Chapter 4
Non-Tradable Goods and Relative Prices*

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1 Introduction

Thus far we have only looked at models with tradable goods. In reality, however, many of the goods consumed in an open economy are non-tradable. The presence of non-tradable goods introduces a key relative price in the economy: the price of non-tradable goods in terms of tradable goods (the inverse of what is commonly defined as the real exchange rate). This is, by far, the most important relative price in a small open economy as it provides the main adjustment mechanism to both demand and supply shocks.

The key difference between tradable and non-tradable goods lies in their supply elasticity: the supply elasticity of tradable goods is in effect infinite since a small open economy can buy/sell as many tradable goods as it wants at a given world price (subject of course to its resource constraint). In sharp contrast, non-tradable goods must be produced at home. In the case of an endowment economy, this implies that the supply elasticity is zero. As a

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result, the adjustment to demand shocks will need to be met by a change in relative prices. Even when production is endogenous, a, say, excess demand for non-tradable goods relative to tradable goods will require an increase in their relative price to elicit a shift in productive resources from the tradable to the non-tradable sector. In this case, the adjustment will come about through both prices and quantities.

This supply side asymmetry between tradable and non-tradable goods and the resulting need for relative price changes is at the core of any macroeconomic adjustment in developing countries. Consider, for instance, a typical boom-bust cycle in a developing country. During the boom, consumption of both tradable and non-tradable goods is high. What is the production pattern that will meet this high demand? Since non-tradables must be produced at home – whereas tradable goods can be imported – more resources will be needed in the non-tradable goods sector. To entice this shift in resources, the relative price of non-tradable goods must be high. The good times will thus be characterized by high consumption, trade and current account deficits, relatively low production of tradables, relative high production of non-tradables, and an appreciated real exchange rate. At some point, however, consumption will need to adjust to satisfy the economy's intertemporal resource constraint. Put differently, at some point the economy will need to run a trade surplus to repay the external debt incurred in the good times. To run a trade surplus, production of tradable goods must exceed consumption of tradable goods. How does this adjustment come about? It is effected via a fall in the relative price of non-tradable goods (in response to an excess supply of home goods at the pre-crisis relative price). This fall (a real depreciation) will induce resources to shift from the non-tradable goods sector to the tradable goods sector. The bad times will thus be characterized by low consumption, trade and current account surpluses, relatively high production of tradables, relatively low production of non-tradables and a depreciated real exchange rate. Our very simple model will explain precisely this adjustment mechanism. One can add many bells and whistles to this basic adjustment mechanism (as many researchers have done, and we will examine some of the more relevant ones in the rest of the book) but, at its core, the essential mechanism will be fully uncovered in this chapter.

The chapter proceeds as follows. Section 2 begins by adding non-tradable goods to the endowment model studied in Chapter 1. This gives us our basic model with non-tradable goods, which will be the workhorse for the rest of this chapter. A key feature of the basic model with non-tradable goods is that the consumption smoothing result obtained in Chapter 1 (i.e., in the absence of any friction, consumption will be flat over time regardless of the path of output) no longer holds. By affecting the marginal utility of tradable goods, a fluctuating path of consumption of non-tradable goods may induce the consumer to vary consumption of tradable goods over time. In this vein, this section explores in detail how consumers react to changes in relative prices.

Section 3 puts our basic model to work and derives the first key message of this chapter: whether in response to supply or demand shocks, trade deficits go hand in hand with real appreciation and trade surpluses go hand in hand with real depreciation. There is no sense, however, in which a real appreciation "causes" a trade deficit (or a real depreciation a trade surplus), a commonly-held view among practitioners. In the model, the real exchange rate and the trade balance are simply responding to some exogenous shock. The logic behind this result is as simple as it is compelling. Suppose there is an excess demand for both tradable and non-tradable goods (as a result of either a negative supply shock or a positive demand shock). On the tradable side, this excess demand is met by importing more tradable goods from abroad (a trade deficit). On the non-tradable side, this cannot happen since the supply of non-tradables is fixed. Hence, the relative price of non-tradable goods must rise to clear the market (real appreciation). In sum, a trade deficit and real appreciation (or trade surplus and real depreciation) are just two sides of the same coin and thus must necessarily go hand in hand.

Section 4 brings fiscal policy into the picture, by introducing government spending and taxes into our basic model with non-tradable goods. The rationale for doing so is that some of the most important policy questions in developing countries revolve around fiscal policy, the real exchange rate, and trade imbalances. In particular, will a fiscal contraction depreciate the real exchange rate? Will a fiscal expansion lead to a trade deficit? This section clarifies the conditions under which the answer to both question is affirmative. If government spending is biased towards non-tradable goods (relative to the private sector's spending patterns), then a reduction in government spending will indeed depreciate the real exchange rate. A fiscal contraction brought about through a temporary increase in tax rates will also lead to a real depreciation. This section then shows how temporary increases in government spending may lead to trade deficits, thus validating the commonly held view that fiscal deficits may cause trade deficits (the so-called "twin deficits" hypothesis).

Up to this point in the chapter, the supply side has been kept out of the picture by assuming that the economy is endowed with some amount of tradable and non-tradable goods. Section 5 endogeneizes production by introducing labor supply into the model. This will enable us to get a fuller picture of how the economy adjusts to different shocks. We pay particular attention to how the economy adjusts to a boom-bust cycle caused by demand shocks and the key role played by the relative price of non-tradable goods in bringing about the needed adjustment.

Finally, Section 6 tackles two important and related questions regarding the response of the real exchange rate: How does the real exchange rate respond to a terms of trade shock? How does a trade liberalization (e.g., a reduction in import tariffs) affect the real exchange rate? To answer these questions, we modify the model by distinguishing between importable and exportable goods. In this context, we find that a terms of trade improvement (i.e., a rise in the price of exportables in terms of importables) will always lead to an increase in the relative price of non-tradable goods (real appreciation) through two effects: a wealth effect and an intertemporal substitution effect. We also show that a temporary trade liberalization (like the one studied in Chapter 3) also leads to an increase in the relative price of non-tradable goods.

2 The basic model with non-tradable goods

Except for the addition of non-tradable goods, the model is the same as the basic endowment model developed in Chapter 1. Consider a small open economy inhabited by a large number of identical, infinitely-lived consumers, who are endowed with perfect foresight. There exist two physical goods: a tradable and a non-tradable good (both non-storable). The tradable good is the numeraire. The endowment path of both goods is exogenously-given. There is no government. Perfect capital mobility prevails in the sense that consumers can buy or sell bonds that are denominated in terms of the tradable good (the numeraire) at a constant interest rate r.

2.1 Consumer's problem

The consumer's lifetime utility is given by

$$\int_0^\infty u(c_t^T, c_t^N)e^{-\beta t}dt, \tag{1}$$

where $\beta(>0)$ is the subjective discount rate, c^T and c^N denote consumption of tradable and non-tradable goods, respectively, and the function $u(.)$ is assumed to be strictly increasing and strictly concave:[1]

$$u_{c^T} > 0, \quad u_{c^N} > 0, \quad u_{c^T c^T} < 0, \quad u_{c^T c^T} u_{c^N c^N} - u_{c^T c^N}^2 > 0.$$

In addition, we will also assume that both c_t^T and c_t^N are normal goods.[2] Normality for c^T and c^N, respectively, requires that (see Appendix):

$$u_{c^N} u_{c^T c^N} - u_{c^T} u_{c^N c^N} > 0, \tag{2}$$

$$u_{c^T} u_{c^T c^N} - u_{c^N} u_{c^T c^T} > 0. \tag{3}$$

Let b denote net foreign assets held by the consumer. The flow constraint is given by

$$\dot{b}_t = rb_t + y_t^T + p_t y_t^N - c_t^T - p_t c_t^N, \tag{4}$$

where y_t^T and y_t^N denote the endowment of tradable and non-tradable goods in period t, respectively, and p_t is the relative price of non-tradable goods in terms of tradable goods.

A brief detour on real exchange rate terminology is warranted. The standard convention – and the one that we will follow in this book – is to define the real exchange rate as the inverse of p. In other words, the real exchange rate is the relative price of tradable goods in terms of non-tradable goods. An increase in p (i.e., an increase in the relative price of non-tradable goods) or, equivalently, a fall in the real exchange rate is referred to as a real appreciation. A fall in p or, equivalently, an increase in the real exchange rate, is referred to as a real depreciation.[3] [4]

[1] Note that, taken together, the following two conditions below for strict concavity imply that $u_{c^N c^N} < 0$.

[2] Recall that a normal good is one whose income efffect is positive (as opposed to a so-called inferior good, whose income effect is negative). At this level of aggregation, normality is all but guaranteed.

[3] It could be argued that it only makes sense to use the label "real exchange rate" in a monetary model, since this label obviously follows from the idea that by deflating the nominal exchange rate one gets a real exchange rate (as discussed in chapter 6). In an abuse of language, however, we will often use the term "real exchange rate" in this chapter to refer to the inverse of the relative price of non-tradable goods.

[4] We often encounter in the literature the expression "real exchange rate appreciation"

Integrating forward (4) and imposing the condition

$$\lim_{t\longrightarrow\infty} e^{-rt}b_t = 0,$$

we obtain the consumer's intertemporal budget constraint:

$$\int_0^\infty (c_t^T + p_t c_t^N)e^{-rt}dt = b_0 + \int_0^\infty (y_t^T + p_t y_t^N)e^{-rt}dt. \tag{5}$$

The consumer's problem consists in choosing $\{c_t^T, c_t^N\}_{t=0}^\infty$ to maximize lifetime utility (1), subject to the intertemporal constraint (5), for a given path of y_t^T, y_t^N, p_t, and r and a given b_0.

In terms of the Lagrangian,

$$\begin{aligned}\mathcal{L} \;=\; & \int_0^\infty u(c_t^T, c_t^N)e^{-\beta t}dt + \lambda\left[b_0 + \int_0^\infty (y_t^T + p_t y_t^N)e^{-rt}dt\right. \\ & \left. - \int_0^\infty (c_t^T + p_t c_t^N)e^{-rt}dt\right]\end{aligned}$$

The corresponding first order conditions are given by:

$$e^{-\beta t}u_{c^T}(c_t^T, c_t^N) = \lambda e^{-rt}, \tag{6}$$

$$e^{-\beta t}u_{c^N}(c_t^T, c_t^N) = \lambda e^{-rt}p_t. \tag{7}$$

Assume, for the reasons discussed in Chapter 1, that $\beta = r$. We can then rewrite the first-order conditions as

$$u_{c^T}(c_t^T, c_t^N) = \lambda, \tag{8}$$

$$u_{c^N}(c_t^T, c_t^N) = \lambda p_t. \tag{9}$$

Equation (8) says that the marginal utility from consuming tradable goods will be constant over time. This is, of course, the same result that we encountered in Chapter 1. The key difference, however, is that since

or "real exchange rate depreciation." This terminology is, strictly speaking, incorrect because the real exchange rate can go up and down but does not really "appreciate" or "depreciate." It is the currency (in monetary models of course) that can appreciate or depreciate in real terms.

there was only one (tradable) good in Chapter 1, a constant marginal utility implied a constant consumption path over time. This is no longer true in a model with non-tradable goods. In fact, it is already apparent from condition (8) that if consumption of non-tradable goods fluctuates over time, then consumption of tradable goods will not be constant over time. First-order condition (9) in turn makes clear that if p_t is not constant over time, then the marginal utility from consuming non-tradable goods will be affected. To see this formally, consider a small change in p along a perfect foresight equilibrium path (PFEP) and totally differentiate first-order conditions (8) and (9) with respect to p to obtain:[5]

$$\frac{dc^T}{dp} = -\lambda \frac{u_{c^T c^N}}{u_{c^T c^T} u_{c^N c^N} - u^2_{c^T c^N}} \begin{cases} = 0, & \text{if } u_{c^T c^N} = 0, \\ > 0, & \text{if } u_{c^T c^N} < 0, \\ < 0, & \text{if } u_{c^T c^N} > 0. \end{cases}, \qquad (10)$$

$$\frac{dc^N}{dp} = \lambda \frac{u_{c^T c^T}}{u_{c^T c^T} u_{c^N c^N} - u^2_{c^T c^N}} < 0, \qquad (11)$$

where the denominator in both expressions is positive due to the assumption of strict concavity of the utility function. Equation (11) says that whenever p is, say, low along a perfect foresight path, the consumption of non-tradable goods will be high. The intuition is the same as we saw in chapter 3: a low p (relative to other periods) indicates that non-tradable goods are relatively cheaper, which induces consumers to engage in intertemporal consumption substitution.

In turn, we can see from (10) that the response of c^T to a change in p depends on the sign of the cross-derivative of the utility function. In other words, the response of c^T to changes in p depends on whether c^T and c^N are Edgeworth substitutes, independent, or complements.[6] There are three possible cases:[7]

[5] Of course, in general equilibrium p will be determined endogenously as analyzed below. Here we are looking at the consumer's response to changes in p (i.e., partial equilibrium).

[6] Two goods are said to be Edgeworth substitutes, independent, or complement depending on whether the cross derivative of the utility function is negative, zero, or positive, respectively. At a microeconomic level, an example of Edgeworth complements would be coffee and sugar and of Edgeworth substitutes Coke and Pepsi.

[7] Exercise 1 at the end of this chapter asks the reader to work out a specific example with CES preferences which shows how Edgeworth substitutability/complementarity depends on the relation between the intra- and intertemporal elasticities of substitution.

- Tradables and non-tradables are Edgeworth independent (i.e., $u_{c^T c^N} = 0$). In this case, changes in p have no effect on the path of c_t^T. We are thus back to the world of Chapter 1 and consumption of tradable goods is fully smoothed over time.

- Tradables and non-tradables are Edgeworth substitutes (i.e., $u_{c^T c^N} < 0$). In this case, additional consumption of non-tradables reduces the marginal utility of consuming tradable goods and hence tradable goods consumption is lower. Hence, when p is relatively low along a perfect foresight path, c_t^N will be relatively high and c_t^T will be relatively low. In other words, c^T and c^N move in opposite directions.

- Tradables and non-tradables are Edgeworth complements (i.e., $u_{c^T c^N} > 0$). In this case, additional consumption of non-tradables increases the marginal utility of consuming tradables goods and hence tradable goods consumption is higher. In this case, c^T and c^N move in the same direction.

In addition to these effects of changes in the level of p, changes in p *over time* will also have an impact on how consumption evolves over time. To see this formally, first take logarithms and then totally differentiate first-order conditions (6) and (7) with respect to time to obtain:

$$-\frac{u_{c^T c^T}}{u_{c^T}}\dot{c}_t^T - \frac{u_{c^T c^N}}{u_{c^T}}\dot{c}_t^N = r - \beta, \tag{12}$$

$$-\frac{u_{c^T c^N}}{u_{c^N}}\dot{c}_t^T - \frac{u_{c^N c^N}}{u_{c^N}}\dot{c}_t^N = r^d - \beta, \tag{13}$$

where

$$r^d \equiv r - \frac{\dot{p}_t}{p_t} \tag{14}$$

is, by definition, the *domestic real interest rate*. Naturally, the LHS of equations (12) and (13) tells how the marginal utility of tradables and non-tradables, respectively, evolves over time. As equation (13) makes clear, the domestic real interest rate is the relevant real interest rate in determining the time profile of the marginal utility of non-tradable goods, as opposed to the world real interest rate, r, which is the relevant real interest rate in determining the profile of the marginal utility of tradable goods (as reflected in equation (12)).

To see more clearly the role of the domestic real interest rate, let us focus for a moment on the separable case; that is, suppose that $u_{c^T c^N} = 0$. In this particular case, equations (12) and (13) reduce to:

$$\frac{-u_{c^T c^T}}{u_{c^T}} \dot{c}_t^T = r - \beta, \tag{15}$$

$$\frac{-u_{c^N c^N}}{u_{c^N}} \dot{c}_t^N = r^d - \beta. \tag{16}$$

In the separable case, then, equation (16) tells us that the time profile of consumption of non-tradable goods is fully determined by the difference between the domestic real interest rate and the discount rate. Instead, for the case of tradable goods, the time profile is determined by the difference between the world real interest rate and the discount rate, as was the case in Chapter 1. Furthermore, it also follows from equation (16) – recalling the definition of r^d given in (14) – that even under our usual assumption that $r = \beta$, consumption of non-tradable goods will not be constant over time if the relative price of non-tradable goods is changing over time (i.e., if $\dot{p}_t$ is different from zero).[8]

What is the intuition behind the domestic real interest rate being the relevant real interest rate for non-tradable goods consumption? Intuitively, suppose that you forego one unit of non-tradable consumption today. The market value of that unit (in terms of the numeraire) is p_t. Hence, you can buy tradable bonds for a value of p_t. Since the return on these bonds is r, the next instant you will have a gross return of rp_t. However, if the relative price of non-tradable goods has increased over time (i.e., $\dot{p}_t > 0$), you will suffer a capital loss because your tradable bond will buy fewer non-tradable goods. Therefore, your net return will be $rp_t - \dot{p}_t$. To obtain the rate of return, you need to divide this net return by your initial investment, p, which yields the rate of return $r - \frac{\dot{p}_t}{p_t}$.[9] Naturally, if the relative price of non-tradable goods does not vary over time (i.e., if $\dot{p}_t = 0$), then $r^d = r$.

[8] In monetary models with sticky prices (Chapter 8), we will see examples in which this is the case.

[9] Incidentally, notice how expression (14) tells us that the real interest rate burden for small open economies will be particularly high in bad times. In effect, bad times are typically accompanied by real depreciation (i.e., $\dot{p} < 0$) which, according to (14), will increase the real interest rate in terms of non-traded goods.

Since the real interest rates relevant for the consumption decision of tradable and non-tradable goods differ, it should be intuitively clear that the real interest rate relevant for a consumption aggregate (i.e., an average of tradable and non-tradable goods) will be some average of r and r^d. This real interest rate is referred to in the literature as the "consumption-based real interest rate". Exercise 2 at the end of this chapter asks you to derive this expression in a discrete-time counterpart of this model. The main insight to be extracted from this exercise – which should be clear from the above discussion – is that if the relative price of non-tradable goods is not constant over time, then aggregate consumption will not be constant over time even if the discount factor equals $1/(1+r)$.

