Chapter 14
Capital flows
Answer Key to Exercises[1]

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1. ## Unanticipated sudden stop

   Consider the 3-period model of Section 4. Initially, the economy is characterized by the unconstrained solution discussed in the text. When period 2 comes along, however, there is an unanticipated sudden stop that requires that the current account in periods 2 and 3 be non-negative. In this context:

   (a) Solve for consumption of tradables, the real exchange rate, the trade balance, and the current account in periods 2 and 3.

   (b) How does this solution compare to the solution for an anticipated sudden stop discussed in the text? Explain the differences, if any.

   ### Answer

   [TO BE WRITTEN.]

2. ## Shocks under flexible exchange rates

   To gain intuition behind the results illustrated in Figures 8 and 9, this exercise asks you to solve the continuous-time version of the model of Section 5.

   Assume foreign inflation is zero. Let preferences be given by

$$\int_0^\infty \alpha_t \left[\log(c_t^T) + \log(c_t^N)\right] e^{-\beta t} dt. \tag{60}$$

   The intertemporal budget constraint takes the form:

$$a_0 + \int_0^\infty \left(y_t^T + \frac{y_t^N}{e_t} + \tau_t \right) e^{-rt} dt = \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t} + i_t m_t \right) e^{-rt} dt. \tag{61}$$

   The cash-in-advance constraint is given by

$$m_t = c^T + \frac{c^N}{e_t},$$

[1] This answer key is part of a graduate textbook on "Open Economy Macroeconomics in Developing Countries", currently under preparation by the author (to be published by MIT Press) and should be cited accordingly. The equation numbering of this answer key continues that of Chapter 14.

where, for notational simplicity, we have set to one the parameter associated with the continous-time cash-in-advance constraint. The supply side of the model follows Chapter 8 with Calvo-pricing. In this context:

(a) Derive the first-order conditions and set-up a dynamic system in n and π.

(b) Analyze the effects of an unanticipated and temporary increase in α_t.

(c) Analyze the effects of an unanticipated and temporary fall in r_t.

Answer

(a) The Lagrangean is given by

$$\begin{aligned}\mathcal{L} &= \int_0^\infty \alpha_t \left[\log(c_t^T) + \log(c_t^N)\right] e^{-\beta t} dt \\ &+\lambda \left[a_0 + \int_0^\infty \left(y_t^T + \frac{y_t^N}{e_t} + \tau_t\right) e^{-rt} dt - \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t}\right)(1+i_t)e^{-rt} dt\right].\end{aligned}$$

The first-order conditions with respect to c_t^T and c_t^N are given by, respectively,

$$\begin{aligned}\frac{\alpha_t e^{-\beta t}}{c_t^T} &= \lambda(1+i_t)e^{-rt}, \\ \frac{\alpha_t e^{-\beta t}}{c_t^N} &= \frac{\lambda}{e_t}(1+i_t)e^{-rt}.\end{aligned}$$

Rearranging and combining the two expressions, we obtain

$$\frac{\alpha_t}{c_t^T} = \lambda(1+i_t)e^{-(r-\beta)t}, \tag{62}$$

$$c_t^T = \frac{c_t^N}{e_t}. \tag{63}$$

Substituting the last expression into the cash-in-advance constraint:

$$m_t = 2c_t^T. \tag{64}$$

Since $e_t m_t = n_t$ and $c_t^T = c_t^N/e_t$, we can also write

$$n_t = 2c_t^N. \tag{65}$$

To derive our differential equation in i, recall that $\dot{m}_t/m_t = \mu - \varepsilon_t$ and use (62) and the interest parity condition to obtain

$$\dot{i}_t = (1+i_t)(i_t - \beta - \mu).$$

As in Chapter 8, the dynamic system is given by

$$
\begin{aligned}
\dot{n}_t &= n_t(\bar{\mu} - \pi_t), \qquad (66)\\
\dot{\pi}_t &= \theta\left(y_f^N - c_t^N\right). \qquad (67)
\end{aligned}
$$

