Chapter 12
Real Anchors*

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1 Introduction

As discussed in Chapter 5, the "traditional" nominal anchors for an open economy are either the nominal money supply or the nominal exchange rate. We refer to them as "anchors" because the rate of growth of either the money supply or the nominal exchange rate will determine (i.e., anchor) the long-run inflation rate. While, in the short or medium-run, the inflation rate may deviate from the rate of money growth or the rate of devaluation, it will eventually converge to those values in the long-run. A newer nominal anchor – and clearly the most widely used in recent times – is the nominal interest rate, as covered in Chapter 9. While a policy based on controlling the level of the nominal interest rate may lead to indeterminacies, we have seen that, under certain conditions, such a policy can lead to a well-defined long-run inflation rate. Long-run inflation targets may also provide a nominal anchor in the context of inflation-targeting rules.

Given the above, it seems clear that policymakers need to set *some* nominal anchor (i.e., the money supply, the exchange rate, the nominal interest rate, or an inflation target) to anchor long-run inflation. Economic agents'

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expectations will adjust accordingly. In the absence of a nominal anchor, there would be nothing in the model to tie down long-run inflation. In practice, we take this to mean that inflation would be determined just by expectations (which, in turn, would have nothing to be based on) and hence would be highly unstable.

In spite of such strong theoretical prescriptions, policymakers around the world have often resorted to real, as opposed to nominal, anchors. After all, the reasoning goes, if one is interested in real variables (such as the real exchange rate or the real interest rate) that have a direct bearing on growth and consumption, why not try to bypass nominal variables altogether and set the value of these real variables? Hence, and in spite of dire warnings from neo-classically inclined economists about the perils of foregoing a nominal anchor, policymakers have time and time again tried to conduct monetary policy by attempting to control real variables such as the real exchange rate or the real interest rate.

Being a key relative price in any open economy, the real exchange rate is arguably the most popular real target among policymakers. A policy of "real exchange rate targeting" typically aims at controlling (at least in the short and medium-term) the level of the real exchange rate in an effort to keep it roughly constant in the face of external or domestic shocks and/or engineering a more depreciated level of the real exchange rate to keep exports competitive and achieve a smaller deficit/larger surplus in the trade balance. A particular case of real exchange rate targeting are the so-called purchasing-power-parity (PPP) rules that typically adjust the nominal exchange rate by past inflation in an effort to keep the real exchange rate roughly constant. As discussed in Box 1, these PPP rules were highly popular in Latin America, starting with Chile in 1965 and Brazil in 1968.

From a macroeconomic point of view, two main questions arise regarding real exchange rate targeting. First, is it effective? In other words, can policymakers "control" the level of the real exchange rate, at least temporarily? Second – and in light of the above discussion on nominal anchors – does real interest rate targeting lead to higher (and possibly more variable) inflation?

As always, our first instinct is to go back to the simplest model and see what we can learn from it. To this effect, Section 2 focuses on real exchange rate targeting in the simple cash-in-advance model with flexible prices introduced in Chapter 5. As two useful conceptual extremes, we first focus on perfect capital mobility and then on financial autarky. Under perfect capital mobility, we first show that, in a stationary equilibrium, the real exchange rate is fully determined by consumption of tradables and non-tradables goods and cannot be affected by (permanent) changes in policy. Policymakers can, however, affect the level of the real exchange rate temporarily. A (temporarily) higher real exchange rate (i.e., a more depreciated level of the domestic currency in real terms) can be achieved by a temporarily higher rate of devaluation (inflation) that makes tradable consumption more expensive. Under no capital mobility, we show that a temporarily higher level of the real exchange rate can also be achieved. Remarkably, while this does not require higher inflation, it does call for ever increasing domestic real interest rates, which are engineered by a tight money policy. Since, in practice, most economies fall somewhere in between the extremes of perfect capital mobility and no capital mobility, this section's overall message is that, while policymakers can succeed in setting a temporarily higher level of the real exchange rate (i.e., a more depreciated level of the currency in real terms), it will come at the cost of some combination of higher inflation and higher domestic real interest rates.

Before leaving the world of flexible prices, Section 3 illustrates the perils of PPP-type rules in the context of our basic model. Suppose policymakers set the (future) rate of devaluation as a function of some future target for the real exchange rate relative to today's level. If today's real exchange rate is, say, low (i.e., relatively appreciated) relative to the future target, policymakers will set a high rate of devaluation. We show that this economy completely loses its nominal anchor in the sense that the rate of devaluation is undetermined. Whatever is the path of the real exchange rate expected by the private sector, there will exist a rate of devaluation that is consistent with such a path. There is nothing in the model to tie down the future rate of devaluation.

Section 4 focuses on a sticky-inflation economy. Policymakers follow a PPP-type rule whereby the rate of devaluation is set taking into account the current inflation rate of non-tradable goods. In this context, think of a shock – such as increase in the supply of tradable goods – that requires a fall in the relative price of tradable goods (i.e., a real appreciation). In the absence of a PPP rule, this real appreciation would take place over time, with the non-tradable goods inflation being above the (constant) rate of devaluation. By setting a higher rate of devaluation, the PPP rule delays this process in the short-term but, of course, cannot prevent the required real appreciation in the long-run. The higher rate of devaluation requires, however, an even higher rate of inflation to effect the required real appreciation. Hence, the presence of the PPP rule is clearly inflationary.

We then switch our attention to real interest rate targeting. A prime example of using a real interest rate as the main policy instrument is Chile during the period 1985-2001. Section 5 makes use of the sticky-inflation economy developed in Section 4 to shed light on this issue. Our first result is that a pure real interest rate targeting whereby policymakers set a given level of the real interest rate leads to an undetermined inflation rate. There is nothing in the model to tie down the level of inflation. We then study a Taylor-type rule whereby the real interest rate is changed according to the deviation of inflation from an inflation target. If credible, the inflation target anchors long-term inflation expectations and thus provides a nominal anchor to the economy.

Finally, Section 6 illustrates the perils of the so-called "threshold rules" whereby policymakers pre-announce policy measures conditional on some target. For instance, policymakers could announce that if the current account deficit reaches a certain threshold, then a tariff will be imposed. We show how this kind of rules may lead to multiple equilibria situations in which the mere announcement of such a rule precipitates the precise scenario that policymakers are trying to avoid.

2 Real exchange rate targeting in a flexible prices model

We begin our journey by looking at real exchange rate targeting in the context of our flexible prices model of Chapters 5 and 7. Specifically, consider the cash-in-advance, two good model (tradables and non-tradables) analyzed in Chapter 7. Unlike in Chapter 7, however, we will also consider the case of no capital mobility. We will therefore set up the model in such a way that we can then specialize it to the cases of perfect capital mobility or no capital mobility.[1] We will assume that the economy is operating under predetermined exchange rates and that the foreign inflation rate is zero.

[1] We follow Calvo, Reinhart, and Vegh (1995).

2.1 Consumers

Let preferences be given by:

$$\int_0^\infty [u(c_t^T) + v(c_t^N)]e^{-\beta t}dt. \tag{1}$$

The country faces a given world real interest rate, r.[2] The domestic real interest rate in terms of tradable goods, ρ, may differ from r due to imperfect capital mobility.[3] The domestic discount factor (in terms of tradable goods) is then given by

$$D_t = e^{-\int_0^t \rho_s ds}. \tag{2}$$

Notice that in the particular case in which $\rho_s = r$, then $D_t = e^{-rt}$.

The intertemporal constraint is given by (see Appendix 8.1)[4]

$$\int_0^\infty \left(y^T + \frac{y^N}{e_t} + \tau_t \right) D_t dt = \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t} + i_t m_t \right) D_t dt. \tag{3}$$

Money is introduced through a cash-in-advance constraint:

$$m_t = \alpha \left(c_t^T + \frac{c_t^N}{e_t} \right). \tag{4}$$

Substituting the cash-in-advance constraint (4) into the intertemporal budget constraint

$$\int_0^\infty \left(y^T + \frac{y^N}{e_t} + \tau_t \right) D_t dt = \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t} \right) (1 + \alpha i_t) D_t dt. \tag{5}$$

The consumer chooses $\{c_t^T, c_t^N\}_{t=0}^\infty$ to maximize (1) subject to (5). Setting up the Lagrangean:

$$\begin{aligned} \mathcal{L} &= \int_0^\infty [u(c_t^T) + v(c_t^N)]e^{-\beta t}dt \\ &+\bar{\lambda} \left[a_0 + \int_0^\infty \left(y^T + \frac{y^N}{e_t} + \tau_t \right) D_t dt - \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t} \right) (1 + \alpha i_t) D_t dt \right], \end{aligned}$$

[2] We assume, as usual, that $\beta = r$.

[3] Note that ρ is not to be confused with r^d, introduced later in the chapter, which denotes the domestic real interest rate in terms of *non-tradable* goods.

[4] For simplicity, and with no loss of generality, we assume initial real financial assets equal to zero.

where we have denoted by $\bar{\lambda}$ the multiplier associated with constraint (5).

The first-order conditions are given by

$$u'(c_t^T)e^{-\beta t} = \bar{\lambda}(1+\alpha i_t)D_t, \tag{6}$$

$$v'(c_t^N)e^{-\beta t} = \frac{\bar{\lambda}}{e_t}(1+\alpha i_t)D_t. \tag{7}$$

These are, of course, the familiar first-order conditions from Chapter 7. The only difference is that, because of the possible endogeneity of the real interest rate under capital controls, the term $e^{-\beta t}$ is not necessarily equal to D_t. Naturally, under perfect capital mobility, these two terms would cancel out (see below).

2.2 Government

The government's budget constraint takes its usual form:

$$\dot{h}_t = rh_t + \dot{m}_t + \varepsilon_t m_t - \tau_t. \tag{8}$$

2.3 Equilibrium conditions

Equilibrium in the home goods market requires that

$$c_t^N = y^N. \tag{9}$$

As shown in Appendix 8.1, the economy's resource constraint is given by

$$\frac{y^T}{r} = \int_0^\infty c_t^T e^{-rt} dt. \tag{10}$$

2.4 Perfect capital mobility

Under perfect capital mobility, $\rho_t = r$ for all t and the model reduces to our standard cash-in-advance model introduced in Chapter 7. First-order conditions (6) and (7) can be expressed as (recall that $\beta = r$)

$$u'(c_t^T) = \bar{\lambda}(1+\alpha i_t), \tag{11}$$

$$e_t = \frac{u'(c_t^T)}{v'(y^N)}. \tag{12}$$

Consider first a perfect foresight equilibrium path (PFEP) with a constant real exchange rate. Condition (12) makes clear that for e_t to be constant over time, c_t^T must be constant. From the resource constraint (10), this constant value of c_t^T would be given by

$$\overline{c^T} = y^T.$$

From equation (12), the constant value of the real exchange rate would thus be given by

$$\bar{e} = \frac{u'(y^T)}{v'(y^N)}.$$

From (11), the constancy of c_t^T requires a constant nominal interest rate which, by the interest parity condition, necessitates a constant rate of devaluation. In fact, notice that as long as the rate of devaluation is constant, its level is irrelevant. Hence, a constant real exchange rate is independent of the level of the devaluation rate. In other words, policymakers cannot influence the real exchange rate by permanent changes in the rate of devaluation.

Suppose now that policymakers want to implement a temporarily higher (i.e., more depreciated) level of the real exchange rate, perhaps to accumulate reserves through a surplus in the current account. Specifically, assume that policymakers aim at setting a level $e^1 (> \bar{e})$ of the real exchange rate for $t \in [0, T)$ (Panel A in Figure 1). To achieve this, the level of consumption of tradable goods, denoted by $(c^T)^1$, would need to be given by

$$e^1 = \frac{u'((c^T)^1)}{v'(y^N)}. \tag{13}$$

Clearly, $(c^T)^1 < \overline{c^T}$. For given values of $(c^T)^1$ and T, the economy's resource constraint will determine a unique value of $(c^T)^2$ (see Panel B). The value of the nominal interest rate for $t \in [0, T)$ necessary to implement the level $(c^T)^1$ follows from using expression (11) to derive the Euler equation:

$$\frac{u'((c^T)^1)}{u'((c^T)^2)} = \frac{1 + \alpha i^1}{1 + \alpha i^2}. \tag{14}$$

For a given i^2 – chosen by policymakers – equation (14) determines the value of i^1 required to set a given level of $(c^T)^1$ and hence, through equation (13), of e^1.

[Figure 1 here]

We thus conclude that it is possible to target a temporarily higher real exchange rate at the cost of higher inflation. From a welfare point of view, however, it should be clear that this policy is sub-optimal since the first-best involves a flat consumption path.[5] As discussed in Box 2, the empirical evidence seems to suggest that episodes of real exchange rate targeting – typically involving an attempt on the part of policymakers to engineer a higher level of the real exchange rate (i.e., a more depreciated currency in real terms) – have indeed led to temporarily higher inflation.

2.5 Capital controls

To illustrate the case of imperfect capital mobility, let us focus on the extreme case of no capital mobility (financial autarky), as in Chapter 2. With no loss of generality, we assume that the rate of devaluation set by policymakers is zero. Again, let us assume that policymakers wish to set a level e^1 for the real exchange rate for $t \in [0, T)$. As before, this requires the level of consumption of tradable goods given by (13). We will now show that policymakers can manipulate the money supply so as to induce that path.

It is easy to show that since the economy will be stationary from T onwards, $\rho_t = r$ for all $t \geq T$. Hence, from equation (6),

$$u'((c^T)^1) = \lambda_t(1 + \alpha i_t), \qquad (15)$$

$$u'((c^T)^2) = \lambda_T(1 + \alpha r), \qquad (16)$$

where

$$\lambda_t \equiv \begin{cases} \bar{\lambda} D_t e^{\beta t}, & 0 \leq t < T, \\ \lambda_T, & t \geq T. \end{cases} \qquad (17)$$

Differentiating (17) with respect to time (recalling that $i_t = \rho_t$):

$$\dot{\lambda}_t = \lambda_t(\beta - i_t), \qquad 0 \leq t < T. \qquad (18)$$

[5] Interestingly, however, one can construct examples in which a policy of real exchange rate targeting is actually optimal. One such example can be found in Exercise 1 at the end of the chapter. If foreign inflation is positive, a non-flat path of i_t^* introduces an intertemporal distortion, à la Chapter 3. By setting a non-flat path of ε_t, policymakers can offset this distortion and restore the first-best equilibrium.