2.2 Equilibrium conditions

Since, by definition, the economy cannot import or export non-tradable goods, equilibrium in the home goods market requires that the consumption of non-tradable goods be equal to the supply of non-tradable goods in every period:

$$c_t^N = y_t^N. \tag{17}$$

Substituting condition (17) into (4), we obtain the current account:

$$\dot{b}_t = rb_t + y_t^T - c_t^T. \tag{18}$$

Note that the presence of non-tradable goods does not change the expression for the current account that we encountered in Chapter 1. This should clearly be the case since, by definition, non-tradable goods do not constitute either a claim or an obligation vis-a-vis the rest of the world.

Substituting condition (17) into (5) yields the economy's intertemporal constraint (i.e., the resource constraint):

$$b_0 + \int_0^\infty y_t^T e^{-rt} dt = \int_0^\infty c_t^T e^{-rt} dt. \tag{19}$$

2.3 Perfect foresight equilibrium

The path of consumption of non-tradable goods is fully determined by the path of non-tradable goods endowment, as equation (17) makes clear. Substitute the home goods market equilibrium condition into the first-order conditions (8) and (9) to obtain:

$$u_{c^T}(c_t^T, y_t^N) = \lambda, \tag{20}$$
$$u_{c^N}(c_t^T, y_t^N) = \lambda p_t. \tag{21}$$

These two conditions, together with the intertemporal constraint (19) and the exogenous paths of y_t^T and y_t^N, fully determine the perfect foresight paths of c_t^T and p_t and the unique value of λ.

It follows from (20) that the path of y_t^N will determine the time *profile* of c_t^T (as opposed to the particular level which will be determined by the intertemporal budget constraint, (19)). If the path of y_t^N is flat over time, then the path of c_t^T will also be flat over time. In this case – and as in Chapter 1 – the economy will run trade surpluses when the endowment of tradables is high and deficits when it is low. However, if the path of y_t^N fluctuates over time, then the path of c_t^T will also fluctuate over time. Formally, differentiate equation (20) along a perfect foresight equilibrium path (i.e., for a given λ) to obtain:

$$\frac{dc^T}{dy_t^N} = -\frac{u_{c^T c^N}(c_t^T, y_t^N)}{u_{c^T c^T}(c_t^T, y_t^N)} \begin{cases} = 0, & \text{if } u_{c^T c^N} = 0, \\ < 0, & \text{if } u_{c^T c^N} < 0, \\ > 0, & \text{if } u_{c^T c^N} > 0. \end{cases} \tag{22}$$

It follows that whether c^T is high or low when the endowment of non-tradables is, say, high depends on whether c^T and c^N are Edgeworth substitutes, independent, or complements, as illustrated in Figure 1.[10] The figure plots condition (20), for different values of y^N. In this light, consider the following three cases regarding the behavior of the trade balance:[11]

[Figure 1]

- Tradables and non-tradables are Edgeworth independent (i.e., $u_{c^T c^N} = 0$). In this case, the path of y_t^N has no effect on the path of c_t^T. In

[10] This is, of course, the same channel studied above but in a general equilibrium context.

[11] Exercise 1 at the end of this chapter asks the reader to work out a specific example with CES preferences which shows how Edgeworth substitutability/complementarity depends on the relation between the intra- and intertemporal elasticities of substitution. The relation between these two parameters will thus determine the behavior of the trade balance along a perfect foresight path with a fluctuating path of non-tradables.

terms of Figure 1, the equilibrium is always at point A, regardless of the path of y^N. We are thus back to the world of Chapter 1: the path of c_t^T is flat over time and the economy will run trade surpluses when the endowment of tradables is high and deficits when the endowment of tradables is low, regardless of the path of y_t^N.

- Tradables and non-tradables are Edgeworth substitutes (i.e., $u_{c^T c^N} < 0$). In this case, when y_t^N is relatively high along a perfect foresight path, c_t^T is relatively low. In terms of Figure 1, when y^N is high, the curve $u_{c^T}(c_t^T, y_t^N)$ shifts to the left and the equilibrium moves from point A to point B. If the path of y^T is relatively smooth (in Figure 2 it has been drawn as flat), then the economy will run trade surpluses when output of non-tradables is high and trade deficits when output of non-tradables is low, as illustrated in Figure 2.

- Tradables and non-tradables are Edgeworth complements (i.e., $u_{c^T c^N} > 0$). In terms of Figure 1, when y^N is high, the curve $u_{c^T}(c_t^T, y_t^N)$ shifts to the right and the equilibrium moves from point A to point C. In this case, if the path of y^T is flat, then the economy will run trade surpluses when y_t^N is low and deficits when y_t^N is high.

[Figure 2]

In general equilibrium – and given the exogenously given path of y_t^N – the path of p_t will adjust so as to induce the consumer to choose the existing path of consumption of non-tradable goods. From equations (20) and (21), the path of p_t is given by

$$p_t = \frac{u_{c^N}(c_t^T, y_t^N)}{u_{c^T}(c_t^T, y_t^N)}. \tag{23}$$

To figure out how fluctuations in y^N will affect p, totally differentiate (23) with respect to y^N and use (22) to obtain:

$$\frac{dp}{dy^N} = \frac{1}{u_{c^T} u_{c^T c^T}} \underbrace{(u_{c^N c^N} u_{c^T c^T} - u_{c^T c^N}^2)}_{+} < 0, \tag{24}$$

where the term in brackets is positive due to the strict concavity of the utility function. It follows that if, along a perfect foresight equilibrium, the endowment of non-tradable goods increases, the relative price of home goods will fall.[12] Intuitively, at an unchanged relative price, there would be an excess supply of non-tradable goods. To clear the market, the relative price needs to fall.

In sum – and relative to Chapter 1 – we have identified a new channel that affects an economy's desire to run trade imbalances. This channel is related to the effects that changes in the endowment of non-tradable goods have on the desire to consume tradable goods. Hence, in a general case in which both the endowment of tradable goods and non-tradable goods fluctuate over time, trade imbalances will be dictated by (i) the consumption smoothing motive analyzed in Chapter 1 and (ii) the effects of fluctuations in the endowment of non-tradable goods on tradable-goods consumption.

Given this new channel, a natural question that arises is the following: could fluctuations in the supply of non-tradable goods provide a possible explanation for the countercyclicality of the trade balance shown by the data? In other words – and in the world of Chapter 1 – could we dispense with investment and still explain the countercyclical behavior of the trade balance? As Exercise 1 at the end of this chapter makes clear, the answer is negative: under the most plausible parametrization of preferences, good times (i.e., times of high endowment of non-tradable goods) will be associated with trade surpluses and bad times with trade deficits.

3 External deficits and the real exchange rate

An important question to ask in the presence of non-tradable goods is the following: what is the relation between external imbalances (i.e., trade and current account imbalances) and the real exchange rate? In other words, will external deficits be accompanied by an appreciated real exchange rate? We will answer this question in the context of the basic model developed in Section 2 with the only modification that, in order to simplify the presentation, we will focus on the separable and logarithmic case.[13] The experiment that

[12] This is, of course, fully consistent with the partial equilibrium effects of p_t that we analyzed above. In other words, when y_t^N increases, p_t falls, and the consumer reacts in the way described by (10).

[13] Exercise 3 at the end of the chapter shows that Edgeworth substitutability is a sufficient (though not necessary) condition for the same results to go through for a general utility function.

we will undertake will be an unanticipated and temporary reduction in the endowment of both goods.

Let lifetime utility be given by:

$$W = \int_0^\infty \left[\gamma \log(c_t^T) + (1-\gamma)\log(c_t^N)\right] e^{-\beta t} dt. \tag{25}$$

The flow and intertemporal constraints continue to be given by equations (4) and (5).

Under these preferences, first-order conditions (8) and (9) reduce to:

$$\frac{\gamma}{c_t^T} = \lambda, \tag{26}$$

$$\frac{1-\gamma}{c_t^N} = \lambda p_t. \tag{27}$$

Combining these two conditions we obtain:

$$\frac{c_t^N}{c_t^T} = \left(\frac{1-\gamma}{\gamma}\right)\frac{1}{p_t}. \tag{28}$$

As made clear below, this condition can be interpreted as a demand function for non-tradable goods relative to tradable goods. As in standard consumer theory, this demand function depends negatively on the relative price of non-tradables, p_t.

The economy's equilibrium continues to be given by equations (17), (18), and (19).

3.1 Initial equilibrium

Suppose that the endowment of both tradables and non-tradables is constant over time (i.e., $y_t^T = y^T$ and $y_t^N = y^N$). Equation (26) then tells that c^T will be constant over time at a level given by (19)

$$c_t^T = rb_0 + y^T. \tag{29}$$

From (17), the equilibrium value of non-tradable goods is given by the constant value of the endowment:

$$c_t^N = y^N. \tag{30}$$

Combining (28), (29) and (30) yields the reduced form for the relative price of home goods:

$$p = \left(\frac{1-\gamma}{\gamma}\right)\frac{rb_0 + y^T}{y^N}. \tag{31}$$

The determination of the relative price of non-tradable goods, p, is best understood in terms of a familiar supply and demand diagram. Figure 3 depicts the supply and demand for non-tradable *relative* to tradables goods. The demand function is given by (28) which, expressed in more familiar terms, takes the form:

$$D(p) = \left(\frac{1-\gamma}{\gamma}\right)\frac{1}{p}.$$

[Figure 3]

Since there is no production in this economy, the supply function is a vertical line, given by

$$S(p) = \frac{y^N}{rb_0 + y^T}.$$

Notice that since the initial stock of net foreign assets, b_0, constitutes a claim for tradables goods on the rest of the world, it must be part of the "supply" of tradable goods. The intersection of these two curves, $D(p) = S(p)$, yields the equilibrium value of p (point A in Figure 3).

3.2 Comparative statics

There are several simple experiments that we can perform that will hone our understanding of the determination of the relative price of non-tradable goods.

Consider first an unanticipated and permanent increase in the supply of tradable goods. Since the change is permanent, the equilibrium discussed above remains valid (for the new value of y^T). As expected, consumption of tradable goods will increase one-for-one with the increase in y^T, as equation (29) makes clear. The equilibrium relative price of non-tradable goods will also increase, as follows from (31). Intuitively – and in terms of Figure 3 – a permanent increase in y^T shows up as a leftward shift of the vertical supply schedule. At the initial value of p, there is an excess demand for non-tradable goods. Hence, the relative price of non-tradable goods must increase to clear the market once again. In the new equilibrium (point B), the relative price is higher than initially.

Consider an unanticipated and permanent increase in the supply of non-tradable goods. In terms of Figure 3, this shock would show up as a rightward shift of the vertical supply schedule. At the initial relative price, there would be an excess supply of non-tradable goods. Hence, p needs to fall to increase the demand for non-tradable and clear the market.

Consider an unanticipated and permanent increase in the demand for tradable goods (i.e., an increase in γ). In terms of Figure 3, this shock would manifest itself as a leftward shift of the demand curve, which would lead to a lower p. Intuitively, at the initial relative price, there would an excess supply of non-tradables, which calls for a lower equilibrium relative price. Conversely, a fall in γ would lead to an increase in p. [14]

3.3 Temporary fall in supply

While the permanent shocks just studied are quite helpful for understanding how different shocks affect the real exchange rate, they cannot shed any light on the relation between external imbalances and the real exchange rate. For these purposes, we need to study temporary shocks.

Suppose that the economy is initially in the stationary equilibrium described above. Suppose, for simplicity, that $b_0 = 0$. At $t = 0$, there is an unanticipated and temporary (equiproportional) fall in the endowment of both goods (i.e., both y_t^T and y_t^N fall but the ratio y_t^T/y_t^N remains unchanged), as depicted in Figure 4, Panel A.[15]

[Figure 4]

What happens to the endogenous variables at time T? Clearly, c^T remains constant at T as follows from (26). We also know from (27) – taking into

[14] Exercise 5 at the end of the chapter illustrates a case in which tradables act as an input in the production of non-tradables (which are the only consumption good). In that case, an exogenous increase in the demand for non-tradables also leads to an increase in the relative price of non-tradables.

[15] We study an equiproporcional change to abstract from effects caused by changes in the *relative* supply of tradable and non-tradable goods.

account (30) – that py^N remains constant at T. Hence, since y^N increases at time T, p falls at time T.

Given that the path of c^T is flat from time 0 onward and that the present discounted value of tradable resources has fallen, we know that c^T will fall at time 0 (Figure 4, Panel B). The path of c^N simply follows the path of y^N (Figure 4, Panel C).

What happens to p on impact? From (28), we know that

$$p_0 = \left(\frac{1-\gamma}{\gamma}\right)\frac{c_0^T}{c_0^N}.$$

It follows that p will increase at time $t = 0$ because while c_0^N falls by the same amount as y^N does, c_0^T falls by less (i.e., it falls by the annuity value of the fall in y^T). At time T, p falls below its initial level (Figure 4, Panel D).[16]

The path of the trade balance follows from the paths of y^T and c^T (see Figure 4, Panel E). The economy runs a trade deficit between time 0 and time T and a surplus afterwards. The path of the current account follows immediately from that of the trade balance. At $t = 0$, the current account goes into deficit and then the deficit increases over time as the economy accumulates debt. At $t = T$, the increase in y^T is such that the current account becomes balanced.

The punchline of this experiment is that trade deficits go hand in hand with real appreciation while trade surpluses go hand in hand with real depreciation. This association between trade deficits and real appreciation is intuitively clear: on impact, there is an excess demand for both goods, which leads to a trade deficit (the economy imports from abroad the desired tradable goods) and a real appreciation (the relative price of non-tradable goods must increase to clear the market). Put differently, the excess demand for tradable goods is fully met by a quantity adjustment whereas the excess demand for non-tradable goods is fully met by a relative price adjustment.

An important corollary is that, in the context of this paradigm, the commonly-held view that real appreciation "causes" trade deficits (or, conversely, that real depreciation "causes" trade surpluses) would be simply incorrect. Here trade deficits and real appreciation (or trade surpluses and real depreciation) are simply an equilibrium response to a common shock;

[16] We know this because at time T, c^N has returned to its pre-shock value, while c^T remains below its pre-shock value.

there is no causality whatsoever.[17]

The same association between trade imbalances and the real exchange rate would emerge in response to a temporary demand shock that increases the demand for both tradable and non-tradable goods (Exercise 4 at the end of this Chapter asks you to work out this case). Intuitively, on impact a positive demand shock increases the demand for both tradable and non-tradable goods. While the excess demand for tradable goods can be met by importing goods from abroad (i.e., by running a trade deficit), the excess demand for non-tradable goods must be choked off by an increase in the relative price of non-tradable goods (i.e., a real appreciation).

4 Fiscal policy, trade imbalances, and the real exchange rate

Will a fiscal contraction lead to a real depreciation? Will a fiscal expansion cause a trade deficit? This section will seek to answer these key policy questions regarding the effects of fiscal policy.

4.1 Government spending with lump sum taxation

Let us incorporate government spending into our basic model with logarithmic preferences. To isolate the effects of changes in government spending on the real exchange rate, we will assume in this subsection that government spending is financed by lump-sum taxation. The consequences of relaxing this assumption and considering distortionary taxation will be addressed below.

4.1.1 Consumers

We continue to work with the logarithmic preferences given in (25). As far as the consumer is concerned, the only modification of the model is that he/she is subject to lump-sum taxes, τ_t. The flow constraint, (4), will now read as

$$\dot{b}_t = rb_t + y_t^T + p_t y_t^N - c_t^T - p_t c_t^N - \tau_t. \tag{32}$$

[17] To see models in which one could meaningfully argue that real appreciations "cause" trade deficits, we will need to wait until Chapter 8 where we will revisit some of these issues in the context of a monetary model with sticky prices.

By the same token, the intertemporal constraint will now be given by

$$b_0 + \int_0^\infty (y_t^T + p_t y_t^N - \tau_t) e^{-rt} dt = \int_0^\infty (c_t^T + p_t c_t^N) e^{-rt} dt. \tag{33}$$

It should be clear that the introduction of lump-sum taxation does not affect the consumer's first-order conditions, which continue to be given by (26) and (27).

4.1.2 Government

Up to this point in this chapter, the government has played no role. It now comes into action. The government spends on both tradables (g_t^T) and non-tradable goods (g_t^N) and finances this spending with lump-sum taxes, τ_t. While g_t^T and g_t^N are assumed to be policy variables (i.e., they are exogenous variables), the path of τ_t will be endogenously determined so as to satisfy the government budget constraint. To simplify the presentation we will assume that the government has no initial debt and that it balances its budget period by period:

$$\tau_t = g_t^T + p_t g_t^N. \tag{34}$$

4.1.3 Equilibrium conditions

Equilibrium in the non-tradable goods sector requires that

$$c_t^N + g_t^N = y_t^N. \tag{35}$$

Combining the consumer's flow constraint (32) with the government's (34), we obtain the flow constraint for the economy as a whole (i.e., the current account):

$$\dot{b}_t = r b_t + y_t^T - c_t^T - g_t^T. \tag{36}$$

Similarly, by substituting the government's constraint (34) into the consumer's intertemporal budget constraint (33) and imposing equilibrium in the non-tradable goods market (condition (35)), we obtain the economy's resource constraint:

$$b_0 + \int_0^\infty y_t^T e^{-rt} dt = \int_0^\infty (c_t^T + g_t^T) e^{-rt} dt. \tag{37}$$

4.1.4 Perfect foresight equilibrium

Along a perfect foresight equilibrium, consumption of tradable goods will be constant, as follows from condition (26). Taking into account the resource constraint (37), we obtain a reduced form for c^T:

$$c^T = r\left[b_0 + \int_0^\infty \left(y_t^T - g_t^T\right)e^{-rt}dt\right]. \tag{38}$$

The path of consumption of non-tradable goods follows directly from the equilibrium in the non-tradable goods market, equation (35):

$$c_t^N = y_t^N - g_t^N. \tag{39}$$

From (28), (38), and (39), we obtain a reduced form for p_t:

$$p_t = \left(\frac{1-\gamma}{\gamma}\right)\frac{r\left[b_0 + \int_0^\infty \left(y_t^T - g_t^T\right)e^{-rt}dt\right]}{y_t^N - g_t^N}. \tag{40}$$

Some important observations follow from this characterization of the perfect foresight equilibrium. First, equation (40) makes clear that the path of p_t depends only on the present discounted value of g_t^T, and not on the particular time path of g_t^T. The reason is that, from the consumer's standpoint, he/she only cares about how much tradable resources the government is taking away from him/her in present value terms. The path of g_t^T is irrelevant because, as in Chapter 1, the consumer will smooth out this shock over time. In contrast, the path of g_t^N will affect the relative price of non-tradable goods. All else equal, when g_t^N is high (low), p will also be high (low). Intuitively, periods of, say, high g_t^N imply a reduced supply of non-tradable goods available to the private sector. The resulting excess demand for non-tradable goods must lead to an increase in their relative price. In terms of Figure 3, a high g_t^N would imply a leftward shift of the supply schedule.

