

# Chapter 2

## Capital Markets Imperfections\*

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### 1 Introduction

The basic model analyzed in Chapter 1 yields the key result that, along a perfect foresight equilibrium path, a small open economy will smooth out consumption over time regardless of the output path. Three key assumptions underlie this stark prediction. First, the economy has perfect access to international capital markets (i.e., it can borrow from/lend to the rest of the world at a given and constant real interest rate). Second, the output path is known with certainty. Third, the country is able to precommit to repaying its debt. How would the results change if these assumptions were relaxed?

This chapter analyzes the consequences of relaxing each of the above assumptions. Section 2 addresses the case of financial autarky, in which the economy has no financial links to the rest of the world. While extreme, this scenario provides a useful illustration of economies with severe restrictions on capital flows. As one would expect, an economy in financial autarky

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behaves exactly as a closed economy and is forced to consume its endowment every period. Since, in bad times, the economy cannot borrow from abroad, domestic real interest rates will have to be higher to induce households to curtail consumption. Real interest rates – rather than the trade balance as in Chapter 1 – thus become the key adjusting variable. In practice, of course, developing countries’ access to international capital markets generally falls somewhere in between the extreme cases of perfect capital mobility – analyzed in Chapter 1 – and financial autarky. In such cases, a temporarily lower level of output will lead to both trade deficits and higher real interest rates.

Section 3 introduces uncertainty into the model by assuming that period 2 output is a stochastic variable. In the presence of uncertainty, the menu of financial claims available to the economy becomes critical. If asset markets are incomplete in the sense that the economy has no access to state contingent claims (i.e., only a risk-free asset is available), then the economy will be unable to smooth consumption over time. There will thus be a positive correlation between output and consumption. In contrast, if asset markets are complete, the economy will be able to fully smooth out consumption over time by buying state-contingent claims. Since developing countries have access to a more limited menu of financial instruments, the model predicts that the correlation between output and consumption should be higher in developing countries than in industrial countries. In terms of welfare, incomplete markets impose social costs by preventing the economy from smoothing consumption. These costs will be larger the higher is the variability of output.

Finally, Section 4 relaxes the assumption that the economy can precommit to repaying its foreign debt. We capture in this way the presence of *sovereign risk*, which refers to the fact that since sovereign countries are not subject to standard bankruptcy laws as would individuals or corporations within a country, creditors face more risk than when lending domestically. We show that the presence of sovereign risk leads to the emergence of credit ceilings, beyond which foreign lenders will not be willing to lend at any interest rate. Hence, faced with a temporarily lower level of output, the economy may not be able to borrow as much as it would need to keep consumption constant over time and domestic real interest rates will need to rise to induce lower consumption. While providing some key insights, this benchmark model of sovereign risk with no uncertainty exhibits two unattractive features: (i) the supply of funds is flat until the credit ceiling is reached and (ii) we never observe default in equilibrium. By introducing uncertainty regarding period

2 output, the model generates an upward-sloping supply of funds and default in equilibrium.

The general message to come out of this chapter is that imperfections in international capital markets will severely limit the ability of a small open economy to smooth consumption over time. From a positive point of view, the inability to fully smooth consumption introduces a positive correlation between output and consumption, which is consistent with the empirical evidence. From a normative point of view, we conclude that such imperfections in capital markets are socially costly which justifies efforts to improve developing countries' access to external financing in bad times.

On the methodological front, this chapter introduces a two-period version of the basic model developed in Chapter 1. As will become clear below, this two-period model is particularly tractable for examining issues related to imperfect access to capital markets and uncertainty.

## 2 Financial autarky

Consider a small open economy inhabited by a large number of identical individuals who live for two periods and are blessed with perfect foresight. There exists only one (tradable and non-storable) good. The consumer receives an endowment of the good in every period. We depart from the basic model of Chapter 1 by assuming that, while the consumer perceives him/herself as being able to borrow as much as he/she wants at a constant real interest rate (subject, of course, to his/her intertemporal constraint), the economy as a whole cannot borrow from the rest of the world (financial autarky).

### 2.1 Consumer's problem

#### 2.1.1 Budget constraints

Let  $b_i^d$ ,  $i = 1, 2$ , stand for the level of net *domestic* assets at the end of period  $i$ . These domestic bonds are denominated in terms of the tradable good and have a face value of unity. The flow budget constraint for period 1 is thus given by:

$$b_1^d = y_1 - c_1. \tag{1}$$

As in Chapter 1, we constrain the consumer not to “die” with outstanding

debt (i.e., we impose the condition that  $b_2^d \geq 0$ ). In addition, given a strictly increasing utility function, it would not be optimal to “die” with a positive level of assets (which implies that  $b_2^d \leq 0$ ). Hence, net domestic assets at the end of period 2 must be zero (i.e.,  $b_2^d = 0$ ). In period 2, the consumer gets interest payments on his/her bond holdings. The flow budget constraint for period 2 is thus:

$$c_2 = (1 + \rho)b_1^d + y_2, \quad (2)$$

where  $\rho$  is the *domestic* real interest rate. Given that the domestic bond is non-tradable,  $\rho$  may differ from the international real interest rate,  $r$ , as will become clear below.

Combining (1) and (2) yields the consumer’s intertemporal budget constraint:

$$y_1 + \frac{1}{1 + \rho}y_2 = c_1 + \frac{1}{1 + \rho}c_2. \quad (3)$$

### 2.1.2 Utility maximization

The lifetime utility of the representative household is given by:

$$W = u(c_1) + \beta u(c_2), \quad (4)$$

where  $c_i$ ,  $i = 1, 2$ , denotes consumption at the beginning of period  $i$ , and  $\beta$  ( $1 > \beta > 0$ ) is the discount factor.<sup>1</sup>

The consumer’s problem consists in choosing  $c_1$  and  $c_2$  to maximize lifetime utility, given by (4), subject to the intertemporal budget constraint, given by (3), taking as given  $\rho$ ,  $y_1$  and  $y_2$ . We can set up the following Lagrangean:

$$\mathcal{L} = u(c_1) + \beta u(c_2) + \lambda \left( y_1 + \frac{1}{1 + \rho}y_2 - c_1 - \frac{1}{1 + \rho}c_2 \right).$$

The first order conditions are given by:

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<sup>1</sup>The discount factor,  $\beta$ , is, by definition, equal to  $1/(1 + \delta)$ , where  $\delta$  is the rate of time preference. The higher the rate of time preference (i.e., the higher the rate at which the future is discounted), the smaller the discount factor.

$$u'(c_1) = \lambda, \tag{5}$$

$$\beta u'(c_2) = \frac{\lambda}{1 + \rho}. \tag{6}$$

Combining (5) and (6) yields:

$$u'(c_1) = \beta(1 + \rho)u'(c_2), \tag{7}$$

which is the Euler equation.

To establish the appropriate analogy with the basic model of Chapter 1, notice that if the economy has full access to international capital markets, then  $\rho = r$  which, given the assumption that  $\beta(1 + r) = 1$ , implies that

$$u'(c_1) = u'(c_2). \tag{8}$$

We are thus back to the world of Chapter 1, where the consumption path is flat regardless of the output path.

## 2.2 Equilibrium conditions

### 2.2.1 Bond market equilibrium

Since the economy is closed in bond markets, the aggregate stock of domestic bonds circulating in the economy at the end of period 1 must be equal to zero. Formally,

$$b_1^d = 0. \tag{9}$$

Since the aggregate stock of bonds is zero, the domestic real interest rate,  $\rho$ , will need to adjust to clear the bond market. Notice that, unlike the economy of Chapter 1 which faces a completely elastic supply of foreign funds at a constant real interest rate, this economy faces a completely vertical supply of foreign funds (at zero).

### 2.2.2 Aggregate constraints

To obtain the flow constraints for the economy as a whole, substitute the bond market equilibrium (9) into the household's flow constraints (1) and (2) to obtain:

$$c_1 = y_1, \tag{10}$$

$$c_2 = y_2. \tag{11}$$

The trade balance in this economy ( $TB \equiv y - c$ ) is therefore always equal to zero. The same is true of the current account.

### 2.3 Solution of the model

The model has three endogenous variables:  $c_1$ ,  $c_2$ , and  $\rho$ . From (10) and (11), it follows immediately that, in equilibrium, the economy must consume its endowment in every period. This is to be expected since, due to financial autarky, the economy cannot run a trade imbalance.

To determine  $\rho$ , substitute (10) and (11) into (7) to obtain:

$$1 + \rho = \frac{u'(y_1)}{\beta u'(y_2)}. \tag{12}$$

Equation (12) makes clear that, in financial autarky, the domestic real interest rate is solely determined by the path of output (as would be the case in a closed economy).

### 2.4 Stationary equilibrium

As a benchmark, suppose that the output path is flat; that is,  $y_1 = y_2 = y$ . It then follows from (10) and (11) that  $c_1 = c_2 = y$ . From (12), it follows that  $1 + \rho = 1/\beta$ . This is, of course, the same equilibrium involving full consumption smoothing that would obtain in the basic model of Chapter 1 under perfect capital mobility. The reason is simply that, faced with a constant path of output, households have no incentive whatsoever to save or dissave. Hence, being shut off from international capital markets is not a binding constraint.

### 2.5 Non-stationary equilibrium

Suppose now that output is lower in the first period; that is,  $y_1 < y_2$ . From (10) and (11), it follows that  $c_1 = y_1 < c_2 = y_2$ . Since  $c_1 < c_2$ , it follows from (12) that  $1 + \rho > 1/\beta$ . Under our maintained assumption that  $\beta(1 + r) = 1$ ,

this implies that  $\rho > r$ . In other words, the domestic real interest rate is higher than the international one.

Intuitively, faced with a lower output in the first period, individual consumers would like to dissave (i.e., to sell bonds) in order to smooth consumption over time. Since the aggregate supply of bonds is zero, however, this excess supply of bonds must lead to a fall in their price (i.e., an increase in  $\rho$ ).<sup>2</sup> In equilibrium, the price of bonds must fall (i.e., the domestic real interest rate must rise) up to the point at which consumers are content with consuming the available endowment. In other words, the high domestic real interest rate reflects the scarcity of goods in period 1 relative to period 2.

It should be intuitively clear that financial autarky is costly since welfare would be higher if the economy were able to smooth consumption over time. To show this formally, notice that under perfect capital mobility, consumption would be constant over time (i.e.,  $c_1 = c_2 \equiv \bar{c}$ ) and given by

$$\bar{c} = \frac{1+r}{2+r} \left( y_1 + \frac{y_2}{1+r} \right). \quad (13)$$

Welfare under perfect capital mobility would thus be higher than under financial autarky if

$$\underbrace{u(\bar{c}) + \beta u(\bar{c})}_{\text{welfare under perfect capital mobility}} > \underbrace{u(y_1) + \beta u(y_2)}_{\text{welfare under financial autarky}}. \quad (14)$$

Rewrite this inequality as:

$$u(\bar{c}) > \frac{1}{1+\beta} u(y_1) + \frac{\beta}{1+\beta} u(y_2). \quad (15)$$

Since, by assumption,  $\beta(1+r) = 1$ , it follows from (13) that  $\bar{c} = [1/(1+\beta)]y_1 + [\beta/((1+\beta))]y_2$ . Hence, by strict concavity of  $u(\cdot)$ , inequality (15) holds for any pair of values  $y_1 \neq y_2$ .

Figure 1 provides a graphical illustration of the welfare costs of financial autarky for the case at hand ( $y_1 < y_2$ ). The left-hand side of inequality (15) is given by point A while the right-hand side is given by point B. Clearly, welfare at point A is higher than welfare at point B. In addition, the figure provides a graphical demonstration of the fact that a mean-preserving spread of the

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<sup>2</sup>Notice that the price of a bond in terms of period 2 consumption is  $1/(1+\rho)$ . There is thus an inverse relationship between the price of the bond in terms of period 2 consumption and the domestic real interest rate.

output distribution (from  $(y_1, y_2)$  to  $(\bar{y}_1, \bar{y}_2)$ ) reduces welfare from point B to point C. Hence, the more variable is output, the higher the costs of financial autarky.

## 2.6 Conclusions

We have just seen that, in the presence of financial autarky, a temporarily lower level of output will lead to reduced consumption, a higher real interest rate and, of course, no change in the trade balance. In the previous chapter, we saw that, in the presence of perfect capital mobility, a temporarily lower level of output results in a deficit in the trade balance but no change in consumption. In reality, the world lies somewhere in between these two extremes (and, of course, the degree of capital mobility varies from country to country). Hence, based on the analysis of these two extreme cases, our model predicts that, in practice, a temporarily lower level of output should be reflected in a combination of lower consumption, a higher trade deficit, and a higher domestic real interest rate. In particular, the more open the capital account, the more a temporarily lower level of output should be reflected in a larger current account deficit, whereas the more closed the capital account, the more the shock should be reflected in a higher domestic real interest rate and lower consumption.