Using (15) to solve for i_t and substituting into (18), we obtain

$$\dot{\lambda}_t = \left(\frac{1}{\alpha} + \beta\right)\lambda_t - \frac{u'((c^T)^1)}{\alpha}, \qquad 0 \le t < T. \tag{19}$$

By definition – recall (17) and (2) – λ is continuous at time T. Hence, the equilibrium path of λ_t solves differential equation (19) with a terminal condition for $t = T$ given by (16). Appendix 8.2 shows that the path of λ_t that satisfies these conditions is the one depicted in Figure 1, Panel E.

Let us now derive the path of ρ $(= i)$.[6] The fact that λ_t decreases over time implies, from (15), that ρ increases over time. Furthermore, since λ_t is continuous at time T, equations (15) and (16) imply that ρ must fall at time T. Further, we can also show that $\rho_0 > \beta$. To this effect, notice that since $\dot{\lambda}_0 < 0$, it follows from (19) that

$$(1 + \alpha\beta)\,\lambda_0 < u'((c^T)^1).$$

Given (15), this implies that

$$(1 + \alpha\beta)\,\lambda_0 < \lambda_0(1 + \alpha i_0),$$

which implies that $i_0 > \beta$.

How do policymakers engineer this path of the domestic real interest rate? By tight money. To see this, use the non-tradable goods market equilibrium (9) to rewrite the cash-in-advance constraint (4) as

$$m_t = \alpha\left(c_t^T + \frac{y^N}{e_t}\right).$$

Since during $t \in [0, T)$, c_t^T is constant and relatively low and e_t is constant and relatively high, m will be constant as well and relatively low. This implies that the nominal money supply is lower during $[0, T)$ than afterwards. At time T, the nominal money supply goes up to support higher consumption of tradables and a lower e_t. Given that the exchange rate is fixed, the path of m_t dictates the path of M_t. The nominal money stock is thus relatively low during $t \in [0, T)$ and rises at time T. The tight money policy in place during $t \in [0, T)$ restricts the amount of liquidity available in the economy and forces low consumption of tradable goods and low demand for non-tradables, which translates into a relatively low price of non-tradables (i.e., high e_t).

[6] By definition, $i = \rho + \varepsilon + \pi^*$. But since we are assuming that $\varepsilon = \pi^* = 0$, then $i = \rho$.

3 Real exchange rate rules in a flexible prices model

This section shows an example, inspired by Uribe (2003), in which a real exchange rule leads to multiple equilibria.[7] The model remains the same as in Subsection 2.4 (i.e., the case with perfect capital mobility) with the only modification that, to simplify the exposition, we will assume logarithmic preferences. We continue to use the superscript 1 to denote values for $t \in [0, T)$ and superscript 2 for values of $t \geq T$.

Suppose that policymakers set $\varepsilon^1 = 0$ but that they will set ε^2 according to the following real exchange rate rule:

$$\varepsilon^2 = \frac{1 + \alpha r}{\alpha} \left(\frac{e^2}{e^1} - 1 \right). \tag{20}$$

To interpret this rule, think of policymakers as having some "long-run" target, e^2, for the real exchange rate. If the current real exchange rate differs from that target, they will adjust ε^2. For instance, if $e^1 < e^2$, then they will set a positive rate of devaluation.

With the logarithmic specification, and taking account conditions (13), (14), and (10), the equilibrium paths are determined by:

$$\begin{aligned}
\frac{(c^T)^1}{(c^T)^2} &= \frac{1 + \alpha(r + \varepsilon^2)}{1 + \alpha r}, && (21) \\
e^1 (c^T)^1 &= y^N, && (22) \\
e^2 (c^T)^2 &= y^N, && (23) \\
\frac{y^T}{r} &= \frac{(c^T)^1}{r} e^{-rT} + \frac{(c^T)^2}{r} (1 - e^{-rT}). && (24)
\end{aligned}$$

We will show that if the central bank follows rule (20), ε^2 is undetermined.

[7] See Uribe (2003) for a fuller analysis. In a similar model, Lahiri (2001) analyzes the use of taxes on foreign bonds to target the real exchange rate (or, equivalently, the trade balance) and shows that it leads to unstable dynamics. Though in a very different model, this section's message is reminiscent of Dornbuch's (1982) early contribution, where he argues that PPP rules may lead to increased instability of prices and output. In a broader context, Bruno (1993, Chapter 3) analyzes how the lack of a nominal anchor may lead to a process of "shocks and accommodation" whereby inflation acquires a life of its own (i.e., unrelated to the underlying fiscal deficit).

In other words, any path of the real exchange rate that satisfies (22), (23), and (24) is a perfect foresight equilibrium path.

To show this, pick a given e^1. Then, condition (22) determines a given value of $(c^T)^1$. This value of $(c^T)^1$ determines, through the resource constraint (24), a value of $(c^T)^2$. This value, in turn, determines through (23), a value of e^2. Then, equation (21) will give us the value of ε^2 that is consistent with the ratio $(c^T)^1/(c^T)^2$. This value of ε^2 is validated by policy rule (20). Hence, any value of e^1 expected by the public is validated by some corresponding ε^2. [8]

Intuitively, if the consumer expects for today a relative low real exchange rate (i.e., a low e^1 relative to e^2), then, following rule (20), policymakers will react by setting a positive ε^2. This introduces an intertemporal distortion (recall that $\varepsilon^1 = 0$) by making today's consumption cheaper than future consumption. As a result, consumers substitute consumption away from the future and towards the present period. The higher demand for non-tradables today relative to the future leads to a low e^1 relative to e^2, thus validating initial expectations.

We have thus shown that the adoption of rule (20) implies that the future rate of devaluation is undetermined. The economy has no nominal anchor.[9]

4 RER targeting in a sticky-inflation model

In the last two sections, we have looked at real exchange rate targeting under flexible prices. While such a framework provided us with some useful insights, it does not lend itself to studying real exchange rate rules of the type often encountered in practice. To this end, this section focuses on real exchange rate rules in the context of a sticky-inflation model.

Consider a small open economy perfectly integrated into goods and capital markets. Foreign inflation is taken to be zero (and we normalize the foreign nominal price to one). Unless otherwise noticed, we continue to use the same notation as above.

[8] Mechanically, notice that since expression (20) follows from combining (21), (22), and (23), we have a system of four equation in five unknows (e^1, e^2, $(c^T)^1$, $(c^T)^2$, and ε^2). The system is therefore undertermined.

[9] With endogenous production and a productive role for inflation (because of, say, real money balances entering the production function), this indeterminacy would extend itself to the real sector of the economy.

4.1 Consumers

Let preferences be given by

$$\int_0^\infty [\log(c_t^T) + \log(c_t^N) + \log(m_t)] e^{-\beta t} dt. \tag{25}$$

The flow constraint reads as

$$\dot{a}_t = r a_t + y_t^T + \frac{y_t^N}{e_t} + \tau_t - c_t^T - \frac{c_t^N}{e_t} - i_t m_t. \tag{26}$$

The corresponding intertemporal constraint is given by

$$a_0 + \int_0^\infty \left(y^T + \frac{y^N}{e_t} + \tau_t \right) e^{-rt} dt = \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t} + i_t m_t \right) e^{-rt} dt. \tag{27}$$

The first-order conditions take the familiar form:

$$\frac{1}{c_t^T} = \lambda, \tag{28}$$

$$\frac{1}{c_t^N} = \frac{\lambda}{e_t}, \tag{29}$$

$$\frac{1}{m_t} = \lambda i_t. \tag{30}$$

Combining (28) and (29), we obtain the standard intratemporal condition:

$$c_t^T = \frac{c_t^N}{e_t}. \tag{31}$$

Our standard money demand follows (28) and (30):

$$m_t = \frac{c_t^T}{i_t}. \tag{32}$$

For further reference, recall from Chapter 8 that we can define real money balances in terms of non-tradable goods (n) as

$$n_t \equiv \frac{M_t}{P_t^N}.$$

Since, as can be easily checked, $n = em$, we can combine first-order conditions (29) and (30) to yield:

$$n_t = \frac{c_t^N}{i_t}. \tag{33}$$

4.2 Supply side

The supply of tradables is assumed to be exogenous and constant over time. The output of non-tradable goods, on the other hand, is demand-determined. The rate of inflation of non-tradables is assumed to be sticky and is governed by the following differential equation:

$$\dot{\pi}_t = \theta(c_t^N - y_f^N) + \gamma(\varepsilon_t - \pi_t), \tag{34}$$

where θ and γ are positive parameters. The inflation rate of non-tradables increases when aggregate demand is above its full-employment level (first term on the RHS of equation (34)) and when the current inflation rate is below the rate of devaluation (second term on the RHS).

4.3 Government

The government's accounting is standard. The government's flow constraint is given by:

$$\dot{h}_t = rh_t + \dot{m}_t + \varepsilon_t m_t - \tau_t. \tag{35}$$

The corresponding intertemporal constraint is

$$h_0 + \int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt = \int_0^\infty \tau_t e^{-rt} dt. \tag{36}$$

4.4 Policy rule

As an example of a policy rule that reflects policymakers' concerns with the real exchange rate, suppose that the actual rate of devaluation is set according to the following rule:

$$\varepsilon_t = \eta\bar{\varepsilon} + (1-\eta)\pi_t, \qquad 0 < \eta \leq 1, \tag{37}$$

where $\bar{\varepsilon}$ may be viewed as the long-run devaluation rate (i.e., the steady-state rate of devaluation and inflation). The rate of devaluation is now an endogenous variable. If $\eta = 1$, the rate of devaluation acts as a "pure" nominal anchor in the sense that it does not respond to the current state of the economy. If $\eta < 1$, then the rate of devaluation responds to the current inflation rate. The lower is η, the more the current devaluation rate responds to current inflation, reflecting policymakers' attempt to prevent the real exchange rate from appreciating too rapidly.

4.5 Equilibrium conditions

Given that perfect capital mobility prevails, the interest parity condition holds:

$$i_t = r + \varepsilon_t. \tag{38}$$

Equilibrium in the non-tradable goods market requires that

$$c_t^N = y_t^N. \tag{39}$$

By definition, $e = E/P^N$. Hence,

$$\frac{\dot{e}_t}{e_t} = \varepsilon_t - \pi_t. \tag{40}$$

This dynamic equation simply states that if, say, non-tradables goods inflation is higher than tradable goods inflation (given by ε_t), the relative price of tradable goods will be falling over time.

Combining the consumers' flow constraint (given by (26)) with the government's (given by (35)) and imposing condition (39), we obtain

$$\dot{k}_t = rk_t + y^T - c_t^T,$$

where $k(\equiv b + h)$ denotes the economy's total net foreign assets.

By the same token, combining the consumer's and the government's intertemporal constraints – given by (27) and (36), respectively – and imposing equilibrium in the home goods market, we obtain

$$k_0 + \frac{y^T}{r} = \int_0^\infty c_t^T e^{-rt} dt. \tag{41}$$

For further reference, notice that, from (28) and (41), c_t^T will be constant along a PFEP and given by

$$c_t^T = rk_0 + y^T. \tag{42}$$

4.6 Dynamic system

We will set up a two differential equation dynamic system in π and e. To this effect, substitute policy rule (37) into equations (34) and (40) and use (31) to obtain, respectively:

$$\dot{\pi}_t = \theta(e_t c_t^T - y_f^N) + \gamma\eta(\bar{\varepsilon} - \pi_t), \tag{43}$$
$$\dot{e}_t = e_t \eta(\bar{\varepsilon} - \pi_t). \tag{44}$$

To establish the system's steady-state, set $\dot{\pi}_t = \dot{e}_t = 0$ in equations (43) and (44) to obtain:

$$\pi_{ss} = \bar{\varepsilon}, \tag{45}$$
$$e_{ss} = \frac{y_f^N}{c^T}. \tag{46}$$

Notice that the value of c^T will be determined outside this system (and will be constant over time).

Linearizing the system around the steady-state, we obtain:

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{e}_t \end{bmatrix} = \begin{bmatrix} -\gamma\eta & \theta c^T \\ -e_{ss}\eta & 0 \end{bmatrix} \begin{bmatrix} \pi_t - \pi_{ss} \\ e_t - e_{ss} \end{bmatrix}. \tag{47}$$

The trace and the determinant of the matrix associated with the linear approximation are, respectively:

$$\text{Trace} = -\gamma\eta < 0,$$
$$\Delta = e_{ss}\eta\theta c^T > 0.$$

A negative trace indicates that at least one of the roots is negative. A positive determinant then implies that both roots have the same sign. We thus conclude that both roots are negative (or have a real negative part) and, hence, that the dynamic system is globally stable.

As shown in Appendix 8.3, roots will be real if

$$\eta \geq \frac{4\theta y_f^N}{\gamma^2}. \tag{48}$$

All else equal, roots are more likely to be complex the lower is η. This is intuitive because, from policy rule (37), a lower η implies that policymakers put a larger weight on current inflation in setting the devaluation rate.[10]

As usual, we proceed to characterize the qualitative behavior of this dynamic system by constructing a phase diagram. We first draw the $\dot{\pi}_t = 0$ and $\dot{e}_t = 0$ loci. To this end, set $\dot{\pi}_t = 0$ in equation (43) and $\dot{e}_t = 0$ in equation (44) to obtain, respectively,

$$\theta(e_t c_t^T - y_f^N) = -\gamma\eta(\bar{\varepsilon} - \pi_t), \tag{49}$$

$$\pi_t = \bar{\varepsilon}. \tag{50}$$

The last equation implies that the $\dot{e}_t = 0$ locus is a horizontal line, as depicted in Figure 2. From (49), it follows that the slope of the $\dot{\pi}_t = 0$ locus, which is given by

$$\left.\frac{d\pi}{de}\right|_{\dot{\pi}_t=0} = \frac{\theta c^T}{\gamma\eta} > 0,$$

is positive (see Figure 2). Proceeding as in previous chapters, we can draw the laws of motion depicted in Figure 2. Since the system is globally stable, whatever the initial conditions, it will converge to the steady-state.[11]

[Figure 2 here]

4.7 Permanent increase in supply of tradable goods

Suppose that just before time $t = 0$, the economy is in a steady-state. At $t = 0$, there is an unanticipated and permanent increase in the endowment of tradables goods, y^T (Figure 3, Panel A). From (42), it follows that consumption of tradable goods adjusts instantaneously to its new steady-state (Panel B).