Second, given the exogenous paths of g_t^T and g_t^N and the path of p_t determined by equation (40), the government's flow constraint determines the level of lump-sum taxes:

$$\tau_t = g_t^T + p_t g_t^N. \tag{41}$$

4.1.5 Comparative statics

To analyze the effects of permanent changes in government spending, let us first derive the perfect foresight path corresponding to a constant path of the exogenous variables. Suppose then that $y_t^T = y^T$, $y_t^N = y^N$, $g_t^T = g^T$ and $g_t^N = g^N$. Given these constant paths, the perfect foresight equilibrium characterized by equations (38), (39), and (40) reduces to (set also $b_0 = 0$):

$$
\begin{aligned}
c^T &= y^T - g^T, && (42)\\
c^N &= y^N - g^N, && (43)\\
p &= \left(\frac{1-\gamma}{\gamma}\right)\frac{y^T - g^T}{y^N - g^N}, && (44)\\
\tau &= g^T + \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{y^T - g^T}{y^N - g^N}\right)g^N. && (45)
\end{aligned}
$$

What will be the effects of unanticipated and permanent changes in g^T and g^N? The answer is, of course, straightforward since the new perfect foresight equilibrium will also be characterized by equations (42)-(45). A permanent increase in g^T will thus lead to a one-for-one reduction in c^T and a fall in p. To understand this result intuitively, we can resort again to Figure 3 and think of the vertical supply schedule as depicting $\left(y^N - g^N\right)/(y^T - g^T)$. In this case, an increase in g^T would imply a rightward shift of the vertical supply schedule. At the initial value of p, there would be an excess supply of non-tradable goods which requires a fall in p. Similarly, an unanticipated and permanent increase in g^N leads to a one-for-one fall in c^N and a rise in p. In terms of Figure 3, the higher g^N would lead to a leftward shift of the vertical supply schedule. Hence, at the initial relative price, there is an excess demand for non-tradable goods, which requires an increase in the relative price, p.

In practice, however, governments normally change *total* government spending, as opposed to spending on either tradables or non-tradables. What will be the effect of an increase in total government spending on the real exchange rate? To answer this question in a simple and illuminating way, suppose that the government sets an overall level of spending equal to g (expressed in terms of tradable goods) and spends in fixed proportions; namely:

$$g^T = \alpha g, \tag{46}$$

$$pg^N = (1-\alpha)g, \tag{47}$$

where $0 \leq \alpha \leq 1$ denotes the proportion of government spending spent on tradable goods and $1-\alpha$ the proportion spent on non-tradable goods. Substituting (46) and (47) into (44), it follows that

$$p = \frac{1-\gamma}{\gamma}\frac{y^T}{y^N} + \frac{g}{y^N}\left(1 - \frac{\alpha}{\gamma}\right).$$

In response to an unanticipated and permanent increase in g, the change in p will be given by

$$\frac{dp}{dg} = \frac{1-\frac{\alpha}{\gamma}}{y^N} = \begin{cases} 0 & \alpha = \gamma, \\ < 0 & \alpha > \gamma, \\ > 0 & \alpha < \gamma. \end{cases}$$

The effect of g on p thus depends on the *composition* of government spending relative to the *composition* of private spending. To see this, notice that we can rewrite the intratemporal condition (28) as

$$\frac{c_t^T}{p_t c_t^N} = \frac{\gamma}{1-\gamma},$$

which implies that the private sector spends a fraction γ of consumption expenditures on tradables and a fraction $1-\gamma$ on non-tradables. If government spending is biased towards tradable goods (i.e., $\alpha > \gamma$), then an increase in g would lead to a lower p. If government spending is biased towards non-tradable goods (i.e., $\alpha < \gamma$), then an increase in g would lead to a higher p. If the government spends in the same proportion as the private sector ($\alpha = \gamma$), then there is no change in p.

What is the intuition behind these results? When the government increases spending, it takes away those resources from the private sector. Given this fall in income, the private sector reduces the demand for tradable and non-tradable goods in a proportion given by $\gamma/(1-\gamma)$. If the government, in turn, spends in the same proportion, the market for non-tradable goods will clear at the initial relative price. If, in contrast, the government spends a higher fraction of these resources on non-tradables (which is the case when $\alpha < \gamma$), then there will be an excess demand for non-tradable goods, which will require a rise in p to clear the market. If the government spends a lower fraction on non-tradables ($\alpha > \gamma$), then there will an excess supply for non-tradable goods, which will lead to a fall in the relative price.

In conclusion, there is indeed a theoretical presumption that a, say, reduction in government spending will lead to a fall in the relative price of non-tradable goods. But, from a policy point of view, the question remains: will the *quantitative* effect of a fiscal contraction be large enough to merit such an action? This is, of course, an important empirical question that has been addressed in the literature (see Box 1). While estimates vary from study to study, the overall message is that this policy action could indeed have some significant effects on the real exchange rate.

4.1.6 Fiscal expansions and trade deficits

An important policy issue is the "twin deficits" hypothesis, which refers to the idea that budget deficits and trade deficits tend to occur simultaneously because, to some extent, the former causes the latter. While, by assumption, the budget is balanced, we can investigate one aspect of this issue: do increases in government spending lead to trade deficits? To answer this question, we need to look at temporary changes in government spending. We will first consider changes in g^T and then in g^N.

Temporary changes in government spending on tradable goods Suppose that the economy is initially in the stationary equilibrium characterized by equations (42), (43) and (44). At time 0, there is an unanticipated and temporary increase in g^T (Figure 5, Panel A).

[Figure 5]

Given first-order condition (26), we know that the path of c^T will be constant along the new perfect foresight equilibrium path. The level of c^T will be given by equation (38). Since the present discounted value of g^T has increased, the new level of c^T will be lower than initially (Figure 5, Panel B).

The path of consumption of non-tradable goods remains flat (Figure 5, Panel C) because there has been no change in the endowment path of non-tradables. As a result, the relative price of non-tradable goods falls on impact and remains flat thereafter (Figure 5, Panel D). Intuitively, due to the fall in the present discounted value of tradable resources available to the private sector, there is a excess demand for tradable goods relative to non-tradable goods which requires, in equilibrium, a higher relative price of tradables goods (i.e., a lower relative price of non-tradable goods).

What happens to the trade balance? Recall that the trade balance is given by:

$$TB_t = y^T - c_t^T - g_t^T.$$

Since c^T falls at time 0 by less than g^T increases (notice from (38) that c^T falls only by the permanent component of the increase in g^T), the trade balance worsens at time 0 (Figure 5, Panel E). At time T, the trade balance improves as g^T falls but c^T does not change. The corresponding path of the current account is illustrated in Figure 5, Panel F.

We thus conclude that a temporary increase in g^T leads indeed to a trade deficit. Intuitively, an increase in g^T is like a negative supply shock in Chapter 1. To smooth out consumption, the consumer will run a trade deficit. Exercise 6 at the end of this chapter shows that it is straightforward to extend this result and establish a link between a budget deficit and a trade deficit thus validating the twin deficits hypothesis.

Temporary changes in government spending on non-tradable goods
Would the same association between increases in government spending and trade deficits hold if the increase in government spending were on non-tradable goods? To answer this question – and for reasons that will become clear below – we need to consider the general preferences specified in equation (1), rather than those given by (25). We will further assume that tradables and non-tradables are Edgeworth substitutes; that is, $u_{c^T c^N} < 0$. Under these general preferences, the consumer's first-order conditions are given by equations (8) and (9). The rest of the model remains unchanged.

Imposing the equilibrium condition in the non-tradable goods markets (assuming a constant path for the endowment of non-tradables, given by equation (35)), we can rewrite the first-order conditions as:

$$u_{c^T}(c_t^T, y^N - g_t^N) = \lambda, \tag{48}$$

$$u_{c^N}(c_t^T, y^N - g_t^N) = \lambda p_t. \tag{49}$$

Combining these two equations, we obtain:

$$p_t = \frac{u_{c^N}(c_t^T, y^N - g_t^N)}{u_{c^T}(c_t^T, y^N - g_t^N)}. \tag{50}$$

Suppose that, starting from a stationary equilibrium, there is an unanticipated and temporary increase in g^N at time 0 (Figure 6, Panel A). Naturally, the path of c^N follows directly from the path of g^N and the non-tradable goods market equilibrium, as illustrated in Figure 6, Panel B.

[Figure 6]

What happens at time T? To find out, totally differentiate equation (48) to obtain

$$\left.\frac{dc^T}{dg^N}\right|_{t=T} = \frac{u_{c^T c^N}}{u_{c^T c^T}} > 0. \tag{51}$$

Notice that the sign of this expression crucially depends on the assumption that tradable and non-tradable goods are Edgeworth substitutes (i.e., $u_{u_{c^T c^N}} < 0$). Hence, at time T, when g^N falls back to its initial level, c^T will also fall. Since the present discounted value of tradable resources has not changed relative to the pre-shock level, we infer that c^T rises at $t = 0$ and then follows the path illustrated in Figure 6, Panel C.

How does p change at time T? Totally differentiate optimality condition (50) and use (51) to obtain

$$\left.\frac{dp}{dg^N}\right|_{t=T} = -\frac{1}{u_{c^T c^T} u_{c^T}} \underbrace{\left(u_{c^T c^T} u_{c^N c^N} - u_{c^T c^N}^2\right)}_{+} > 0, \tag{52}$$

by strict concavity of the utility function.[18] Hence, when g^N falls at time T, p also falls.

How will the level of p at time T compare to the pre-shock level? Since we know that, at time T, g^N is back to its pre-shock level and c^T is lower, we can simply differentiate the optimality condition (50) with respect to c^T holding constant g^N to obtain:

$$\left.\frac{dp_t}{dc^T}\right|_{\text{constant } g^N} = \frac{1}{u_{c^T}^2} \underbrace{\left(u_{c^T} u_{c^T c^N} - u_{c^N} u_{c^T c^T}\right)}_{+} > 0,$$

where the sign of the term in brackets follows from the assumption that c^N is a normal good (recall condition (3)). Hence, p at time T will be lower than before the shock. Intuitively, the lower level of c^T (compared to its pre-shock

[18] Notice that, as we should have expected, this expression is the same as (24) but with the opposite sign.

level) needs to be accommodated, in equilibrium, by a higher relative price of tradable goods (i.e., a lower p).

How will p react on impact? To figure this out, differentiate (50) with respect to c^T and g^N to obtain:

$$\left.\frac{dp}{dg^N}\right|_{t=0} = \frac{1}{u_{c^T}^2}\left[\underbrace{(u_{c^N}u_{c^Tc^N} - u_{c^T}u_{c^Nc^N})}_{+} + \underbrace{(u_{c^T}u_{c^Nc^T} - u_{c^N}u_{c^Tc^T})}_{+}\underbrace{\left.\frac{dc^T}{dg^N}\right|_{t=0}}_{+}\right] > 0,$$

where, as indicated, the sign of the terms in brackets follows from the assumption that c^T and c^N are normal goods (recall conditions (2) and (3)) and, from Figure 6, Panel C, we know that dc^T/dg^N at $t = 0$ is positive.[19] Hence, on impact, the relative price of non-tradable goods will increase, as illustrated in Figure 6, Panel D. Intuitively, there are two forces acting upon p at $t = 0$ that reinforce each other. First, the increase in g^N leads to a reduction in the supply of non-tradable goods available to the private sector. The resulting excess demand for non-tradables would lead to an increase in p. Second, the rise in c^T means that, at the pre-shock relative price, there is also an excess demand for c^N (to keep the marginal rate of substitution constant), which should also lead to an increase in p.

The path of the trade balance (Figure 6, Panel E) follows directly from that of c^T. The corresponding path of the current account is illustrated in Figure 6, Panel F.

We thus conclude that, provided that the consumption of tradable and non-tradable goods are Edgeworth substitutes (i.e., $u_{c^Tc^N} < 0$), a temporary increase in g^N leads to a trade deficit.[20] The channel through which this happens is precisely the one that we studied in Section 2 above when we discussed the effects that fluctuations in the endowment of non-tradable goods may have on the trade balance. Intuitively, an increase in g^N reduces non-tradable resources available to the private sector and reduces c^N. If c^T and c^N are Edgeworth substitutes, then a reduction in c^N increases the marginal utility of consuming tradable goods, which leads to a higher c^T and, hence, to a trade deficit.

[19] Notice that we cannot use expression (51) – which we use to derive equation (52) – because expression (51) only holds at time T.

[20] It is worth noticing, though, the the path of p is not dependent on the sign of the cross-derivative.

4.2 Effects of taxation on the real exchange rate

We have analyzed above the effects of changes in government spending on the real exchange rate. To isolate such effects, we abstracted from the effects of distortionary taxes by assuming that the government could resort to lump-sum taxes (recall equation (41)). In that case, the government sets the spending level on both goods and allows lump-sum taxes to be endogenously determined so as to satisfy the flow budget constraint. In this subsection, we will assume the opposite: the government will set tax rates and let the level of government spending adjust endogenously. In addition – and to abstract from unwanted wealth effects – we will assume that government spending (which is only on tradable goods) takes the form of a transfer to households, which implies that the level of government spending will not affect the economy's resource constraint. In this way, we can isolate the effects of distortionary taxation without clouding the picture with unnecessary features.

4.2.1 Consumers

Let preferences be given by (25). Let θ_t^T and θ_t^N denote the tax rate on tradable and non-tradable goods, respectively. The consumer's flow constraint then becomes

$$\dot{b}_t = rb_t + y_t^T + p_t y_t^N + g_t^T - (1+\theta_t^T)c_t^T - (1+\theta_t^N)p_t c_t^N. \tag{53}$$

The term g^T shows up in the consumer's flow constraint because we are assuming that government spending is transferred to the consumer. The corresponding intertemporal constraint is given by

$$b_0 + \int_0^\infty (y_t^T + p_t y_t^N + g_t^T)e^{-rt}dt = \int_0^\infty \left[(1+\theta_t^T)c_t^T + (1+\theta_t^N)p_t c_t^N\right] e^{-rt}dt. \tag{54}$$

The first-order conditions are now given by:

$$\frac{1}{c_t^T} = \lambda(1+\theta_t^T), \tag{55}$$

$$\frac{1}{c_t^N} = \lambda(1+\theta_t^N)p_t. \tag{56}$$

First-order condition (55) brings us back to the world of Chapter 3: if the tax rate varies over time, there will be an intertemporal distortion and consumption of tradables goods will not be constant over time.

Combining these two conditions we obtain:

$$\frac{c_t^T}{c_t^N} = p_t \left(\frac{1+\theta_t^N}{1+\theta_t^T} \right). \tag{57}$$

This equation allows us to introduce the key concept of an *intratemporal* distortion. In contrast to the concept of *intertemporal* distortion introduced in Chapter 3 (which, as noted above, reemerges in equation (55)), an intratemporal distortion is a static distortion that affects the consumer's choice between two goods at a particular point in time. In this case, if θ_t^T is different from θ_t^N, then the effective relative price of non-tradable goods faced by the consumer, given by $p_t(1+\theta_t^N)/(1+\theta_t^T)$, is different from p. As result, the consumer's choices will not correspond to a first-best equilibrium.

In sum, there are two types of distortion that may result from tax policy:

- Intertemporal distortion. To isolate this case, think of uniform taxation across goods (i.e., $\theta_t^T = \theta_t^N = \theta_t$). A non-constant path of θ_t would impose an intertemporal distortion and induce a sub-optimal choice of consumption paths, along the lines examined in Chapter 3.

- Intratemporal distortion. To isolate this case, think of constant tax rates over time (i.e., $\theta_t^T = \theta^T$ and $\theta_t^N = \theta^N$) but different from one another (i.e., $\theta^T \neq \theta^N$). In this case, the effective relative price faced by the consumer would differ from the market price, p, and hence the consumer's choices will be sub-optimal.

4.2.2 Government

The government is assumed to spend only on tradable goods and, as before, runs a budget balance. The government's flow constraint is now given by

$$g_t^T = \theta_t^T c_t^T + \theta_t^N p_t c_t^N. \tag{58}$$

It is worth stressing that in this case the tax rates, θ_t^T and θ_t^N, are policy variables (i.e., exogenous variables) and g^T is endogenously determined.

4.2.3 Equilibrium conditions

As before, equilibrium in the non-tradable goods market requires that condition (35) hold. Combining the consumer's flow constraint (equation (53)) with the government's (equation (58)) and imposing non-tradable goods market equilibrium yields the current account, which is still given by expression (**??**).[21] Finally, combining the consumer's intertemporal constraint with the government's flow constraint and imposing non-tradable goods market equilibrium yields the economy's resource constraint, given by (19).

4.2.4 Initial stationary equilibrium

Let us solve for a perfect foresight equilibrium with constant paths of the exogenous variables, y_t^T, y_t^N, θ_t^T, and θ_t^N at the levels y^T, y^N, θ^T, and θ^N, respectively. From first-order condition (55), we infer that c_t^T will be constant over time. Hence, from the resource constraint (19), the constant value of c^T is given by

$$c_t^T = rb_0 + y^T. \tag{59}$$

Home-goods market equilibrium indicates that

$$c_t^N = y^N. \tag{60}$$

Given the constant values of c^T, c^N, θ^T and θ^N, the equilibrium value of p follows from equation (57):

$$p_t = \frac{c^T(1+\theta^T)}{y^N(1+\theta^N)}. \tag{61}$$

4.2.5 Static effects of changes in tax rates

How do unanticipated and permanent changes in tax rates affect the relative price of non-tradable goods? Let us start by an unanticipated and permanent increase in θ^T. Clearly, consumption of tradable goods is still given by (59). Hence, it follows from (61) that p will increase as a result of the increase in θ^T. Intuitively, an increase in θ^T makes tradable goods more expensive, thus leading to an excess supply of tradables (or, equivalently an excess

[21] Since g^T is transferred to the consumer, it does not show up in the economy's flow constraint.

demand for non-tradables). The relative price of non-tradable goods must thus increase to clear the market.

This experiment can be analyzed graphically with the help of Figure 3. In the presence of distortionary taxes, the relevant demand curve for non-tradables relative to tradables goods becomes (as follows from (57))

$$D(p) = \frac{1}{p}\left(\frac{1+\theta^T}{1+\theta^N}\right).$$

In terms of Figure 3, a permanent increase in θ^T is tantamount to a rightward shift of the demand curve because, for a given p, the demand for non-tradables increases. In the new equilibrium (not drawn), supply and demand would intersect at a higher p.