Using (66), we can rewrite the last equation as

$$\dot{\pi}_t = \theta\left(y_f^N - \frac{n_t}{2}\right).$$

(b) Suppose the system is in an initial steady-state and that there is an unanticipated and temporary increase in α_t from a low value to a high (and constant) value. For the purposes of this experiment, we can assume that $\beta = r$. First-order condition (62) then becomes

$$\frac{\alpha_t}{c_t^T} = \lambda(1 + i_t). \qquad (68)$$

Let us establish the changes in endogenous variables at time T. Notice first that c^T cannot jump at T because of equation (64). Since α jumps down at time $\not T$ and c^T does not jump, then the last expression tells us that i_t will jump down as well. Since i_t is governed by an unstable differential equation, it follows that i will jump up at time 0, increase over time until time T and then jump down at T back to its pre-shock value. From (68), it follows that c_t^T is going to fall during $[0, T)$. Given the economy's resource constraint, this implies that c^T will jump up at $t = 0$, fall over time to a level below its preshock value at time T. This is precisely the path depicted for the discrete time model in Figure 8, Panel B. Given equation (64), real money balances m follow exactly the same path as c^T.

Notice that the temporary increase in α does not affect at all the system given by equations (66) and (67). Hence, π and c^N are not affected. Given that c^N is not affected, equation (63) indicates that e_t follows the path depicted in Figure 8, Panel G for the discrete time case.

(c) Suppose now that there is an unanticipated and temporary fall in r_t (that is, r_t, which is initially equal to β, falls below β during $[0, T)$ and then returns to the value β at time T). We use condition (62) to establish the jumps at T. At T, r increases. c^T cannot jump because of the cash-in-advance constraint. The jump in r implies that $e^{-(r-\beta)t}$ jumps down (from a number larger than than to one, since $r_T = \beta$). It follows that i needs to increase at T.

Since i jumps up at time T, it follows that i_t falls on impact, falls during the transition and jumps up at time T. What about the path of consumption? The fact that i falls over time implies that c^T becomes cheaper over time, so c^T should increase over time in this regard. But the lower r calls for a decreasing path of consumption. So it is not obvious how c^T will change over time.

Following the same logic as in b), we conclude that the paths depicted in 9 obtain.

3. Inflation dynamics and capital inflows

Using the Matlab programs posted on the book website, perform the following experiments:

(a) Temporary reduction in rate of money growth under flexible exchange rates for the separable case

(b) Temporary reduction in rate of money growth under flexible exchange rates for the non-separable case

(c) Temporary reduction in rate of devaluation rate under predetermined exchange rates for the non-separable case

Answer

See, respectively, Figures A1, A2, and A3.

4. Contractionary fiscal policy as a policy response to capital inflows

This exercise illustrates the use of contractionary fiscal policy as a way of alleviating the real exchange rate appreciation that accompanies episodes of capital inflows. Consider a small-open economy operating under predetermined exchange rates with fixed endowment of tradables (y^T) and non-tradables (y^N) and perfectly integrated into world goods and capital markets. Preferences are given by

$$\int_0^\infty [\log(c_t^T) + \log(c_t^N)]e^{-\beta t}dt,$$

where c^T and c^N denote consumption of tradables and non-tradables, respectively, and β is the positive discount rate. The intertemporal constraint takes the form:

$$a_0 + \int_0^\infty \left(y^T + \frac{y^N}{e_t} - \tau_t\right) e^{-rt}dt = \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t} + i_t m_t\right) e^{-rt}dt,$$

where a_0 are initial real financial assets, $r(=\beta)$ is the world real interest rate, e_t is the real exchange rate (i.e., the relative price of tradables in terms of non-tradables), τ_t denotes lump-sum taxes, i is the nominal interest rate, and m are real money balances in terms of tradable goods.

