</td>
<td>(0.493)</td>
<td>(0.110)</td>
<td>(0.417)</td>
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<td></td>
<td>Post-merger</td>
<td>t-stat</td>
<td>-0.020</td>
<td>-0.002</td>
<td>0.018</td>
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<tr>
<td>t-stat</td>
<td>-1.242</td>
<td>-2.752</td>
<td>1.552</td>
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</table>

Note: The t-stat measures the significance of the difference of post-merger and pre-merger efforts. A common threshold for significance at 5% is 1.96.
times. The numbers in column 6 is the average winning signals, or the average realized profit, with the standard deviation recorded in parenthesis. According to the values presented, inefficiency does exist, and the average realized profit is significantly lower after a merger, but the magnitude of decrease is small (less than 1%). The frequency of inefficient outcome is shown in column 7 (around 10%).

The last column measures the integrated effect on the net surplus. It shows that in all case\textsuperscript{11} a merger increases the net surplus and that the increase is statistically significant if there is a small number of firms premerger. This is aligned with previous findings because the reduction of total effort outweighs the decrease in profit when the number of firms is small. It may further imply that two firms is optimal in industries which are innovation-driven and require intensive R&D investment. Such a result, distinct from the implication of the innovation theory of harm as summarized in Federico (2017), calls for an attention to the welfare analysis where firms shall also be included as part of the economy.

Table 2 presents a comparison between symmetric and asymmetric mergers through which triopoly becomes duopoly. The first row, which can be found in Table 1 as well, is the case of symmetric firms becoming asymmetric after a merger. The second row shows the case where asymmetric firms merging to become symmetric. The third row shows the case where firms become even more asymmetric after a merger. Please note that I omit the simulated total effort, because it can be inferred directly from winner’s signal and net surplus. Since the number of firms is still small, merger leads to higher expected effort level per firm and reduces total effort as expected. Moreover, a net surplus gain is realized in all cases.

4.2 Sensitivity

One calibration issue is whether it is sensitive to the chosen density function, i.e. the uniform distribution. Without a detailed analysis, I examine the sensitivity with an example. Suppose each firm has two signals from a uniform distribution. Then the highest signal does not follow uniform. Table 3 presents a contrast

\textsuperscript{11}The net surplus does not always increase after a merger. If the number of firms is large enough, for example 70, the loss in efficiency is dominant, and thus reduces the net surplus.
Table 2: Symmetry vs. Asymmetry

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<tr>
<th>Market structure</th>
<th>(1)</th>
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<td>0.5</td>
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<td>0.590</td>
<td>0.795</td>
<td>0.106</td>
<td>0.246</td>
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of calibrated results for triopoly merging to be duopoly. The case of uniform
distribution is in the first row, and the new model the last row.

Table 3: Uniform vs. Non-uniform Distribution

<table>
<thead>
<tr>
<th>Market structure</th>
<th>(1)</th>
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<th>(3)</th>
<th>(4)</th>
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<td>pre-merger</td>
<td>n = 3</td>
<td>1</td>
<td>0.167</td>
<td>0.5</td>
<td>0.755</td>
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<tr>
<td>post-merger</td>
<td>n1 = 1</td>
<td>1</td>
<td>0.190</td>
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<td>0.750</td>
<td>0.106</td>
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The two sets of results do not exhibit qualitative differences. Conclusions in
previous part can be carried over to the non-uniform case, which is a desirable
property since close model shall have close outcomes. It may be extended further
that if firms have similar initial shares of signals, merger would yield to similar
consequences based on my model.

5 Conclusion

This paper develops a model for analyzing mergers in markets where, apart
from price competition, R&D investment decision also plays an important role.
R&D investment is modeled as an all-pay contest, and I give an explicit character-
ization of the unique solution to this multi-player asymmetric all-pay contest.
Simulation shows that each firm is expected to exert more effort after a merger,
but the total effort will be lower after a merger if the industry is concentrated
premerger. Merger may cause inefficiency, but the loss is not large. As an overall estimate of merger effects, the net surplus tends to increase after merger if the number of firms is small.

As an early attempt to analyze merger effects on R&D investment using an incomplete information contest model, there are several directions to extend current work. Firstly, in my model, the winner takes all potential profits, while efforts by the other firms are simply wasted. But it may not be so in real life. Firms, though falling behind, may come up with some second-best substitutes and share a fraction of the signaled profits. It would be interesting to explore models that incorporate the multi-prize feature into merger analysis.