What is the effect on the real exchange rate of an increase in θ^N? The answer follows immediately from equation (61). An increase in θ^N will lead to a fall in p (i.e., a real depreciation). In terms of Figure 3, this experiment would be captured by a leftward shift of the demand curve since for a given p the demand for non-tradables falls. Hence, at the initial p, there would be an excess supply of non-tradable goods which would require a fall in p to equilibrate the market.

4.2.6 Intertemporal effect of changes in tax rates

To analyze the intertemporal distortions introduced by taxation, let us assume that taxation is uniform across goods; that is $\theta_t^T = \theta_t^N = \theta_t$.

Under uniform taxation across goods, optimality conditions (55) and (57) reduce to:

$$\frac{1}{c_t^T} = \lambda(1+\theta_t), \tag{62}$$

$$\frac{c_t^T}{c_t^N} = p_t. \tag{63}$$

Since taxation is uniform across goods, there is no static distortion, as (63) makes clear.

The government flow constraint is now given by (we continue to assume balanced budget)

$$g_t^T = \theta_t(c_t^T + p_t c_t^N).$$

Let us begin by computing the initial perfect foresight equilibrium corresponding to flat paths of y^T, y^N, and θ. From (19) and (62), it follows that the constant level of c^T is still given by (59). The constant level of consumption of non-tradable goods continues to be given by (60). The equilibrium relative price is given by:

$$p_t = \frac{c^T}{y^N}. \tag{64}$$

Suppose now that at time 0, there is an unanticipated and temporary fall in θ (Figure 7, Panel A). Since the Lagrange multiplier will remain constant along the new perfect foresight equilibrium, condition (62) tells us that at time T, c^T will fall in response to the increase in θ. In other words, consumption of tradable goods before T will be higher than after T. Since the present discounted value of tradable resources has not changed, the path of c^T will be as illustrated in Figure 7, Panel B. Naturally, the path of c^N remains flat at the initial level (Figure 7, Panel C).

[Figure 7]

In light of condition (64) and the paths for c^T and c^N just described, the path of p will be as illustrated in Figure 7, Panel D. The relative price of non-tradable goods will rise on impact and then fall at time T. Intuitively, at time 0, there is an increase in the demand for *both* tradables and non-tradables in response to temporarily lower taxes. The higher demand for tradables will be satisfied through a trade deficit (as made clear below). However, the higher demand for non-tradable goods will have to be met by a higher relative price of non-tradable goods, given that the supply is fixed. At time T, the opposite is true.

As for the external accounts, recall that the trade balance is given by

$$TB_t = y^T - c_t^T.$$

The economy thus runs a trade deficit during $[0, T)$ and a surplus afterwards (Figure 7, Panel E). The corresponding path of the current account is illustrated in Figure 7, Panel F.

We conclude that a fiscal expansion engineered via a temporary reduction in tax rates will lead to a trade deficit and real appreciation. Conversely, a fiscal contraction brought about by a temporary increase in tax rates would result in a trade surplus and real depreciation. We should stress that these effects are due exclusively to intertemporal consumption substitution because we have purposely abstracted from wealth effects.

5 Resource reallocation and relative prices

This section modifies the basic model introduced in Section 2 by endogeneizing production. We will assume that there is one scarce factor of production (labor) that must be allocated to the production of either tradable or non-tradable goods. Labor services are non-tradable internationally. To simplify the exposition – and since it is not central to this section's message – we will also assume that labor supply is exogenous (i.e., leisure does not enter the utility function).

The key feature that we wish to highlight in this section is the role of the relative price of non-tradable goods in guiding the allocation of the scarce factor among sectors.

5.1 Consumer's problem

The lifetime utility of the representative individual is given by:

$$\int_0^\infty (\log c_t^T + \log c_t^N) e^{-\beta t} dt. \tag{65}$$

The flow constraint of the consumer is given by

$$\dot{b}_t = r b_t + y_t^T + p_t y_t^N - c_t^T - p_t c_t^N, \tag{66}$$

where y^T and y^N denote the production of tradable and non-tradable goods, respectively.

Production takes place according to the following technologies:

$$y_t^T = Z_t^T (n_t^T)^\alpha, \qquad 0 < \alpha < 1, \tag{67}$$

$$y_t^N = Z_t^N n_t^N, \tag{68}$$

where Z^T and Z^N are (time-varying) productivity parameters, n^T and n^N denote the amount of labor used in each sector, and α captures decreasing returns to scale in the production of tradable goods.[22]

Finally, there is the labor supply constraint:

$$n_t = n_t^T + n_t^N, \tag{69}$$

where n is the (exogenously-given) endowment of labor.

Integrating forward equation (66) we can write the individual's intertemporal budget constraint as

$$\int_0^\infty (c_t^T + p_t c_t^N) e^{-rt} dt = b_0 + \int_0^\infty (y_t^T + p_t y_t^N) e^{-rt} dt \tag{70}$$

The consumer's problem consists in choosing $\{c_t^T, c_t^N, n_t^T, n_t^N\}$ for all $t \in [0, \infty)$ to maximize lifetime utility (65), subject to (67), (68), (69), and (70), for a given path of Z_t^T, Z_t^N, n_t and a given b_0. In terms of the Lagrangian,[23]

$$\begin{aligned} \mathcal{L} &= \int_0^\infty (\log c_t^T + \log c_t^N) e^{-\beta t} dt + \lambda \left\{ b_0 + \int_0^\infty \left[Z_t^T (n_t^T)^\alpha + p_t Z_t^N n_t^N \right] e^{-rt} dt \right. \\ &\quad \left. - \int_0^\infty (c_t^T + p_t c_t^N) e^{-rt} \right\} dt + \int_0^\infty e^{-rt} \mu_t (n_t - n_t^T - n_t^N) dt. \end{aligned}$$

The first-order conditions are given by:

$$\begin{aligned} \frac{1}{c_t^T} &= \lambda, & (71) \\ \frac{1}{c_t^N} &= \lambda p_t, & (72) \\ \lambda \alpha Z_t^T (n_t^T)^{\alpha-1} &= \mu_t, \\ \lambda p_t Z_t^N &= \mu_t. \end{aligned}$$

[22] To simplify the presentation, we assume that the production technology for non-tradables goods is linear (in practice, the non-tradable sector is indeed more labor intensive than the tradable sector). This is not critical, however, for the results derived in the text. Indeed, as Exercise 7 at the end of this chapter asks you to verify, the same results would go through regardless of which sector is more labor intensive (or if they are equally intensive). As analyzed in the same exercise, labor intensity would matter, however, for the effects of an increase in the labor supply.

[23] We multiply the multiplier μ_t by e^{-rt} so that it is expressed in current value terms (rather than present discounted value terms). Of course, results are the same regardless of how we set it up.

Combining the last two conditions yields the production efficiency condition:

$$\alpha Z_t^T (n_t^T)^{\alpha-1} = p_t Z_t^N. \tag{73}$$

This condition says that, at an optimum, the marginal productivity of labor must be the same across the two sectors. If this were not the case, it would be optimal to reallocate labor from the low marginal productivity sector to the high marginal productivity sector.

5.2 Equilibrium conditions

In equilibrium, the consumption of non-tradables must equal the production of non-tradable goods:

$$c_t^N = y_t^N. \tag{74}$$

Combining equations (66) and (74) yields the economy's current account:

$$\dot{b}_t = rb_t + y_t^T - c_t^T. \tag{75}$$

From (70) and (74), we obtain the intertemporal constraint of the economy,

$$\int_0^\infty c_t^T e^{-rt} dt = b_0 + \int_0^\infty y_t^T e^{-rt} dt. \tag{76}$$

5.3 Perfect foresight equilibrium

Let us characterize a perfect foresight equilibrium for a constant path of the exogenous variables: Z_t^T, Z_t^N, and n_t. Suppose then that $Z_t^T = Z^T$, $Z_t^N = Z^N$, and $n_t = n$.

Showing that the endogenous variables will also be constant along a perfect foresight path is somewhat more involved than in previous occasions. From equation (71), it is easy to establish that c^T will be constant over time. But to proceed further, we need to show that p will be constant over time. To this effect, notice that given the constant paths of Z_t^T and Z_t^N, condition (73) establishes that n^T and p are negatively related along a PFEP. Given the labor supply constraint (69) and the constancy of n, we then infer that n^N and p are positively related along a PFEP. Hence, by the equilibrium condition (74), c^N and p are positively related. It then follows from optimality condition (72) that p must be constant along a PFEP for, if it changed, this condition would be violated.

Having established that p is constant along a PFEP, the rest of the proof is straightforward. If p is constant, then by (72), c^N will also be constant. If c^N is constant, then n^N is also constant, which implies that n^T is constant.

Having shown that if Z^T, Z^N, and n^T are constant over time, then c_t^T, c_t^N, p_t, and n_t^N are also constant over time, we can solve implicitly for this perfect foresight equilibrium:

$$
\begin{aligned}
c^T &= rb_0 + Z^T(n^T)^\alpha, & (77)\\
c^N &= Z^N n^N, & (78)\\
p &= \frac{rb_0 + Z^T(n^T)^\alpha}{Z^N n^N}, & (79)\\
p &= \frac{\alpha Z^T(n^T)^{\alpha-1}}{Z^N}, & (80)\\
n &= n^T + n^N. & (81)
\end{aligned}
$$

For given values of Z^T, Z^N, n, and b_0, this can be viewed as a system of five equations in five unknowns: c_t^T, c_t^N, p_t, n_t^T and n_t^N.

5.4 Wealth effect

To illustrate how a wealth effect leads to a reallocation of resources across sectors, suppose that an instant before time 0, the economy is in the stationary equilibrium just described with net foreign assets given by b_0^L. At time 0, there is an unanticipated and permanent increase in b_0 (say, a foreign grant) to b_0^H ($b_0^H > b_0^L$). Given this unanticipated event, the consumer will reoptimize. Naturally, the new perfect foresight equilibrium will be characterized by the same five equations, (77) through (81), with b_0^H. Hence, from a mathematical point of view, we need to do a comparative statics exercise and find out how c_t^T, c_t^N, p_t, n_t^T and n_t^N respond to a higher b_0.

The best way to proceed is to substitute (80) and (81) into (79) to obtain n^T as a function of Z_t^T, Z_t^N, n and b_0:

$$\alpha n(n^T)^{\alpha-1} - \alpha(n^T)^\alpha - (n^T)^\alpha - rb_0(Z^T)^{-1} = 0. \qquad (82)$$

Totally differentiating with respect to n^T and b_0, we obtain:

$$\frac{dn^T}{db_0} = -\frac{-r/Z^T}{\alpha(\alpha-1)n(n^T)^{\alpha-2} - \alpha^2(n^T)^{\alpha-1} - \alpha(n^T)^{\alpha-1}} < 0. \quad (83)$$

It follows that in response to an increase in b_0, n^T will fall. From (81), it follows that n^N increases, which implies, by (78), that c^N increases. Given the fall in n^T, equation (80) indicates that p rises. The rise in p implies, by equation (79), that c^T not only rises but in fact rises by a higher proportion than c^N.

Intuitively, the higher b_0 implies that the economy has more tradable resources available and will therefore increase consumption of tradable goods. At an unchanged relative price, consumers will want to increase consumption of non-tradable goods by the same proportion. In an endowment economy, the higher demand for non-tradable goods would be reflected entirely in an increase in their relative price. In this economy with endogenous production, the excess demand for non-tradable goods – at the initial relative price – induces a rise in p. This rise in p brings about two effects, both of which tend to reduce the excess demand. First, the higher p increases the marginal productivity of labor in the non-tradable sector. This induces a reallocation of labor from the tradable to the non-tradable sector. Second, the higher p chokes off some of the excess demand for non-tradables. In other words, the combination of more supply and lower demand equilibrates the non-tradable goods market.

This experiment has thus illustrated the key role played by the relative price of non-tradable goods in bringing about a reallocation of resources across sectors that contributes to restoring equilibrium in this economy.

5.5 Demand shock

We will now study a temporary demand shock (which will lead to a boom-bust cycle) to illustrate the role of relative prices in engineering the adjustment needed for this economy to repay its debt. To this end, let us modify the preferences given by (25) to incorporate a demand shock, γ_t:

$$\int_0^\infty \gamma_t(\log c_t^T + \log c_t^N)e^{-rt}dt \quad (84)$$

The rest of the model remains the same. Under the preferences given by (84), the first-order conditions reduce to:

$$\frac{\gamma_t}{c_t^T} = \lambda, \tag{85}$$

$$\frac{\gamma_t}{c_T^N} = \lambda p_t, \tag{86}$$

$$Z_t^T (n_t^T)^{\alpha-1} = p_t Z_t^N. \tag{87}$$

We will characterize a perfect foresight equilibrium path for constant paths of Z^T, Z^N, and n but for a *non-constant* path of γ_t. Specifically, suppose that γ is high until T and low thereafter (Figure 8, Panel A). What will be the corresponding paths of c_t^T, c_t^N, p_t, n_t^T and n_t^N?

[Figure 8]

To answer this question, we need to focus on what happens at time T. From (85), we infer that at T, c_t^T will fall as a result of the fall in γ. From (86), we infer that $p_t c_t^N$ will fall at T. To proceed further, we need to show that p_t will also fall at time T. To this effect, we follow the same logic as above and establish that at time T, c_t^N is an increasing function of p_t (denoted by $\widetilde{c_t^N}(p_t)$) since there is no change in Z^T, Z^N, and n. Using first-order condition (86), we can then write that, at time T,

$$\frac{\gamma_t}{\widetilde{c_t^N}(p_t)} = \lambda p_t, . \tag{88}$$

Since γ_t falls at time T, it cannot be the case that p_t remains constant at time T or that it increases because, in either case, condition (88) would be violated. If p_t remained constant, then the LHS would fall and the RHS would not change and if p_t increased, then the LHS would fall and the RHS would increase. We conclude that p_t must fall at time T.

Since p falls at time T, condition (87) indicates that n^T increases at time T, leading to the path illustrated in Figure 8 Panel B. Hence, by the labor supply constraint, n^N falls at time T (Figure 8, Panel C). Given the paths of n^T and n^N, the path of c^T and c^N are given by Figure 8, Panels D and E, respectively. The path of p, in turn, is illustrated in Figure 8, Panel F. Since the output of tradable goods follows the path of n^T, the economy will run a trade deficit between 0 and T and a surplus after T (Figure 8, Panel G). The corresponding current account path is illustrated in Figure 8, Panel H.

This experiment thus illustrates a small open economy going through a boom-bust cycle, which is typical of developing countries. In the good times, consumption of both tradables and non-tradables is high, the relative price of non-tradable goods is high (real appreciation), production of tradables is low, production of non-tradables is high, and there is a trade deficit. When the situation turns for the worse, consumption of both goods collapses, the relative price of non-tradable goods falls, and the economy runs a trade surplus. In other words, the adjustment of the economy involves a sharp real depreciation, a collapse in consumption, and a switch in resources from the non-tradable to the tradable goods sector.[24] While disaggregated data for tradables and non-tradables is hard to come by, Box 2 illustrates the adjustment in the trade balance, the current account balance and the relative price of non-tradable goods for nine episodes in emerging countries in which consumption fell sharply. As argued in the box, the facts coincide exactly with the predictions of the model.

6 Terms of trade and the real exchange rate

Exports in most developing countries are often concentrated on some key commodities or agricultural products. Commodity prices, in particular, are extremely volatile, which results in large fluctuations in developing countries' terms of trade (i.e., the relative price of exports in terms of imports). The question of how will a, say, improvement in the terms of trade affect the real exchange rate has thus been a perennial one in development macroeconomics. The traditional wisdom has been that a deterioration in the terms of trade will lead to a real depreciation (see, for example, Edwards and Wijnbergen (1989)). To answer this question in our theoretical framework, we need to extend the basic model with non-tradable goods studied in Section 2 to allow for both exportables and importables. We will thus study a small open economy with three non-storable goods (exportables, importables and non-tradables). The exportable good will be the numeraire. There is a given and constant endowment path of exportables and non-tradables. The economy

[24] Several shocks – some of which we will see in more detail in later chapters – would lead to these same dynamics. For instance, the "demand shock" could be due to, say, a temporary exchange rate stabilization program along the lines of Calvo (1986) and Rebelo and Végh (1995) or to a temporary fall in international interest rates, along the lines of Edwards and Végh (1997).

consumes but is not endowed with importables. International bonds are denominated in terms of the exportable good.

6.1 Consumer's problem

The consumer's lifetime utility is given by

$$\int_0^\infty u(c_t^I, c_t^N)e^{-\beta t}dt, \tag{89}$$

where $\beta > 0$ is the subjective discount rate, and c^I and c^N denote consumption of importables and non-tradable goods, respectively.

In a model with three goods, we need to be careful with the definition of the various relative prices. In the model developed in Section 2, we denoted by p the relative price of non-tradable goods in terms of the numeraire (tradable goods). This relative price was the relevant one for consumption decisions. In the current model, the relevant relative price for consumption decisions is the relative price of non-tradable goods in terms of the importable good (denoted by p^I). There is, of course, another relative price: the relative price of the non-tradable good in terms of the exportable (denoted by p^x). Since p^I is the only relative price that affects marginal conditions, we will use it as our measure of the (inverse of) real exchange rate.[25] We will denote by q the relative price of importables in terms of exportables (i.e., the inverse of the terms of trade). Then, by definition,

$$p_t^I = \frac{p_t^x}{q_t}. \tag{90}$$

Our main question will be: how do changes in q (i.e., the inverse of the terms of trade) affect the relative price of non-tradable goods (p^I)?