The cash-in-advance constraint is given by

$$m_t = \alpha\left(c_t^T + \frac{c_t^N}{e_t}\right).$$

where α is a positive parameter. The government's flow budget constraint is given by

$$\dot{h}_t = rh_t + \dot{m}_t + (\varepsilon_t + \pi_t^*)m + \tau_t - \frac{g_t^N}{e_t}, \tag{69}$$

where g_t^N is government spending on non-tradables (which we take as the policy instrument). The government sets a constant rate of devaluation, $\bar{\varepsilon}$. Lump-sum taxes adjust endogenously to ensure that the government's budget constraint holds.

Equilibrium in the non-tradable goods market requires that

$$y^N = c_t^N + g_t^N.$$

Interest parity holds:

$$i_t = i_t^* + \bar{\varepsilon}.$$

In the context of this model:

(a) Suppose that, starting from an initial stationary equilibrium, there is an unanticipated and temporary reduction in i_t^* between time 0 and time T. (Assume g^N is constant.) Show that this will lead to a consumption boom in the tradable sector and to real appreciation. Plot the time paths of the different variables. Explain the intuition behind the results.

(b) Suppose now that, in response to the temporary reduction in i_t^*, the government reduces g^N temporarily by as much as needed to keep the real exchange rate between time 0 and time T at its pre-shock level. Compute a reduced form solution for the level of g^N that will keep e constant. Plot the time paths of the different variables. Explain the intuition behind the results.

Answer

(a) Denote by i^1 and i^2 the nominal interest rate during $[0, T)$ and after T, respectively (where $i^1 < i^2$). Then, from the first-order condition for c^T:

$$\begin{aligned} \frac{1}{(c^T)^1} &= \lambda(1 + \alpha i^1) \\ \frac{1}{(c^T)^2} &= \lambda(1 + \alpha i^2) \end{aligned}$$

which implies that $c_1^T > c_2^T$. From the first-order condition for c^N and non-tradable goods equilibrium:

$$\begin{aligned} e^1 &= \frac{y^N - g^N}{c_1^T}, \\ e^2 &= \frac{y^N - g^N}{c_2^T}, \end{aligned}$$

where g^N is the constant level of government spending on non-tradables. Hence, $e^1 < e^2$.

The intuition is the same as in Chapter 7. The lower foreign nominal interest rate introduces an intertemporal distortion and leads to increased demand for c^T and c^N. The increased demand for c^T translates into a trade deficit and the increased demand for c^N results in a increase in their relative price.

(b) From equilibrium in the non-tradable goods markets and the fact that $c^T = c^N/e$, it follows

$$g_t^N = y^N - e_t c_t^T.$$

Denote by e_{0^-} the pre-shock level of the real exchange rate. The level of g^N needed to keep $e^1 = e_{0^-}$ will be given by

$$\left(g^N\right)^1 = y^N - e_{0^-}\left(c^T\right)^1 \tag{70}$$

It follows that since $\left(c^T\right)^1 > c_{0^-}^T$, then $\left(g^N\right)^1 < g_{0^-}^N$. In other words, and as expected, government spendign on non-tradable goods needs to be reduced to keep the real exchange rate constant. Using the resource constraint, we can compute $\left(c^T\right)^1$:

$$c_1^T = \frac{r\left(b_0 + ry^T\right)}{1 - e^{-rT} + \frac{1+\alpha i^1}{1+\alpha i^2}e^{-rT}}.$$

Substituting this last equation into equation (70) gives us a reduced form for the precise level of g^N needed to keep e constant as a function of T and i^1.

Intuitively, the government is reducing goverment spending between time 0 and time T exactly by the amount that the private sector is increasing its demand. In other words, the increased demand is matched exactly by an increase in supply, and therefore the relative price does not need to change.

The above ensures that e does not change at $t = 0$. But what happens at time T? Notice that if at T, g^N returns to its pre-shock level, then e will increase (because while g^N has come back to its pre-shock level and hence the net supply of non-tradables is the same as before the shock, consumption of tradables is lower than before time 0). In sum, e will be constant until T (at the pre-shock level) and then jump up.

On the other hand, g^N at time T could be set a higher level than before the shock such that the real exchange after T will be the same as it was before the shock. In this case, the path of e will be completely flat throughout.