[Figure 3 here]

[10] For the purposes of the theoretical analysis below, we will assume that both roots are real.

[11] Graphically, the role of condition (48) in determining whether roots are real or not can be seen in Figure 2 by noting that the lower is η the more vertical will the $\dot{\pi}_t = 0$ locus be, which makes it less likely that the system will converge without fluctuating.

In terms of the dynamic system, let point A in Figure 2 denote the pre-shock equilibrium. As (46) makes clear, e_{ss} falls as a result of the increase in y^T. Let point B in Figure 2 denote the new steady-state (and $e_{ss}^{'}$ denote the new steady-state value of the real exchange rate). Since both π and e are predetermined variables, the transition begins at point A. The system then travels along the arrowed path until it finally converges to point B. The corresponding paths of the real exchange rate and inflation of non-tradable goods are depicted in Figure 3, Panels C and D, respectively.

The path of c_t^N (Figure 3, Panel E) follows from equation (31). Since e_t does not jump at $t = 0$ while c_t^T jumps up, c_t^N also rises up on impact. Then c_t^N will move in the same direction as e_t.

The path of ε follows from equation (37). Differentiating with respect to time:

$$\dot{\varepsilon}_t = (1 - \eta)\, \dot{\pi}_t.$$

The path of ε is depicted in Panel F.

Intuitively, the increase in y^T requires a fall in the steady-state relative price of non-tradables (i.e., a real appreciation). Since the economy is operating under predetermined exchange rates (and, hence, the nominal exchange rate cannot adjust), the required real appreciation can only occur gradually over time through a rate of inflation that is higher than the rate of devaluation.

4.8 Inflationary consequences of real exchange rate targeting

A PPP rule such as (37) that attempts to smooth out the real appreciation will inevitably lead to higher inflation. How much inflation is generated critically depends on the value of η, which measures the extent of real exchange rate targeting. To illustrate this idea, Figure 4 illustrates the paths of the inflation rate and the real exchange rate rate in response to a permanent increase in y^T from 1 to 2 for three different values of η $(1, 0.5$, and $0.2)$.[12]

[12] Parameters are as follows: $\theta = 0.2$, $\gamma = 1$, $y^T = y_f^N = 1$, and $\bar{\varepsilon} = 0.5$. From Appendix 8.3, recall that the condition for roots to be real is $\eta \geq 4\theta y_f^N/\gamma^2$. For the above parameters, $4\theta y_f^N/\gamma^2 = 0.8$. Hence roots are real for $\eta = 1$ and complex for $\eta = 0.5$ and 0.2.

[Figure 4 here]

In response to the increase in tradable goods, the real exchange rate must fall across steady-states from an initial value of 1 to a value of 0.5. How it gets there, of course, depends on the particular value of η. The lower is η, the more inflation is needed for the real exchange rate rate to fall to its new steady-state value. We thus see inflation reaching its highest level for $\eta = 0.2$ and its lowest value for $\eta = 1$. The counterpart is that, early on (i.e., until about period 10), the level of the real exchange rate is highest (i.e., the most depreciated) for $\eta = 0.2$ and the lowest (i.e., the most appreciated) for $\eta = 1$. Policymakers are thus able to have a less appreciated level of the real exchange rate but at the cost of higher inflation.

5 Real interest rate targeting

A real interest rate has often served as a real anchor as well. The best-known example is the case of Chile where, from August 1985 to July 2001, the main instrument of monetary policy was an interest rate on an indexed bond (see Box 3).

This chapter looks at real interest rate targeting in the context of the sticky-inflation model of Section 4. The only change is that, since we will assume that the economy is operating under flexible exchange rates, the law of motion for the inflation rate now takes the form:

$$\dot{\pi}_t = \theta(c_t^N - y_f^N) + \gamma(\mu_t - \pi_t). \tag{51}$$

In other words, and compared to equation (34), μ_t has taken the place of ε_t.

5.1 Pure real interest rate targeting

We will first show that if policymakers target a given level of the domestic real interest rate, the inflation rate is undetermined. Recall from Chapter 8 that the domestic real interest rate is defined as

$$r_t^d = i_t - \pi. \tag{52}$$

Differentiating (29) with respect to time, using (40) and noting that $r^d = r + \varepsilon - \pi$:

$$\frac{\dot{c}_t^N}{c_t^N} = r_t^d - r. \tag{53}$$

Clearly, for a steady-state to exist, policymakers need to target the level r for the real interest rate (denote by $\overline{r^d}$ the target):

$$\overline{r^d} = r.$$

It then follows from the Euler equation (53) that $\dot{c}_t^N = 0$ for all $t \geq 0$. Then, from (51),

$$\dot{\pi}_t = \gamma(\mu_t - \pi_t). \tag{54}$$

By definition, real money balances in terms of non-tradable goods (n) are given by $n \equiv M/P^N$. Then,

$$\frac{\dot{n}_t}{n_t} = \mu_t - \pi_t.$$

Using (33), taking into account that $\dot{i}_t = \dot{\pi}$, we can rewrite this last equation as

$$\mu_t - \pi_t = -\frac{\dot{\pi}_t}{i_t}.$$

Substituting this last equation into (54),

$$\dot{\pi}_t \left(1 + \frac{\gamma}{i_t}\right) = 0.$$

Along a PFEP, $\dot{\pi}_t = 0$. Otherwise, n would diverge over time. Intuitively, if π were increasing over time, the nominal interest rate would also be increasing over time (since $r_t^d = \overline{r^d}$). Since c_t^N is constant over time, it follows from (33) that n would be falling over time. The opposite is true if π were falling over time.

Let us denote the constant level of inflation of non-tradable goods by $\tilde{\pi}$. Hence, $\mu_t = \tilde{\pi}$, $i_t = r + \tilde{\pi}$, and $c_t^N = y_f^N$. But what ties down $\tilde{\pi}$? The answer is nothing. To see this, suppose that, for whatever reason, the public came to expect that the inflation rate will be $2\tilde{\pi}$. Then, $\mu_t = 2\tilde{\pi}$ and $i_t = r + 2\tilde{\pi}$. The demand for n would go down, of course, but this would be accommodated by a fall in the nominal stock of money.[13] Any other value of constant inflation would be similarly accommodated by policymakers. There is absolutely nothing tying down $\tilde{\pi}$.

5.2 A real interest rate rule

We will now show that if the real interest rate targeting is complemented with an inflation target, the indeterminacy discussed above disappears.

Suppose that policymakers set an inflation target, $\bar{\pi}$, and follow the rule

$$\dot{r}^d = \psi(\pi_t - \bar{\pi}). \tag{55}$$

The domestic real interest rate is thus raised (lowered) whenever actual inflation is above (below) the inflation target.

To solve this system, we will set-up a three differential equation system in r^d, c_t^N, and π_t. To this effect, differentiate equation (32) with respect to time, taking into account that $\dot{m}_t/m_t = \mu_t - \varepsilon_t$, the interest parity condition, and (52) to obtain:

$$\mu_t = i_t - r - \left(\frac{\dot{r}_t^d + \dot{\pi}_t}{i_t}\right).$$

Substituting this equation into (51) and rearranging terms, we obtain

$$\dot{\pi}_t = \frac{\theta}{1+\frac{\gamma}{i_t}}(c_t^N - y_f^N) + \frac{\gamma}{1+\frac{\gamma}{i_t}}\left(r_t^d - r\right) - \frac{\frac{\gamma}{i_t}\psi}{1+\frac{\gamma}{i_t}}(\pi_t - \bar{\pi}). \tag{56}$$

Equations (55), (53), and (56) constitute a three differential equation system in r^d, c_t^N, and π_t.[14] In the steady-state,

$$\begin{aligned} r_{ss}^d &= r, \\ c_{ss}^N &= y_f^N, \\ \pi_{ss} &= \bar{\pi}. \end{aligned}$$

[13]Notice, of course, that the nominal money supply is endogenous even though we are operating under flexible exchange rates because the monetary authority is not setting the path of the money supply.

[14]Note that even though equation (56) contains terms in i_t, these terms will not be part of the linear approximation because they are multiplied by terms which, in the steady-state, will be equal to zero.

Linearizing this system around the steady-state, we obtain:

$$\begin{bmatrix} \dot{r}^d \\ \dot{c}_t^N \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & \psi \\ y_f^N & 0 & 0 \\ \frac{\gamma}{1+\frac{\gamma}{i_{ss}}} & \frac{\theta}{1+\frac{\gamma}{i_{ss}}} & -\frac{\frac{\gamma}{i_{ss}}\psi}{1+\frac{\gamma}{i_{ss}}} \end{bmatrix} \begin{bmatrix} r_t^d - r \\ c_t^N - y_f^N \\ \pi_t - \bar{\pi} \end{bmatrix},$$

where $i_{ss} = r + \bar{\pi}$. The trace and determinant of the matrix associated with the linear approximation are given by

$$\begin{aligned} \text{Trace} &= -\frac{\frac{\gamma}{i_{ss}}\psi}{1+\frac{\gamma}{i_{ss}}} < 0, \\ \Delta &= \psi \frac{\theta}{1+\frac{\gamma}{i_{ss}}} y_f^N > 0. \end{aligned}$$

Since the trace is the sum of the roots, a negative trace indicates that at least one of the three roots is negative. A positive determinant, in turn, implies that either the system has three positive roots or two negative and one positive roots. We conclude therefore that the system has two negative and one positive roots. Since there are two non-jumping variables (r_t^d and π_t), the system exhibits saddle-path stability. For given initial values of r_t^d and π_t, the value of c_t^N will be such so as to position the system along its unique perfect foresight equilibrium path.

Let δ_i, $i = 1, 2$ denote the two negative roots with $\delta_1 < \delta_2$. Let h_{ij}, $j = 1, 2, 3$ denote the elements of the eigenvector associated with root δ_i. Hence, for $i = 1, 2$, it follows that

$$\begin{bmatrix} -\delta_i & 0 & \psi \\ y_f^N & -\delta_i & 0 \\ \frac{\gamma}{1+\frac{\gamma}{i_{ss}}} & \frac{\theta}{1+\frac{\gamma}{i_{ss}}} & -\frac{\frac{\gamma}{i_{ss}}\psi}{1+\frac{\gamma}{i_{ss}}} - \delta_i \end{bmatrix} \begin{bmatrix} h_{i1} \\ h_{i2} \\ h_{i3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

It then follows that

$$\frac{h_{i3}}{h_{i1}} = \frac{\delta_i}{\psi} < 0.$$

As will become clear below, this will provide a critical piece of information when it comes to solving the dynamic behavior of this system.

Setting to zero the constant corresponding to the unstable root, the solution to this dynamic system takes the form

$$
\begin{aligned}
r^d - r &= \omega_1 h_{11} \exp(\delta_1 t) + \omega_2 h_{21} \exp(\delta_2 t), \\
c_t^N - y_f^N &= \omega_1 h_{12} \exp(\delta_1 t) + \omega_2 h_{22} \exp(\delta_2 t), \\
\pi_t - \bar{\pi} &= \omega_1 h_{13} \exp(\delta_1 t) + \omega_2 h_{23} \exp(\delta_2 t).
\end{aligned}
$$

Hence,

$$\lim_{t\to\infty} \frac{r^d - r}{\pi_t - \bar{\pi}} = \lim_{t\to\infty} \frac{\omega_1 h_{11} \exp\left[(\delta_1 - \delta_2)t\right] + \omega_2 h_{21}}{\omega_1 h_{13} \exp\left[(\delta_1 - \delta_2)t\right] + \omega_2 h_{23}}.$$

Since, by assumption, $\delta_1 - \delta_2 < 0$, it follows that

$$\lim_{t\to\infty} \frac{r^d - r}{\pi_t - \bar{\pi}} = \frac{h_{21}}{h_{23}} < 0.$$

This implies that as t becomes large, the domestic real interest rate and the inflation rate of non-tradables will converge to their steady-state values from opposite directions. Put differently, the "dominant eigenvector ray," which is illustrated in Figure 5, is negatively sloped (see Calvo (1987)). Graphically, the system must converge asymptotically to the dominant eigenvector ray. From (55), we know that when $\pi_t > \bar{\pi}$, $\dot{r}^d > 0$ and when $\pi_t < \bar{\pi}$, $\dot{r}^d < 0$. The corresponding directional arrows are drawn in Figure 5.

[Figure 5 here]

Let us now study how the economy responds to a reduction in the inflation target. Suppose that the initial steady-state corresponds to point A in Figure 5, where $\pi_{ss} = \bar{\pi}^H$ and $r^d = r$. At time 0, there is an unanticipated and permanent reduction in the inflation target from $\bar{\pi}^H$ to $\bar{\pi}^L$. The new steady-state is denoted by point B in Figure 5.

How does the economy adjust to the new equilibrium? The arrowed path in Figure 5 indicates the transition from point A to point B.[15,16] Figure 6, Panels B and C, illustrates the corresponding paths of inflation and the domestic real interest rate, respectively.

[Figure 6 here]

[15] Since, as we will show above, c^N falls on impact, it follows from (56) that $\dot{\pi}_0 < 0$. The system must head in a southeastern direction as indicated in Figure 5.

[16] Note that we are assuming that roots are real and, therefore, that the adjustment is non-cyclical.

To derive the path of c_t^N, notice that since r_t^d is always greater than r during the transition, it follows from equation (53) that $\dot{c}_t^N > 0$ for all $t \geq 0$. Since c_t^N does not change across steady-states, it must fall on impact and then rise gradually toward its unchanged steady-state (Figure 6, Panel D).

What about real money balances, n? From (33), we infer that (i) n increases across steady-states because the nominal interest rate is lower in the new steady-state and consumption of non-tradable goods is the same, and (ii) n jumps down on impact because i does not change while c^N falls. We also know that $\dot{n}_t > 0$ from t_1 onwards.[17] Hence, n will increase, on average, during the transition.

Finally, regarding the nominal interest rate, note from (52) that it will fall across steady-states. Further, since neither r^d nor π jump on impact, i will not change on impact. Hence, on average, i will fall during the transition.