Let b denote net foreign assets held by the consumer. The flow constraint is given by

$$\dot{b}_t = rb_t + y^x + p_t^x y^N - q_t c_t^I - p_t^x c_t^N, \tag{91}$$

where y^x and y^N denote the constant endowments of exportable and non-tradable goods, respectively. Taking into account (90), we can rewrite the flow constraint as:

[25] If exportables were consumed (or produced), it would make sense to define the real exchange rate in terms of some index for tradable goods (see Buffie (1999)).

$$\dot{b}_t = rb_t + y^x + q_t p_t^I y^N - q_t c_t^I - q_t p_t^I c_t^N, \tag{92}$$

Integrating forward (92), we obtain the intertemporal budget constraint:

$$b_0 + \int_0^\infty (y^x + q_t p_t^I y^N) e^{-rt} dt = \int_0^\infty (q_t c_t^I + q_t p_t^I c_t^N) e^{-rt} dt. \tag{93}$$

The consumer's problem consists in choosing paths of c_t^I and c_t^N to maximize lifetime utility (89), subject to (93), for a given path of p_t^I and q_t, and given values of y^x, y^N and b_0. The Lagrangian is given by

$$\begin{aligned} \mathcal{L} &= \int_0^\infty u(c_t^I, c_t^N) e^{-\beta t} dt \\ &\quad + \lambda \left[b_0 + \int_0^\infty (y^x + q_t p_t^I y^N) e^{-rt} dt - \int_0^\infty (q_t c_t^I + q_t p_t^I c_t^N) e^{-rt} dt \right]. \end{aligned}$$

The corresponding first order conditions (after assuming $\beta = r$) are:

$$u_{c^I}(c_t^I, c_t^N) = \lambda q_t, \tag{94}$$

$$u_{c^N}(c_t^I, c_t^N) = \lambda q_t p_t^I. \tag{95}$$

Combining these two first-order conditions, we obtain:

$$\frac{u_{c^N}(c_t^I, c_t^N)}{u_{c^I}(c_t^I, c_t^N)} = p_t^I. \tag{96}$$

Equation (94) says that, at an optimum, consumers equate the marginal utility of consuming each good to the shadow value of wealth times the relative price. Condition (96) says that, at an optimum, the consumer equates the marginal rate of substitution between non-tradables and importables to the corresponding relative price.

6.2 Equilibrium Conditions

The market of non-tradable goods clears:

$$c_t^N = y^N. \tag{97}$$

Imposing the equilibrium condition (97) into (91) yields the economy's flow constraint (i.e., the current account):

$$\dot{b}_t = rb_t + y^x - q_t c_t^I. \tag{98}$$

The corresponding resource constraint is given by:

$$b_0 + \int_0^\infty y^x e^{-rt} dt = \int_0^\infty q_t c_t^I e^{-rt} dt. \tag{99}$$

6.3 Perfect foresight equilibrium

Let us characterize a perfect foresight equilibrium for a constant path of q. It then follows from (94) that consumption of importables will be constant over time. From the resource constraint (99), this constant level will be given by

$$c^I = \frac{rb_0 + y^x}{q}. \tag{100}$$

Consumption of non-tradables will be equal to the endowment:

$$c^N = y^N. \tag{101}$$

Substituting the last two equations into (96),

$$p_t^I = \frac{u_{c^N}(c^I, y^N)}{u_{c^I}(c^I, y^N)}. \tag{102}$$

6.4 Permanent change in terms of trade

Suppose that, starting from the stationary equilibrium just derived, there is an unanticipated and permanent improvement in the terms of trade (i.e., a fall in q) at $t = 0$. Since there has been an unanticipated shock, consumers reoptimize at $t = 0$. Given that q will still be constant along the new perfect foresight equilibrium path, equation (100) remains valid. Clearly, since q has fallen, c^I will be higher in the new equilibrium. The increase in c^I reflects the wealth effect due to the fact that the same amount of exportables now buys a larger quantity of imports.

What happens to p^I? To find this out, assume that the change in q is small. We can then use (100) to compute the change in c^I:

$$\frac{dc^I}{dq} = -\frac{rb_0 + y^x}{q^2} < 0$$

Totally differentiating (102), we obtain

$$\frac{dp^I}{dq} = \frac{dc^I}{dq}\left(\frac{u_{c^I}u_{c^Ic^N} - u_{c^N}u_{c^Ic^I}}{u_{c^I}^2}\right) < 0,$$

since the term in brackets is positive if goods are normal and $dc^I/dq < 0$. This result implies that a permanent fall in q leads to an increase in p^I (a real appreciation).

Intuitively, an improvement in the terms of trade makes the consumer wealthier. At the pre-shock relative price, the consumer would like to consume more of both importables and non-tradables. Importables can be procured from the rest of the world. Non-tradable goods, however, are in fixed supply. At the pre-shock relative price, there is thus an excess demand for non-tradable goods that must be met by an increase in their relative price.

6.5 Change in terms of trade along a perfect foresight path

Having just isolated the effect on the relative price of non-tradable goods brought about by the wealth effect stemming from an improvement in the terms of trade, we now abstract from the wealth effect and study the effect arising from intertemporal consumption substitution. To this effect, consider an improvement in the terms of trade along a perfect foresight path. In other words, consider a perfect foresight path along which q falls at time $t = T$ (Figure 9, Panel A). What will be the effect on p^I?

[Figure 9]

Since λ is constant along a perfect foresight path, it follows from first-order condition (94) that if q falls at T, c^I will increase (Figure 9, Panel B). Intuitively – and as we saw in Chapter 3 – if importables become relatively cheaper at T, consumers will engage in intertemporal consumption substitution and consume more importables.

To find out what happens to p^I, impose the non-tradable goods equilibrium condition (101) in (96) to obtain:

$$\frac{u_{c^N}(c_t^I, y^N)}{u_{c^I}(c_t^I, y^N)} = p_t^I. \tag{103}$$

Totally differentiating this condition at time T yields:

$$\left.\frac{dp_t^I}{dc_t^I}\right|_{t=T} = \left(\frac{u_{c^I} u_{c^I c^N} - u_{c^N} u_{c^I c^I}}{u_{c^I}^2}\right) > 0,$$

where, again, the numerator is positive due to the assumption of normality. The increase in c^I at time T will be accompanied by an increase in p^I (Figure 9, Panel C). Intuitively, at an unchanged relative price, and given the increase in consumption of importables at time T, consumers would like to consume more non-tradables. The excess demand for non-tradables leads to an increase in their relative price, p^I.

In general, the behavior of the trade balance and p^x will depend on preferences. In the case of logarithmic preferences (i.e., $u(c_t^I, c_t^N) = \log(c_t^I) + \log(c^N)$), it is easy to see from (94) that $q_t c_t^I$ is constant along a perfect foresight equilibrium path and hence, that the trade balance (equal to $y^x - q_t p_t^I$) would also be constant over time (Figure 9, Panel D). Similarly, p^x would be constant over time since, from (95) and (101), $q_t p_t^I$ would be constant over time and, from (90), so would p^x.

We thus conclude that an improvement in the terms of trade along a perfect foresight path will bring about an increase in the relative price of non-tradable goods in terms of importables due to the intertemporal substitution effect on consumption of importables. Together with the results derived in the previous sub-section, we conclude that both the wealth and intertemporal substitution effect associated with an improvement in the terms of trade will lead to a real appreciation. We thus infer – as can be easily verified by the reader – that an unanticipated and temporary improvement in the terms of trade at time 0 will lead to an initial increase in p^I since both the wealth effect and the intertemporal substitution effect will increase consumption of importables.[26]

[26] See Mendoza (1995) for a quantitative analysis of the effects of terms of trade shocks on the real exchange rate.

6.6 Trade liberalizations

We can use the same three-good model just developed to consider another important question in development macroeconomics: how will a trade liberalization affect the real exchange rate? Conventional wisdom holds that a trade liberalization will lead to a real exchange rate depreciation (see Edwards (1989), Chapter 2).[27] As the analysis below will make clear, however, our simple model should lead us to expect precisely the opposite.

To reinterpret the above model as applying to a trade liberalization, assume (like in Chapter 3) that the international terms of trade are equal to one and think of q as the tariff-inclusive relative price of importables in terms of exportables. Hence, $q-1$ represents the import tariff. In the context of a trade liberalization, the more natural assumption is that the government returns to consumers the tariff proceeds in a lump-sum way (as in Chapter 3). We thus modify the intertemporal constraint (93) to read

$$b_0 + \int_0^\infty (y^x + q_t p_t^I y^N + \tau_t) e^{-rt} dt = \int_0^\infty (q_t c_t^I + q_t p_t^I c_t^N) e^{-rt} dt, \qquad (104)$$

where τ_t denotes lump-sum transfers from the government. It should be obvious that this modification does not alter the consumer's first-order conditions, which continue to be given by (94), (95), and, hence, (96).

The government's role is to impose the tariff and to return the proceeds from the tariff in a lump-sum way. Hence,

$$\tau_t = (q_t - 1) c_t^I. \qquad (105)$$

To obtain the economy's resource constraint, substitute (105) into (104) – taking into account the equilibrium condition in the non-tradable goods market – to obtain:

$$b_0 + \int_0^\infty y^x e^{-rt} dt = \int_0^\infty c_t^I e^{-rt} dt. \qquad (106)$$

A comparison of (106) and (99) reveals that the critical difference with the terms of trade case is that q_t does not enter the resource constraint. This was to be expected since, by assumption, the terms of trade are equal to one.

[27] The traditional argument runs as follows: a reduction in tariffs will increase consumption of importables, which leads to a trade deficit. This requires a real depreciation for equilibrium to be restored.

6.6.1 Permanent reduction in tariffs

Consider a perfect foresight equilibrium for constant paths of y^x, y^N, and q. Taking into account the non-tradable goods market equilibrium condition (97), the first-order condition (94) and the resource constraint (106) imply that

$$c^I = rb_0 + y^x. \tag{107}$$

From (103) and (107),

$$p_t^I = \frac{u_{c^N}(rb_0 + y^x, y^N)}{u_{c^I}(rb_0 + y^x, y^N)}. \tag{108}$$

Since neither c^I nor p^I depend on q, it should be clear that an unanticipated and permanent reduction in q will have no effects. This was to be expected since q continues to be constant along the new perfect foresight equilibrium path and changes in q do not have wealth effects.[28]

6.6.2 Temporary reduction in tariffs

In contrast to the permanent case, an unanticipated and temporary reduction in tariffs will have real effects. From (94) and (106), it follows that c^I will rise at time 0 and then fall at T below its initial value (since the present discounted value of c^I remains unchanged).[29]

Given the intratemporal condition (96), the path of c^I fully determines the path of p^I. Specifically, substitute the non-tradable goods equilibrium condition into (96) to obtain:

$$p_t^I = \frac{u_{c^N}(c_t^I, y^N)}{u_{c^I}(c_t^I, y^N)}.$$

Totally differentiating at any given point in time:

$$\frac{dp^I}{dc^I} = \left(\frac{u_{c^I} u_{c^I c^N} - u_{c^N} u_{c^I c^I}}{u_{c^I}^2} \right) > 0,$$

[28] Of course, if exportables were consumed, a permanent change in q would have real effects through consumptions substitution between exportables and importables. See Edwards (1989), Edwards and Van Wijnbergen (1987) and Buffie (1999) for detailed analysis.

[29] This was, of course, the same result that we obtained in Chapter 3 for a temporary fall in tariffs.

where the numerator is positive by the condition of normality. It follows that p^I increases on impact and falls at time T. Since c^I is below its initial value after time T, p^I is also below its initial value.

We conclude that, in the context of our simple model, a temporary trade liberalization leads to an increase in the relative price of non-tradable goods in terms of importables (a real appreciation). Intuitively, importables become cheaper which leads to a higher consumption of importables. At the initial relative price, consumers do not wish to change the relative consumption of importables and non-tradable goods and hence would demand more non-tradables as well. The excess demand for non-tradables leads to an increase in their relative price. Our result is the exact opposite of what the conventional wisdom would hold. [30]

7 Final remarks

This chapter has introduced non-tradable goods into the basic model developed in Chapter 1. Once non-tradable goods enter the picture, the full consumption smoothing result obtained in Chapter 1 needs to be qualified since a fluctuating path of non-tradable goods may lead to a non-constant path of tradable goods consumption. The presence of non-tradable goods introduces a key relative price – the relative price of non-tradable goods – which plays a key role in the adjustment of a small open economy to various shocks.

We analyzed how, in response to a temporary negative supply shock, trade and current account deficits co-exist with a high relative price of non-tradable goods (real appreciation) while surpluses co-exist with real depreciation. There is no sense, however, in which the changes in the real exchange rate "cause" the external imbalances; they are both equilibrium responses to a common shock.[31]

[30] If exportables were consumed (and/or produced), substitution in both consumption and production would affect how the relevant real exchange rate (which, in such a model, would be best defined as the inverse of the relative price of non-tradable goods in terms of some index of tradable goods) is affected by changes in tariffs (see Edwards (1989) and Buffie (1999)). In particular, Buffie (1999) shows that, under the more plausible parameter configuration, a temporary liberalization will lead to a real exchange rate appreciation.

[31] As we will see in Chapter @, it is only in a monetary model with sticky prices that we may argue that the real exchange rate causes external imbalances in a meaningful sense.

We also emphasized the role of the relative price of non-tradable goods in bringing about the adjustment required when a small open economy goes from boom to bust. The fall in the relative price of non-tradable goods that accompanies the shift from boom to bust is key in engineering the shift in resources from the non-tradable to the tradable goods sector that will be needed to repay the external debt accumulated in the good times. We will encounter this adjustment mechanism time and time again in the rest of the book.

8 Appendix

8.1 Conditions for normality

Since some of the results of this chapter depend on the assumption that c^T and c^N are normal goods, it will prove useful to derive the conditions that ensure normality.[32] To do so, consider the static problem corresponding to our dynamic problem:

$$\begin{aligned}&\underset{\{c^T,c^N\}}{\text{Max}}\; u(c^T,c^N)\\&\text{subject to}\\&c^T+pc^N=y,\end{aligned}$$

where y ($\equiv y^T+py^N$) denotes the value of the endowment.

The first-order conditions are given by:

$$u_{c^T}(c^T,c^N) = \lambda \tag{109}$$

$$u_{c^N}(c^T,c^N) = \lambda p \tag{110}$$

$$c^T+pc^N = y,$$

where λ is the Lagrange multiplier associated with the budget constraint.

Totally differentiating the above system with respect to c^T, c^N, λ and y, we obtain (in matrix form):[33]

[32] For more details, see, for example, Silbergerg and Suen (2000).

[33] We are implicitly assuming that y increases due to a change in y^T or y^N (and not p).

$$\begin{bmatrix} u_{c^T c^T} & u_{c^T c^N} & -1 \\ u_{c^N c^T} & u_{c^N c^N} & -p \\ -1 & -p & 0 \end{bmatrix} \begin{bmatrix} dc^T \\ dc^N \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} dy.$$

Letting Δ denote the determinant associated with the 3x3 matrix above, we obtain, by Cramer's rule:

$$\begin{aligned} \frac{dc^T}{dy} &= \frac{1}{\Delta} \begin{vmatrix} 0 & u_{c^T c^N} & -1 \\ 0 & u_{c^N c^N} & -p \\ -1 & -p & 0 \end{vmatrix} = \frac{u_{c^T c^N} p - u_{c^N c^N}}{\Delta}, \\ \frac{dc^N}{dy} &= \frac{1}{\Delta} \begin{vmatrix} u_{c^T c^T} & 0 & -1 \\ u_{c^N c^T} & 0 & -p \\ -1 & -1 & 0 \end{vmatrix} = \frac{u_{c^N c^T} - p u_{c^T c^T}}{\Delta}, \end{aligned}$$

where $\Delta > 0$ (by the sufficient second order conditions for a maximum). Since, from (109) and (110), we know that $p = u_{c^N}/u_{c^T}$, we can rewrite these expressions as:

$$\begin{aligned} \frac{dc^T}{dy} &= \frac{u_{c^N} u_{c^T c^N} - u_{c^T} u_{c^N c^N}}{u_{c^T} \Delta}, \\ \frac{dc^N}{dy} &= \frac{u_{c^T} u_{c^N c^T} - u_{c^N} u_{c^T c^T}}{u_{c^T} \Delta}. \end{aligned}$$

For these expressions to be positive (which implies that consumption of both goods increases in response to an increase in output), it must be the case that:

$$\begin{aligned} u_{c^N} u_{c^T c^N} - u_{c^T} u_{c^N c^N} &> 0, \\ u_{c^T} u_{c^N c^T} - u_{c^N} u_{c^T c^T} &> 0, \end{aligned}$$

which are conditions (2) and (3) in the text.

Exercises[34]

1. **Comovements between output and trade balance**

   This exercise uses a specific class of preferences to illustrate the effect that fluctuations in the endowment of non-tradable goods have on tradable goods consumption and hence on the trade balance.

   Consider the same economy analyzed in Section 2. Let preferences be given by [35]

$$u(c_t^T, c_t^N) = \frac{c^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}, \tag{111}$$

   where c is a CES consumption composite defined as:

$$c = z^{\frac{\rho}{\rho-1}}, \tag{112}$$

   where

$$z = q(c^T)^{\frac{\rho-1}{\rho}} + (1-q)(c^N)^{\frac{\rho-1}{\rho}}. \tag{113}$$

   The parameter $\sigma > 0$ is the *intertemporal* elasticity of consumption substitution. The parameter $\rho > 0$ captures the *intratemporal* elasticity of substitution between tradable and non-tradable goods.

   (a) Suppose that the endowment of tradable goods is flat over time and that the endowment of non-tradable goods fluctuates over time. Show that whether the economy runs trade surpluses or deficits during periods of high non-tradable endowment depends on the relation between σ and ρ. [Hint: Differentiate the marginal condition for tradable goods along a perfect foresight path.]

   (b) Available empirical evidence for developing countries suggests that $\rho > \sigma$ (see Ostry and Reinhart (1992)).[36] What does the model

[34] An answer key is available from the author upon request.

[35] In many instances, these CES preferences are simply written as $c^{1-\frac{1}{\sigma}}/(1-\frac{1}{\sigma})$. The reason for adding the term minus one on the numerator is to have the Cobb-Douglas formulation as a particular case. To see this, take the limit of (111) when $\sigma = 1$ and apply L'Hopital rule to obtain $\log(c)$ as the result.

[36] Based on data for 13 developing countries, Ostry and Reinhart (1992) estimate the intratemporal elasticity of substitution between tradable and non-tradable goods to be in the range 1.22-1.27, whereas the intratemporal elasticity is estimated to be in the range 0.38-0.50 (which is consistent with the evidence provided in Chapter 3).

predict in terms of the relation between good times (i.e., high non-tradable endowment) and the trade balance? Does the sign of this comovement match the stylized facts described in Box 1 in Chapter 1?

(c) Compute the cross derivative for the CES preferences in (111) and show that it depends on the relation between σ and ρ. Relate this finding to the expression that you derived in item a) above.

2. Consumption-based real interest rate (based on Dornbusch (1983))

This exercise derives the so-called "consumption-based real interest rate", an influential concept popularized by Dornbusch (1983).