Figure A.1. Temporary fall in rate of money growth under flexible exchange rates (separable case)

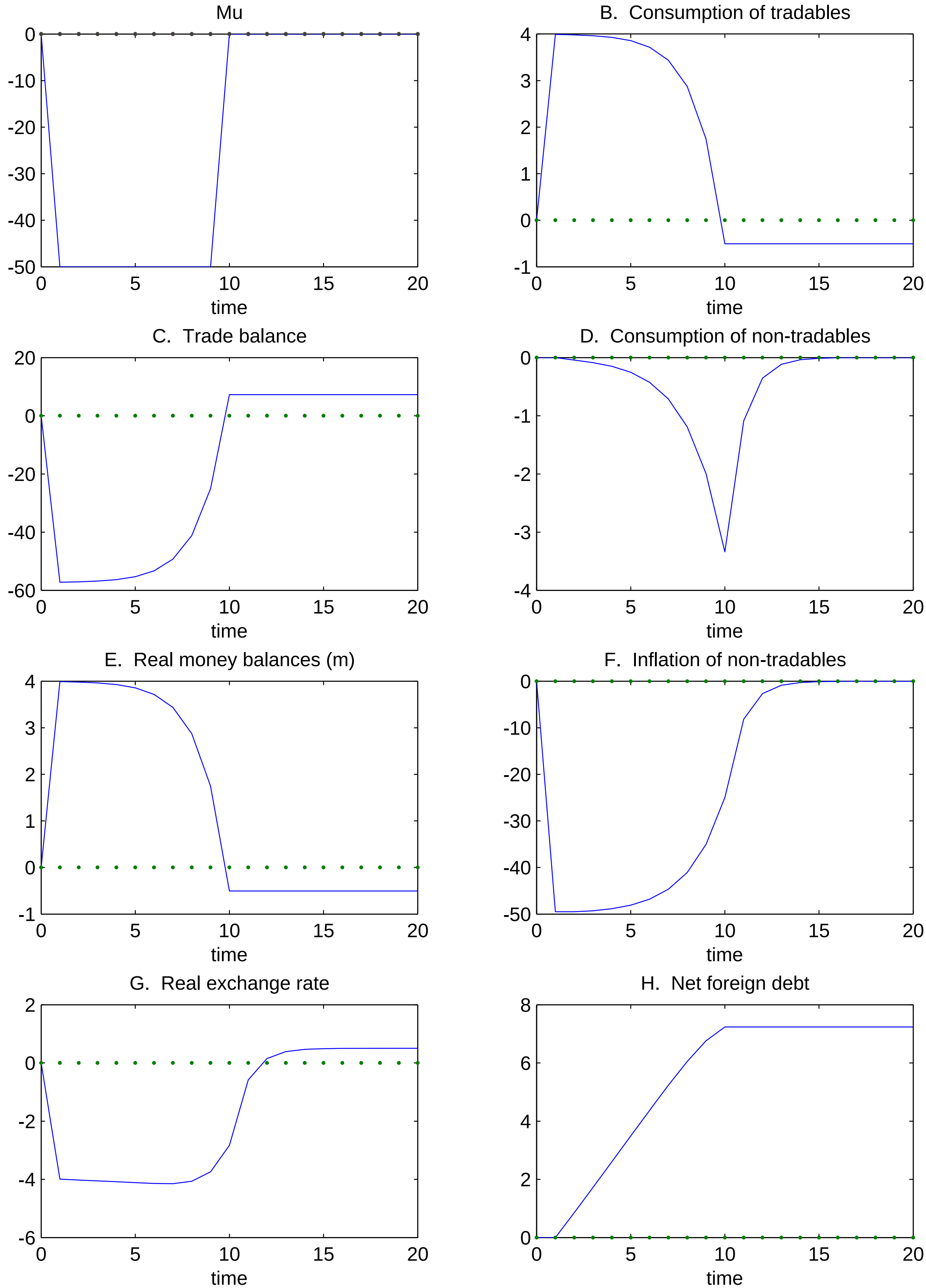

Figure A2. Temporary fall in money growth rate under flexible exchange rates (non-separable case)