We conclude that a real interest rate rule like (55) does provide a nominal anchor to the economy. Of course, for this to be true, the inflation target must be fully credible on the part of the public. If that is the case, our model suggests that a real interest rate could be used effectively as a policy instrument.[18]

6 Real targeting and multiple equilibria

As discussed in the introduction, countries have often followed threshold rules whereby some policy measure will be enacted if some macroeconomic variable reaches a certain threshold. The Central Bank of Chile, for instance, has traditionally had the mandate of maintaining a "sustainable" current account deficit, which has been taken to mean a deficit no larger than 4-5 percent of GDP, at trend terms of trade.[19] (see Medina and Valdes (2002)). One of the major perils associated with threshold rules that target real variables is the possibility of multiple equilibria. We will illustrate this idea in a simple two-period model. Consider an economy with constant endowments of

[17] We also know, from (51), that n is increasing for $t > t_2$. The reason is that $\dot{\pi}_t > 0$ for $t > t_2$ and $c_t^N - \bar{y}^N$ is always negative. Hence, $\mu_t - \pi_t$ must be positive for $t > t_2$.

[18] In fact, as shown in exercise 2 at the end of the chapter, we can find some basic equivalences between real interest rules, nominal interest rules, and controlling the money supply. The rules, however, become more complex as we move from traditional instruments (the money supply) to less conventional ones (such as the real interest rate).

[19] See Medina and Valdes (2002) and Morande (2002).

traded (y^T) and non-traded goods (y^N). The economy is small and perfectly integrated into world goods and capital markets.

The representative consumer's preferences are given by

$$U = \log(c_1^T) + \log(c_1^N) + \beta\left[\log(c_2^T) + \log(c_2^N)\right],$$

where c^T and c^N denote consumption of traded and non-traded goods, respectively, and $u(.)$ and $v(.)$ are strictly increasing and strictly concave functions.

The intertemporal constraint takes the form:

$$y^T + p_1 y^N + \frac{y^T}{1+r} + \frac{p_2 y^N}{1+r} + \tau = c_1^T + p_1 c_1^N + \frac{(1+\theta)(c_2^T + p_2 c_2^N)}{1+r}, \qquad (57)$$

where p is the relative price of non-traded goods in terms of traded goods, r is the world real interest rate, θ is a consumption tax, and τ are lump-sum transfers.

Suppose that, for reasons not modeled explicitly, the government is concerned about trade deficits. As a result, it announces that if the trade deficit is greater than a certain level, a consumption tax $\bar{\theta}$ (> 0) will be imposed on second-period consumption:

$$\theta = \begin{cases} 0, & \text{if } TB_1 \geq \overline{TB}, \\ \bar{\theta}, & \text{if } TB_1 < \overline{TB}. \end{cases} \qquad (58)$$

Formally, we proceed in the following way. We will first solve for the case in which households do not expect the above rule to bind (i.e., they expect that no tariff will be imposed in the second period). We will then establish the range of parameter values for which the rule does not indeed bind. We will then solve for the case in which households expect the rule to bind (i.e., they expect that a tariff will be imposed in the second period). We will then establish the range of parameter values for which the rule will indeed bind.

6.1 Households do not expect the threshold rule to bind

If households do not expect that a tariff will be imposed, the Lagrangian is given by

$$\begin{aligned}\mathcal{L} &= \log(c_1^T) + \log(c_1^N) + \beta\left[\log(c_2^T) + \log(c_2^N)\right] \\ &+\lambda\left[y^T + p_1 y^N + \frac{y^T}{1+r} + \frac{p_2 y^N}{1+r} - c_1^T - p_1 c_1^N - \frac{c_2^T}{1+r} - \frac{p_2 c_2^N}{1+r}\right].\end{aligned}$$

Since the government will not levy a tariff in the second period, lump-sum transfers will be zero.

The first-order conditions are given by[20]

$$\frac{1}{c_1^T} = \lambda, \tag{59}$$

$$\frac{1}{c_1^N} = \lambda p_1, \tag{60}$$

$$\frac{\beta}{c_2^T} = \frac{\lambda}{1+r}, \tag{61}$$

$$\frac{\beta}{c_2^N} = \frac{\lambda p_2}{1+r}. \tag{62}$$

Combining first-order conditions (59) and (61):

$$\frac{1}{c_1^T} = \frac{\beta(1+r)}{c_2^T}. \tag{63}$$

Since $\beta(1+r) < 1$, then

$$c_1^T > c_2^T.$$

Imposing equilibrium in the non-traded goods markets, the intertemporal constraint (57) reduces to

$$\left(\frac{2+r}{1+r}\right) y^T = c_1^T + \frac{c_2^T}{1+r}. \tag{64}$$

[20] To generate a trade deficit in the first period, we will assume that $\beta(1+r) < 1$.

Combining (63) and (64), we can solve for first-period consumption:

$$c_1^T = y^T \frac{2+r}{(1+r)(1+\beta)}. \tag{65}$$

By definition, the trade balance is given by

$$TB_1 = y_1^T - c_1^T. \tag{66}$$

Substituting (65) into the last equation,

$$TB_1 = y^T \left[1 - \frac{2+r}{(1+r)(1+\beta)}\right] < 0,$$

since $[(2+r)/(1+r)(1+\beta)] > 1$.

From threshold rule (58), we know that the government will not impose a tariff as long as the trade balance is above $\overline{TB}$. Hence, the condition

$$y^T \left[1 - \frac{2+r}{(1+r)(1+\beta)}\right] \geq \overline{TB} \tag{67}$$

ensures that this case is a rational expectations equilibrium. In other words, if parameter values satisfy condition (67), then it is rational for consumers to expect that the government will not impose a tariff.

6.2 Households expect the threshold rule to bind

If households expect that a tariff will be imposed, the Lagrangian is given by

$$\begin{aligned} \mathcal{L} &= \log(c_1^T) + \log(c_1^N) + \beta\left[\log(c_2^T) + \log(c_2^N)\right] \\ &\quad + \lambda\left[y^T + p_1 y^N + \frac{y^T}{1+r} + \frac{p_2 y^N}{1+r} + \tau - c_1^T - p_1 c_1^N - \frac{(1+\bar{\theta})(c_2^T + p_2 c_2^N)}{1+r}\right]. \end{aligned}$$

First-order conditions are given by

$$\begin{aligned}
\frac{1}{c_1^T} &= \lambda, \\
\frac{1}{c_1^N} &= \lambda p_1, \\
\frac{\beta}{c_2^T} &= \frac{\lambda(1+\bar{\theta})}{1+r}, \\
\frac{\beta}{c_2^N} &= \frac{\lambda p_2(1+\bar{\theta})}{1+r}.
\end{aligned}$$

From the first and third first-order conditions, it follows that

$$c_2^T = c_1^T \frac{\beta\,(1+r)}{1+\bar{\theta}}.$$

In this case, the government's budget constraint is given by

$$\tau = \frac{\bar{\theta}(c_2^T + p_2 c_2^N)}{1+r}$$

Using the last equation and the equilibrium in the non-traded goods market ($c_1^N = c_2^N = y^N$), we can solve for c_1^T from the intertemporal budget constraint (57):

$$\widetilde{c_1^T} = y^T \left(\frac{2+r}{1+r}\right) \frac{1}{1+\frac{\beta}{1+\bar{\theta}}}, \tag{68}$$

where we are using a tilda superscript to denote equilibrium values in the consumption tax case. Comparing $\widetilde{c_1^T}$ (65) and (68), we conclude that, as expected,

$$\widetilde{c_1^T} > c_1^T$$

for $\bar{\theta} > 0$. If the consumption tax is positive in period 2, intertemporal substitution leads to higher consumption in period 1.

Given (68), the trade balance is given by

$$\widetilde{TB_1} = y^T \left[1 - \left(\frac{2+r}{1+r}\right) \frac{1}{1+\frac{\beta}{1+\bar{\theta}}}\right].$$

Given the threshold rule (58), the following must hold for this to be a rational expectations equilibrium:

$$y^T\left[1-\left(\frac{2+r}{1+r}\right)\frac{1}{1+\frac{\beta}{1+\theta}}\right]<\overline{TB_1} \qquad (69)$$

To summarize, we have derived the conditions that ensure that a tax will not be imposed (given by equation (67))) and that a tax will be imposed (given by condition (69)). We can rewrite these two conditions as

$$\begin{aligned} y^T-\widetilde{c_1^T}-\overline{TB} &< 0, \\ y^T-c_1^T-\overline{TB} &\geq 0. \end{aligned}$$

Figure 7 plots these two expressions as a function of the threshold $\overline{TB}$. Consider first the line $y^T-\widetilde{c_1^T}-\overline{TB}$, which intersects the horizontal axis at point A (where $\overline{TB}=y^T-\widetilde{c_1^T}$). For any point to the right of point A, a tax would always be imposed because the trade balance would be below the threshold. Consider then the line $y^T-c_1^T-\overline{TB}$, which intersects the horizontal axis at point B. For any point to the left of point B, no tax would ever be imposed because the trade balance would always be above the threshold. Three regions are then defined, labeled I (to the left of point A), II (between points A and B), and III (to the right of point B). For large values of $\overline{TB}$ (i.e., above point B), a tax will always be imposed. For low values of $\overline{TB}$ (i.e., to the left of point A), a tax will never be imposed. For intermediate values of $\overline{TB}$ (i.e., between points A and B), there is multiple equilibria. If agents expect a consumption tax rate to be imposed tomorrow, their consumption will be such that the threshold will be hit and the tax rate will indeed be imposed. If agents do not expect a consumption tax rate to be imposed, they will consume less today and the threshold will not be hit, which implies that a tax rate will indeed not be imposed. Ironically, then, the mere existence of threshold rule (58) may trigger the precise scenario that policymakers are trying to avoid to begin with!

[Figure 7 here]

Finally, notice that we could express the threshold rule as a function of the real exchange rate (i.e., the inverse of p) because for each value of TB

there is a corresponding value of p_1. Formally, from (59) and (60), and imposing non-tradable goods market equilibrium, we obtain

$$c_1^T = p_1 y^N.$$

Substituting this expression into the definition of the trade balance (equation (66)) and solving for p_1:

$$p_1 = \frac{y^T - TB_1}{y^N}$$

The rule could then be expressed as

$$\theta = \begin{cases} 0, & \text{if } p_1 \leq \overline{p}, \\ \bar{\theta}, & \text{if } p_1 > \overline{p}. \end{cases} \tag{70}$$

In this case, the announcement of this rule could trigger the precise real appreciation that the rule is presumably intended to avoid.

7 Concluding remarks

Policymakers often use real variables either as policy targets or policy instruments. An example of a real policy target would be a so-called PPP rule whereby the rate of devaluation is set as a function of the past differential between domestic and foreign inflation. An example of a real policy instrument would be the use of an interest rate on an indexed bond as the main monetary policy instrument. In principle, either scenario carries the risk of the economy losing its nominal anchor, which could lead to high and/or volatile inflation. This chapter has analyzed whether those fears are warranted.

By and large, our theoretical results suggest that such fears are indeed justified. While the details differ depending on the model and the type of experiment, the main message that emerges from our analysis is that depriving an economy of a "traditional" nominal anchor (in the form of the money supply, the exchange rate, or a nominal interest rate) will seed the sows of nominal instability and/or leave the economy at the hands of private sector's expectations. When such policies do work, it is typically due to some implicit nominal anchor like a fully credible inflation target. In practice, however, such targets are unlikely to carry the level of credibility needed to function as a nominal anchor.

8 Appendix

8.1 Derivations of constraint in the model of Section 2[21]

Under capital controls (modeled as a dual exchange rate regime), the consumer's flow constraint is given by

$$\dot{a}_t = \rho_t s_t b_t + y^T + \frac{y^N}{e_t} + \tau_t - c_t^T - \frac{c_t^N}{e_t} - \varepsilon_t m_t, \tag{71}$$

where b is the stock of net foreign bonds; s is the domestic price of real bonds; $a(\equiv m + sb)$ denotes real financial assets; and τ are real transfers from the government.[22] The intertemporal constraint (3) in the text follows from multiplying (71) by D_t, integrating forward, imposing the standard transversality condition, and assuming that $a_0 = 0$ (note that $i = \rho + \varepsilon$).

To derive the resource constraint (41), combine the government's flow constraint (equation (8) in the text) with the consumer's flow constraint (71) to obtain (after imposing condition (9) and taking into account that arbitrage implies that $\rho_t = r/s_t + \dot{s}_t/s_t$):

$$\dot{h}_t + s_t \dot{b}_t = y^T - c_t^T + r(h_t + b_t). \tag{72}$$

Under perfect capital mobility, $s \equiv 1$. Under capital controls, $b_t = b_0$ for all t so that $\dot{b}_t = 0$. In either case, equation (41) in the text follows from integrating forward (72), imposing the appropriate transversality condition, and assuming that $h_0 + b_0 = 0$.

8.2 Path of λ_t

We first show that λ decreases over time. We proceed by contradiction.

(i) Suppose that λ_t were constant over time; that is, $\lambda_t = \lambda_0$ for all $t \in$

[21] This appendix follows Guidotti and Vegh (1992).

[22] Notice that, in a regime of capital controls, the domestic real price of bonds, s, may differ from the international price (one). Further, the stock of net foreign bonds will be constant since the private sector cannot acquire bonds from either abroad or the central bank.

$[0, T)$. Then, since $\dot{\lambda}_t = 0$, it follows from (19) that

$$\lambda_0 = \frac{u'((c^T)^1)}{(1+\alpha\beta)}.$$

From (16), however, $\lambda_T \neq \lambda_0$, which is a contradiction.

(ii) Suppose that λ_t increased over time. Then, given that $\dot{\lambda}_0 > 0$, it follows from (19) that

$$(1+\alpha\beta)\,\lambda_0 > u'((c^T)^1).$$

Then, since $\lambda_T > \lambda_0$, it follows that

$$(1+\alpha\beta)\,\lambda_T > u'((c^T)^1).$$

And since $(c^T)^1 < (c^T)^2$,

$$(1+\alpha\beta)\,\lambda_T > u'((c^T)^2).$$

This last expression, however, contradicts (16).

We thus conclude that λ_t decreases over time.

8.3 Roots of system for real exchange targeting under sticky prices

Let δ_i, $i = 1, 2$ denote the roots of the differential equation system given by (47) in the text. To derive the characteristic equation, we need to solve for

$$\begin{vmatrix} -\gamma\eta - \delta_i & \theta c^T \\ -e_{ss}\eta & -\delta_i \end{vmatrix} = 0.$$

This can be rewritten as:

$$\delta_i^2 + \gamma\eta\delta_i + e_{ss}\eta\theta c^T = 0.$$

Roots are thus given by

$$\delta_i = \frac{-\gamma\eta \pm \sqrt{(\gamma\eta)^2 - 4e_{ss}\eta\theta c^T}}{2}.$$

Roots are real if

$$\eta \geq \frac{4\theta y_f^N}{\gamma^2},$$

where we have used the fact that $e_{ss}c^T = y_f^N$.