On the normative side, financial autarky is socially costly since welfare would be higher if the economy could smooth consumption over time. The costs of financial autarky are higher the more variable is output.

## 3 Uncertain output path

As mentioned in the introduction, a second key assumption of the basic model of Chapter 1 is that the output path is known with certainty. In other words, because households know exactly the present discounted value of their resources, they can keep consumption smooth by borrowing today while having the certainty that they will have the resources to repay their creditors tomorrow. But suppose households did not know for sure what their future endowment will be. How would they decide whether to save or dissave? This section addresses this important question.

Consider the following modification of the two-period model presented in section 2. The first-period endowment is  $y_1$  with certainty. The second-

period endowment, however, is stochastic:

$$y_2 = \begin{cases} y_2^H & \text{with probability } p, \\ y_2^L & \text{with probability } 1 - p, \end{cases} \quad (16)$$

where  $y_2^H > y_2^L$ . Furthermore – and to capture a situation in which the economy is going through rough times in period 1 relative to period 2 – suppose that the expected value of  $y_2$  is higher than  $y_1$ . Formally:

$$E(y_2) = py_2^H + (1 - p)y_2^L > y_1. \quad (17)$$

Clearly, in a perfect-foresight version of this set-up in which consumers were endowed with the average level of output in period 2, they would borrow in the first period (i.e., run a current account deficit) to fully smooth consumption over time as analyzed in Chapter 1.

We will first study the case of incomplete markets and then the case of complete markets.

### 3.1 Incomplete capital markets

Consider the following financial environment. This small economy can borrow from/lend to the rest of the world at a constant world real interest rate,  $r$ , as in Chapter 1. However, it has no access to contingent debt; in other words, it cannot borrow contingent on the realization of output in the second period. In this sense, markets are incomplete.<sup>3</sup>

#### 3.1.1 Budget constraints

Let us begin by looking at the first period flow constraint. Assume that initial net foreign assets are zero. Then, since there is no uncertainty in period 1:

$$b_1 = y_1 - c_1. \quad (18)$$

The second-period flow constraint will depend on the realization of output. Let  $c_2^H$  and  $c_2^L$  denote second-period consumption in the high-output state of nature and low-output state of nature, respectively. Then, since the budget constraint must hold in every state of nature:

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<sup>3</sup>See Blanchard and Fischer (1989, Chapter 6) for a detailed analysis of consumption under uncertainty.

$$0 = (1+r)b_1 + y_2^H - c_2^H, \quad (19)$$

$$0 = (1+r)b_1 + y_2^L - c_2^L. \quad (20)$$

Combining equations (18), (19), and (20), we obtain an intertemporal budget constraint for each of the two possible output paths:

$$y_1 + \frac{1}{1+r}y_2^H = c_1 + \frac{1}{1+r}c_2^H, \quad (21)$$

$$y_1 + \frac{1}{1+r}y_2^L = c_1 + \frac{1}{1+r}c_2^L. \quad (22)$$

### 3.1.2 Utility maximization

As is standard, we will assume that consumers maximize expected lifetime utility, given by:

$$W = u(c_1) + \beta E\{u(c_2)\}. \quad (23)$$

Taking into account the distribution of output (given by (16)), we can rewrite (23) as:

$$W = u(c_1) + \beta[p u(c_2^H) + (1-p)u(c_2^L)]. \quad (24)$$

The consumer's problem consists in choosing  $c_1$ ,  $c_2^H$ ,  $c_2^L$  to maximize (24) subject to (21) and (22). We set up the Lagrangian to obtain:

$$\begin{aligned} \mathcal{L} = & u(c_1) + \beta[p u(c_2^H) + (1-p)u(c_2^L)] \\ & + \lambda^H \left( y_1 + \frac{1}{1+r}y_2^H - c_1 - \frac{1}{1+r}c_2^H \right) \\ & + \lambda^L \left( y_1 + \frac{1}{1+r}y_2^L - c_1 - \frac{1}{1+r}c_2^L \right). \end{aligned} \quad (25)$$

The first-order conditions with respect to  $c_1$ ,  $c_2^H$ , and  $c_2^L$  are given by, respectively:

$$\begin{aligned}
u'(c_1) &= \lambda^H + \lambda^L, \\
\beta p u'(c_2^H) &= \frac{\lambda^H}{1+r}, \\
\beta(1-p)u'(c_2^L) &= \frac{\lambda^L}{1+r}.
\end{aligned}$$

Combining these three equations (and imposing the condition that  $\beta(1+r) = 1$ ), we get:

$$u'(c_1) = pu'(c_2^H) + (1-p)u'(c_2^L), \quad (26)$$

which can be rewritten as:

$$u'(c_1) = E\{u'(c_2)\}. \quad (27)$$

This is the stochastic Euler equation (i.e., the counterpart of equation (8)).

## 3.2 Equilibrium

The solution of the model will critically depend on the sign of  $u'''(c)$ . Two pieces of information will be important in the derivations below.

First, notice that  $u'(c)$  is linear, strictly convex, or strictly concave depending on whether  $u'''(c) = 0$ ,  $u'''(c) > 0$ , or  $u'''(c) < 0$ , respectively. It follows that:

$$pu'(c_2^H) + (1-p)u'(c_2^L) = u'[pc_2^H + (1-p)c_2^L], \quad \text{if } u'''(c) = 0, \quad (28)$$

$$pu'(c_2^H) + (1-p)u'(c_2^L) > u'[pc_2^H + (1-p)c_2^L], \quad \text{if } u'''(c) > 0, \quad (29)$$

$$pu'(c_2^H) + (1-p)u'(c_2^L) < u'[pc_2^H + (1-p)c_2^L], \quad \text{if } u'''(c) < 0. \quad (30)$$

Second, notice that by multiplying (21) by  $p$  and (22) by  $1-p$ , we can derive an intertemporal constraint in expected values:<sup>4</sup>

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<sup>4</sup>A general point is in order. Since the intertemporal constraint must hold for *each* state of nature, it will always hold in expected value. The reverse, however, is *not* true. It would therefore be incorrect to solve a model with uncertainty using an intertemporal constraint that holds only in expected value.

$$c_1 + \frac{E\{c_2\}}{1+r} = y_1 + \frac{E\{y_2\}}{1+r}. \quad (31)$$

We now consider each case individually.

**Case 1 (certainty equivalence):**  $u'''(c) = 0$  This case corresponds to quadratic preferences. Using (28), we can rewrite the Euler equation (26) as

$$u'(c_1) = u'[pc_2^H + (1-p)c_2^L],$$

or, equivalently,

$$u'(c_1) = u'(E\{c_2\}).$$

Since  $u'(c)$  is a strictly decreasing function (recall that  $u''(c) < 0$ ), it follows that

$$c_1 = E\{c_2\}. \quad (32)$$

In an expected value sense, therefore, the economy smooths consumption over time.<sup>5</sup> As will become clear below, however,  $c_1$  will differ from actual period-2 consumption and hence the economy is not able to smooth actual consumption over time, which will negatively affect welfare.

To compute a reduced form for consumption, combine the Euler equation for the quadratic case (equation (32)) with the intertemporal constraint in expected value (equation (31)) to obtain:

$$c_1 = \frac{1+r}{2+r} \left[ y_1 + \frac{E\{y_2\}}{1+r} \right]. \quad (33)$$

With quadratic preferences, certainty equivalence holds in the sense that consumption in the first period is the same as it would have been if consumers had been endowed in the second period with the average value of  $y_2$ . This implies that, with quadratic preferences, consumption in period 1 does not depend on the variance of output in period 2.

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<sup>5</sup>This is Robert Hall's (1978) celebrated result that, under quadratic utility and  $\beta(1+r) = 1$ , consumption follows a random walk. In other words, the best predictor of tomorrow's consumption is today's consumption since, under rational expectations, news about permanent income are unforecastable.

To derive the current account balance in period 1 (which, given the assumption of zero initial net foreign assets is the same as the trade balance), combine (18) and (33) to obtain:

$$b_1 = \frac{1}{2+r} [y_1 - E\{y_2\}] < 0.$$

As expected, the economy runs a current account deficit. In fact, the current account deficit is the same as it would have been if second period's output were  $E\{y_2\}$ . Substituting this expression for the current account into (19) and (20), we obtain reduced-form expressions for  $c_2^H$  and  $c_2^L$ :

$$c_2^H = y_2^H + \frac{1+r}{2+r} [y_1 - E\{y_2\}] < y_2^H, \quad (34)$$

$$c_2^L = y_2^L + \frac{1+r}{2+r} [y_1 - E\{y_2\}] < y_2^L. \quad (35)$$

Two observations are worth making. First, consumption in period 2 will always be lower than actual output because of the need to repay the debt incurred in period 1. Second – and as we should have expected – consumption in period 2 depends on the actual realization of output, which implies that  $c_2^H > c_2^L$ . Incomplete markets thus introduce a positive correlation between period 2 consumption and output. In fact, it follows from (34) and (35) that

$$E\{c_2\} = E\{y_2\} + \frac{1+r}{2+r} [y_1 - E\{y_2\}], \quad (36)$$

$$Var\{c_2\} = Var\{y_2\}. \quad (37)$$

By definition, the correlation between  $c_2$  and  $y_2$  is given by

$$Corr(c_2, y_2) \equiv \frac{E\{(c_2 - E\{c_2\})(y_2 - E\{y_2\})\}}{\sqrt{Var\{c_2\}}\sqrt{Var\{y_2\}}}.$$

Using (34), (35), and (36), it follows that

$$E\{(c_2 - E\{c_2\})(y_2 - E\{y_2\})\} = E\{(y_2 - E\{y_2\})^2\}. \quad (38)$$

Hence, given (37) and (38), we find that

$$Corr(c_2, y_2) = 1.$$

There is thus a perfect correlation between consumption and output in the second period.

Finally, notice that the fact that the economy is not able to smooth actual consumption over time has negative welfare consequences (compared to the certainty case). To show this, notice that in the certainty case, period-1 consumption would be given by  $c_1$  (i.e., the same as in the quadratic case) while period-2 consumption would be given by  $E\{c_2\}$ . Hence, we would like to show that

$$\underbrace{u(c_1) + \beta u(E\{c_2\})}_{\text{welfare under certainty}} > \underbrace{u(c_1) + \beta[pu(c_2^H) + (1-p)u(c_2^L)]}_{\text{welfare under uncertainty}},$$

which, given that first-period utility is the same, simplifies to

$$u(E\{c_2\}) > pu(c_2^H) + (1-p)u(c_2^L).$$

This inequality holds in light of the strict concavity of  $u(\cdot)$ . (In fact, suitably relabeled in an obvious way, Figure 1 would apply to this case as well with point A denoting period-2 utility under certainty and point B denoting period-2 utility under uncertainty.)

Furthermore, it is also easy to show that the more variable is output, the higher are the welfare costs. To see this, consider a mean-preserving spread of output; that is, suppose that the possible realizations of output are  $\tilde{y}_2^H (> y_2^H)$  and  $\tilde{y}_2^L (< y_2^L)$  but the expected value is the same. By analogy with (34) and (35), we know that in this case second-period consumption would be given by

$$\tilde{c}_2^H = \tilde{y}_2^H + \frac{1+r}{2+r} [y_1 - E\{y_2\}], \quad (39)$$

$$\tilde{c}_2^L = \tilde{y}_2^L + \frac{1+r}{2+r} [y_1 - E\{y_2\}]. \quad (40)$$

Then, by strict concavity of  $u(\cdot)$ ,

$$pu(\tilde{c}_2^H) + (1-p)u(\tilde{c}_2^L) < pu(c_2^H) + (1-p)u(c_2^L).$$

In terms of Figure 1, period-2 utility for the mean-preserving spread would correspond to point C. Hence, welfare is lower than under the previous, less-spread, distribution (point B).

**Case 2 (precautionary savings):**  $u'''(c) > 0$ . Using (29), we can rewrite the Euler equation (26) as

$$u'(c_1) > u'[pc_2^H + (1-p)c_2^L].$$

It follows that

$$c_1 < E\{c_2\}. \quad (41)$$

Unlike the previous case, consumers do *not* smooth consumption in an expected value sense. Using (31) and (41), it follows that

$$c_1 < \frac{1+r}{2+r} \left[ y_1 + \frac{E\{y_2\}}{1+r} \right].$$

Consumption is thus lower than it would be in the certainty (or certainty equivalence) case. In other words, the economy engages in *precautionary savings*. The uncertainty regarding second-period output induces consumers to be more prudent and dissave less than they would otherwise. Hence, the current account deficit will be smaller than in the previous case (i.e., the quadratic case).<sup>6</sup> As before, the presence of uncertainty prevents consumers from achieving full consumption smoothing and introduces a positive correlation between output and consumption in period 2.