Secondly, the current model considers R&D investment for only one round. It is reasonable to extend it to multiple rounds as R&D projects can be progressive. Moreover, it is possible for a temporary laggard to catch up and even surpass the leader. Thus, to develop methods for estimating merger effects in the long run is important, yet challenging as well.

Thirdly, current analysis is under the independent private value framework. Since my model in this paper is more appropriately applied to industries with high concentration, firms’ profit signals or their costs for R&D activity are likely to be correlated or affiliated. How merger outcomes would change when affiliation is taken into consideration remains an open question.

Lastly, the focus of this paper is the likely effects of merger on R&D investment decisions, while the more direct effects on price competition is captured by the signals for simplicity. It is, thus, worthwhile to develop a comprehensive model which unifies both price and R&D competitions. All of these, however, are left for future research.
A Proof of Theorem 1

The existence and uniqueness of equilibrium is proved in Parreiras and Rubinchik (2009) in a more general sense. It is then sufficient for me to prove the equilibrium using necessary conditions.

Before proving, I would like to define the effort distribution function $M(a) := F(\alpha^{-1}(a))$ and $N(b) := G(\beta^{-1}(b))$ as in Amann and Leininger (1996). I present some general results concerning the equilibrium below. These are of the same essence as those in Amann and Leininger (1996), but the proofs are different from theirs.

Lemma 1 (Common Support). $\text{supp}(M) = \text{supp}(N)$.

Proof. Suppose $a = \alpha(x) \in \text{supp}(M)$, yet $a \notin \text{supp}(N)$. Then there is an open neighborhood of $a$, $U(a)$, such that for all $a' \in U(a)$, $N(a') = N(a)$. Suppose $N(a) > 0$. Then if type 1 firms all lower their effort from $a$ to $a' < a$ but $a' \in U(a)$, their probability of winning does not change, while they may save cost through reducing effort from $a$ to $a'$. This improves type 1 firms’ payoffs and, thus, $a$ cannot be optimal, which contradicts $a = \alpha(x)$. Consequently, $N(a) = 0$.

A similar argument holds for type 2 firms. \qed

Lemma 2 (Full Support).

$$\text{supp}(M) = [0, \max_{x \in [0,1]} \alpha(x)],$$
$$\text{supp}(N) = [0, \max_{y \in [0,1]} \beta(y)].$$

Proof. Suppose there is a “hole” $(s, t)$, $0 < s < t < \max_y \beta(y)$, over which $M$ is constant, while $s$ and $t$ belong to the support of $M$. Then, by Lemma 1, $N$ must be constant over $(s, t)$. Since $N(s) = N(t)$, it can never be optimal for type 2 firms to exert effort $b = t$ by the same argument as in Lemma 1. Hence, such a hole in the interior of $[0, \max_{x \in [0,1]} \alpha(x)]$ cannot exist. Neither can it exist in the interior of $[0, \max_{y \in [0,1]} \beta(y)]$. \qed
Lemma 3 (Monotonicity). Let $x > x'$, $a = \alpha(x)$, and $a' = \alpha(x')$. Then $N(a) \geq N(a')$.

Proof. By definition of equilibrium, we must have

$$
\Pi_1(a, x; \alpha, \beta) \geq \Pi_1(a', x; \alpha, \beta),
\Pi_1(a', x'; \alpha, \beta) \geq \Pi_1(a, x'; \alpha, \beta).
$$

Plug in the expression of $\Pi_1$ and rearrange the terms, we get

$$(xM^{n_1-1}(a) - x'M^{n_1-1}(a'))(N^{n_2}(a) - N^{n_2}(a')) \geq 0.$$ 

By definition of effort distribution, $M(a) = F(\alpha^{-1}(a)) = F(x) > F(x') = M(a')$. Then the first term is positive and, thus, the second term must be non-negative, which implies $N(a) \geq N(a')$. \qed

Lemma 4 (No Atoms). $M$ is continuous on $[0, \beta(1)]$ with $\beta(1) \leq 1$, $N$ is continuous on $[0, \alpha(1)]$ with $\alpha(1) \leq 1$.

Proof. Suppose $N$ is not continuous at $z$; i.e., let $z \in (0, \alpha(1)]$ and $\delta > 0$ such that $N(z) > N(z - \varepsilon) + \delta$ for all $\varepsilon < \varepsilon_1(z, \delta)$.