Exercises[23]

1. ### Shocks to the foreign nominal interest rate

   Consider the model of Section 2 under perfect capital mobility and with a positive foreign inflation rate. In this context:

   (a) Solve the model for a PFEP along which i_t^* is first low and then high.

   (b) Show that by an appropriate choice of the path of ε, policymakers can restore the first-best equilibrium.

2. ### An equivalence proposition

   (This exercise follows Vegh (2002)). Consider a one-good closed economy with preferences given by

   $$\int_0^\infty u(c_t)e^{-\beta t}dt,$$

   where c denotes consumption. Consumers hold two assets: a bond (indexed to the price level and in zero net supply) and money. Let a_t denote the household's real financial wealth:

   $$a_t = b_t + m_t,$$

   where b_t and m_t denote the real stocks of bonds and money, respectively. The nominal interest rate on the bond is denoted by i_t. As in Chapter 7, money is introduced into the model through a transactions costs technology $v(m)$, where $v'(m) < 0$ and $v''(m) > 0$.[24] The consumer's flow constraint is given by

   $$\dot{a}_t = r_t a_t + y_t + \tau_t - c_t - i_t m_t - v(m_t),$$

   where y_t denotes output of the good and τ_t are lump-sum transfers from the government. Further, assume that the transactions costs technology takes the form:

   $$v(m_t) = m_t \left[\frac{1}{\sigma}\log(m_t) - \chi\right].$$

[23] An answer key is available from the author upon request.

[24] For simplicity, and unlike chapter 7, we assume that the transactions technology does not depend on consumption, which implies that the corresponding real money demand will only depend on the nominal interest rate.

The government plays no active role. It gives back to consumers as lump-sum transfers the proceeds from money creation and transactions costs. The government's constraint is thus

$$\tau_t = \mu_t m_t + v(m_t).$$

The fact that $v(m_t)$ appears in the government's flow constraint reflects the assumption that $v(m_t)$ is a private cost for consumers but not a social cost. Formally, one can think of some federal agency providing (at zero cost) the transactions costs needed by consumers. The profits of this federal agency are returned to households as lump-sum transfers. This assumption is made to eliminate wealth effects associated with changes in inflation which would unnecessarily complicate the analysis.

Output is endogenous and assumed to be demand-determined; that is, $y_t = c_t$. As in Section 4, the inflation rate is assumed to be predetermined at each point in time. The change in the inflation rate is given by

$$\dot{\pi}_t = \gamma(\mu_t - \pi_t) + \alpha(c_t - y_f^N),$$

where y_f^N denotes the "full-employment" level of output. Equation (51) says that the rate of inflation will increase whenever the rate of inflation is below the rate of money growth, μ_t, or aggregate demand exceeds full-employment output.

In the context of this model, show that the following three monetary policy rules are exactly equivalent:

(a) policymakers set a fixed-money growth rule:

$$u_t = \bar{\mu};$$

(b) policymakers announce an inflation target, $\bar{\pi}$ (equal to $\bar{\mu}$), and follow the nominal interest rate rule

$$\dot{i}_t = \theta(\pi_t - \bar{\mu}), \qquad \theta = \frac{1}{\sigma};$$

(c) policymakers announce an inflation target, $\bar{\pi}$ (equal to $\bar{\mu}$), and follow the real interest rate rule

$$\dot{r}_t = \theta^1(\pi_t - \bar{\mu}) + \theta^2(c_t - \bar{y}), \qquad \theta^1 = \gamma + \frac{1}{\sigma},\ \theta^2 = -\alpha.$$

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BOX 1. PPP rules in practice: How did it all begin?

Purchasing power parity rules refer to monetary policy rules that set the rate of devaluation as a function of some measure of the differential between domestic and foreign inflation with the aim of keeping a relatively stable real exchange rate. As such, they are a particular case of "crawling pegs," a term coined by John Williamson in a 1965 article to refer to predetermined exchange rate regimes in which the peg is adjusted in small or gradual steps, as opposed to the so-called adjustable pegs that prevailed under the Bretton Woods regime, in which currencies would be devalued only occasionally but by a large magnitude (and, if some capital mobility was allowed, in conjunction with a loss of international reserves and/or capital outflows). The key qualifier of PPP rules as crawling pegs is that they are *passive* crawling pegs because the rate of devaluation is set in reaction to past inflation differentials, in sharp contrast to *active* crawling pegs of the type described in Chapter 13 in which the rate of devaluation is set with the objective of influencing future expectations of inflation.

The fist country to adopt a formal PPP rule appears to have been Chile in April 1965. The idea – as related in personal communication with the author by Carlos Massad – was to abandon the existing regime whereby the exchange rate would only be adjusted when exchange rate pressures had become unbearable and replace it by a system of frequent and small adjustments, in such a way as to discourage speculative attacks and keeping a relatively stable real exchange rate.[25] Table 1 provides details on the number, frequency, and rates of adjustment. This PPP rule was implemented in the context of unified exchange rate markets and other market-oriented measures. The rule ended in November 1970 when President Salvador Allende took power and returned to exchange rate controls and widespread regulation of the economy.[26] The idea of frequent but small devaluations – in contrast to periods of fixed exchange rates followed by large devaluations – comes across clearly in Figure 8, which plots the nominal exchange rate in Chile during the period 1960-1971.

[Figure 8 here]

[25] Carlos Massad (born in 1932), a Chilean economist, was President of the Central Bank of Chile during 1967-1970 and 1996-2003. In 1964-1965, when the Chilean PPP rule was born, he was Vice-President of the Central Bank (a position he held from 1964 to 1967) and part of President Eduardo Frei's economic team.

[26] For a detailed account of this period, see Ffrench-Davis (1981). Chile returned to a PPP rule in the period 1985-1992, as discussed in Box 2.

[Table 1 here]

In August 1968 – and partly influenced by the Chilean experience – Brazil adopted a PPP rule as well, officially described as a regime of "mini-desvalorizacoes", the Portuguese expression for "mini-devaluations."[27] Up to that point, Brazil used to devalue once a year, with devaluations exceeding 15 percent. The devaluation would typically be the result of domestic inflation exceeding world inflation and the pressure on international reserves brought about by the anticipation of yet another large devaluation. According to Bacha (1979), from the inception of the system until December 1976, the currency was devalued 81 times, or about once every 38 days. The mean devaluation was 1.5 percent. Unlike the Chilean case, the adoption of a PPP rule in Brazil went hand-in-hand with rather stringent exchange rate controls. With occasional deviations (due to terms of trade shocks or short-lived inflation stabilization programs), Brazil followed PPP rules – even if those rules were not made explicit – until the implementation of the Real Plan in July 1994.

In a similar vein, and after a very serious exchange rate crisis in 1967, Colombia implicitly adopted a PPP rule in 1967, whereby the currency was devalued once or twice a week in response to the differential between domestic inflation and that of major trading partners. Such a regime continued essentially in place until 1991 (see Cardenas (2007), Chapter 6). As recounted by Urrutia (1981), Colombia's PPP rule was never made explicit when originally implemented and for many years the government maintained the fiction that the minidevaluations reflected supply and demand considerations. Like Brazil, Colombia's implicit PPP rule was implemented in the midst of severe exchange rate controls.

In sum, the birth of PPP rules can be traced back to Latin America (in particular, Chile, Brazil, and Colombia) in the mid to late 1960s. The motivation for the adoption of a regime of mini-devaluations was to escape the vicious cycle (theoretically described in Chapter 16) in which a high rate of domestic inflation – inconsistent with a fixed exchange rate – would lead eventually to a costly exchange rate crisis and a large devaluation. Policymakers thought of avoiding these recurrent crisis by devaluing frequently and by small amounts. At least from a conceptual point of view, this policy change was arguably for the better. If one takes as given that domestic

[27] See Fendt (1981) for an account of the Brazilian experience.

inflation will be higher than foreign inflation due to monetary financing of underlying fiscal deficits, then setting a rate of devaluation consistent with that rate of inflation (and have a "sustainable" regime, to use the language of Chapter 5) is clearly better than the alternative of living in a world in which, with daunting regularity, the economy goes through a full-fledge exchange rate crisis.[28] The perils – as stressed in the text – is that the economy may lose its nominal anchor and/or that shocks that require a real depreciation will be more inflationary than would otherwise be the case.

[28]Theoretically, in terms of our cash-in-advanced model, a constant rate of devaluation is better in terms of welfare than a zero rate of devaluation followed at time T by a discrete jump.

BOX 2. Is real exchange rate targeting inflationary?

Sections 2 through 4 in the text have emphasized the potential inflationary consequences of targeting the real exchange rate. But what does the empirical evidence say? This box focuses on the experience of Brazil, Chile, and Colombia, countries with a long history of real exchange rate targeting and PPP rules. Figure 9 provides some relevant data for analyzing the experiences of these three countries.[29] The left panels show the evolution of both the logarithm of the nominal exchange rate and the logarithm of the PPP exchange rate.[30] Vertical bars indicate periods during which either explicit or implicit PPP rules were in effect, as discussed below. The idea is that, if a PPP rule was being pursued, the actual exchange rate should be close to its PPP value. The right panels show the evolution of the real exchange rate and the inflation rate for each country.[31]

[Figure 9 here]

In July 1985, Chile established an exchange rate band whose central parity was adjusted at daily intervals according to the differential between domestic and foreign inflation. Remarkably, there were no deviations from this rule until January 1992, when strong capital inflows led to a revaluation of 5 percent (see left-hand panel in Figure 9, Row A). In the right-hand side panel, we can see that inflation remains relative high and variable during the PPP period and begins to fall systematically only after Chile gives up the PPP rule in 1992.[32] Earlier, Chile also provides an excellent example of policies aimed at achieving a higher level (i.e., more depreciated) level of the real exchange rate. Indeed, in the period 1982-1985, Chile pursued a policy of

[29] Data come from the IMF's International Financial Statistics (IFS) and Global Financial Data (GFD).

[30] Following Calvo, Reinhart, and Vegh (1995), the PPP exchange rate was computed as the ratio of the domestic CPI to that of the U.S., and set equal to the actual value of the exchange rate in 78.01 for Brazil, 85.07 for Chile, and 86.01 for Colombia. In the case of Brazil, the base date coincides with the beginning of the sample; in the case of Chile and Colombia, the base date marks the beginning of a period during which a PPP rule was in effect (see below).

[31] The real exchange rate is defined as EP^*/P, where E is the nominal exchange rate, P^* is the U.S. CPI and P is the domestic country CPI.

[32] Needless to say, we are not controlling for a myriad of factors, so these figures should be seen only as suggestive. More formal evidence on the link between real exchange rate targeting and inflation is summarized below.

very aggressive nominal devaluation in order to engineer a real depreciation. This episode is clear from the left-hand panel in Row A.

As discussed in Box 1, Brazil implemented a PPP rule in August 1968 and with, occasional interruptions, essentially maintained this policy until the Real Plan in July 1994. The left-hand side panel in Figure 9, Row B, which shows the actual nominal exchange rate tracking very closely the PPP exchange rate, is clearly consistent with this interpretation. The right-hand side panel shows how inflation became high and variable in response to various external shocks, starting with the oil price shock of 1979. The lack of a nominal anchor allowed inflation to take a life of its own.

As also mentioned in Box 1, Colombia first implemented a PPP rule in 1967 and essentially kept such a regime in place until 1991. Once again, the left-hand side panel in Row C of Figure 9 is consistent with this view. Notice also an attempt in 1985 to rapidly depreciate the currency to engineer a real depreciation. The right-hand side panel suggests that inflation remained high and variable until the PPP rule regime was abandoned in 1991.

Calvo, Reinhart, and Vegh (1995) then proceed to test for these three countries the idea that there should be a positive correlation between the temporary components of inflation and the real exchange rate. To this effect, they decompose the real exchange rate, which is non-stationary in all three countries, into its permanent and temporary component using the Beveridge-Nelson decomposition. In all three cases, the correlation has the expected sign and is statiscally different from zero, with values ranging from 0.26 to 0.42. The formal evidence thus supports the idea that targeting a more depreciated real exchange rate or preventing the real exchange rate from appreciating in response to a positive shock is inflationary.

BOX 3. Real interest rate targeting: A look at the Chilean experience

In August 1985, Chile adopted as its main policy instrument the interest rate on bonds issued by the Central Bank of Chile and denominated in UFs, an indexed unit of account.[33] By all accounts, the peculiar choice of what was effectively a real interest rate as the main instrument of monetary policy was simply a recognition of the importance that the UF had acquired as the unit of account for most financial instruments. During the first two years, bonds of different maturities were offered at a fixed interest rate by the Central Bank. By 1987, the main instrument had become the 90-day PRBC.[34] As indicated in Table 2, in 1995 the Central Bank of Chile switched from the 90-day PRBC to targeting an overnight interest rate denominated in UFs. This real interest rate target policy was in effect until 2001, when the Central Bank switched to an overnight nominal interest rate as its main policy instrument.

[Table 2 here]

How did this policy fare in practice? Figure 10 shows the evolution over time of the reference interest rate and the inflation rate. Inflation remained high until the early 90s and then fell steadily over time. Importantly, the Central Bank of Chile became autonomous in 1989 and in 1990 proceeded to announce inflation targets. Our theoretical discussion of Section 5 suggests that while a pure real interest rate target would leave the inflation rate undetermined, a real interest rate rule based on a *credible* inflation target would provide a perfectly sensible way of conducting monetary policy. The evidence offered in Valdés (1997) is fully consistent with this interpretation. Valdés estimates a vector-autoregression model to empirically account for the monetary transmission mechanisms operating in Chile during this period. He concludes that while changes in the 90-day PRBC affect the output path, they only influence inflation indirectly. Specifically, changes in the 90-day PRBC

[33] In January 1967 – and in the midst of high and chronic inflation – Chile introduced the UF ("unidad de fomento"), a unit of account constructed using CPI values for the previous months. It was originally intended to be used to index the price of houses, but soon became a popular way of protecting savings from inflation. By the early 1980s, the UF had become a cornerstone of the financial sector, with most loans and deposits denominated in UFs.

[34] PRBC is the Spanish acronym for "Pagarés Reajustables del Banco Central," which stands for "Central Bank readjustable obligations."

affect the differential between the inflation target and the actual inflation rate but not the level of the inflation rate itself. This suggests that the key nominal anchor in Chile during the disinflation period of the 1990s illustrated in Figure 10 was a credible inflation target.