Two important observations are worth making at this point. First, it is important to distinguish between the concepts of “risk aversion” and “prudence” (see Kimball (1990)). Risk aversion refers to the fact that consumers dislike uncertainty, while prudence refers to the idea that consumers are prepared to save in anticipation of an uncertain outcome. The degree of risk aversion is measured by the concavity of  $u(\cdot)$ , whereas the degree of prudence is measured by the convexity of  $u'(c)$  or, equivalently, the concavity of  $-u'(c)$ .

Second, standard preferences – such as constant relative (or absolute) risk aversion – are characterized by  $u'''(\cdot) > 0$  and will therefore exhibit precautionary saving. Exercise 1 at the end of this chapter examines the case in which preferences exhibit constant absolute risk aversion and period-2

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<sup>6</sup>In fact, it may even be possible that the introduction of uncertainty induces consumers to *save* in the first period (i.e., run a current account surplus) even if  $y_1 < E\{y_2\}$ . You should check that this is indeed a possibility by constructing numerical examples using, say, a CRRA function.

output follows a normal distribution. In this case, we can compute a reduced-form for consumption in period 1. As one might expect, period 1 consumption is lower the higher is the variance of period-2 output. Greater uncertainty thus makes consumers more prudent and leads to more precautionary savings. Intuitively, recall the stochastic Euler equation – given by (27) – and notice that  $u'''(c) > 0$  implies that the marginal utility of consumption is a *convex* function. Hence, a mean-preserving spread – which, by construction, leaves expected consumption in period 2 unchanged – increases the corresponding expected marginal utility and induces consumers to decrease consumption in period 1 (i.e., induces consumers to choose a steeper consumption path in an expected value sense). Finally, welfare is strictly decreasing in the variance of output.<sup>7</sup> Exercise 4 asks you to show numerically that the same results follow for the case of constant relative risk aversion.

**Case 3:**  $u'''(c) < 0$  Using (30), we can rewrite the Euler equation (26) as

$$u'(c_1) < u'[pc_2^H + (1-p)c_2^L].$$

It follows that

$$c_1 > E\{c_2\}. \tag{42}$$

In this case, consumers are “imprudent” in the sense that they choose to dissave more than they would otherwise and will in fact consume more in period 1 than in the certainty case. Therefore, the current account deficit in period 1 will be higher than in the certainty case. It is still the case that consumption is not smooth over time and that this is costly from a welfare point of view.

While utility functions that exhibit a negative third derivative are much less common in economic theory, an example would be the following:

$$u(c) = ac - bc^3, \quad c < \sqrt{\frac{a}{3b}}.$$

As can be easily checked,  $u'(c) > 0$ ,  $u''(c) < 0$ , and  $u'''(c) < 0$ . This consumer is thus risk averse but *imprudent*, which illustrates the fact that risk aversion does not necessarily imply prudence. Exercise 4 at the end of the chapter

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<sup>7</sup>See Jacobs, Pallage, and Robe (2005) for an empirical estimation of the welfare costs of incomplete markets using state-level data for the United States.

asks you to compute first-period consumption and welfare as a function of a mean-preserving spread in the distribution of second-period output. As expected,  $c_1$  increases due to the fact that the consumer is imprudent but welfare decreases since the consumer is still risk averse. Intuitively, recall once again the stochastic Euler equation – given by (27) – and notice that the marginal utility of consumption is now a concave function. Hence, a mean-preserving spread will *reduce* the expected marginal utility of period-2 consumption and thus induce consumers to increase  $c^1$  (i.e., to choose a less steep consumption path).

### 3.3 Complete capital markets

Suppose now that there are complete asset markets in the sense that households can buy contingent claims in international capital markets. In other words, households may buy a claim that promises to pay one unit of output in the good state of nature for the price (as of period 1)  $q^H/(1+r)$  and a claim that promises to pay a unit of output in the bad state of nature for the price  $q^L/(1+r)$ . In addition, there exists (as before) a risk free asset that promises to pay one unit of output regardless of the state of nature, which can be acquired at the price  $1/(1+r)$ .<sup>8</sup>

#### 3.3.1 Budget constraints

Assume that initial net foreign assets are zero. Denote by  $b_1^j$ ,  $j = H, L$ , the number of claims purchased in period 1 that promise to pay one unit of output in the second period in state of nature  $j$ . The flow budget constraint for period 1 is thus:

$$\frac{q^H}{1+r}b_1^H + \frac{q^L}{1+r}b_1^L = y_1 - c_1. \quad (43)$$

As in the incomplete markets case, the second-period flow budget constraint will depend on the realization of output. We continue to use  $c_2^H$  and  $c_2^L$  to denote second-period consumption in the high-output state of nature and low-output state of nature, respectively. Then:

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<sup>8</sup>Notice, however, that as far as the consumer is concerned, the risk free asset is redundant because it can be replicated with contingent claims (see below). For simplicity, therefore, the consumer's problem below assumes that consumers only purchase contingent claims.

$$0 = b_1^H + y_2^H - c_2^H, \quad (44)$$

$$0 = b_1^L + y_2^L - c_2^L. \quad (45)$$

Substituting (44) and (45) into (43), we obtain:

$$y_1 + \underbrace{\frac{q^H y_2^H + q^L y_2^L}{1+r}}_{\text{value of claims that can be sold}} = c_1 + \underbrace{\frac{q^H c_2^H + q^L c_2^L}{1+r}}_{\text{value of claims that can be bought}}. \quad (46)$$

As indicated, the second term on the LHS captures the value of all the claims on output that can be sold, whereas the second term on the RHS denotes the value of all claims on consumption that need to be bought.

### 3.3.2 Utility maximization

Consumers choose  $c_1$ ,  $c_2^H$ ,  $c_2^L$  to maximize (24) subject to (46). In terms of the Lagrangian:

$$\begin{aligned} \mathcal{L} = & u(c_1) + \beta[pu(c_2^H) + (1-p)u(c_2^L)] \\ & + \lambda \left( y_1 + \frac{q^H y_2^H + q^L y_2^L}{1+r} - c_1 - \frac{q^H c_2^H + q^L c_2^L}{1+r} \right). \end{aligned}$$

The first-order conditions with respect to  $c_1$ ,  $c_2^H$ , and  $c_2^L$  are given by, respectively,

$$u'(c_1) = \lambda, \quad (47)$$

$$\beta pu'(c_2^H) = \lambda \frac{q^H}{1+r}, \quad (48)$$

$$\beta(1-p)u'(c_2^L) = \lambda \frac{q^L}{1+r}. \quad (49)$$

Combining (48) and (49), we obtain:

$$\frac{pu'(c_2^H)}{(1-p)u'(c_2^L)} = \frac{q^H}{q^L}, \quad (50)$$

which says that the consumer equates the marginal rate of substitution across states of nature to the relative price of the corresponding contingent claims. If prices are actuarially fair, it will be the case that <sup>9</sup>

$$\frac{q^H}{q^L} = \frac{p}{1-p}. \quad (51)$$

Imposing (51) into (50), it follows that  $c_2^H = c_2^L$ . Hence, under actuarially fair prices, the consumer equates period 2 consumption across states of nature.

To show that consumption will be smoothed across time, we need to derive an arbitrage condition between a riskless bond and the price of contingent claims. Notice that a riskless bond that pays one unit of output in period 2 can be bought in period 1 by the price  $1/(1+r)$ . The same outcome can be achieved by buying one unit of  $b^H$  at the price  $q^H/(1+r)$  and one unit of  $b^L$  at the price  $q^L/(1+r)$ . Since these are just two different ways of receiving one unit of output in period 2, they should have the same period-1 price:

$$\frac{1}{1+r} = \frac{q^H}{1+r} + \frac{q^L}{1+r},$$

which implies that  $q^H + q^L = 1$ .<sup>10</sup> Combining the latter with (51) and the first-order conditions (47), (48), and (49) implies that (recalling that  $\beta(1+r) = 1$ )

$$c_1 = c_2^H = c_2^L.$$

We conclude that, *with complete markets, consumption is fully smoothed* even when the output path is uncertain. Hence – and in contrast to the incomplete markets case – the correlation between period 2 output and consumption will be zero. Complete markets thus allow the economy to fully smooth consumption regardless of the path of output as in Chapter 1.<sup>11</sup>

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<sup>9</sup>Actuarially fair prices means that the market is offering a fair gamble (i.e., a gamble with zero expected value at the prevailing market prices). See Hirshleifer and Riley (1992) for a detailed analysis.

<sup>10</sup>This arbitrage condition could be explicitly derived from the consumer's problem by allowing him/her to also purchase a riskless bond in period 1.

<sup>11</sup>As Exercise 2 analyzes, in a two-country model, the assumption of complete markets would imply perfect correlation between consumption growth rates in both countries. In particular, this correlation should be higher than the correlation between consumption and domestic output. This prediction is at the core of most empirical tests of market completeness (see Box 1).

## 4 No precommitment

The basic model of Chapter 1 assumes that if a country has borrowed during bad times, it will repay its debt in good times. In other words, the model assumes that the country can precommit itself to repaying its debt. This is quite a strong assumption since casual observation suggests that sovereign countries often default on their external debt. It follows that, in practice, precommitment mechanisms are, at best, tenuous and countries tend to default if they find it convenient to do so.

### 4.1 Implications of no precommitment

The best way of understanding the implications of no precommitment is to first consider the two-period model discussed above under perfect capital mobility and commitment. In this case, the representative consumer chooses  $c_1$  and  $c_2$  to maximize (4) subject to the following intertemporal constraint:

$$y_1 + \frac{1}{1+r}y_2 = c_1 + \frac{1}{1+r}c_2. \quad (52)$$

It immediately follows from the first-order conditions that  $c_1 = c_2$ . Let us denote this constant level of consumption by  $\bar{c}$  (i.e.,  $c_1 = c_2 = \bar{c}$ ). From (52), this implies that

$$\bar{c} = \frac{1+r}{2+r} \left( y_1 + \frac{1}{1+r}y_2 \right). \quad (53)$$

Suppose, again, that  $y_1 < y_2$ . How much does this economy borrow in period 1? Substituting (53) into the flow constraint for period 1 (given by  $b_1 = y_1 - \bar{c}$ ) and solving for  $b_1$ , we obtain:

$$b_1 = \frac{y_1 - y_2}{2+r} < 0. \quad (54)$$

Let  $d_1$  denote net debt (i.e.,  $d_1 \equiv -b_1$ ). Then,

$$d_1 = - \left( \frac{y_1 - y_2}{2+r} \right) > 0$$

Under commitment, the economy pays  $(1+r)d_1$  in period 2. Consumption in period 2 is thus  $y_2 - (1+r)d_1 < y_2$ . In the absence of precommitment,

however, the economy would be clearly better off by not repaying its debt and instead consuming the entire second-period endowment. But, of course, rational creditors would anticipate this and not lend to this economy in period 1 to begin with. Hence, in the absence of commitment, this economy would not be able to borrow at all in period 1. The economy would thus be in a state of financial autarky and forced to consume its endowment in every period (as in Section 2 above).

## 4.2 Cost of default

The example just examined assumes that there are no costs associated with defaulting. In practice, however, there will be some costs stemming from possible sanctions that creditors may impose. For simplicity, suppose that there is some exogenous cost of defaulting in the second period. Formally, this cost is a fraction  $\phi \in [0, 1)$  of  $y_2$ .<sup>12</sup> We assume that the country will repay its debt only if it is in its own benefit to do so. Formally, the country will repay its debt in period 2 if and only if consumption under no default (LHS in the expression below) is higher than consumption under default (RHS):

$$y_2 - (1 + r)d_1 \geq (1 - \phi)y_2.$$

Simplifying this no-default condition, we obtain:

$$d_1 \leq \frac{\phi y_2}{1 + r}, \tag{55}$$

which says that if debt is higher than  $\phi y_2 / (1 + r)$ , then the consumer will choose to default. Since creditors, however, internalize the debtor country's incentives to default, they will not lend over and above  $\phi y_2 / (1 + r)$ . Hence, this no-default condition effectively imposes an upper bound on the feasible amount of borrowing in the first period. In other words – and as illustrated in Figure 2 – the country faces a completely elastic supply of funds from abroad at a constant interest rate  $r$  until  $d_1 = \phi y_2 / (1 + r)$ , at which point the supply of funds becomes vertical. If  $\phi = 0$ , then this upper bound is zero (no lending from the rest of the world). At the other extreme, for high

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<sup>12</sup>This fraction  $\phi$  is intended to capture various costs associated with default, such as trade disruptions, seizure of assets, and exclusion from future borrowing (in a model with more than two periods). See Eaton and Gersovitz (1981), Sachs (1984), and Sachs and Cohen (1982) for a detailed analysis.

enough values of  $\phi$ , this constraint will not bind and perfect smoothing will obtain. In this sense, *high costs of default are beneficial as a precommitment device*.