Using the monotonicity in Lemma 3, we have

$$
\Pi_1(z, x; \alpha, \beta) - \Pi_1(z - \varepsilon, x; \alpha, \beta) = xM^{n_1-1}(z)N^{n_2}(z) - z - [xM^{n_1-1}(z - \varepsilon)N^{n_2}(z - \varepsilon) - (z - \varepsilon)] \\
> xM^{n_1-1}(z)[N^{n_2}(z) - N^{n_2}(z - \varepsilon/2)] - \varepsilon/2 \\
> xM^{n_1-1}(z)[N^{n_2}(z) - (N(z) - \delta)^{n_2}] - \varepsilon/2 \\
> \frac{\varepsilon}{2}
$$

for any $\varepsilon < xM^{n_1-1}(z)[N^{n_2}(z) - (N(z) - \delta)^{n_2}] = \varepsilon_2(x, z, \delta)$.

If we define $\bar{\varepsilon} = \min\{\varepsilon_1(z, \delta), \varepsilon_2(x, z, \delta)\}$, then for all $\varepsilon < \bar{\varepsilon}$,

$$
\Pi_1(z, x; \alpha, \beta) - \Pi_1(z - \varepsilon, x; \alpha, \beta) > \frac{\varepsilon}{2} > 0,
$$

18
which means that type 1 firm will not exert any effort in the range of \([z - \bar{\varepsilon}, z]\). As a consequent, \(M(\cdot)\) is constant for \([z - \bar{\varepsilon}, z]\). However, by the same argument as in Lemma 1, \(z\) cannot be a best response for type 2 firms, which is in contradiction to \(z = \alpha(x)\) for some \(x \in [0, 1]\). Thus, \(N\) must be continuous.

A symmetric argument works for \(M\).

**Lemma 5.** If \(F(0) = G(0) = 0\), then \(\min \{M(0), N(0)\} = 0\).

**Proof.** Suppose \(M(0) = s > 0\) and \(N(0) = t > 0\). Then for any \(x \neq 0\), take some \(\epsilon < \frac{n-1}{n}x^{n_1-1}t^{n_2}\),

\[
\Pi_1(\epsilon, x; \alpha, \beta) = xM^{n_1-1}(\epsilon)N^{n_2}(\epsilon) \\
\geq xs^{n_1-1}t^{n_2} - \epsilon \\
> \frac{1}{n}xs^{n_1-1}t^{n_2} \\
= \Pi_1(0, x; \alpha, \beta).
\]

However, this means that for almost any type 1 firm (except those with signal 0), a strictly positive effort would yield a better payoff than no effort, which is a contradiction to equilibrium. Thus, either \(M(0) = 0\) or \(N(0) = 0\), whichever completes the proof. □

Lemmas 1, 2 and 3 together implies that \(\alpha(1) = \beta(1)\). Also, based on the 5 Lemmas, \(k(x)\) is well defined on \((0,1]\) and maps \([0,1]\) to \([0,1]\). Moreover, \(k(x)\) is strictly increasing except possibly on \(k^{-1}(0)\).

We are now ready to prove Theorem 1.

**Proof of Theorem 1.** The expected payoffs for firms of each type are

\[
\Pi_1(a, x; \alpha, \beta) = xF^{n_1-1}(\alpha^{-1}(a))G^{n_2}(\beta^{-1}(a)) - a \\
\Pi_2(b, y; \alpha, \beta) = yF^{n_1}(\alpha^{-1}(b))G^{n_2-1}(\beta^{-1}(b)) - b
\]

where I will suppress subscripts \(i\) and \(j\) for succinctness. Let \(f\) and \(g\) be the
density function of $F$ and $G$ respectively. Then, the first-order conditions are

\[
x (n_1 - 1) F^{n_1 - 2}(x) f(x) G^{n_2} (\beta^{-1}(\alpha(x))) \frac{1}{\alpha'(x)} + n_2 F^{n_1 - 1}(x) G^{n_2 - 1} (\beta^{-1}(\alpha(x))) g(\beta^{-1}(\alpha(x))) \frac{1}{\beta'(\beta^{-1}(\alpha(x))))} = 1 \quad (1)
\]

and

\[
y (n_1 F^{n_1 - 1}(x) f(\alpha^{-1}(\beta(y))) G^{n_2 - 1}(y) \frac{1}{\alpha'(\alpha^{-1}(\beta(y)))} + (n_2 - 1) F^{n_1}(x) G^{n_2 - 2}(\beta^{-1}(y)) g(y) \frac{1}{\beta'(y)}) = 1. \quad (2)
\]