[Figure 10 here]

The decision to "nominalize" the system in August 2001 did not come without much debate on the pros and cons of doing so (see Morandé (2002) and Fuentes *et al* (2003)). There were concerns about whether the new policy would be as effective as the old one in affecting the target real variables (output and inflation) and whether it would increase the variability of interest rates on UF-denominated assets (which, presumably, would increase the variability of the real interest rate relevant for economic decisions). On the other hand, nominalizing the system would likely get rid of inflation inertia due to backward-looking indexation mechanisms embedded in the calculation of the UF. As it turns out, even though the variability of UF interest rates does appear to have increased after the policy change, Fuentes *et al* (2002) argue that no major changes occurred in the transmission mechanism because of arbitrage between indexed and nominal rates.[35] Theoretically, one would not expect a large difference in transmission mechanisms between a nominal interest rate rule or a real interest rate rule, as long as they are combined with a credible inflation target (Végh (2002)).

[35] See also Chumacero (2002).

Figure 1. Real exchange rate targeting under flexible prices

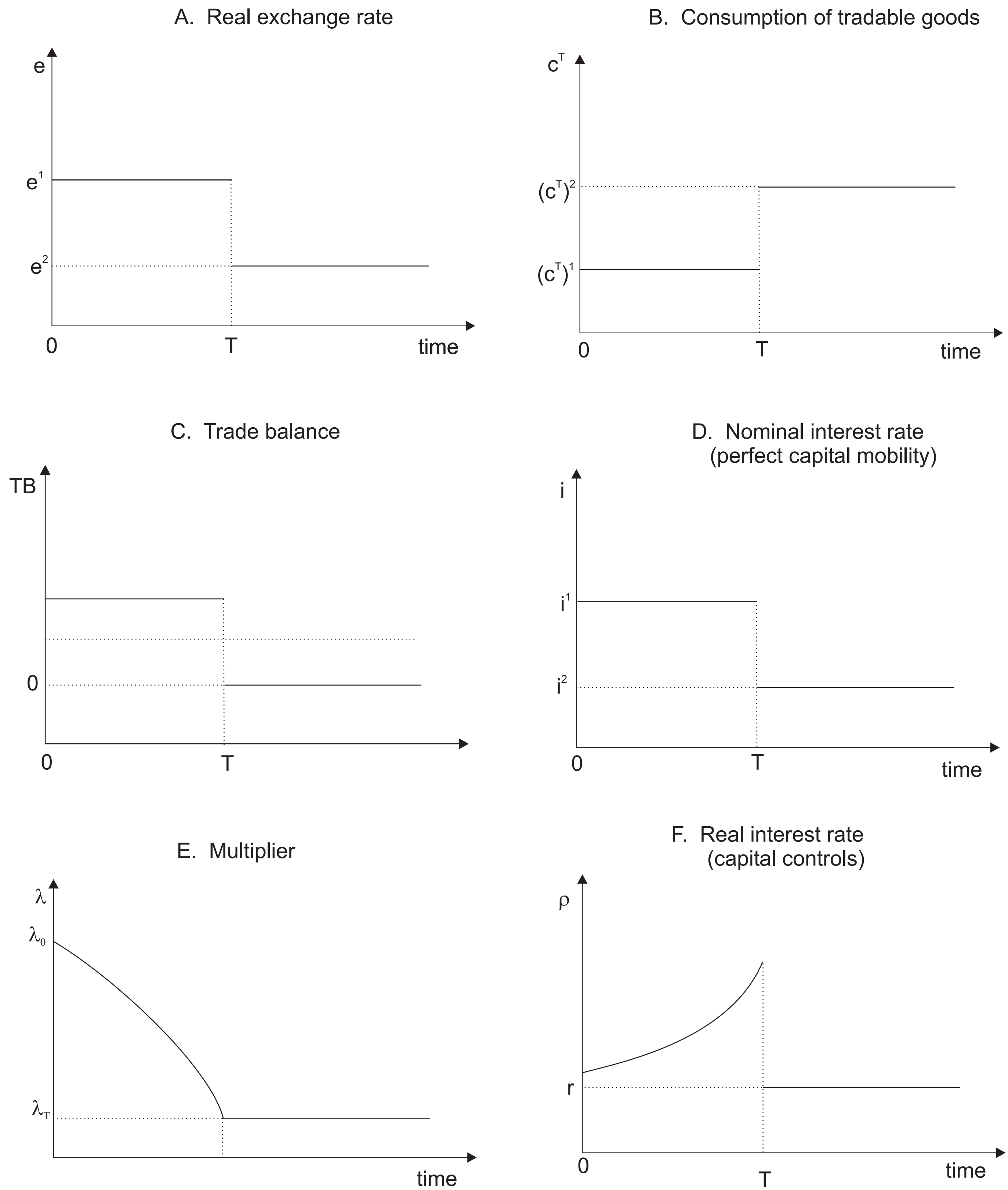


Figure 2. Real exchange rate targeting in a sticky inflation model: Phase diagram