Formally, the maximization problem gets simplified by the fact that we know that in equilibrium we will never observe default (which implies that condition (55) will hold) and that, as a result, the intertemporal constraint (52) will also hold. Hence, the consumer's maximization problem consists in choosing  $c_1$  and  $c_2$  to maximize (4) subject to the intertemporal constraint (52) and the no-default condition (55). Since  $d_1$  would show up in the Lagrangian through the no-default condition, it proves convenient to re-state the maximization problem in terms of  $d_1$ . To this effect, use the flow constraints

$$c_1 = y_1 + d_1, \tag{56}$$

$$c_2 = y_2 - (1+r)d_1, \tag{57}$$

to write the Lagrangean as follows:

$$\mathcal{L} = u(y_1 - b_1) + \beta u[y_2 - (1+r)d_1] + \chi \left( \frac{\phi y_2}{1+r} - d_1 \right).$$

The Kuhn-Tucker conditions for  $c_1$  and  $\chi$  are given by:

$$\begin{aligned} u'(c_1) &= u'(c_2) + \chi, \\ \frac{\phi y_2}{1+r} - d_1 &\geq 0, \quad \chi \left( \frac{\phi y_2}{1+r} - d_1 \right) = 0. \end{aligned} \tag{58}$$

There are two possible solutions for this problem depending on what would be the level of borrowing for the unconstrained case (i.e., the case in which condition (55) is not present).

#### 4.2.1 Non-binding constraint

Suppose that  $y_1$  and  $y_2$  are such that the unconstrained problem calls for a level of first-period borrowing that satisfies condition (55)). Then, by construction, constraint (55) does not bind and  $\chi = 0$ . Hence, from (58),

$$u'(c_1) = u'(c_2),$$

and full consumption smoothing would obtain.

### 4.2.2 Binding constraint

Suppose that  $y_1$  and  $y_2$  are such that the unconstrained problem calls for a level of first-period borrowing that does not satisfy condition (55). In other words, the fluctuations in endowment are such that the economy would like to borrow in the first period more than what international creditors are willing to lend. Then, condition (55) is binding, which implies that  $\chi > 0$ . Hence, from (58),

$$u'(c_1) > u'(c_2),$$

which implies that  $c_1 < c_2$ . In fact, from (56) and (57), consumption will be given by

$$c_1 = y_1 + \bar{d}_1, \tag{59}$$

$$c_2 = y_2 - (1+r)\bar{d}_1, \tag{60}$$

where  $\bar{d}_1 (= \phi y_2 / (1+r))$  is the maximum amount of borrowing. Hence, the consumer cannot smooth consumption over time and is forced to consume less in bad times than in good times.<sup>13</sup> This simple example thus illustrates the idea that *sovereign risk may severely limit the ability of a small open economy to smooth out consumption over time by imposing constraints on its ability to borrow during bad times*. Furthermore, the more output fluctuates (i.e., the larger the difference between  $y_1$  and  $y_2$ ), the more likely it is that the no-default condition will bind and hence that sovereign risk will prevent the economy from achieving full consumption smoothing. Since output fluctuations are much more pronounced in developing than in industrial countries, sovereign risk is particularly relevant for developing countries.<sup>14</sup>

### 4.3 Default risk under uncertainty

The certainty model just analyzed delivers quite insightful, but somewhat extreme, implications. First, the supply of funds is kinked (recall Figure 2)

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<sup>13</sup>Notice that, for the same reasons as in section 2 above, the domestic real interest rate will be higher than the international one. In fact, the domestic real interest rate will be given by an equation such as (12) with  $c_1$  and  $c_2$  given by (59) and (60).

<sup>14</sup>According to figures reported in Talvi and Vegh (2005), output in developing countries is more than twice as volatile as in OECD countries.

but nowhere upward sloping. Second, while the threat of default generates a debt ceiling, there will never be default in equilibrium. We will now see how introducing uncertainty into the model (i.e., an uncertain level of period 2 output) – along the lines of Sachs and Cohen (1983) – generates both an upward sloping supply of funds and default in equilibrium.

### 4.3.1 Supply of international credit

Suppose that period 2 output ( $y_2$ ) is a random variable drawn from a uniform distribution with support  $[0, y_2^H]$ . The probability density function is thus given by:

$$f(y_2) = \begin{cases} \frac{1}{y_2^H}, & 0 < y_2 < y_2^H, \\ 0, & \text{otherwise.} \end{cases} \quad (61)$$

The corresponding cumulative distribution function, given by  $F(\alpha) = \int_{-\infty}^{\alpha} f(y_2) dy_2$ , is thus:

$$F(\alpha) = \begin{cases} 0, & \alpha \leq 0, \\ \frac{\alpha}{y_2^H}, & 0 < \alpha < y_2^H, \\ 1, & \alpha \geq y_2^H. \end{cases} \quad (62)$$

Denote by  $r^s$  the real interest rate charged by creditors. As before, we assume that in the second period the country will repay its debt only if the cost of repaying (given by  $d_1(1+r^s)$ ) is smaller than the cost of not repaying (given by  $\phi y_2$ ). Hence, the default decision is given by:

$$\begin{aligned} \text{default if} & \quad y_2 \leq y_2^*, \\ \text{no default if} & \quad y_2 > y_2^*, \end{aligned}$$

where

$$y_2^* \equiv \frac{d_1(1+r^s)}{\phi} \quad (63)$$

is the default threshold. The country will thus choose to default for low values of period 2 output (i.e., it chooses to default in bad times) and choose to repay for high values of period 2 output (i.e., it chooses not to default in high times).

What is the probability that a country will default? Denote the probability of default by  $\pi$ . Then, using (62) and (63),

$$\pi = \Pr(y_2 < y_2^*) = F\left[\frac{d_1(1+r^s)}{\phi}\right]. \quad (64)$$

For a given value of  $r^s$ , the probability of default depends on the level of debt. For “high” levels of debt (i.e.,  $d_1(1+r^s)/\phi \geq y_2^H$ ) the economy will *always* default because the benefits of doing so outweigh the costs for any possible realization of output.<sup>15</sup> On the other hand, if the economy is a creditor (i.e.,  $d_1 \leq 0$ ), the economy will obviously never default. For intermediate levels of debt (i.e.,  $0 < d_1(1+r^s)/\phi \leq y_2^H$ ) the probability of default is an increasing function of the debt level because the higher the level of debt, the more likely that the realization of output will be such that the economy will find it in its advantage to default.

We will now derive an explicit functional form for the supply of funds; that is, the interest rate at which creditors will be willing to lend a given amount of funds. To this effect, suppose that international lenders are risk neutral and that their opportunity cost of funds is  $r$ . Then,

$$1+r = (1-\pi)(1+r^s). \quad (65)$$

Hence, from (64) and (65),

$$r^s = \frac{1+r}{1-F(y_2^*)} - 1. \quad (66)$$

Taking into account (62) and (63), we can rewrite (66) as

$$r^s = \frac{1+r}{1 - \frac{(1+r^s)d_1}{\phi y_2^H}} - 1. \quad (67)$$

This equation implicitly defines  $r^s$  as a function of  $d_1$ . In fact, we can solve (67) explicitly to obtain:<sup>16</sup>

$$r^s = \begin{cases} r, & d_1 \leq 0, \\ \frac{2(1+r)d_1^{\max}}{d_1} \left(1 - \sqrt{1 - \frac{d_1}{d_1^{\max}}}\right) - 1, & 0 < d_1 \leq d_1^{\max}, \end{cases} \quad (68)$$

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<sup>15</sup>Of course, in *equilibrium*, we will never observe levels of debt such that  $d_1(1+r^s)/\phi > y_2^H$ . Since the country would default with probability one, lenders will never lend the country this amount of funds to begin with.

<sup>16</sup>Equation (67) leads to a quadratic equation in  $1+r^s$ . There are thus two values of  $r^s$  corresponding to a single value of  $d_1$ . The economically sensible solution is the lowest value of  $r^s$ .

where

$$d_1^{\max} \equiv \frac{\phi y_2^H}{4(1+r)}$$

is the maximum level of debt that can be sustained by international markets.<sup>17</sup> Beyond this level of debt, international lenders cannot be compensated for the risk that they would be taking. Furthermore,  $r^s$  is an increasing function of  $d_1$ :

$$\frac{dr^s}{dd_1} = \frac{(1+r)d_1^{\max}}{d_1^2 \sqrt{1 - \frac{d_1}{d_1^{\max}}}} \left( 2 - \frac{d_1}{d_1^{\max}} - 2\sqrt{1 - \frac{d_1}{d_1^{\max}}} \right) > 0, \quad 0 < d_1 \leq d_1^{\max}.$$

What is the maximum interest rate charged by creditors? To find out, evaluate (68) at  $d_1 = d_1^{\max}$  to obtain:

$$r^s|_{d_1=d_1^{\max}} = 1 + 2r. \quad (69)$$

While the model is certainly not geared to offer quantitative predictions, it is still interesting to point out that it generates a substantial premium over the risk-free international real interest rate. Indeed, equation (69) says that if, say,  $r$  is 3 percent, the maximum interest rate will be 106 percent!

Figure 3 illustrates the upward sloping supply of funds just derived. Intuitively, for a given  $\phi$ , the real interest rate faced by this small open economy,  $r^s$ , is an increasing function of the level of indebtedness ( $d_1$ ), reflecting the fact that the higher the level of debt the more likely that the economy will default. Figure 3 thus offers a simple and insightful rationalization of an upward sloping supply of funds and suggests that developing countries may pay a substantial premium for their international loans due to the implicit default risk.

For future reference, let us compute how the probability of default varies with the level of debt. Using (62) and (64), it follows that

$$\pi = \frac{(1+r^s)d_1}{\phi y_2^H}. \quad (70)$$

Hence, from (70), taking into account (65), it follows that:

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<sup>17</sup>Naturally, equation (67) holds for  $d_1 \geq 0$ . For  $d_1 < 0$ , it follows from (62) and (63) that  $F(y_2^*) = 0$  and, hence, from (66),  $r^s = r$ .

$$\frac{d\pi}{dd_1} = \frac{1+r}{\phi y_2^H (1-2\pi)}. \quad (71)$$

As will become clear below, at an optimum  $\frac{d\pi}{dd_1}$  will always be positive.

### 4.3.2 Demand for international credit

The demand for credit comes from the consumer's side. To simplify the analysis, we will assume that preferences are linear and given by:

$$W = c_1 + \frac{1}{1+\delta} E\{c_2\}, \quad (72)$$

where  $\delta (> 0)$  is the discount rate. To introduce a motive for borrowing in this linear case, we will assume that  $\delta > r$ . Since future utility is discounted at a higher rate than the risk-free world real interest rate, the economy would prefer to consume all of its endowment in the first period rather than in the second.

Denote by  $c_2^D$  and  $c_2^{ND}$  the levels of consumption in period 2 in the event of default ( $D$ ) and non-default ( $ND$ ), respectively. It follows that (assuming net initial assets are zero)

$$c_1 = y_1 + d_1, \quad (73)$$

$$c_2^D = (1-\phi)y_2, \quad (74)$$

$$c_2^{ND} = y_2 - (1+r^s)d_1. \quad (75)$$

We will now express lifetime utility – given by (72) – as a function of  $d_1$ , which we will view as the planner's choice variable. To this effect, notice that, by the law of iterated expectations, we can write

$$E\{c_2\} = E_X\{E\{c_2|X\}\},$$

where  $X$  is the random variable that takes the value  $D$  with probability  $\pi$  and  $ND$  with probability  $1-\pi$ . Hence,

$$E\{c_2\} = \pi E\{c_2^D\} + (1-\pi)E\{c_2^{ND}\}. \quad (76)$$

Substituting equation (76) into (72) (recall that  $\beta \equiv 1/(1+\delta)$ ):

$$W = c_1 + \beta\pi E\{c_2^D\} + \beta(1 - \pi)E\{c_2^{ND}\}. \quad (77)$$

Now take conditional expectations of (74) and (75) and substitute the resulting expressions and (73) into (76) – using (65) – to obtain:

$$W = y_1 + \frac{1}{1 + \delta}E\{y_2\} + \frac{\delta - r}{1 + \delta}d_1 - \beta\pi\phi E\{y_2/D\}. \quad (78)$$

Given that preferences are linear, the planner cares only about expected values, as captured by the first two terms on the RHS of (78).<sup>18</sup> The third term on the RHS of (78) captures the benefits of bringing forward consumption given that second period utility is discounted at a higher rate than the risk-free real interest rate. The last term captures the loss of resources that the economy suffers if it defaults. Notice that the expected output loss in case of default is  $\phi E\{y_2/D\}$ . Since the economy defaults with probability  $\pi$ , the expected resource loss in terms of period 1 utility is  $\beta\pi\phi E\{y_2/D\}$ .

To proceed further, notice that, conditional on the economy having defaulted,  $y_2$  varies uniformly between 0 and  $y_2^*$ . By the same token, conditional on the economy not having defaulted, output varies uniformly between  $y_2^*$  and  $y_2^H$ . Hence:

$$E\{y_2/D\} = \frac{y_2^*}{2} = \frac{\pi y_2^H}{2}, \quad (79)$$

$$E\{y_2/ND\} = \frac{y_2^* + y_2^H}{2} = \frac{(1 + \pi)y_2^H}{2}, \quad (80)$$

where the right-most expressions follow from (63) and (70).