(a) Write a discrete-time version of the model analyzed in Section 2 with the discount factor given by δ and preferences given by (111) with z given by (113) and c, the consumption composite, given by

$$c \equiv \left(c^T\right)^{\alpha} \left(c^N\right)^{1-\alpha}. \tag{114}$$

(b) Show that the domestic real interest rate, r^d, is given by $r^d \equiv (1+r)\frac{p_t}{p_{t+1}} - 1$. Interpret intuitively this real interest rate.

(c) Show that you can combine the first-order conditions to obtain the following Euler equation for the consumption aggregate:

$$\frac{c_{t+1}}{c_t} = [\delta(1+r_t^c)]^{\sigma},$$

where

$$r_t^c \equiv (1+r)\left(\frac{p_t}{p_{t+1}}\right)^{1-\alpha} - 1$$

is the consumption-based real interest rate. Notice how in determining the time profile for the consumption aggregate the consumer compares the discount factor, δ, to r_t^c. Further, if the relative price of non-tradable goods does not vary over time, then $r_t^c = r$. In contrast, if the relative price of non-tradable goods is not constant over time, then aggregate consumption will not be constant over time even if $\delta(1+r) = 1$.

3. External deficits and the real exchange rate

This exercise illustrates the fact that Edgeworth substitutability is a sufficient (though not necessary) condition for the association between trade deficits (trade surpluses) and real appreciation (real depreciation) derived in Section 3 to hold for the general preferences given by equation (1).

Consider the model of Section 3 with preferences given by (1). Assume that goods are Edgeworth substitutes. In this context, analyze the effects of a temporary and equiproportional fall in the endowment of both goods. In particular, show that trade deficits go hand in hand with real appreciation.

4. Demand shocks in an endowment economy

This exercise analyzes the effects of demand shocks in the endowment model developed in Section 2. Suppose that preferences are given by

$$\int_0^{\infty} \left[\alpha_t^T \log(c_t^T) + \alpha_t^N \log(c_t^N)\right] e^{-\beta t} dt,$$

where $\alpha_t^T > 0$ and $\alpha_t^N > 0$ are parameters meant to capture "demand shocks" to consumption of tradables and non-tradables, respectively.

(a) Suppose that $\alpha_t^N = 1 - \alpha_t^T$.

i. Characterize the perfect foresight equilibrium of this economy for a constant path of all exogenous variables. Does the relative price of non-tradable goods depend on α^T? Explain the intuition behind this result.

ii. Analyze the effects of (i) an unanticipated and permanent increase in α^T and (ii) an unanticipated and permanent fall in α^T. Show the results both analytically and graphically (i.e., in terms of Figure 3). Explain the intuition behind the results.

(b) Suppose $\alpha_t^T = \alpha_t^N = \alpha_t$.

i. Characterize the perfect foresight equilibrium of this economy for a constant path of all exogenous variables. Does the relative price of non-tradable goods depend on α? Explain the intuition behind this result.

ii. Show that an unanticipated and permanent increase in α_t does not affect the economy's equilibrium.

iii. Analyze the effects of an unanticipated and temporary increase in α_t. In particular, show that high demand periods will be characterized by a consumption boom, real appreciation, and trade deficits while low demand periods will be associated with low consumption, real depreciation, and trade surpluses.

5. Demand shocks in a production economy

Suppose that households consume only non-tradable goods and that non-tradable goods are produced using tradable goods as an input. (If it helps you, think of an economy importing candies (which are not directly consumed), wrapping them domestically, and selling them domestically for consumption.)

Specifically, consider a small open economy fully integrated into the world economy. Preferences are given by

$$\int_0^\infty \gamma_t u(c_t^N) e^{-\beta t} dt,$$

where $u' > 0$, $u'' < 0$, c_t^N is consumption of non-tradable goods and $\gamma_t > 0$ is a parameter that captures demand shocks.

Non-tradables goods are produced using tradables goods as an input:

$$y_t^N = \frac{(c_t^T)^\alpha}{\alpha}, \qquad \alpha < 1.$$

There is an exogenous and constant endowment of tradable goods, y^T.

(a) State the household's flow and intertemporal budget constraints. (Assume that households also carry productive activities.)

(b) Derive the first-order conditions. Explain the intuition.

(c) Suppose that, starting from an initial stationary equilibrium, there is an unanticipated and temporary increase in γ_t. Derive the time path for all endogenous variables (and plot them against time). Explain the intuition behind your results.

6. The twin deficits revisited

The purpose of this exercise is to show that once we allow the government to borrow/lend over time, we can establish a link between fiscal deficits and trade deficits. In other words, we will see how a temporary increase in g^T leads to both a primary fiscal deficit and a trade deficit.

Consider the same model analyzed in subsection 4.1 but, for simplicity, assume that the government spends only on tradable goods. More importantly, relax the assumption that the government must balance its budget period by period (reflected in equation (34)). Instead, assume that the government's flow constraint takes the form

$$\dot{b}_t^G = r b_t^G + p b_t,$$

where

$$pb_t \equiv \bar{\tau} - g_t^T \tag{115}$$

denotes the primary balance, $\bar{\tau}$ is a constant tax rate (whose level will be endogenously determined) and b^G denotes net foreign assets held by the government. The corresponding intertemporal constraint is given by

$$b_0^G + \frac{\bar{\tau}}{r} = \int_0^\infty g_t^T e^{-rt} dt,$$

which says that the present discounted value of primary balances must match the initial government debt (given by $-b_0^G$).

In this context:

(a) Characterize the stationary equilibrium corresponding to constant paths of y^T, y^N, and g^T. To simplify the exercise assume that $b_0^G = 0$ and $pb_0 = 0$.

(b) Analyze the effects of an unanticipated and temporary increase in g^T. Does the twin deficits hypothesis holds? Discuss the intuition behind the results.

7. Endogenous supply revisited

In the context of the model in Section 5:

(a) Consider a more general version of this model in which the production of non-tradables is given by $y_t^N = Z_t^N (n_t^N)^\beta$, where β could be greater, equal, or smaller than α. Solve for the effects of an increase in b_0 and a temporary demand shock and show that the same results that we found in the text go through.

(b) Analyze in the more general version the effects of an unanticipated and permanent increase in Z^T. Do the results depend on whether $\alpha \gtreqless \beta$? How do the results relate to the celebrated Balassa-Samuelson effect?

(c) Analyze in the more general version the effects of an unanticipated and permanent increase in n. Do the results depend on whether $\alpha \gtreqless \beta$?

(d) Suppose that production is linear in both sectors; that is, $y_t^T = Z_t^T n_t^T$ and $y_t^N = Z_t^N n_t^N$. Obtain a reduced form for all endogenous variables in the model. How is the real exchange rate determined in this model?

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Box 1. How much does a fiscal contraction depreciate the real exchange rate?

Estimating the impact of changes in fiscal spending on the real exchange rate has proved to be a hard task, as has been the case with most of the determinants of the real exchange rate. This is particularly true in this case given that the magnitude of the effect depends critically on variables that are difficult to quantify.

These limitations notwithstanding, some progress has been done. A summary of the main results are shown in Table 1. Two different methodologies can be found in the literature. The first one is to derive equations from a model and test them econometrically. The link between the model and the regressions, however, varies from study to study and in some cases there is no explicit theoretical framework to accompany the econometric estimation. A second approach consists of calibrating a theoretical model by mapping observable characteristics of the economy into "deep parameters" (the real business cycle approach described in Box 1 of Chapter 1). The calibrated model shows the reaction of the variables of interest to different shocks.

[Table 1]

The first method has been the more popular one and has resulted in a variety of results for different countries. In addition to some measure of government spending, the regressions typically incorporate various control variables, such as terms of trade and productivity differentials among sectors. Overall, the results tend to confirm that in Latin America and the OECD countries there is a negative direct impact of government spending on real exchange rate (i.e., a contraction in government spending leads to a rise in the real exchange rate or a real depreciation). This is, of course, consistent with the result derived in the text under the assumption that government spending is more biased towards non-tradable goods than private spending. This is likely to be the case given the heavy concentration of government spending on health, education, and related expenses. In the studies conducted for Asia, however, no significant relationship has been found. Given the theoretical framework, one might conclude that Asian governments have a stronger preference for spending on tradable goods. As a matter of fact, there is some evidence that points in that direction, since the share of public investment in GDP – a component of spending intensive in tradable goods –

is systematically higher in Asia than in other developing regions such as Latin America (Bouton and Sumlinski, 1996). There are some studies, however, that show the opposite effect of government spending on the real exchange rate. Kim and Roubini (2008) and Monacelli and Perotti (2010) show that in the US, UK, Canada and Australia, the real exchange rate depreciates in response to a rise in government spending. They argue that the key failure of the standard model lies in the equilibrium behavior of private consumption. In the model, as result of a negative wealth effect, private consumption falls in response to a rise in government spending, whereas the opposite is true in the data.

The second approach does not take advantage from data on government expenditure and has not been the subject of much research. However, it provides key insights with respect to the differential effect of permanent and temporary shocks. Arrau (1992) shows for the case of the Chilean economy that a 1 percent temporary increase in government spending tends to appreciate the real exchange rate by a higher amount than permanent changes (1 percent versus. 0.8 percent). This result is also in line with the conceptual framework, as temporary changes in government spending imply a smaller permanent reduction in the consumption of tradables (and thus a higher transitory increase in the relative price of nontradables).

There are some avenues still to be explored in this empirical literature that go beyond the basic benchmark model. For instance, some authors have suggested that effects might vary according to the exchange rate regime. A paper by Kaminsky and Klein (1994), which shows that the channel considered was insignificant in the Gold Standard era, provides some evidence in that direction. Other issues that might need increased attention are the sources of government spending financing and interest rate effects associated with variations in the credibility of the government.

Box 2. How do economies adjust in practice?

In Subsection 5.5, we analyzed how a small open economy adjusts to a negative demand shock (i.e., a fall in the demand for both tradable and non-tradable goods). The model predicts that, in response to such a shock, (i) the trade balance improves; (ii) the current account improves; and (iii) the relative price of non-tradables falls. What does the data show?

To abstract from other factors, it proves quite illuminating to look at the behavior of these macroeconomic variables during crises episodes in which consumption fell substantially. Table 2 presents three-year averages before and after the crisis for nine major crises in developing countries: the Southern-Cone "tablitas" (Argentina, Chile, and Uruguay in 1982), the Mexican 1994 crisis, the South-East Asia 1997 crises (Indonesia, Korea, Malaysia, and Thailand) and the Turkish 2001 crisis.[37] [38] Figure 10 depicts the average behavior of the four variables relative to the year of the crisis. On average, consumption fell by almost 10 percent in the first year after the crisis. This dramatic fall in consumption was accompanied by a shift in the trade balance from a average deficit of 2 percent of GDP to a surplus of 8 percent of GDP and in the current account from a deficit of 5 percent to a surplus of 3 percent. In turn, the relative price of non-tradable goods fell on average by almost 35 percent.[39]

[Table 2]

[Figure 10]

Looking at the individual cases in Table 2, we also get a clear sense of how dramatic the turnaround in the external accounts can be. In Thailand, for

[37] We selected these crises because, in all nine of them, consumption fell in the first year after the crisis. The fall in consumption ranges from 5.5 percent in Argentina 1982 to 16 percent in Malaysia 1997. Notice that the the early 2000's (until the Fall 2008) were generally a period of boom for major emerging markets and the U.S. financial crisis of 2008 had only short-lived repercussions on emerging markets.

[38] All data are from International Financial Statistics (IMF). The relative price of non-traded goods was computed as EP^*/P, with E being the nominal exchange rate, P^* the United States CPI and P the domestic CPI.

[39] While outside of the scope of the model, it is interesting to notice that consumption recovers relatively quickly and by the third year after the crisis it has returned to its initial level. (Output behaves similarly and by the third year has also returned to its initial level.)

example, the shift in the external accounts was in the order of 14 percent of GDP and in Malaysia more than 20 percent of GDP. The fall in the relative price of non-tradable may be equally dramatic, even halving in some cases (Argentina, Uruguay, and Indonesia).

We thus conclude that – while in practice the shock that may have triggered the fall in consumption may differ from case to case – the sharp improvement in the external accounts and the large fall in the relative price of non-tradable goods is fully consistent with the predictions of our simple model.

Box 3. How is the real exchange rate measured in practice?

While, from a theoretical point of view, the definition of the real exchange rate (i.e., the inverse of the relative price of non-tradable goods) is rather straightforward, the question of how to measure this concept in practice is far from trivial. The most direct and obvious way would be to compute:

$$RER_1 = \frac{P_c^T}{P_c^N}, \tag{116}$$

where P_c^T and P_c^N are all goods classified as tradable or nontradable from consumer price indexes or national accounts. Food, beverages, apparel, furniture, and the like are usually considered tradable goods. On the other hand, housing, health, transportation, education, and entertainment are viewed as nontradable goods. However, such a measure would represent at best an approximation to the true relative price P^T/P^N since most goods for which prices are available represent essentially a mix of tradables and nontradables. Tradable goods, for example, have an important nontradable component given mainly by distribution costs.[40] Nontradable goods, on the other hand, usually include tradable goods inputs for their production.

The methodology captured in (116) presents a more fundamental drawback, as should be clear from Section 6. In theory, an increase in the export price should cause an appreciation of non-tradable goods relative to imported goods – the well known "Dutch Disease" phenomenon. The measure (116), however, might not reveal this important effect, as average tradable prices can increase and thus the relative price of nontradables decrease if the export good is important enough in the basket of tradable goods. Taking an index of import prices as a proxy for tradable prices would overcome this problem. However, reliable data for this series are rarely available. Indeed, many countries do not even compute such an index and others merely follow the price of some relevant commodities.

To avoid this problem, the standard practice is to measure tradable goods prices using foreign prices. Under such a procedure, the measured real exchange rate would be

$$RER_2 = \frac{EP^*}{P^d}, \tag{117}$$

[40] See Burstein, Neves, and Rebelo (2003).

where P^* is an index of international prices which are multiplied by the nominal exchange rate (E) to express them in domestic currency and P^d are domestic prices. More specifically, P^* is often computed as a weighted average of tradable prices (proxied by the wholesale price index) among main trading partners, or even main industrial countries. In this setup, nontradable prices are usually substituted by the consumer price index (CPI) for ease of computation. It can be easily shown that this is still a very good approximation to variations of the RER. Specifically, if the CPI can be expressed as:

$$P^d = \alpha P^T + (1-\alpha)P^N,$$

then the RER measure (117) becomes

$$RER_2 = \frac{EP^*}{\alpha P^T + (1-\alpha)P^N}.$$

If we further assume that $EP^* = P^T$, then

$$RER_2 = \frac{1}{\alpha + (1-\alpha)\frac{P^N}{P^T}}$$

which varies inversely with $\frac{P^N}{P^T}$ (as our theoretical measure does, of course).

Measure (117) is often computed with P^* as the consumer price inflation of another country (typically the United States). In such a case, the real exchange rate becomes a measure of relative prices of two countries, which intends to capture competitiveness.[41] Conceptually, however, factors that may affect non-tradables prices in, say, the United States should have no impact whatsoever on the real exchange rate, which is an attribute of a small individual economy facing world markets (Harberger 2004). In other words, the concept of the real exchange rate as the ratio of prices in two countries only makes sense in two-country models, which are essentially irrelevant for developing countries.[42] As a practical matter, however, this measure is widely

[41] Others, among them the IMF, have gone further and compute the real exchange rate as the relative unit labor costs. This measure is even questionable to measure competitiveness, as increase in wages can reflect increases in productivity. See Harberger (2004) for a scathing critique of IMF's measures of the real exchange rate.

[42] In fact, standard textbooks define the real exchange rate as the ratio of the CPI of two countries (see, for instance, Krugman and Obstfeld). For a small open economy facing a myriad of foreign countries, such a definition clearly makes little sense.

used since it has the advantage that the real exchange rate becomes easy to compute and exhibits a high correlation with previous definitions (see Edwards (1989)). This high correlation is highly surprising since in most cases the movement in the real exchange rate for developing countries are dominated by the behavior of domestic prices (i.e., the denominator in equation (117)).