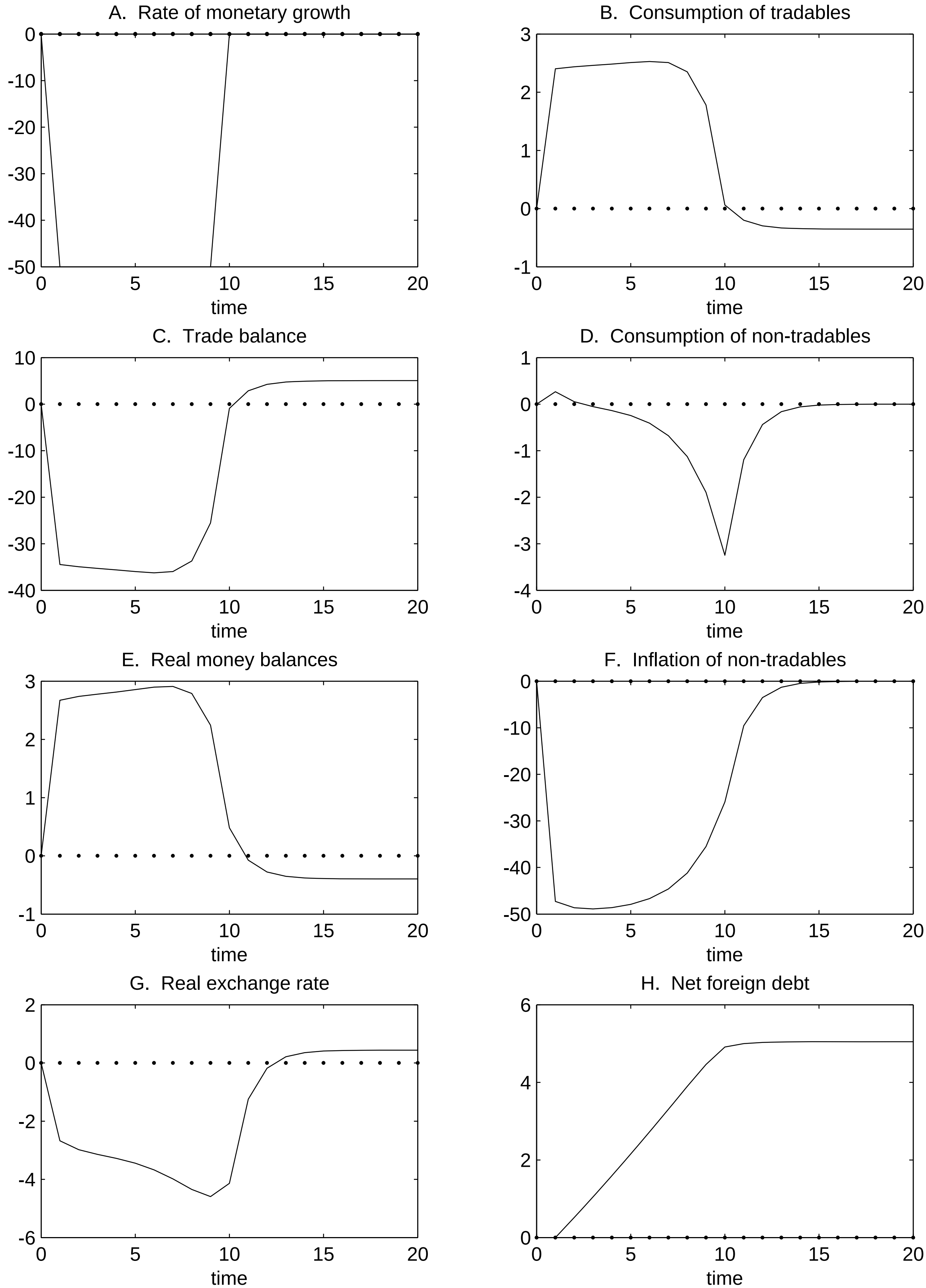

Figure A3. Temporary reduction in devaluation rate under predetermined exchange rates (non-separable case)