Substituting (79) into (78), we obtain:

$$W = y_1 + \frac{1}{1 + \delta}E\{y_2\} + \frac{\delta - r}{1 + \delta}d_1 - \frac{\beta\phi\pi^2 y_2^H}{2}. \quad (81)$$

The planner's problem consists in choosing  $d_1$  to maximize (81). The first-order condition is given by:

$$\delta - r = \phi\pi y_2^H \frac{d\pi}{dd_1}. \quad (82)$$

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<sup>18</sup>Notice that in the standard model with  $\delta = r$  and no sovereign risk ( $\pi = 0$ ), then  $W = y_1 + \beta E\{y_2\}$ .

The LHS of this expression captures the marginal benefit of borrowing to increase period's 1 consumption (net of the risk-free real interest rate). Since the rate of time preference,  $\delta$ , is greater than the risk-free real interest rate,  $r$ , the marginal benefit of borrowing is constant and positive. The RHS captures the marginal cost of borrowing. Recall, from the last term on the RHS of (78), that the cost of borrowing is given by the expected output loss in case of default. Borrowing an additional unit increases the marginal cost of borrowing on two accounts. First, for a given loss  $\phi E\{y_2/D\}$ , an additional unit of borrowing increases the probability of default by  $d\pi/dd_1$  and hence raises the marginal cost of borrowing by  $\phi E\{y_2/D\}(d\pi/dd_1)$ . Second, for a given probability of default, an additional unit of borrowing increases the expected loss in case of default by  $\phi [d(E\{y_2/D\})/d\pi] (d\pi/dd_1)$ . Since default occurs with probability  $\pi$ , the marginal cost of borrowing increases by  $\pi\phi [d(E\{y_2/D\})/d\pi] (d\pi/dd_1)$ . Recalling (79), these two effects add up to the RHS of equation (82).

We can now compute reduced forms for the equilibrium values of  $\pi$ ,  $d_1$ , and  $r^s$ . Using (65), (70), (71), and (82), we obtain:

$$\pi = \frac{\delta - r}{1 + 2\delta - r}, \quad (83)$$

$$d_1 = \frac{\phi y_2^H (1 + \delta)(\delta - r)}{(1 + r)(1 + 2\delta - r)^2}, \quad (84)$$

$$r^s = \frac{1 + r}{1 + \delta}(1 + 2\delta - r) - 1. \quad (85)$$

Several features of the equilibrium solution are worth noting. First, as (83) makes clear, the probability of default is independent of the cost of default,  $\phi$ . The reason is that although, for a given level of borrowing, a higher  $\phi$  makes it more costly to default (and hence decreases the probability of default, as follows from (70)), it also increases the amount that the economy may borrow (thus increasing the probability of default). In equilibrium, these two effects cancel each other out. Second, all  $\pi$ ,  $d_1$ , and  $r^s$  are increasing functions of  $\delta$ . Hence, an economy with a higher  $\delta$  (i.e., a more "impatient" economy) will borrow more, pay a higher real interest rate, and face a higher probability of default.

Even though the probability of default does not depend on  $\phi$ , it can be checked that welfare is an increasing function of  $\phi$ . Substituting (83) and (84) into (81), we obtain:

$$W = y_1 + \frac{1}{1 + \delta} E\{y_2\} + \phi \frac{y_2^H (\delta - r)^2}{2(1 + 2\delta - r)(1 + r)(1 + \delta)}.$$

Welfare is thus a linear function of  $\phi$ . Hence, higher costs of default are welfare improving for this economy as the benefits of more borrowing outweigh the resulting increase in the expected cost of default. It is worth noticing, however, that even for  $\phi = 1$  (which means that all of period 2 endowment would be lost in case of default), this economy cannot replicate the first-best equilibrium (that is, the equilibrium that would obtain under no sovereign risk). To see this, notice that if there were no sovereign risk, this economy would want to consume all of its endowment (in a present-value sense) in period 1. Borrowing would thus be given by

$$d_1|_{\text{no sovereign risk}} = \frac{E\{y_2\}}{1 + r}.$$

With sovereign risk, we see from (84) that the maximum borrowing would obtain for very large values of  $\delta$  and be given by

$$d_1^{\max} = \frac{\phi E\{y_2\}}{2(1 + r)}.$$

Hence, even for  $\phi = 1$ , this economy would be borrowing just half of the amount it would borrow under perfect capital markets. It is thus clear from (81) that welfare under sovereign risk (even for  $\phi = 1$ ) is lower than welfare under no sovereign risk. Sovereign risk thus imposes welfare costs to this economy even under very high costs of default.

Finally, notice that this model generates the possibility of default as an equilibrium outcome. The economy borrows in the first period an amount given by (84) at the interest rate given by (85). In the second period, the economy will default for any realization of  $y_2$  such that the cost of defaulting ( $\phi y_2$ ) is lower than the cost of repaying ( $(1 + r^s)d_1$ ). Hence, bad times (i.e., low realizations of  $y_2$ ) will be associated with more defaults.

### 4.3.3 Upward sloping supply of funds

Assuming some ad-hoc upward sloping supply of funds is common in the literature.<sup>19</sup> Typically, it is just assumed that the real interest rate charged by creditors is an increasing function of the level of debt:

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<sup>19</sup>Bardhan (1967) was a precursor.

$$r^s = r + f(d), \tag{86}$$

where  $f'(d) > 0$ . This should be viewed as an approximation to an endogenously-derived supply of funds, such as the one depicted in Figure 3. A key implicit assumption behind (86) is that the economy is operating on the upward portion of the “true” supply of funds (i.e., the one depicted in Figure 3).

As Problem 3 at the end of this chapter illustrates, an upward sloping supply of funds has important consequences for the optimal choice of borrowing by a small open economy. First, the economy will choose *not* to smooth consumption. Specifically, if  $y_1 < y_2$ , the economy will borrow in the first period but still consume less than in the second period. Given the imperfection in international capital markets, it is in fact socially optimal not to smooth consumption over time since more borrowing increases borrowing costs. Second, a planner would choose to borrow less than the private sector would. In other words, the social marginal cost of borrowing is higher than the private marginal cost. Intuitively, the country is a monopsonist in world capital markets as its borrowing affects the interest rate charged by creditors. The planner fully internalizes this monopsony power and equates, at the margin, the marginal benefit of borrowing with the *marginal* social cost of borrowing. Individual consumers, however, equate the marginal benefit of borrowing with the *average* social cost of borrowing. It would thus be optimal for the government to impose a tax on foreign borrowing that would reduce private borrowing to the socially optimal level. This is an example of a “second-best” policy, wherein an existing distortion (i.e., the upward sloping supply of funds) makes it optimal to introduce a second distortion (i.e., a tax on foreign borrowing).

Problem 3 thus provides the best theoretical case for a so-called “Tobin tax” – so named after Tobin’s (1978) celebrated article advocating throwing some sand in the wheels of excessively efficient international financial markets.<sup>20</sup> Even from a theoretical point of view, however, the argument is weaker than it appears for two reasons. First, our simple model does not take into account well-known microeconomic distortions introduced by taxes on foreign borrowing such as misallocation of resources and rent-seeking activities.<sup>21</sup> Second, even in the simple example examined in Problem 3, the

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<sup>20</sup>See also Harberger (1986).

<sup>21</sup>See De Gregorio, Edwards, and Valdes (2000) for an examination of the practical effectiveness of capital controls.

first-best policy would be to address the initial distortion (i.e., to address the imperfections in capital markets that give rise to the upward-sloping supply of funds) and ensure that the country faces an infinitely elastic supply of funds. Unfortunately, while there have been many proposals involving some sort of international bankruptcy law that would deal with problems of sovereign risk, there has been little progress in addressing the tremendous practical problems of effectively implementing such a proposal.

## 5 Final remarks

This chapter has examined the implications of relaxing the main three assumptions of the basic model presented in Chapter 1: (i) full access to international capital markets, (ii) no uncertainty in the output path, and (iii) the ability to precommit to repaying outstanding debts. We have seen that relaxing each of these assumptions may fundamentally alter the economy's ability to smooth consumption over time. In all three cases – no access to international borrowing, uncertain output path (in the absence of complete markets for contingent claims) and no pre-commitment – the economy is unable to smooth consumption over time, as in the first-best world of Chapter 1. These restrictions have therefore important welfare consequences that will be particularly severe for developing countries in light of the high output variability and often limited access to international financing in bad times.

## 6 Appendices

### 6.1 Closed form solution for consumption in an infinite horizon model

This appendix derives a closed-form solution for consumption in an infinite horizon version of the two-period model in the text. Let preferences be given by

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right]. \quad (87)$$

The flow budget constraint takes the form

$$b_{t+1} = (1+r)b_t + y_t - c_{t+1}. \quad (88)$$

Imposing the transversality condition

$$\lim_{t \rightarrow \infty} \left( \frac{1}{1+r} \right)^t b_t = 0,$$

we can derive the intertemporal constraint

$$(1+r)b_0 + \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t y_t = \sum_{t=s}^{\infty} \left( \frac{1}{1+r} \right)^t c_t. \quad (89)$$

Notice that this lifetime constraint must hold for *every* history of shocks.

To derive the first-order condition, substitute the flow constraint (88) into (87) and differentiate with respect to  $b_{t+1}$  to obtain (assuming that  $\beta(1+r) = 1$ ):

$$u'(c_t) = E[u'(c_{t+1})].$$

Assuming, as in the text, that preferences are quadratic, we obtain:

$$c_t = E\{c_{t+1}\}. \quad (90)$$

This implies that

$$c_0 = E\{c_1\} = E\{c_2\} = \dots \quad (91)$$

Since the intertemporal constraint holds for every history of shocks, it also holds in expectations:

$$(1+r)b_0 + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t E_0\{y_t\} = \sum_{t=s}^{\infty} \left(\frac{1}{1+r}\right)^t E_0\{c_t\}. \quad (92)$$

Using (91), we can rewrite this last expression as

$$c_0 = \frac{r}{1+r} \left[ (1+r)b_0 + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t E_0\{y_t\} \right] \quad (93)$$

As discussed in the text, certainty equivalence holds in the sense that households decide today's consumption as though the future realizations of output were given by its expected value.

To see the permanent income hypothesis in action, so to speak, suppose that output follows a random walk process:

$$y_t = y_{t-1} + \varepsilon_t,$$

where  $\varepsilon_t$  is an i.i.d. shock with mean zero. Then:

$$y_0 = E_0\{y_1\} = E_0\{y_2\} = \dots$$

Using this in (93), we obtain

$$c_0 = rb_0 + y_0.$$

Intuitively, since output is a random walk, any shock to output is permanent. Consumers therefore adjust consumption one-to-one with higher output. Notice that this is exactly the result that we obtained in our perfectly foresight model.

## 6.2 Calibrating our basic model

Given the importance of numerical methods in open economy macroeconomics, this appendix develops a version of our basic model with uncertainty and incomplete markets, calibrates it for Mexico, and computes second moments to compare them to the data.

Two technical issues come up. First, as discussed in Box 3, we need to induce stationarity in the model to be able to have well-defined stochastic distributions. We will do so by assuming that  $\beta(1+r) < 1$ . Second, we need to ensure that the non-Ponzi games condition is not violated. This becomes

an issue when we put the model in the computer because we cannot use the lifetime budget constraint.<sup>22</sup>

### 6.2.1 The model

Agent's preferences are given by

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right], \quad (94)$$

where

$$u(c_t) = \frac{c_t^{1-\theta}}{1-\theta}. \quad (95)$$

The flow budget constraint is given by

$$b_{t+1} = (1+r)b_t + y_t - c_t,$$

where  $\varepsilon_t$  follows a first-order autoregressive process that takes the form

$$y_t = \mu + \rho y_{t-1} + \varepsilon_t, \quad (96)$$

where  $0 < \rho < 1$  and  $\varepsilon_t \sim N(0, \sigma)$ . We will choose  $\mu = 1 - \rho$  so that mean output is one.<sup>23</sup>

Since, from a computational point of view, it would be difficult to incorporate the lifetime budget constraint – as we would do when solving this type of model with pencil and pad – we will impose the so-called “natural debt limit,” which is given by

$$b_t \geq -\frac{y_{\min}}{r}, \quad (97)$$

where  $y_{\min}$  is the worse possible realization of output.<sup>24</sup> As shown by Aiyagari (1993), the natural debt limit can be derived by combining the non-Ponzi

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<sup>22</sup>When we solved our perfect foresight consumption model of Chapter 1 using the King-Plosser-Rebelo linearization method (Appendix 7.6), we are imposing the non-Ponzi games when we specify a steady-state in which  $c = rb + y$ .