Figure 1. Determination of c^T along a perfect foresight path

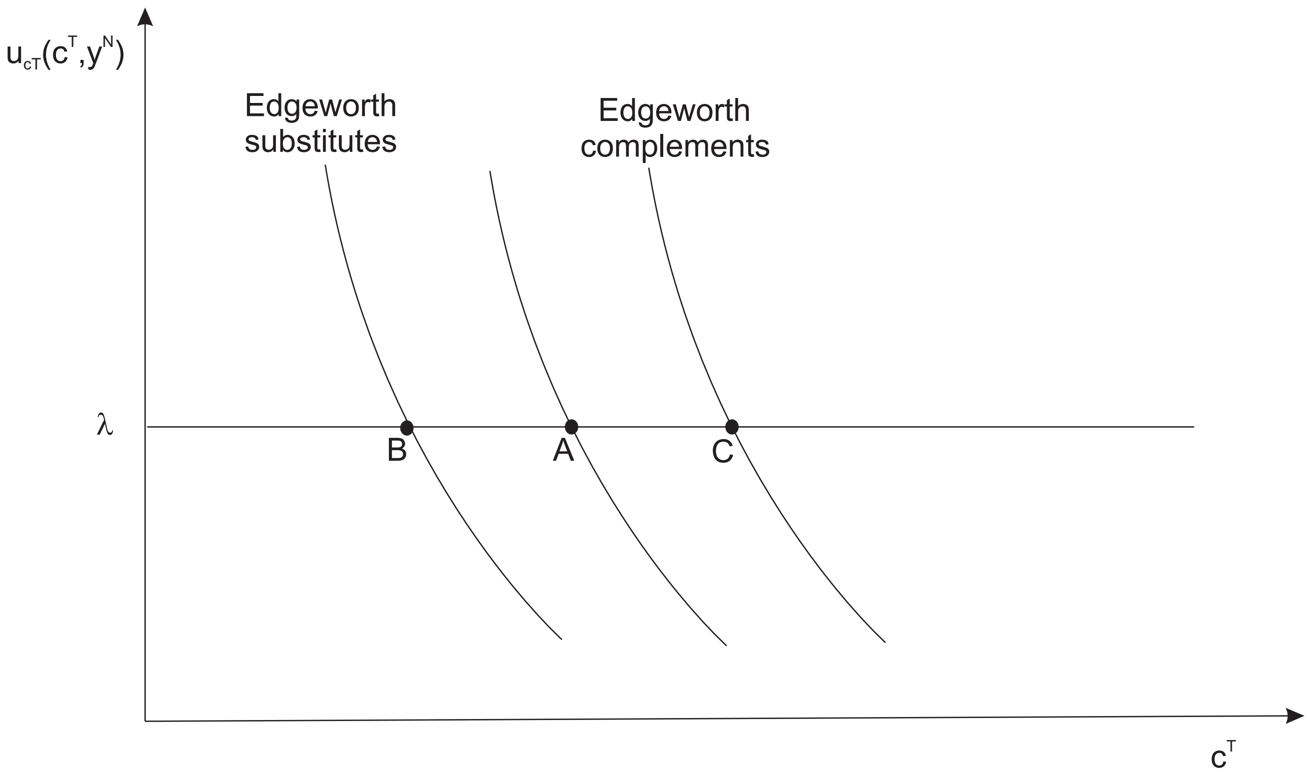


Figure 2. Perfect foresight paths

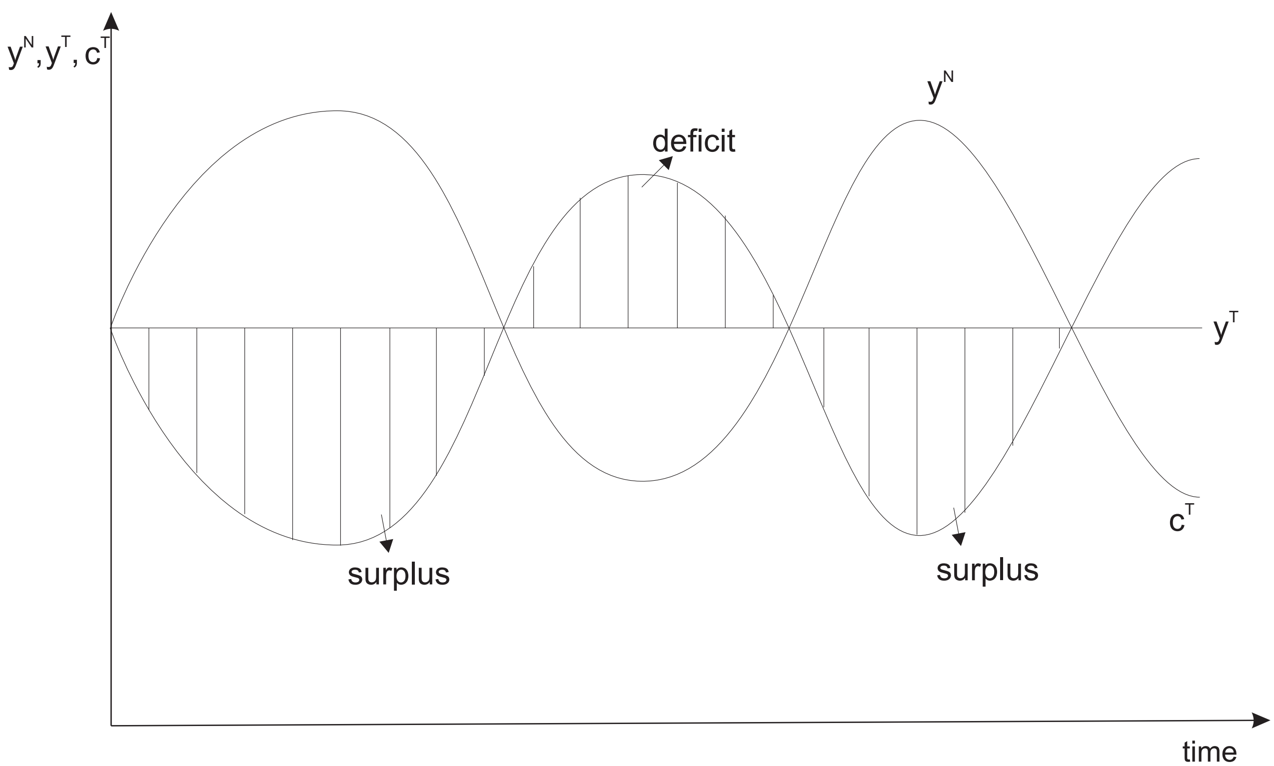


Figure 3. Determination of relative price of non-tradable goods

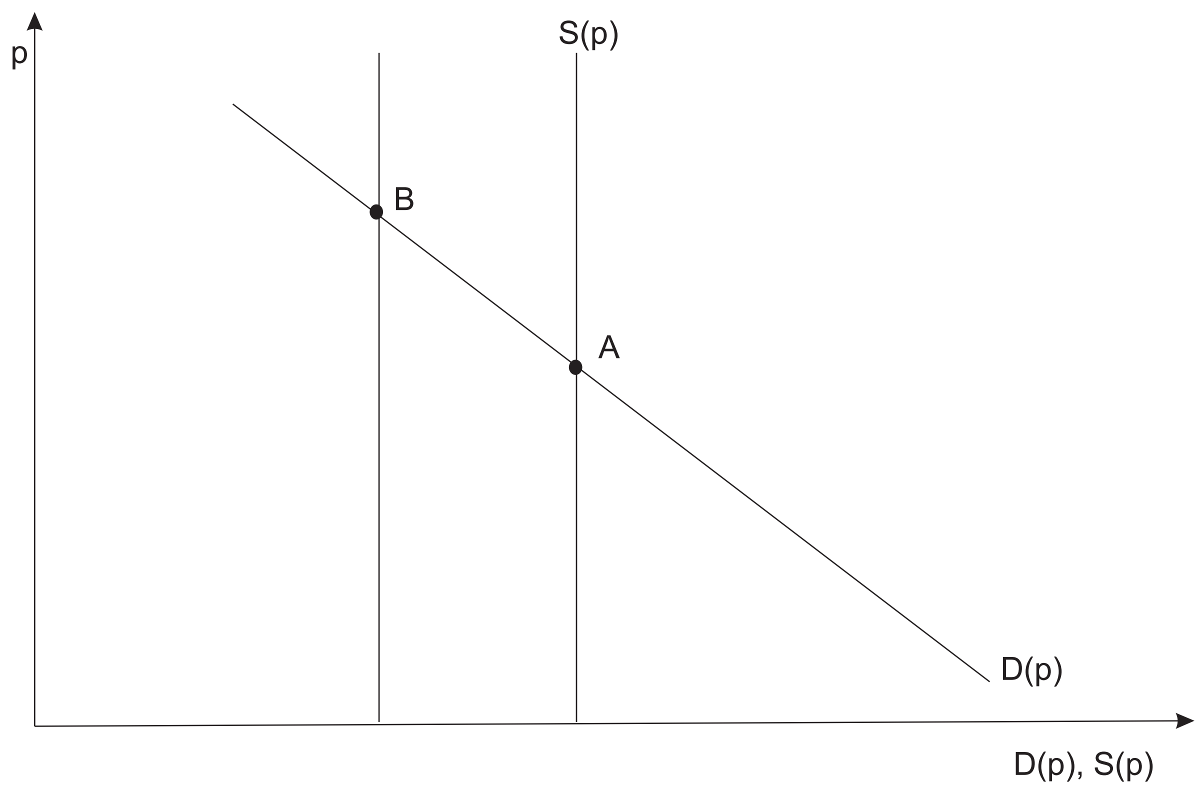


Figure 4. Temporary and proportional fall in output

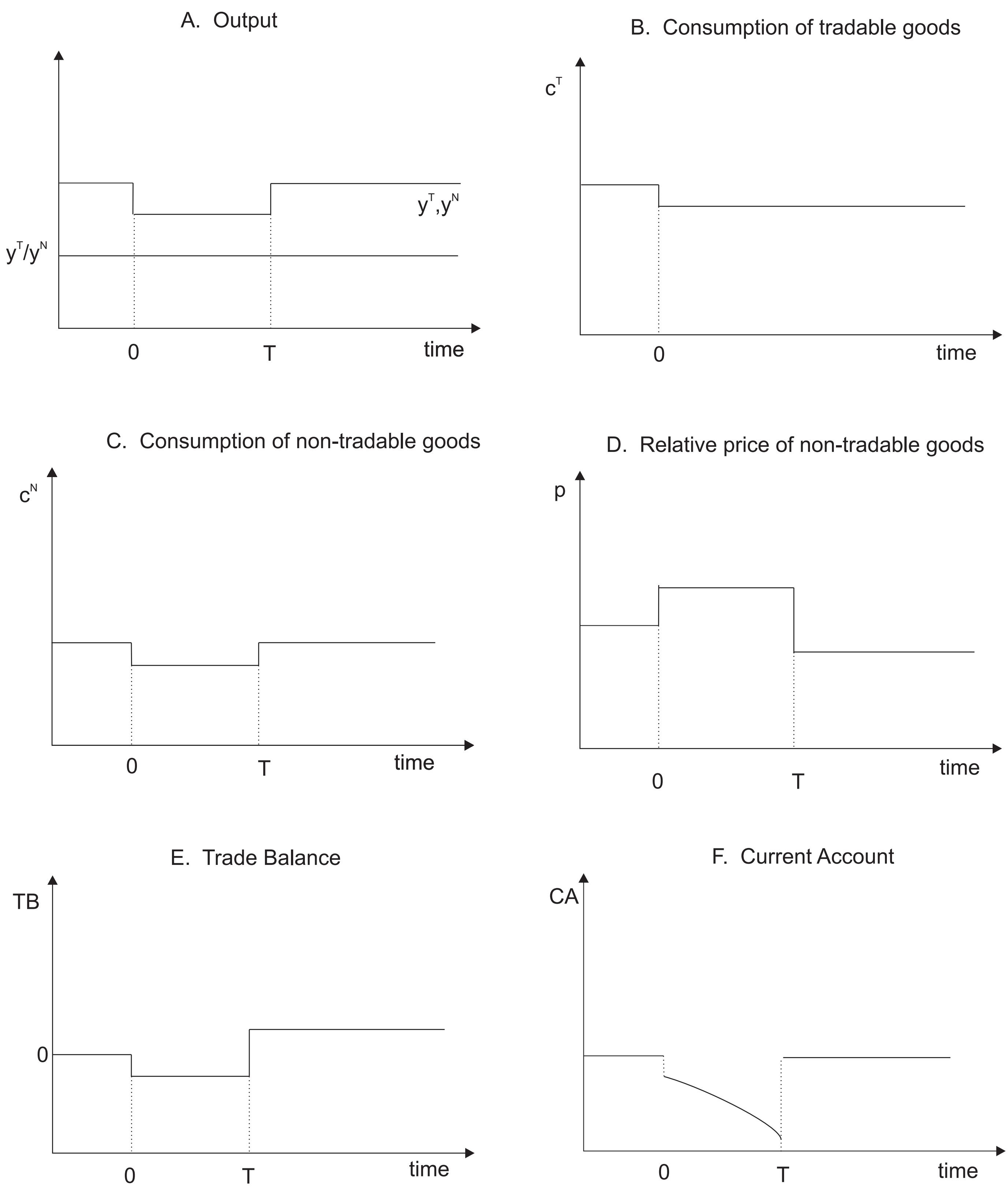


Figure 5. Temporary increase in government expenditure on tradable goods

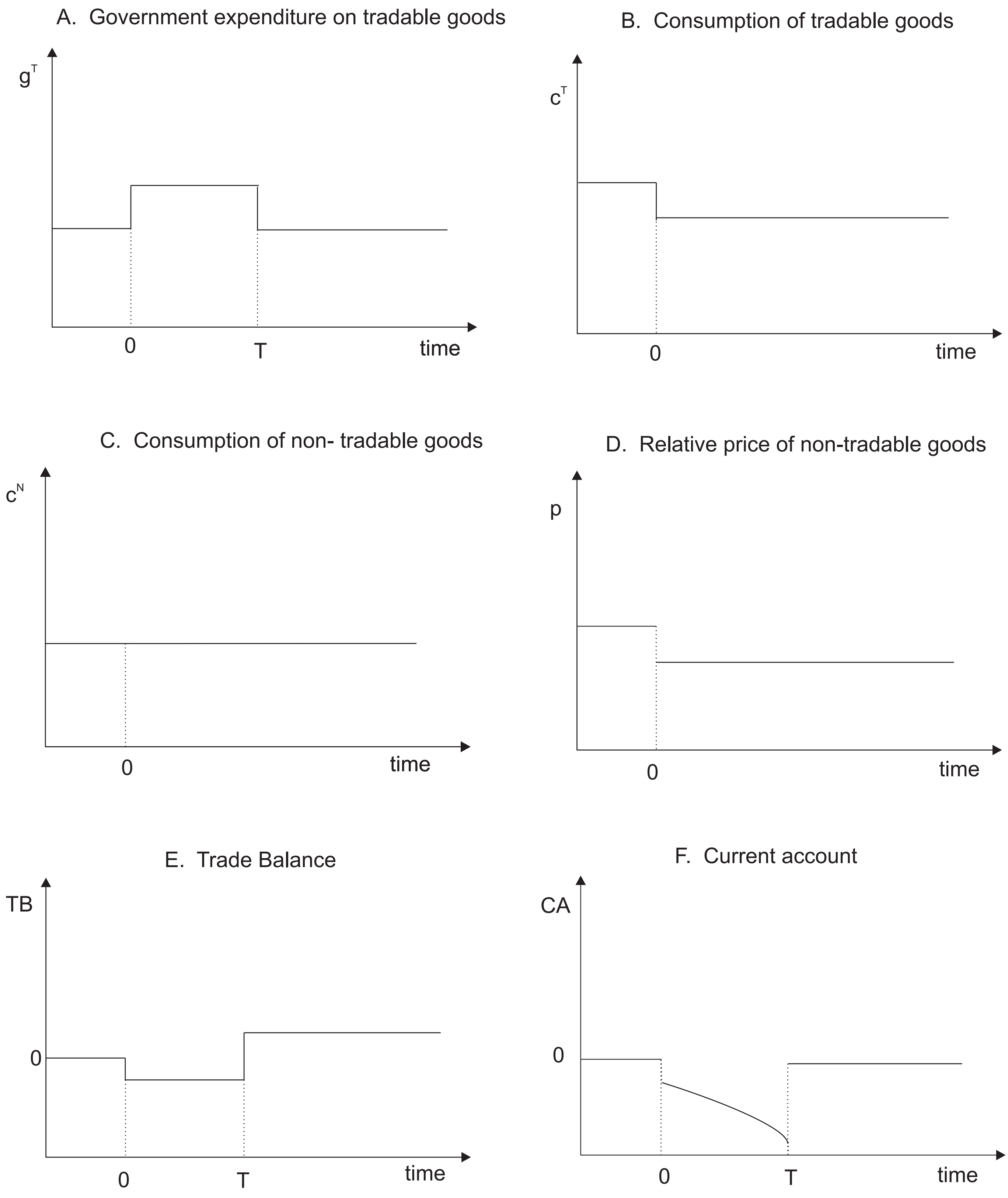


Figure 6. Temporary increase in government expenditure on non-tradable goods

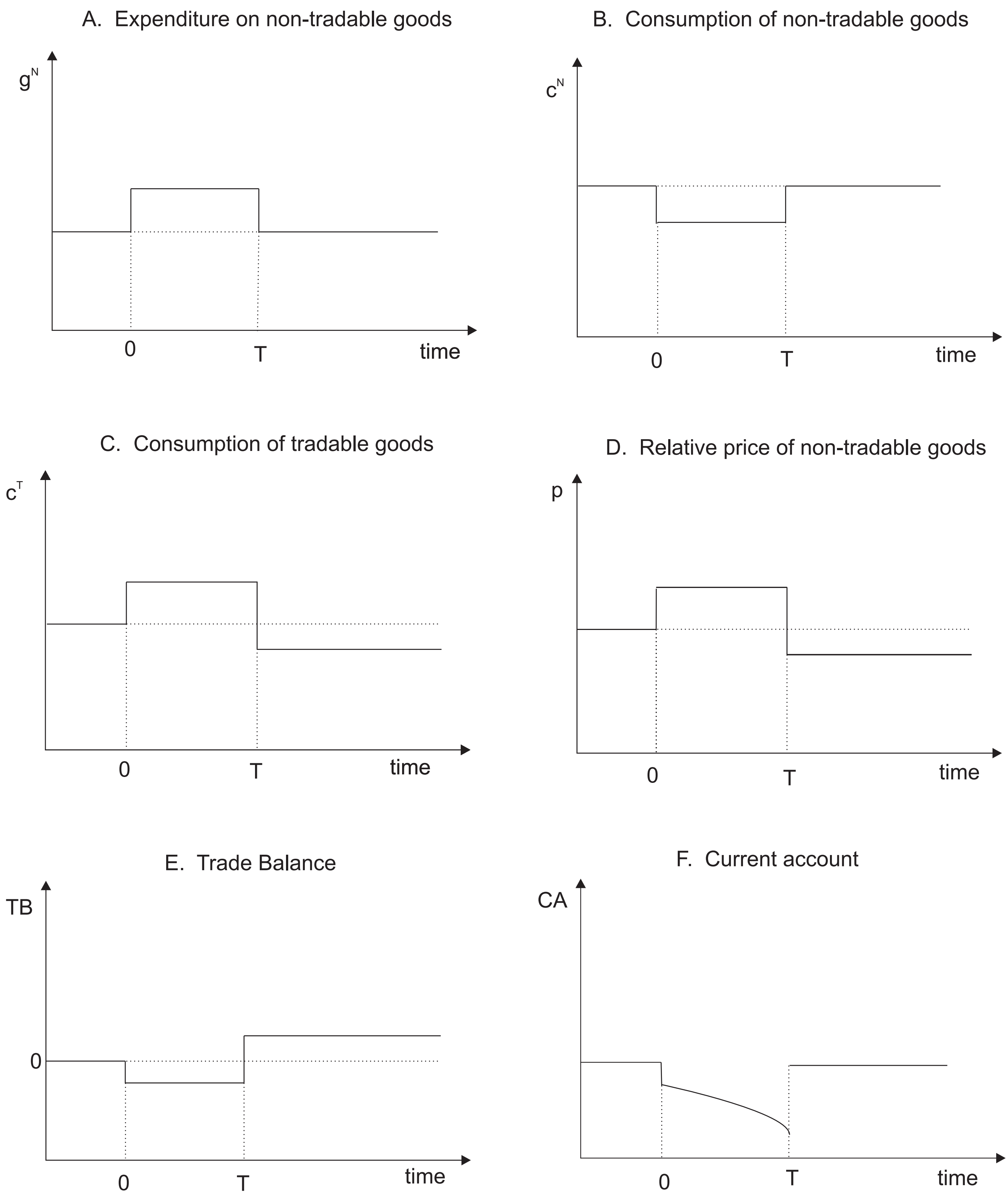


Figure 7. Temporary fall in tax rate

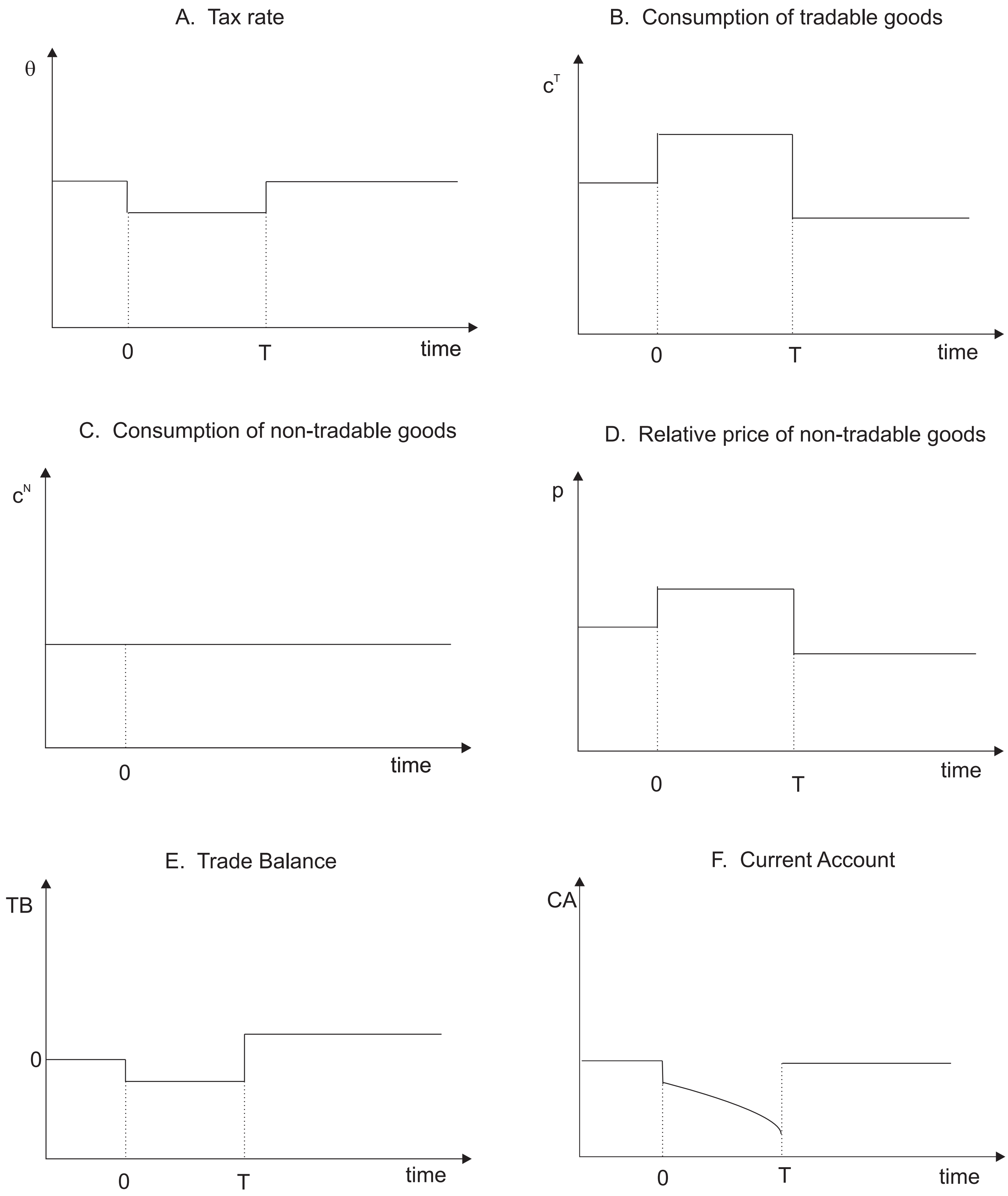


Figure 8. Temporary demand shock

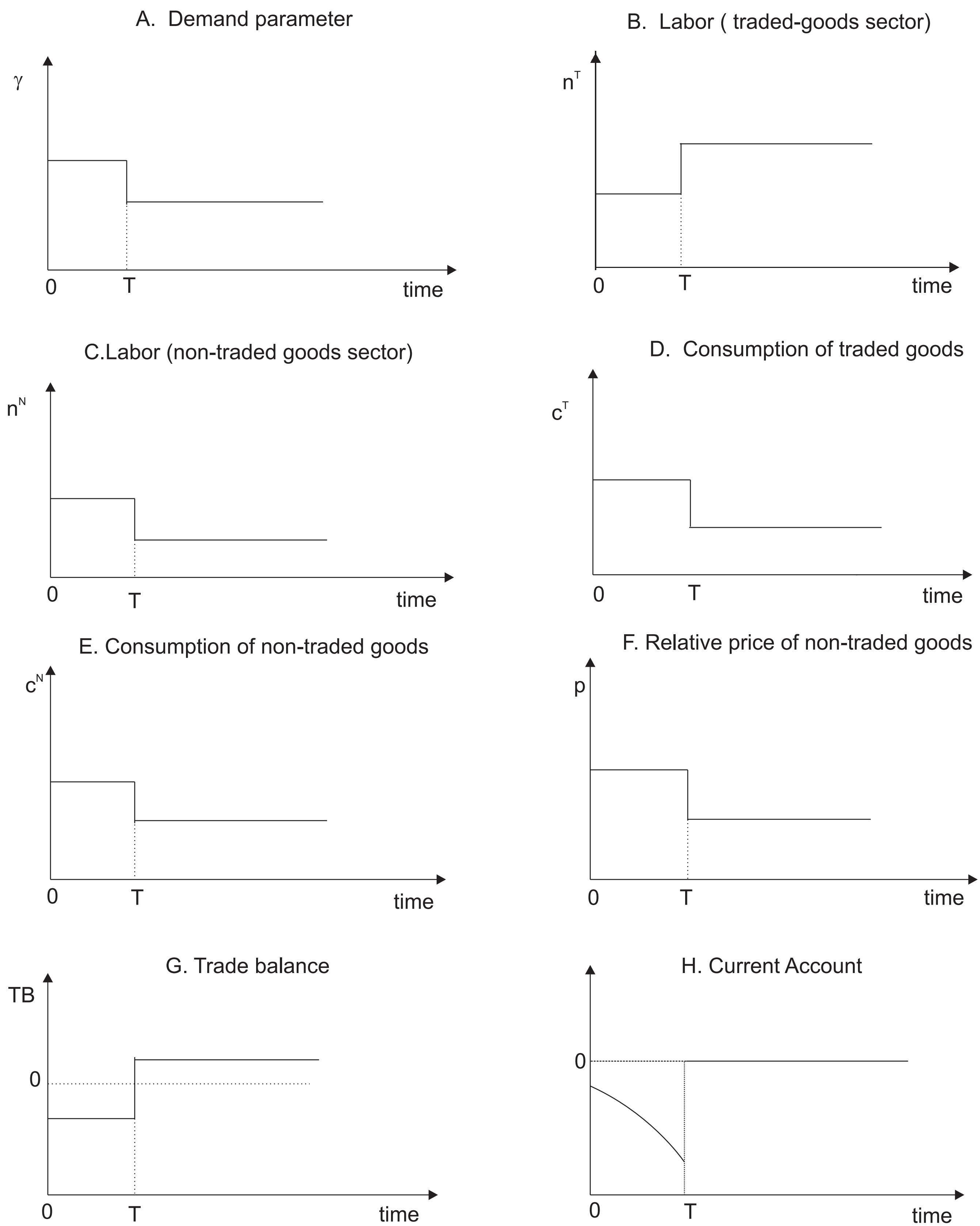


Figure 9. Anticipated improvement in terms of trade

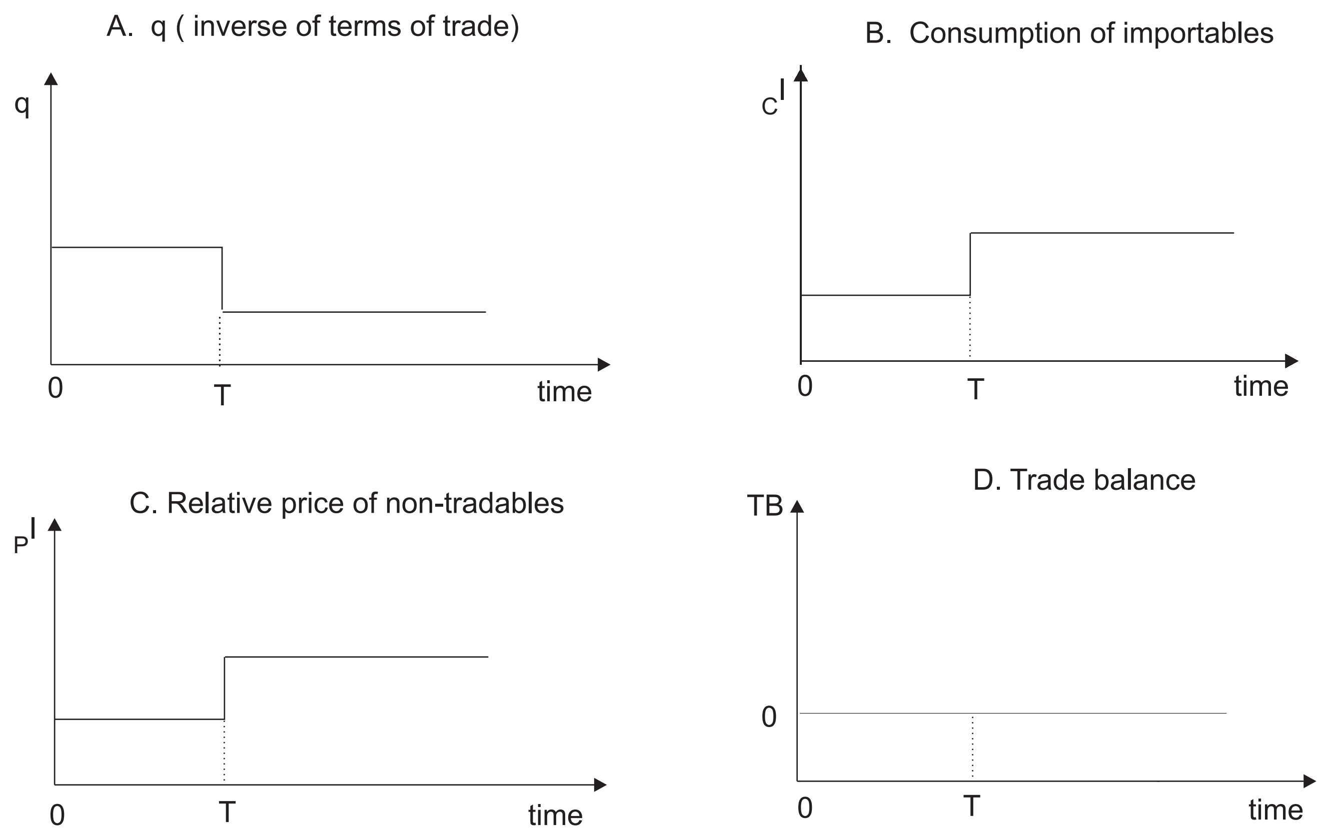
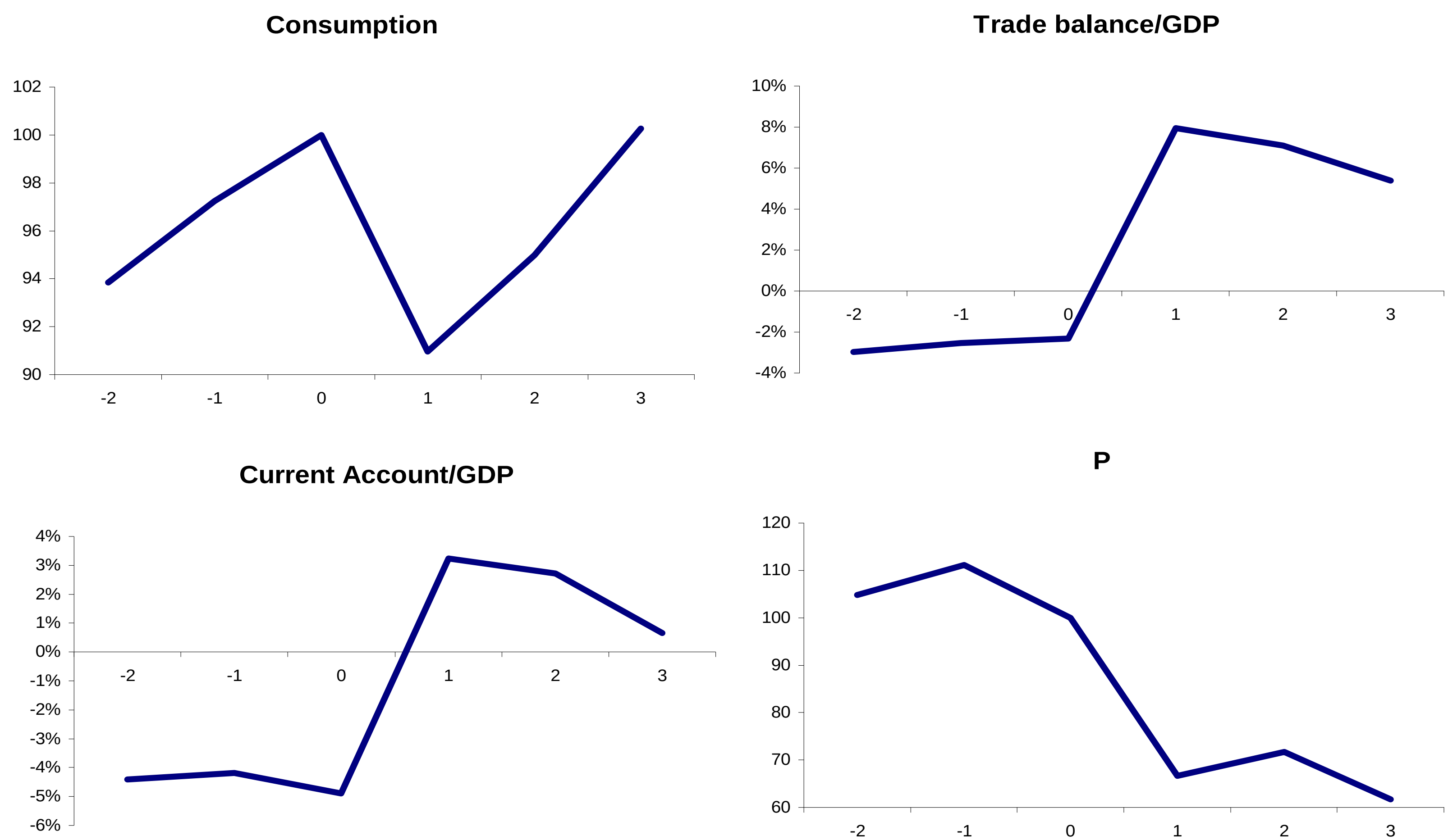


Figure 10. External Accounts and Relative Prices During Crises



Note: Values represent unweighted averages from nine currency crisis: Argentina (82), Chile (82) Uruguay(82), Mexico (94), Thailand (97), Indonesia (97), Korea (97), Malaysia (97) and Turkey (01). Period 1 is defined as the year after the crisis or the year of the crisis if it takes place after june.

Table 1. Effects of government spending on real exchange rate

Country	Point estimation*	Sample and frequency	Estimation method	Source
8 European countries	-2.1 to -3.5	Annual 1979-1989	Panel	Froot and Rogoff (1991)
Japan	Not significant	Quarterly 1975.I-1990.III	Cointegration analysis	Rogoff (1992)
Chile	-0.8	*Calibration*	*Calibration*	Arrau (1992)
Chile	-3.1	Quarterly 1982.I-1993.IV	Cointegration analysis	Arellano and Larrain (1996)