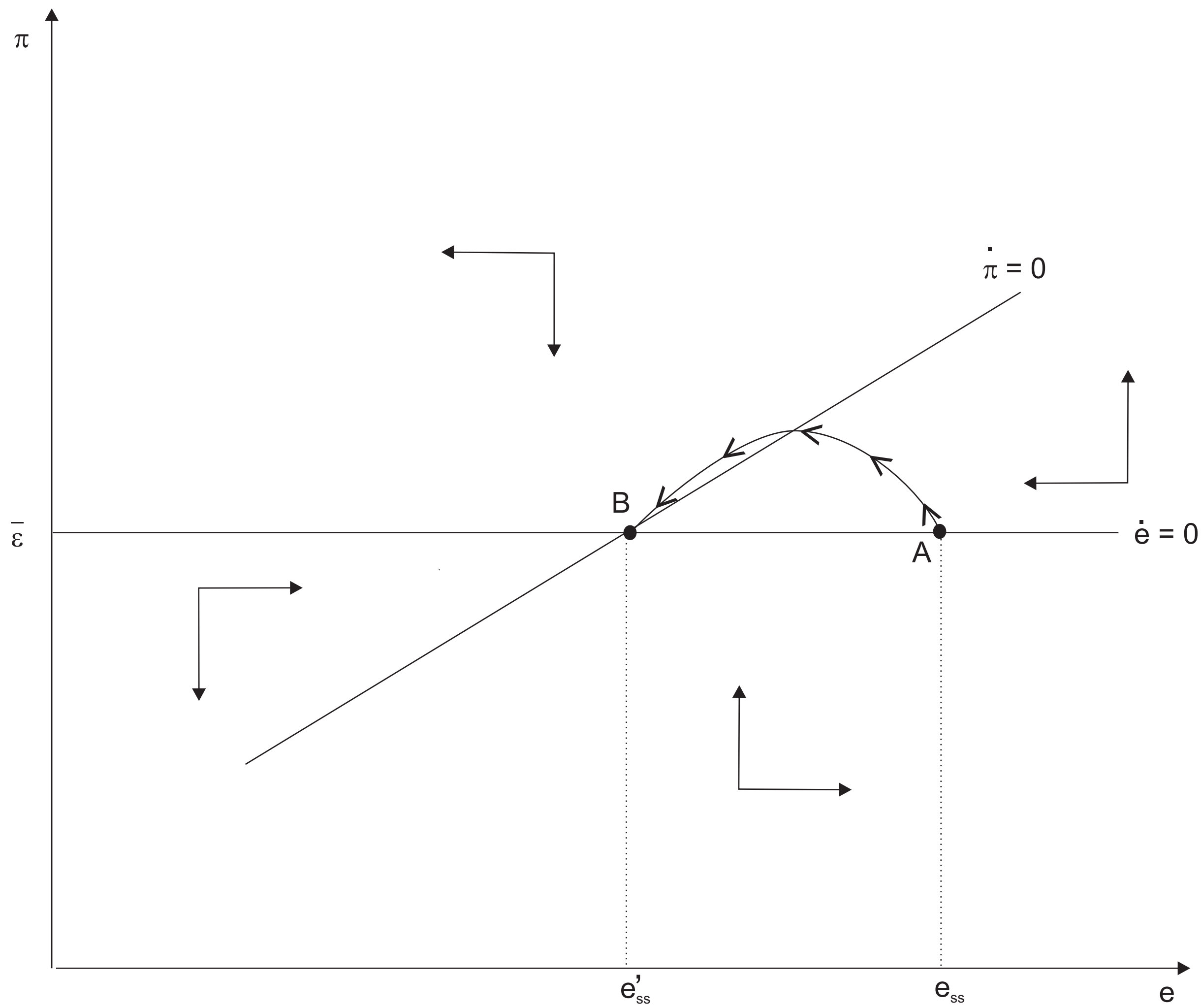


Figure 3. Permanent increase in endowment of tradables

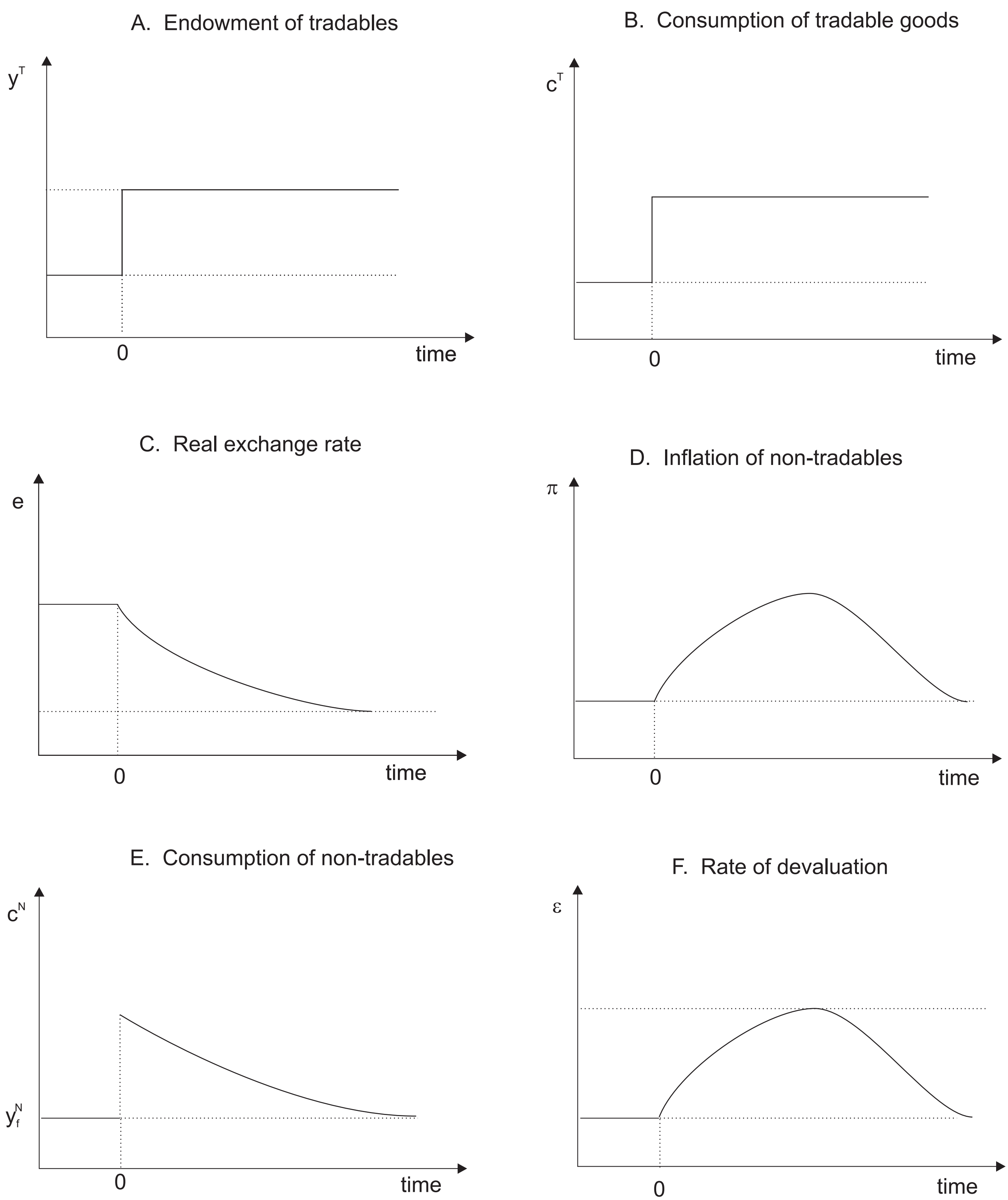


Figure 4. Real exchange rate targeting in a sticky-inflation model

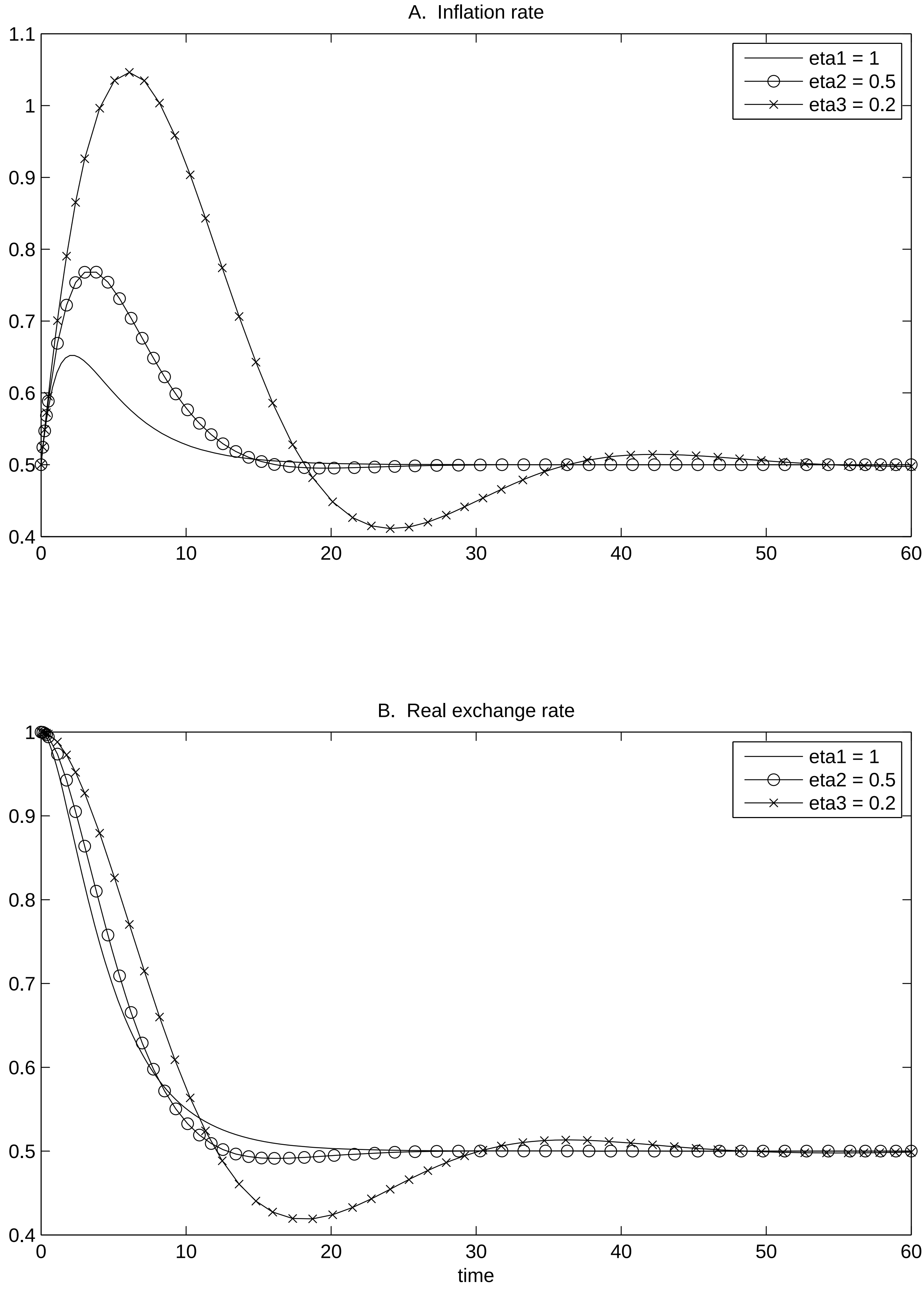


Figure 5. Dynamics in the (r^d,π) plane

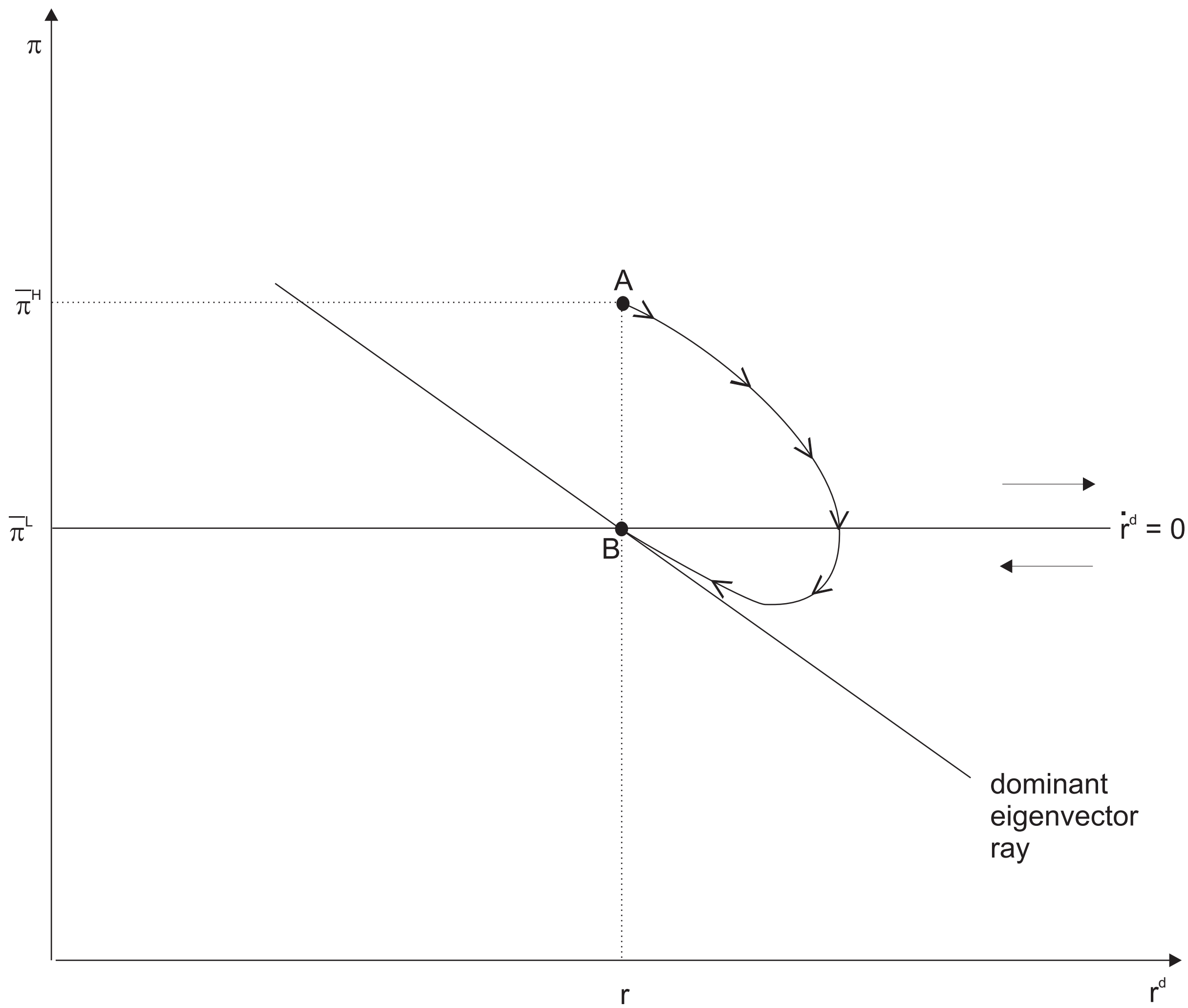


Figure 6. Reduction in inflation target

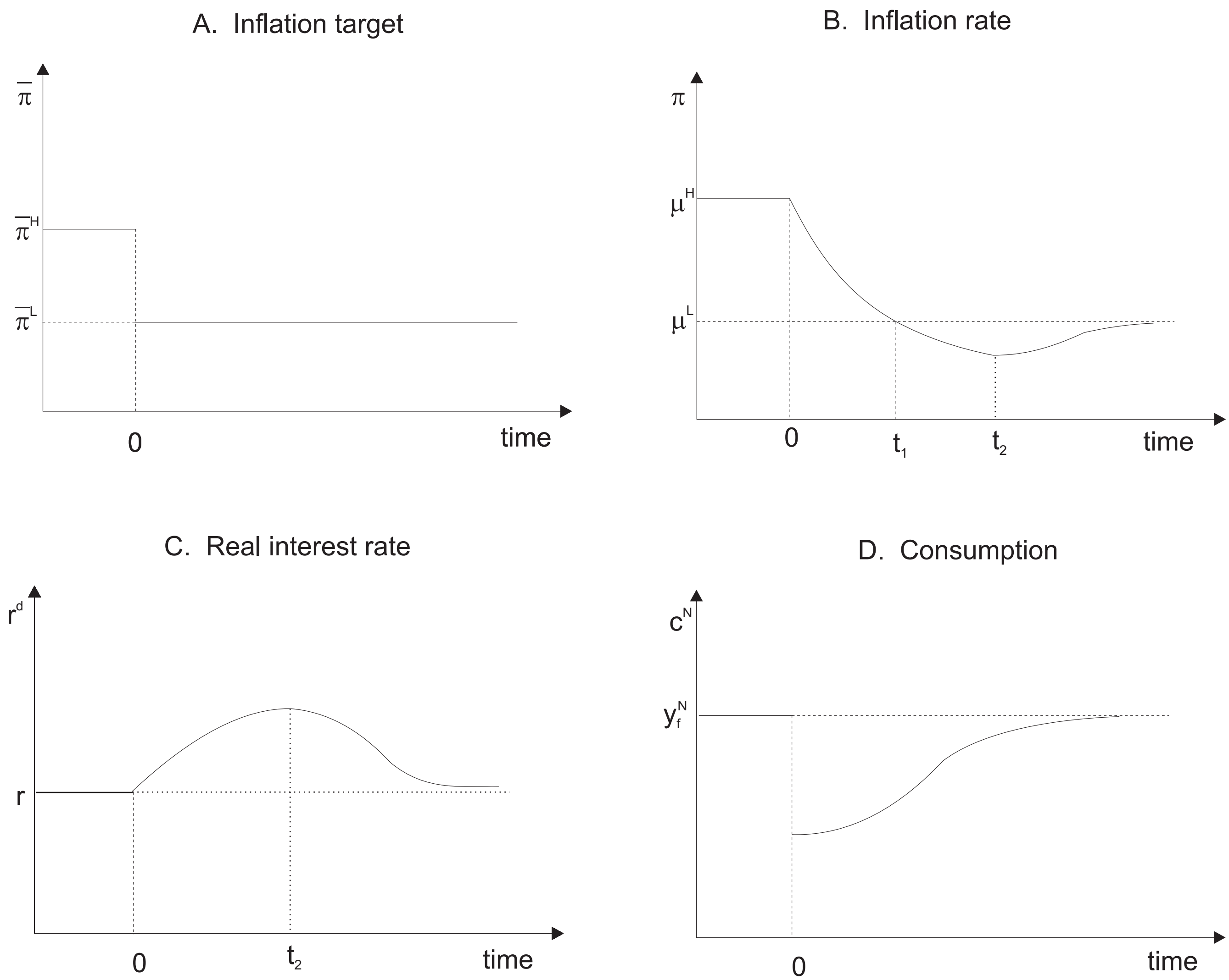
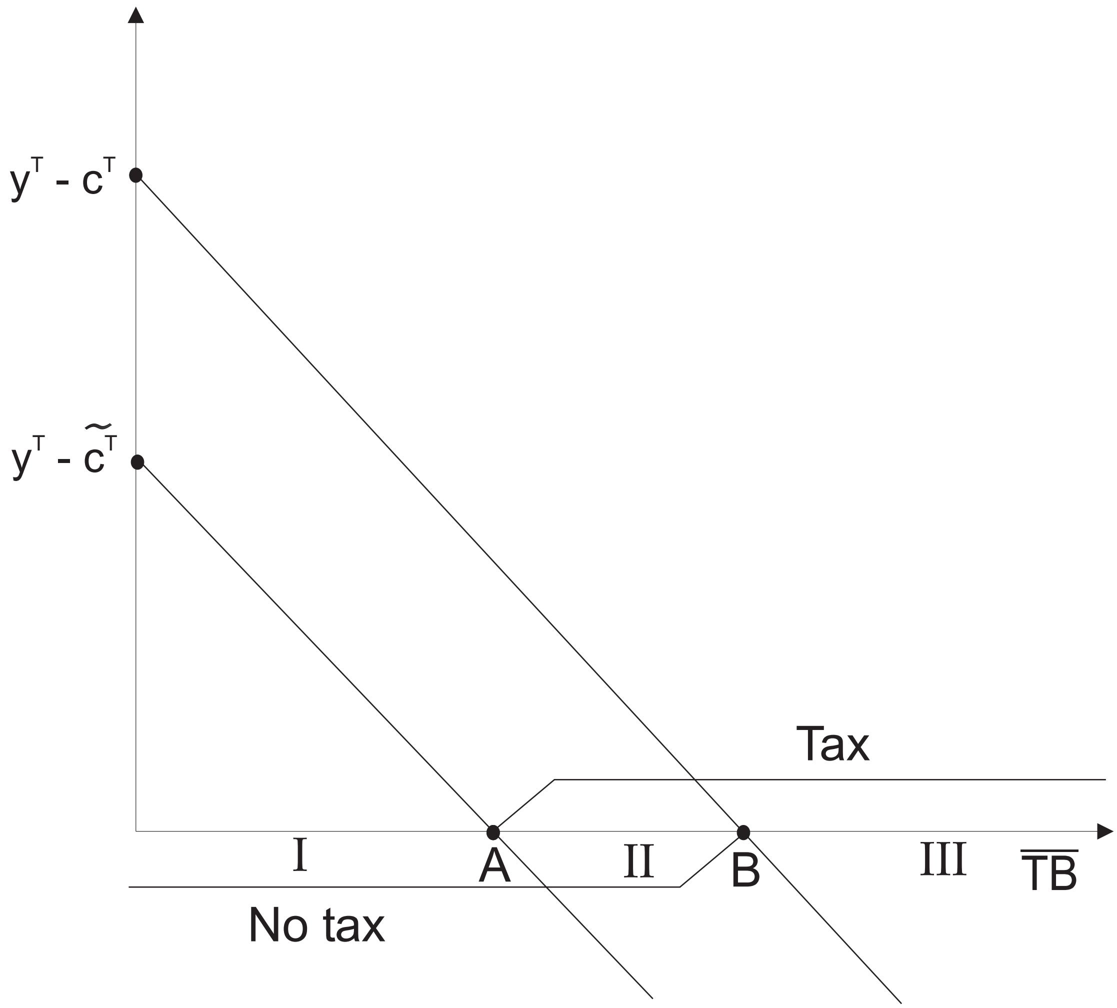