<sup>23</sup>To verify this, take expectations of equation (96) and use the fact that, since the output process is stationary,  $E\{y_t\} = E\{y_{t-1}\}$ .

<sup>24</sup>As shown by Tauchen (1986), even though the distribution of output does not have a finite support, one can approximate an AR(1) process with a first-order Markov process.

games condition with a non-negativity constraint on consumption (which may result from preferences that satisfy the condition that  $u'(c)$  tends to infinity as consumption tends to zero). Intuitively, the constraint (97) ensures that households can service the debt without violating a non-negativity constraint on consumption.

For calibration purposes, we often impose a tighter constraint than the natural debt limit in order to match some observed level of debt to GDP. Formally,

$$b_t \geq \phi \geq -\frac{y_{\min}}{r}, \quad (98)$$

where  $\phi$  is the parameter that is chosen to match the data. If  $\phi > -y_{\min}/r$ , we refer to this constraint as an “ad-hoc debt limit” because it becomes an arbitrary debt limit that does not follow from the non-Ponzi games condition.

The recursive representation of the economy’s problem is:

$$V(b, y) = \max_{b'} \left( \frac{c^{1-\theta}}{1-\theta} + \beta EV(b', y') \right) \quad (99)$$

$$\text{subject to} \quad (100)$$

$$c = (1+r)b + y - b', \quad (101)$$

where superscript primes denote next period’s values. Using this recursive representation of the economy’s problem, it is easy to solve the model using the value function iteration method. This method consists of starting with some arbitrary value function (typically zero) and iterating using the Bellman equation (99) until we reach a fixed point (i.e., until we have achieved convergence of the value function).

Compared to the linearization methods for solving models with the computer described in Chapter 1, recursive methods have some advantages. First, recursive methods are general global methods. Global solutions are important if one is interested in evaluating the effects of changing the variance of the exogenous processes driving the dynamics of the model. Second, by the very fact of reducing the problem to a “two-period” formulation, recursive methods are better suited to dealing with uncertainty. In contrast, optimal control methods, which typically rely on linearizing around a deterministic steady-state, will yield less accurate solutions.

We proceed to calibrating the model for the Mexican economy using the parameter values in Table 5, taken from Durdu, Mendoza, and Terrones

(2009). The values of  $\phi$  and  $\beta$  are chosen so as to match the long-run average of foreign assets and the cyclical standard deviation of consumption in the data. The model was run 200 times; first and second moments were computed for each run, and the corresponding averages are presented in Table 6. As a robustness check, we also computed moments for four other values of  $\theta$ . The model's predictions are as follows:

- A positive correlation between consumption and output (ranging from 0.70 to 0.77)
- A positive correlation between the trade balance and output (ranging from 0.92 to 0.97)
- A lower volatility of consumption relative to output (ranging from 0.25 to 0.42)

What do the data say? We can refer to Table 3 in Chapter 1. For our purposes, the relevant columns are the first (correlation of consumption with output), the third (correlation of net exports with output), and fourth (ratio of consumption volatility to output volatility). These correlations have been computed using the cyclical components obtained with the Hodrick-Prescott filter for 13 emerging markets and 13 developed markets. The main stylized facts are:

- The correlation of consumption with output is positive for every single country and is, on average, 0.72 for emerging markets and 0.66 for industrial countries
- The correlation of net exports is negative for 11 out of 13 emerging markets and 10 out of 13 industrial countries, with averages of -0.51 and -0.17, respectively.
- The volatility of consumption relative to output is, on average, 1.45 for emerging markets and 0.94 for industrial countries.

We thus conclude that the model is successful in predicting the positive and large correlation between consumption and output, but is not successful in reproducing the other two stylized facts. In fairness to the model, however, we should remember that it does not include investment. The correlation between net exports and output is therefore capturing the correlation of saving

with output which, as the data indicate, is overwhelmingly positive (see Box 2 in Chapter 1). To have a chance of replicating a negative correlation between the trade balance and output, we need to introduce investment into the picture along the lines of Chapter 1. In terms of replicating the higher volatility of consumption relative to output, one would need to introduce trend shocks a la Aguiar and Gopinath (2004). The presence of trend shocks implies that a good shock today might imply an even higher shock tomorrow. As a consequence, consumption will react more than actual output, and hence consumption volatility will be larger than output volatility.

## Exercises<sup>25</sup>

### 1. Reduced forms for consumption and welfare under CARA utility

Consider the incomplete markets case analyzed in subsection 3.1 with the following two modifications. First, assume that preferences are given by the constant absolute risk aversion function:

$$u(c) = -\frac{1}{\alpha}e^{-\alpha c}. \quad (102)$$

Second, instead of the Bernoulli distribution for period 2's output specified in (16), suppose that

$$y_2 = y_1 + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2).$$

In this context:

- (a) Show that the coefficient of absolute risk aversion is equal to  $\alpha$ . (Recall that the Arrow-Pratt measure of absolute risk aversion is given by  $-u''/u'$ .)
- (b) Following Kimball (1990), show that the coefficient of absolute prudence (defined as  $-u'''/u''$ ) is equal to  $\alpha$ .
- (c) Derive reduced-form solutions for  $c_1$  and  $c_2$ . In particular, how does a higher  $\sigma^2$  affect  $c_1$  and  $c_2$ ? [Hint: Recall that if  $x \sim N(E\{x\}, \sigma_x^2)$ , then  $E\{e^x\} = e^{Ex + \sigma_x^2/2}$ .]
- (d) Compute the correlation coefficient between  $c_2$  and  $y_2$ .
- (e) Show that welfare is a decreasing function of  $\sigma^2$ .

In all cases, discuss the intuition behind the results.

### 2. A two-country world with complete markets

Consider a two country (domestic and foreign) version of our small open economy model with complete markets analyzed in Subsection 3.3. Suppose that preferences are logarithmic and that countries have the

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<sup>25</sup>An answer key is available from the author upon request.

same discount factor. We will use star superscripts to denote the foreign country variables. Since this is a two-country model, the following world output constraints will hold:

$$\begin{aligned} c_1 + c_1^* &= y_1 + y_1^* \equiv y_1^W, \\ c_2^A + c_2^{A*} &= y_2^A + y_2^{A*} \equiv y_2^{AW}, \\ c_2^B + c_2^{B*} &= y_2^B + y_2^{B*} \equiv y_2^{BW}, \end{aligned}$$

where a superscript “W” denotes world quantities. Notice that we have re-labeled states “high” and “low” as “A” and “B,” respectively. (The reason is that if, for instance, there is no aggregate uncertainty, then when domestic output is high, foreign output will be low and viceversa.)

In this context:

- (a) Derive first-order conditions and show that the ratio of consumption across states of nature is the same at home and abroad.
- (b) Derive reduced forms for  $q^A$  and  $q^B$ .
- (c) Derive a reduced-form solution for the world real interest rate.
- (d) Show that consumption as a proportion of world output is constant across time and states of nature in both countries.
- (e) Show that  $\text{corr}(c_2, c_2^*) = 1$ .
- (f) Show that, if there is no world uncertainty (i.e.,  $y_2^{AW} = y_2^{BW}$ ),  $c_2$  is uncorrelated with domestic output.

### 3. An ad-hoc upward sloping supply of funds

Let preferences be given by:

$$W = \log(c_1) + \beta \log(c_2), \tag{103}$$

where  $\beta (\equiv 1/(1 + \delta))$  is the discount factor and  $\delta$  is the discount rate. Assume that  $\beta(1 + r) = 1$ , where  $r$  is the world real interest rate.

The flow constraints are given by:

$$c_1 = d_1, \tag{104}$$

$$c_2 = y_2 - (1 + r^s)d_1, \tag{105}$$

where  $d_1$  is net external debt and  $y_2 > 0$  is second period's output (notice that, for simplicity, we have assumed that output in the first period is zero).

The economy faces an upward sloping supply of funds of the form:

$$r^s = r + f(d_1), \quad f(0) = 0, \quad f'(d_1) > 0, \quad (106)$$

where  $r^s$  is the real interest rate charged to the country.

In this context:

- (a) Solve the planner's problem. Show that the planner will choose not to smooth consumption over time.
- (b) Solve the consumer's problem (i.e., the free market solution). Show that, relative to the planner's solution, the market solution implies that consumption in the first period is too high relative to the second period.
- (c) Consider a linear version of this model (i.e., linear preferences and linear supply of funds):

$$W = c_1 + \frac{c_2}{1 + \delta}, \quad (107)$$

$$f(d_1) = \alpha d_1. \quad (108)$$

Assume  $\delta > r$ . In this context:

- i. Derive a reduced-form solution for the equilibrium values of  $d_1$  and  $r^s$  for both the planner's problem and the market problem.
- ii. Provide a graphical illustration of how the equilibrium values of  $d_1$  and  $r^s$  are determined and interpret the results intuitively. [Hint: Think of the country as a monopsonist in world capital markets and proceed as in the textbook analysis of a monopsony in factor markets. To this end, you may want to review the analysis of a monopsonist in your favorite undergraduate microeconomics textbook.]
- iii. Show that by imposing a borrowing tax rate (which increases linearly with the amount of borrowing), the government can implement the planner's solution.

#### 4. Numerical example of incomplete markets model with CRRA preferences

Consider the incomplete markets model analyzed in Subsection 3.1 with CRRA preferences of the form

$$u(c) = \frac{c^{1-\theta} - 1}{1-\theta},$$

where  $\theta$  is the coefficient of constant relative risk aversion. In this context:

- (a) Show that the coefficient of relative risk aversion is equal to  $\theta$ . (Recall that the Arrow-Pratt measure of relative risk aversion is given by  $-cu''/u'$ .)
- (b) Following Kimball (1990), show that the coefficient of relative prudence (defined as  $-cu'''/u''$ ) is equal to  $\theta$ .
- (c) Plot  $c_1$ ,  $c_2^H$ ,  $c_2^L$ , current account, and expected utility as a function of the coefficient of risk aversion. Explain the intuition behind the results.
- (d) Plot the same variables as a function of a mean-preserving spread in the output distribution. Explain the intuition behind the results.

#### 5. Numerical example of incomplete markets model with imprudent preferences

Consider the incomplete markets model analyzed in Subsection 3.1 with the following preferences:

$$u(c) = 10c - \frac{1}{5}c^3,$$

for  $c < \sqrt{\frac{50}{3}}$ .

In this context:

- (a) Show that these preferences exhibit risk aversion and *imprudence*.
- (b) Plot  $c_1$ ,  $c_2^H$ ,  $c_2^L$ , current account, and expected utility as a function of the coefficient of risk aversion. Explain the intuition behind the results.

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## Box 1. How incomplete are markets?

When thinking about whether a small open economy faces complete markets or not, it proves insightful to first review the evidence of market completeness at the household level. If households within a financially sophisticated market, such as the United States, do not face complete markets, we would think that, if anything, countries would face even more markets incompleteness because of international credit market imperfections stemming principally from the lack of contract enforcement at a transnational level. The question is then whether households can insure themselves against idiosyncratic shocks – such as illness, loss of job, and other shocks to income – through stock and security markets, borrowing and lending in credit markets, unemployment insurance, and help from family and/or community members.

While there are different econometric methods to test for full insurance, the basic strategy remains the same: reject the complete markets (full insurance) hypothesis if one can find idiosyncratic shocks that affect the marginal utility of consumption, after controlling for aggregate shocks. A common problem with tests for full insurance is the potential endogeneity of right-hand variables. Most studies attempt to address this problem by finding idiosyncratic variables that are genuinely exogenous, rather than choice variables for the household. Others, like Ham and Jacobs (2000) address the endogeneity problem using instrumental variables technique. Table 1 offers a summary of some of the most influential studies. While some of the evidence is mixed, it is fair to conclude that, by and large, available studies suggest that households in the United States are not able to fully insure against idiosyncratic shocks.

At an international level, complete markets would imply that domestic consumption should be very highly correlated with world consumption, rather than with domestic output. An extreme version of this idea can be illustrated in a two-country version of our two period model analyzed in Exercise 2. Given the simple stochastic structure, the model predicts that consumption across countries should be perfectly correlated whereas, in the absence of world uncertainty, the correlation between domestic consumption and domestic output should be zero. As Table 2 details, however, empirical studies tend to reject the hypotheses that domestic and foreign consumption are highly correlated and/or that domestic consumption is less correlated with domestic output.

We thus conclude that, by and large, studies based on both household

data and country data tend to reject the hypothesis of complete markets. However, some studies do find that, as one would expect, markets are less incomplete within a country than across countries (Kose, Prasad, and Terrones (2009)).