Given the definition of $k(x)$, we have $\beta(k(x)) = \alpha(x)$ and

\[
k'(x) = (\beta^{-1})'(\alpha(x)) \alpha'(x) = \frac{\alpha'(x)}{\beta'(\beta^{-1}(\alpha(x)))}.
\]

Then, equation (1) can be rewritten as

\[
\alpha'(x) = x (n_1 - 1) F^{n_1 - 2}(x) f(x) G^{n_2}(k(x)) + n_2 F^{n_1 - 1}(x) G^{n_2 - 1}(k(x)) g(k(x)) k'(x) = x [F^{n_1 - 1}(x) G^{n_2}(k(x))]'.
\]

Let $y = k(x)$ in equation (2) and observe that $\alpha^{-1}(\beta(k(x))) = x,$

\[
\alpha'(x) = k(x) (n_1 F^{n_1 - 1}(x) f(x) G^{n_2 - 1}(k(x)) + (n_2 - 1) F^{n_1}(x) G^{n_2 - 2}(k(x)) g(k(x)) k'(x) = k(x) [F^{n_1}(x) G^{n_2 - 1}(k(x))]'.
\]

Therefore, a necessary condition for $k(x)$ is such that

\[k(x) [F^{n_1}(x) G^{n_2 - 1}(k(x))]' = x [F^{n_1 - 1}(x) G^{n_2}(k(x))]'.
\]

This is an ordinary first-order differential equation, which admits a unique solution with boundary condition $k(1) = 1.$
Then, \( k(x) \) yields the unique equilibrium strategies

\[
\alpha(x) = \int_{\max\{k^{-1}(0)\}}^{x} k(t) d\left[F^{n_1}(t)G^{n_2-1}(k(t))\right]
\]

\[
\beta(x) = \alpha(k^{-1}(x))
\]

where \( \alpha(x) = 0 \) if and only if \( x \in k^{-1}(0) \) by Lemma 5.

\[\square\]

**B Derivation in Merger Analysis**

In this section, I derive the analytical solution to the model introduced in Merger Analysis. The boundary condition that \( k(1) = 1 \) is implied in each of the following solutions.

When the signals follow \( F(x) = x^a \), after a merger, there are \( n_1 = n - 2 \) unmerged firms whose signals follow distribution \( F \). For the merged firm, its highest signal follows \( G(x) = F^2(x) = x^{2a} \). Then, according to Theorem 1,

\[
k(x)[(x^a)^{n-2}]' = x[(x^a)^{n-3}k^{2a}(x)]'
\]

which is, after simplification,

\[
(n - 2)x^{a-1} = (n - 3)k^{2a-1}(x) + 2xk^{2a-2}(x)k'(x).
\]

Several special cases shall be addressed before I give a general solution.

**When** \( a = 1 \), equation \( \text{(3)} \) becomes

\[
k'(x) + \frac{n - 3}{x}k(x) = \frac{n - 2}{x}.
\]

The solution to this first order linear differential equation is

\[
k(x) = \begin{cases} 
1 + \frac{1}{2} \ln x & \text{if } n = 3 \\
1 + \frac{1}{n-3} \left(1 - x^{-\frac{n-3}{2}}\right) & \text{if } n \geq 3.
\end{cases}
\]
When $a = \frac{1}{2}$, equation (3) becomes

$$\frac{k'(x)}{k(x)} + \frac{n - 3}{2x} = \frac{n - 2}{2} x^{-\frac{3}{2}}$$

which is equivalent to

$$d \ln(k(x)) = \left(\frac{n - 2}{2} x^{-\frac{3}{2}} - \frac{n - 3}{2x}\right) dx.$$ 

Therefore, the solution is

$$k(x) = x^{-\frac{n-2}{2}} e^{(n-2)(1-x^{-\frac{1}{2}})}.$$

When $a \neq 1$ or $\frac{1}{2}$, equation (3) is a Bernoulli differential equation. Define $z = k^{1-(2-2a)} = k^{2a-1}$ to transform it into a linear differential equation such that

$$z' + \frac{(2a - 1)(n - 3)}{2x} z = \frac{(2a - 1)(n - 2)}{2} x^{a-2}$$

The solution to $z$ is

$$z(x) = \frac{(2a - 1)(n - 2)}{(2a - 1)(n - 2) - 1} x^{a-1} - \frac{1}{(2a - 1)(n - 2) - 1} x^{-\frac{2a-1}{4}}.$$

and then

$$k(x) = [z(x)]^{\frac{1}{2a-1}}.$$

References