Figure 7. Multiple equilibria



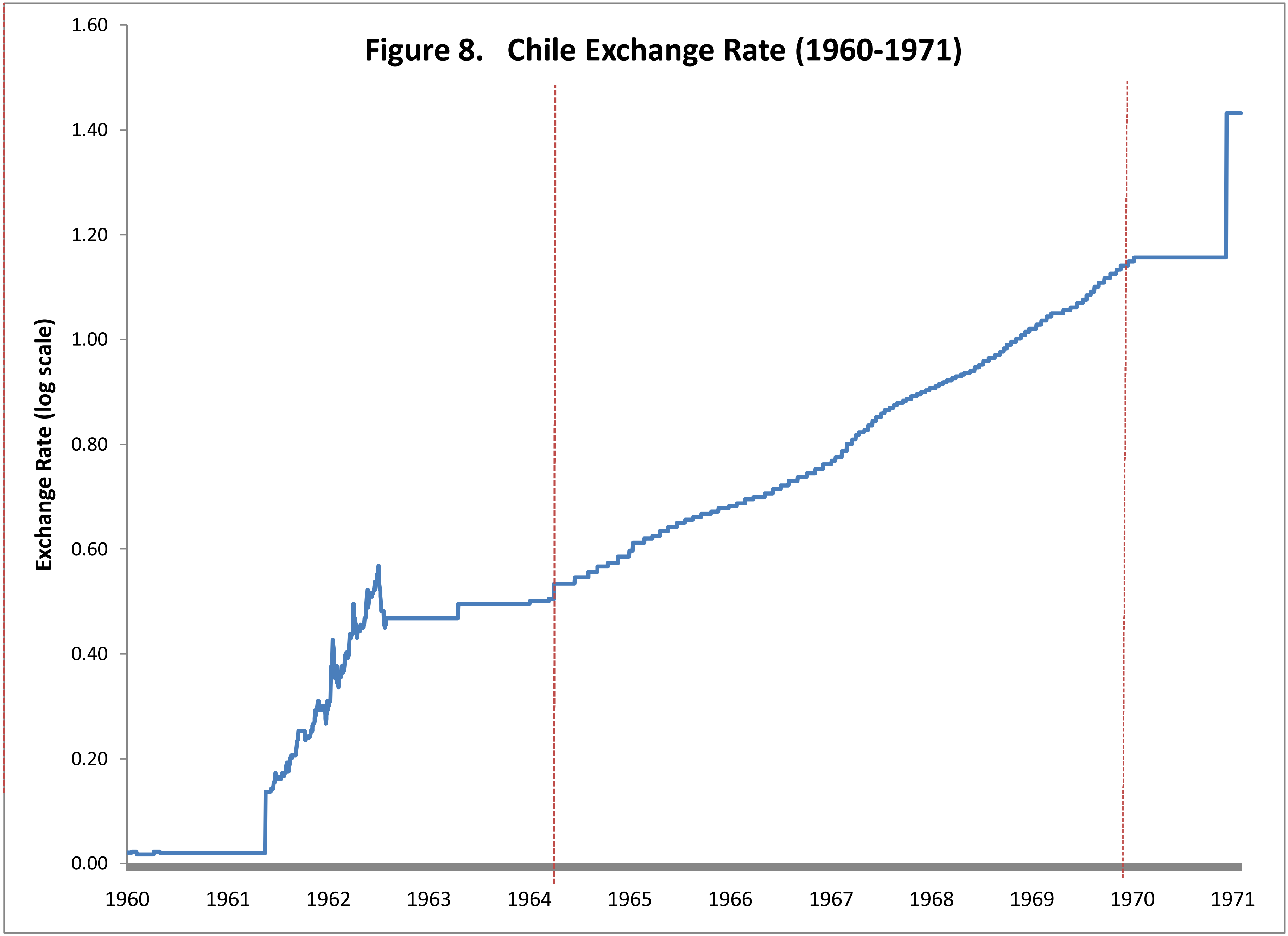
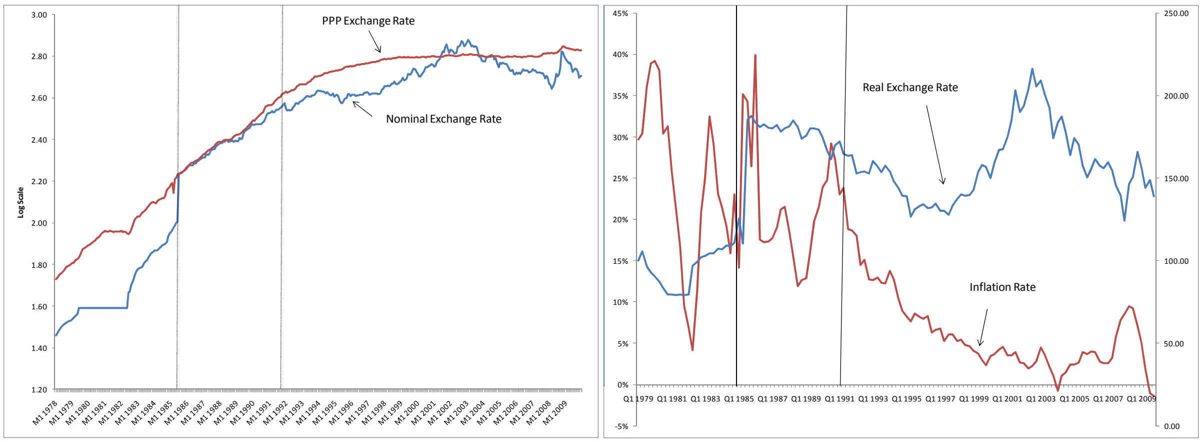


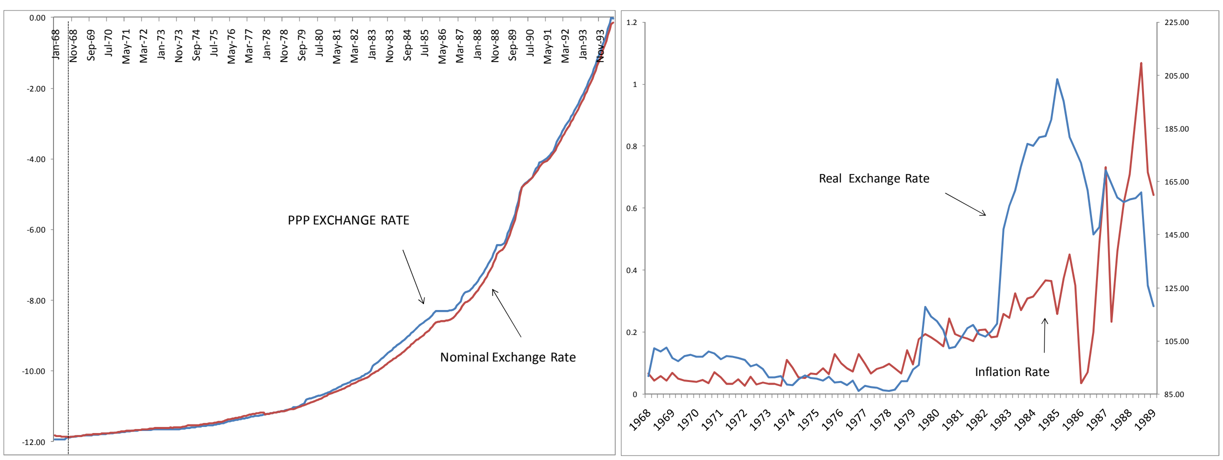
Figure 8. Chile Exchange Rate (1960-1971)
Exchange Rate (log scale)
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
1960
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971

Figure 9. PPP rules in Chile, Brazil, and Colombia

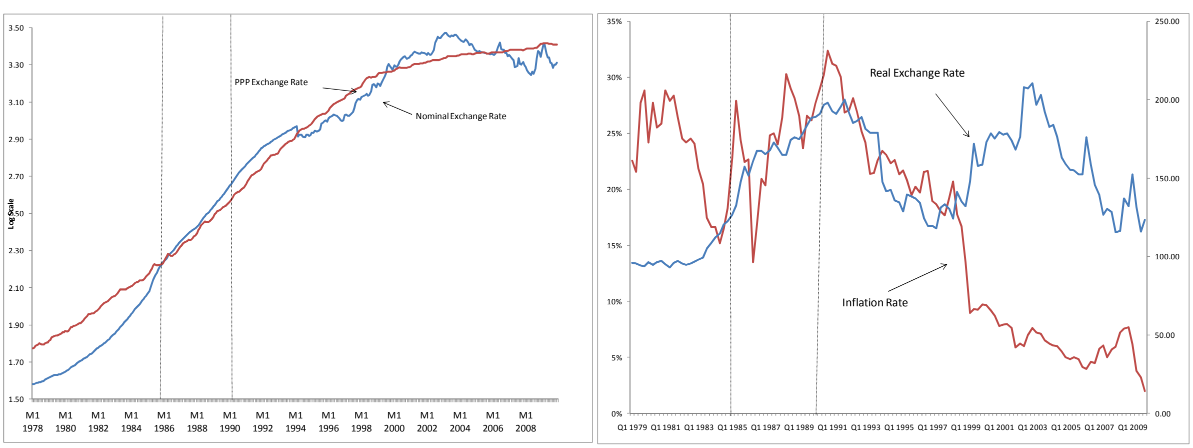
A. CHILE

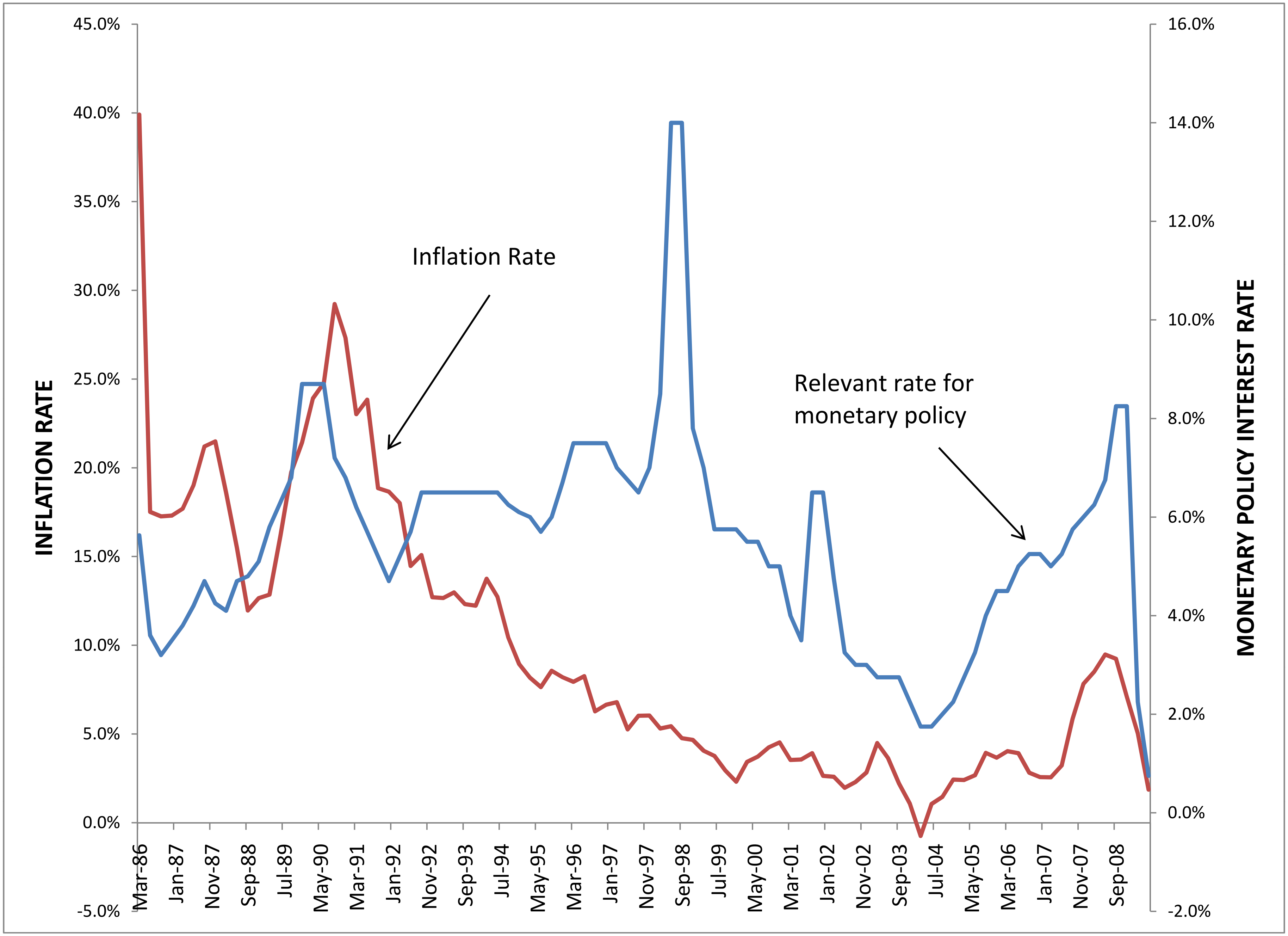


B. BRAZIL



C. COLOMBIA





Inflation Rate
Relevant rate for
monetary policy
INFLATION RATE
MONETARY POLICY INTEREST RATE
45.0%
40.0%
35.0%
30.0%
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
-5.0%
16.0%
14.0%
12.0%
10.0%
8.0%
6.0%
4.0%
2.0%
0.0%
-2.0%
Mar-86
Jan-87
Nov-87
Sep-88
Jul-89
May-90
Mar-91
Jan-92
Nov-92
Sep-93
Jul-94
May-95
Mar-96
Jan-97
Nov-97
Sep-98
Jul-99
May-00
Mar-01
Jan-02
Nov-02
Sep-03
Jul-04
May-05
Mar-06
Jan-07
Nov-07
Sep-08

Table 1. Chile: Adjustments of the nominal exchange rate, 1965-1970

	Number of Adjustments	Average number of days between adjustments	Rates of Adjustment (%)		
	(1)	(2)	Maximum (3)	Average (4)	Minimum (5)
1965 (May-Dec)	8	30.0	2.9	1.5	0.1
1966	12	30.4	2.3	1.8	1.2
1967	16	22.8	2.4	1.8	0.7
1968	24	15.3	1.9	1.2	0.8
1969	19	19.2	1.7	1.4	0.8
1970 (Jan-July)	12	17.6	1.9	1.7	1.4

Source: Ffrench-Davis (1981)

Table 2. Chile: Instruments used to conduct monetary policy, levels, and standard deviation of the relevant rates and the inflation rate, 1985-2002

Period	Instrument	Rate of Monetary Policy in UF* (%)		Inflation Rate (%)	
		Average	Standard Deviation	Average	Standard Deviation
August 1985-April 1995	PRBC-90	5.7	1.4	16.9	5.4
May 1995- December 2000	Interbank rate – 1 day in UF	6.7	1.5	5.5	1.8
January 2001- July 2001	Interbank rate – 1 day in UF	4.0	0.5	3.7	0.5
August 2001- March 2003	Interbank rate – 1 day in pesos	1.4	1.6	2.9	0.7

Note: The rate for monetary policy corresponds to the effective rate expressed in UF for the first three periods. The rate shown for the last period (August 2001- March 2003) corresponds to an ex-ante real interest rate, defined as the effective rate for monetary policy in nominal terms minus the center of the target inflation band (3%)

*UF: Unidad de Fomento.