12 Latin American countries	-0.3	Annual 1962-1984	Panel data, fixed effects	Edwards (1989)
9 Asian countries	Not significant	Annual 1970-1991	Error correction regressions, estimated with nonlinear least squares	Chinn (1997)
14 OECD countries	-2 to -5	Annual 1970-1985	SUR	De Gregorio and Wolf (1994)
US and UK	Not significant	Annual 1779-1914	VAR	Kaminsky and Klein (1994)
US	0.5	Quarterly 1973:1-2004:1	VAR	Kim and Roubini (2008)
US, UK, Canada and Australia	-0.3 to 0.6	Quarterly 1980:I-2006:4	VAR	Monacelli and Perotti (2010)

* Percent response of real exchange rate to a permanent 1 per cent increase in the ratio of government spending to GDP.

Table 2. Crises episodes: Main Macroeconomic Variables

Argentina 82

Variable	Before	After
P	100	49.60
TB/GDP	0.61%	3.80%
CA/GDP	-2.69%	-1.85%
Y	100	98.47
C	100	100.50

Chile 82

Variable	Before	After
P	100.00	70.09
TB/GDP	-3.39%	2.09%
CA/GDP	-7.35%	-7.63%
Y	100	92.19
C	100	95.69

Uruguay 82

Variable	Before	After
P	100.00	49.23
TB/GDP	-4.29%	3.76%
CA/GDP	-5.63%	-2.99%
Y	100	88.39
C	100	87.46

Mexico 94

Variable	Before	After
P	100.00	77.93
TB/GDP	-4.04%	1.53%
CA/GDP	-6.52%	-1.04%
Y	100	116.66
C	100	93.88

Indonesia 97

Variable	Before	After
P	100.00	53.18
TB/GDP	1.60%	14.38%
CA/GDP	-2.78%	4.55%
Y	100	93.69
C	100	103.55

Korea 97

Variable	Before	After
P	100.00	73.70
TB/GDP	-1.48%	7.92%
CA/GDP	-2.63%	7.14%
Y	100	108.96
C	100	110.57

Malaysia 97

Variable	Before	After
P	100.00	71.61
TB/GDP	2.56%	25.38%
CA/GDP	-6.69%	12.85%
Y	100	107.13
C	100	97.02

Thailand 97

Variable	Before	After
P	100.00	74.14
TB/GDP	-6.89%	7.65%
CA/GDP	-5.94%	10.18%
Y	100	94.49
C	100	98.58

Turkey 01

Variable	Before	After
P	100	84.42
TB/GDP	-7.82%	-0.04
CA/GDP	-1.54%	-0.57%
Y	100	102.16
C	100	105.58