## Box 2. How costly is to default?

Since, unlike private debt, sovereign debt repayment is not enforceable, there must be costs associated with default for countries to have an incentive to repay their debts. Empirically, four types of default costs have been identified in the literature (Table 3 summarizes the main empirical findings associated with each type of default cost):

- **Reputational costs:** costs that countries suffer due to loss of access to capital markets (Eaton and Gersovitz (1981)). While, in practice, countries that default are certainly not excluded from international capital markets forever, they face higher borrowing costs once they regain access.
- **International trade cost:** costs that countries suffer due to various disruptions to international trade. There are two main channels involved: (i) direct import sanctions or restrictions, and (ii) damage to exporters' creditworthiness. While the evidence for direct trade sanctions is rare, there is some evidence in favor of the second. When a country is in a situation of external financing distress, it often resorts to exchange controls or capital outflow restrictions, which affects the repayment capacity of all private debtors, especially export-linked credits (Ozler (1993) and Borensztein and Panizza (2006)).
- **Domestic banking sector costs:** since banks in emerging markets hold a significant amount of government bonds, a sovereign default (which, on occasions, may also include a default on domestic debt) will likely weaken banks' balance sheets and perhaps even cause a bank run (Borensztein and Panizza (2008)).
- **Political costs:** A weakened economy and, possibly, a banking system in crisis does not bode well for survival in power of the incumbent party and policymakers (Borensztein and Panizza (2008)).

Naturally, defaulting affects a country's rate of growth through the above channels and possibly others. Available estimates (Sturzenegger and Zettelmeyer (2005), Borensztein and Panizza (2008)) suggest that, in the year of default, growth falls between 0.5 and 2 percent.

### Box 3. What explains the risk premium in practice?

While, in our simple model, the level of indebtedness is the main determinant of the risk premium, in practice there are a host of variables that appear to influence it. The main determinants identified in the empirical literature are as follows:

- Solvency and liquidity variables. The main measures of solvency include the level of debt, the budget deficit, debt service, and the export to GDP ratio. Liquidity is normally measured by the amount of foreign currency reserves held by a country, typically as a proportion of international trade or, more recently, short-term debt (the so-called Guidotti-Greenspan rule).<sup>26</sup> Studies generally find that a higher debt to GDP ratio and lower liquidity increase spreads (Edwards (1986) and Caceres *et al.* (2010)).
- Macroeconomic fundamentals are variables that have an impact on the long-term solvency of a country: among them, inflation rate, public expenditure, and terms of trade. In general, better fundamentals in the form of lower inflation, lower public expenditures, and better terms of trade are associated with lower spreads (Min (1998), Akitoby and Stratmann (2006)).
- Fiscal institutions. Sounder fiscal institutions are associated with a lower lower risk premium. Furthermore, fiscal imbalances matter less for risk premium in countries with better institutions (Hallerberg and Wolff (2006)).
- External factors: global risk aversion, contagion, and international liquidity. Investor's attitude towards risk appears to explain movements of the emerging market bond spread in recent years. For advanced economies, higher global risk aversion generally lowers spread due to flight-to-quality reasons.<sup>27</sup> Contagion, in contrast, is always negative for government bonds (i.e., leads to higher spreads). Finally, higher

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<sup>26</sup>The Guidotti-Greenspan rule states that reserves should equal short-term debt (one year or less maturity), implying a ratio of reserves-to-short term debt of 1.

<sup>27</sup>When global risk aversion increases, capital flows away from risky assets and towards advanced countries' government securities.

(lower) international liquidity, measured by low (high) U.S. interest rates, leads to lower (higher) spreads in emerging markets. See Caceres and al. (2010) and Hartelius *et al.* (2010).

There is evidence of substantial co-movement in sovereign bond spreads over time indicating that spreads do not only capture country specific information but also reflect spillover or contagion from regional and/or global forces. McGuire and Schrijvers (2003) and Ciarlone, Piselli, and Trebeschi (2007) find that common factors account for one third of the total variation. Common factors include (i) the level of short and long term interest rates in advanced economies (mainly US), as measure of global liquidity conditions; (ii) their volatility, as a measure of the uncertainty about the path of monetary policy in advanced economies (Arora and Cerisola, 2001); and (iii) the yield spread between low and high rating corporate bonds or US long-term bonds, as a measure of international investor's appetite for risk (Hartelius *et al* 2010).

## Box 4. To close or not to close the small open economy model?

As Exercise 3 at the end of Chapter 1 made clear, our basic perfect-foresight, small open economy model in continuous time has roots equal to  $r$  and  $r - \beta$ . If  $r = \beta$ , the roots are  $r$  and 0. This would correspond, in a discrete time model and under the assumption that  $\beta(1 + r) = 1$ , to the system having roots  $1 + r$  and 1. In other words, under the assumption that  $\beta(1 + r) = 1$ , our basic small open economy model has a unit root. Intuitively, this simply means that if permanent income increases by one unit, the permanent level of consumption also increases by one unit, a natural and logical implication of the permanent income hypothesis. On occasions, researchers speak of consumption depending on initial conditions because if, say, the initial level of assets were to change exogenously, the permanent level of consumption would change by the permanent income component of the change in the stock of assets, again a very logical implication. Under perfect foresight, therefore, having a unit root presents no formal problem and is, in fact, the natural outcome of Friedman's permanent income hypothesis. We should have been shocked if this had not been the result!

What happens when we introduce uncertainty? If the horizon is finite, uncertainty introduces no problems whatsoever. In infinite horizon problems, things become trickier. Theoretically, however, very little changes. As Appendix 6.1 shows, under quadratic preferences, consumption follows a random walk (the stochastic equivalent of a unit root) and we can compute a reduced form for consumption that exactly replicates our perfect foresight counterpart. Further, specifying a particular stochastic process for output, we can perform all of our main experiments such as analyzing how consumption and the current account respond to different shocks. In this sense, our random walk result is no different from some of the more popular models in macroeconomics that yield the prediction that the main variable of interest – for instance, aggregate consumption or stock prices – should follow a random walk. In sum, the non-stationary of consumption is a fundamental feature of the standard (stochastic) small open economy model and is just the counterpart of the unit root in the perfect foresight version of the model. In fact, if the computer had never been invented (and, hence, quantitative considerations had not become paramount), we would probably be dealing only with these non-stationary models in open economy macroeconomics.

Computers, however, are here to stay, and influenced by a paper by

Schmitt-Grohé and Uribe (2003), open economy researchers often refer to the need to “close” a small open economy model. By “closing” they mean getting rid of the unit root by introducing some friction – such as an upward sloping supply, Uzawa preferences, or cost of adjustment in the stock of net foreign assets – which transforms the unit root into a stable root and renders the model stationary. Since, as argued above, there is nothing wrong with the non-stationary model and hence, there is no formal *need* to “close” a stochastic small open economy model, why may we *choose* to do so? The “problem” is that a non-stationary model has no well-defined second moments. (Recall that the variance of a random walk is infinite.) In other words, non-stationary models do not have a well-behaved stationary stochastic distribution. If our only objective is to solve a small open economy model quantitatively and compute impulse responses (i.e., responses of the endogenous variables to a single shock), we do not need to get rid of the unit root. This is equivalent to studying the response of our perfect foresight economy in Chapter 1 to a temporary shock. More typically, however, researchers that solve models with the computer following the real business cycle methodology seek to compute second moments to capture variances and covariances of the main endogenous variables and compare them with the second moments in the data. Since variances/covariances would not be well-defined in the presence of a unit root, the need arises to get rid of this unit root by “closing” the model.

While “closing” small open economy models is therefore necessary for the purposes of computing variances and covariances, it does come at the cost of blurring some important conceptual foundations. Consider, for instance, Uzawa preferences that force consumption to remain the same across steady-states. This implies that even if your wealth increases by one unit, your steady-state consumption would remain unchanged, a clear violation of the permanent income hypothesis! You would be forced to consume a lot during the transition in order to go back to the same steady-state consumption.

Figure 1. Welfare costs of financial autarky

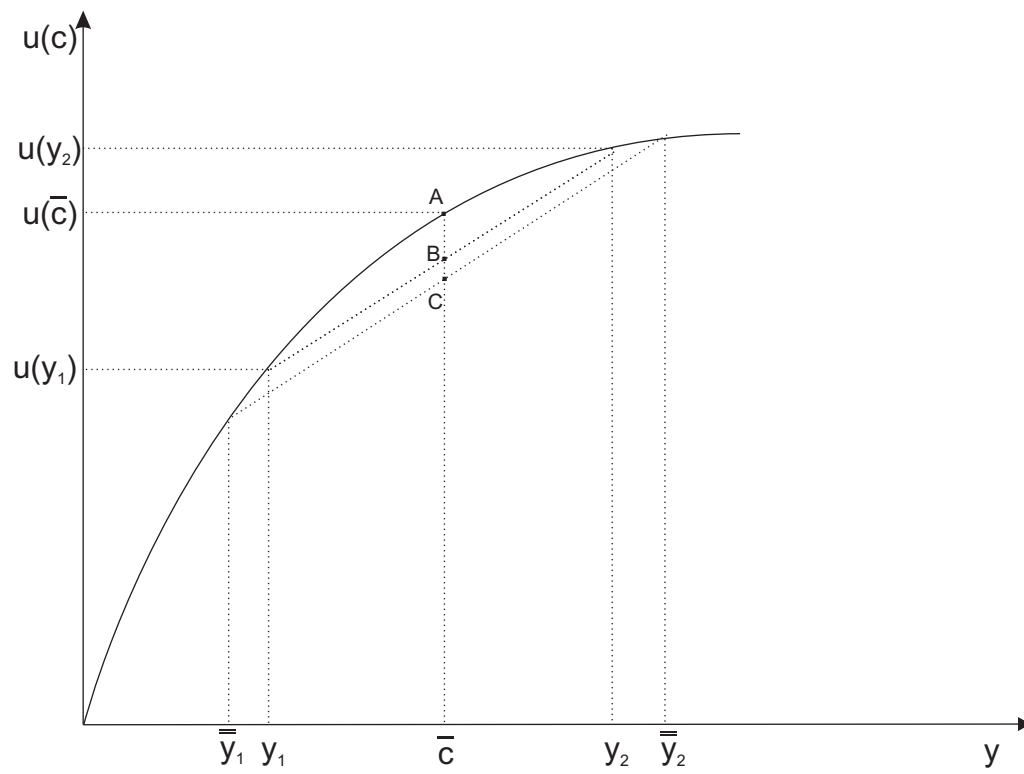


Figure 2. Supply of funds

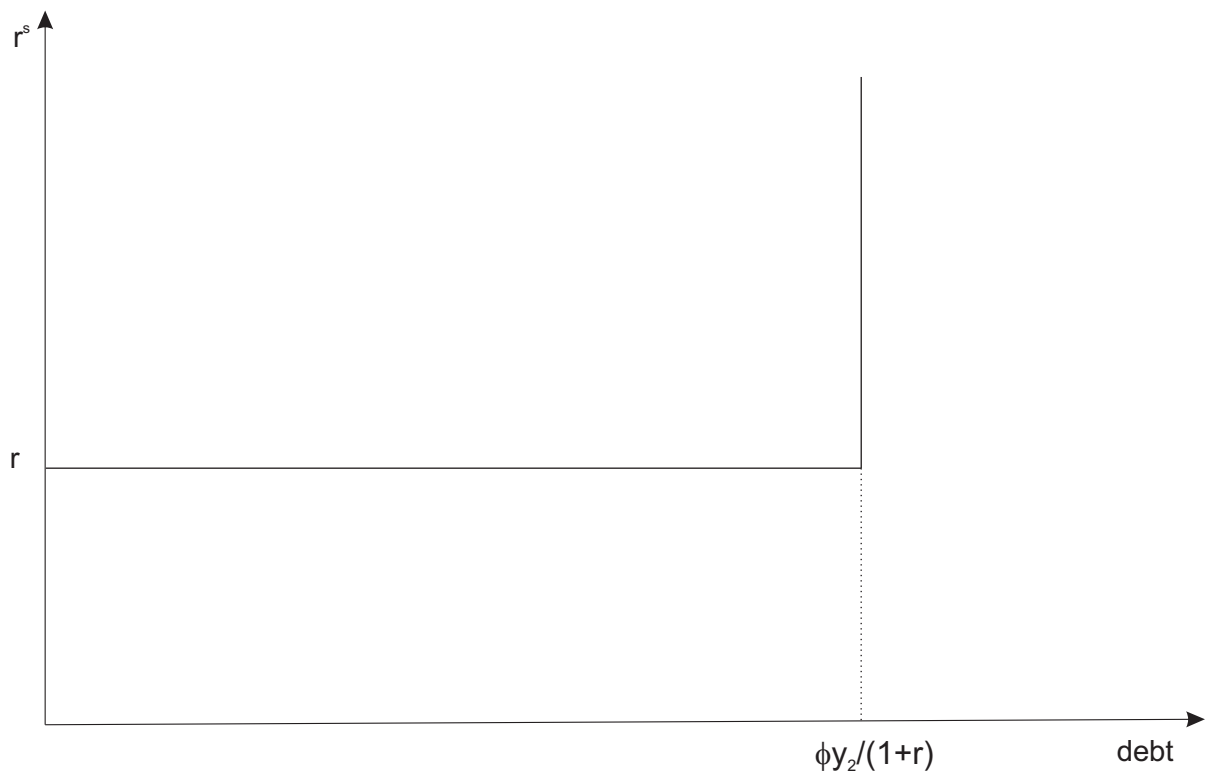
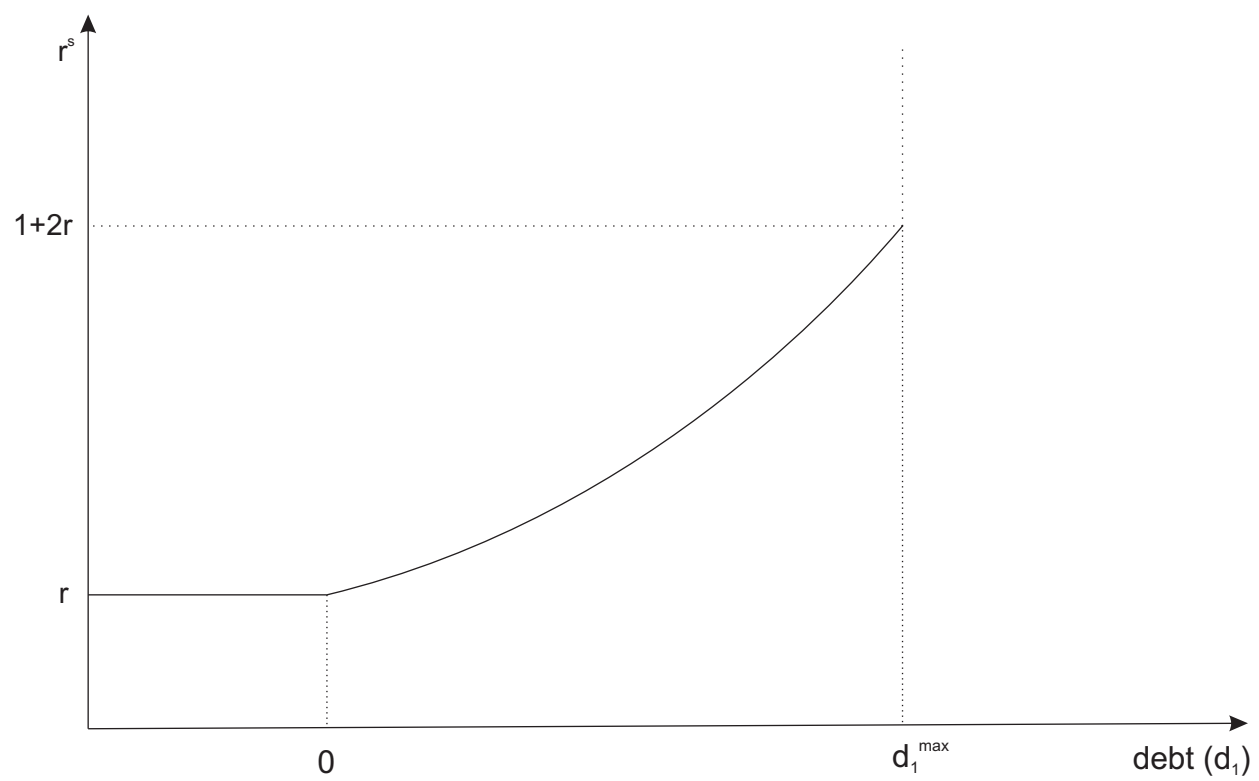


Figure 3. Upward sloping supply of funds



**Table 1. Studies on market completeness based on household data**

Author	Dataset	Testing strategy	Results
Cochrane (1991)	Panel Study of Income Dynamics 1980-1983	Full consumption insurance implies that consumption growth should be independent of idiosyncratic shocks to households. The idiosyncratic shocks are captured by involuntary job loss (sickness, strike, move)	Full insurance is rejected for long illness and involuntary job loss, but not for spells of unemployment, loss of work due to strike, and an involuntary move.
Mace (1991)	Consumer Expenditure Survey 1980-1983	Risk sharing implies that individual consumption varies positively with aggregate consumption only and not with individual income and changes of employment status. For specific preferences, changes in consumption or growth rates of consumption, net of preference shocks, should be equalized across individuals.	Once the change in aggregate consumption is accounted for, the change in household income does not help to explain the change in household consumption. Results for the growth rate specification, however, reject full insurance.
Hayashi, Altonji, and Kotlikoff (1996)	Panel Study of Income Dynamics 1968-1981 1985-1987	Full risk sharing implies that one-year consumption changes are uncorrelated with the wage rate at all leads and lags. This is tested by examining the correlation of long term and one-year changes in consumption and the wage rate.	Both intra- and inter-family risk sharing is rejected, but not self-insurance.
Attanasio and Davis (1997)	Consumption Expenditure Survey and the Current Population Survey 1980-1990	Examines the impact of systematic, publicly observable shifts in the hourly wage structure on the distribution of household consumption.	Relative wage movements among men had large impact on the distribution of household consumption, thus rejecting the hypothesis of consumption insurance.
Ham and Jacobs (2000)	Panel Study of Income Dynamics 1974-1987	To avoid potential endogeneity problems, authors use current unemployment rates in the household head's industry and the head's occupation separately as instruments for idiosyncratic shock.	Strong rejection of full insurance hypothesis. Households not insured against changes in unemployment rate associated with the household head's occupation.

**Table 2. International evidence on market incompleteness**

Author	Dataset	Testing strategy	Results
Atkeson and Bayoumi (1993)	US; 1963-1986  6 members of OECD 1970-1987	Consumers should try to construct capital portfolios generating income that has a negative correlation with their regional income.	Reject full insurance in both panels. Despite opportunities available for consumers to smooth regional risks, a large portion of consumers do not seem to privately insulate their consumption from regional conditions.
Crucini (1999)	US; 1972-1990  Canada; 1973-1991  G-7; 1970-1987	Imperfect risk sharing implies that after controlling for common income shocks, consumption should move together more closely across individuals that engage in more risk sharing.	There is much more risk sharing across the Canadian provinces and the US states than there is across the G7 countries.
Obstfeld (1994)	7 largest industrial countries  1950-1988	In an integrated world asset market with representative national agents, the ex post difference between two countries' intertemporal marginal rates of substitution in consumption should be uncorrelated with any random shock that is insurable.	In the postwar period, there is an increasing comovement between domestic and world consumption growth, but the correlation between them remains far below expected, even in a world of unrestricted international asset trade.
Kose, Prasad and Terrones (2009)	69 countries 1960-2004	Financial integration predicts countries' consumption growth to be more correlated with world output than with country's output growth.	For most countries the correlation between domestic consumption and output is higher than correlation between domestic consumption and world output. The gap between the two measures is much larger for emerging markets and other developing countries than for industrial countries.

**Table 3. Costs of default**

Author	Country/Data	Default cost	Main findings
English (1996)	U.S. states / 19 <sup>th</sup> century	Reputation	Since foreign creditors could not impose trade embargoes on U.S. states, states paid back their debt exclusively for reputational reasons and not because of the threat of sanctions.
Reinhart <i>et al.</i> (2003)	53 countries/ 1979-2000	Reputation	A history of default is associated with lower ratings assigned by the Institutional Investor.
Borensztein and Panizza (2008)	83 countries/ 1972-2000	Reputation	Default episodes do not have a long-term impact on credit ratings; only recent defaults correlate with current credit ratings.
Sule Ozler (1993)	64 countries/ 1968-1981	Reputation - Borrowing cost	Recent defaults influence spreads but size of premium attached to previous defaults fades with time.
Dell' Ariccia <i>et al.</i> (2002)	Russian crisis/ 54 countries	Reputation - Borrowing cost	Defaults have long-lasting effects. Countries involved in Brady exchange faced higher borrowing costs in the late 1990s.
Flandreau and Zumer (2004)	16 countries/ 1880-1914	Reputation - Borrowing cost	Default is associated with a jump in spreads of about 90 basis points in the year that follows the end of a default episode. Effects on spread, however, declines rapidly over time.
Borensztein and Panizza (2008)	31 countries/ 1997-2004	Reputation - Borrowing cost	Significant effect of up to 400 basis point in year following default. Subsequent effects are smaller and not statistically significant.
Rose (2005)	200 countries/ 1948-1997	International Trade	Paris Club debt renegotiations are associated with a decline in bilateral trade that lasts for 15 years and amounts to around 8 percent per year.
Borensztein and Panizza (2008)	Industry-level data / 1980-2000	International Trade	Sovereign defaults are costly for export-oriented industries, but effect tends to be short-lived.
Borensztein and Panizza (2008)	149 countries/ 1975-2000	Domestic banking system	Controlling for banking crisis, defaults do not appear to have an effect on industries that depend more on external finance.
Borensztein and Panizza (2008)	19 countries/ 1980-2003	Political cost	On average, ruling governments in countries that defaulted face a 16 percent reduction in electoral support. In 50 percent of cases, there was a change in the chief of the executive either in the year of the default or in the following year (more than twice the probability in normal times).

**Table 4. Studies explaining sovereign risk premium**

Author	Country/Data	Determinants	Main Results
Edwards (1986)	26 countries/ 1976-1980	Solvency	Risk premium is positively related to debt-to-GDP ratio.
Min (1998)	11 countries/ 1991-1995	Solvency/Liquidity/macroeconomic fundamentals	A 1 percent increase in debt-to-GDP ratio increases the spread by 1.005 percent. A one percent increase in net foreign assets lowers the spread by 1.022 percent. A one percent improvement in the terms of trade leads to a 1.02 percent reduction in spreads. A one percent increase in the inflation rate is associated with 1.016 percent increase in the yield spread.
Caceres <i>et al</i> (2010)	Euro area / mid-2005 to 2010	Solvency	Swap spreads tighten when public debt (in percentage of GDP) rise or budget balance deteriorate.
Akitoby and Stratmann (2006)	32 countries/ 1994-2003	Macroeconomic fundamentals	Reduction in public expenditure is a more powerful tool for reducing spreads than increases in revenues.
Hallerberg and Wolff (2006)	12 EU members/ 1993-2005	Solvency/Fiscal institutions	An increase of the deficit by one percent of GDP increases the spread by 1.5 to 4 basis points. Countries that have institutions better suited to solving deficit biases pay lower risk premia for their current deficits.
Caceres <i>et al</i> (2010)	Euro area / mid-2005 to 2010	External factors	When global risk aversion is on the rise, the spreads between AAA-rated country and lower-rated ones tend to widen significantly. Contagion is always negative for government bonds. When the probability of a credit squeeze in a country rises, swap spreads tighten. High-debt, lower-rated countries tend to exhibit larger sensitivities since these countries are more vulnerable to even remote probabilities of distress among higher-rated countries.
Hartelius <i>et al</i> (2010)	33 emerging countries/ 1991-2007	External factors	An increase of one percent in the expected U.S. policy rate over the next three month window increases spreads by about 5 percent. The VIX is used as proxy for investor's attitude toward risk. An increase of the VIX by one standard deviation increases spreads by about 30 percent.

**Table 5. Parameters for calibration**

Notation	Parameter/Variable	Value
$\beta$	discount factor	0.94
$r$	interest rate	0.059
$\theta$	relative risk aversion	2
$\sigma$	Standard deviation of GDP innovations	0.026
$\rho$	Autocorrelation of GDP	0.597
$\phi$	Ad-hoc debt limit	-1.34

**Table 6. Model**

	<b>Baseline</b>	$\theta=3$	$\theta=4$	$\theta=5$	$\theta=6$
<b>Means</b>					
Output	1.000	1.000	1.000	1.000	1.000
Consumption	0.977	0.979	0.983	0.988	0.988
Foreign Assets	-0.477	-0.440	-0.382	-0.278	-0.278
Trade Balance	0.024	0.021	0.018	0.013	0.013
Current Account	0.000	-0.001	-0.002	-0.001	-0.001
<b>Standard Deviation</b>					
Output	0.025	0.025	0.024	0.025	0.0249
Consumption	0.010	0.009	0.007	0.006	0.0062
Foreign Assets	0.067	0.072	0.074	0.080	0.0801
Trade Balance	0.050	0.053	0.054	0.058	0.0578
Current Account	0.049	0.051	0.053	0.056	0.0563
<b>Output Correlations</b>					
Consumption	0.766	0.727	0.723	0.700	0.6999
Foreign Assets	0.468	0.435	0.422	0.403	0.4032
Trade Balance	0.924	0.943	0.958	0.973	0.9729
Current Account	0.302	0.293	0.292	0.295	0.2949
<b>Autocorrelations</b>					
Output	0.266	0.263	0.265	0.278	0.2779
Consumption	0.364	0.356	0.359	0.401	0.4008
Foreign Assets	0.719	0.730	0.736	0.739	0.739
Trade Balance	0.251	0.256	0.259	0.269	0.2691
Current Account	0.248	0.256	0.256	0.267	0.2673
$\sigma(c)/\sigma(y)$	0.417	0.359	0.303	0.249	0.249