Chapter 5
The Basic Monetary Model*

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1 Introduction

Up to this point, we have only looked at real models that have completely abstracted from monetary considerations. The second part of this book – which comprises Chapter 5 through 9 – looks at the foundations of monetary economics in an open economy. This chapter starts our journey into this monetary world by introducing money into the endowment economy of Chapter 1. To focus exclusively on monetary phenomena (as opposed to focusing on the *interaction* between monetary and real phenomena), we introduce money in such a way that it acts as a "veil" in the sense that the economy's real variables (i.e., consumption and the external accounts) are independent of the path of monetary variables such as the money supply and the exchange rate.

In this context, we look at some basic monetary experiments that will enable us to understand monetary considerations in isolation from the rest of the economy. Section 2 starts by defining the two basic regimes under which an open economy can operate: predetermined exchange rates and flexible exchange rates. The fundamental difference between these two regimes is that while under flexible exchange rates the monetary authority controls the path of the nominal money supply, under predetermined exchange rates the nominal money supply is *endogenously* determined. Section 3 then shows that monetary and exchange rate policy are both *neutral* (changes in the level of the money supply or the exchange rate do not affect the real sector) and *superneutral* (changes in the rate of growth of the money supply or the exchange rate do not affect the real sector). Section 4 then proceeds to show that there is a fundamental equivalence between predetermined and flexible exchange rates. Specifically, for any given

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path of the nominal exchange rate set by the monetary authority under predetermined exchange rates, there will be some endogenously-determined path of the nominal money supply. If, under flexible exchange rates, the monetary authority set this precise path of the nominal money supply, the same equilibrium would obviously obtain.

While this fundamental equivalence is an important conceptual benchmark, it should not be taken to imply that in the real world exchange rate regimes are irrelevant. Consider, for instance, an economy operating under predetermined exchange rates. Given that the economy will be subject to a myriad of monetary and real shocks, the resulting equilibrium path of the nominal money supply will exhibit high variability. Such a highly variable path would not be a path that would be set by the monetary authority under flexible rates. Rather, under flexible rates, the monetary authority would choose a more stable and predictable path since, after all, the whole idea of choosing a nominal anchor is to provide a stable nominal foundation to the economy. Put differently, since, in practice, the monetary authority will choose relatively simple paths of either the nominal exchange rate or the money supply, the economy will react differently to the same shocks.

We illustrate this idea in Section 5 by analyzing the different response of the economy to money shocks depending on the exchange rate regime. Suppose that there is a negative money demand shock that will occur with certainty sometime in the future (say at time T). Under predetermined exchange rates, real money balances will only fall at the moment that the shocks hits. This fall will come about by the public buying foreign bonds from the Central Bank in exchange for domestic money. In sharp contrast, under flexible exchange rates the private sector as a whole has no means of getting rid of unwanted money balances at time T. Further, the nominal exchange rate cannot jump at time T because, if it did, there would be unbounded profit opportunities. As a result, real money balances must begin to fall in anticipation of the negative shock, which requires high inflation *before* the shock. The anticipation of the negative money demand shock will have inflationary consequences under flexible exchange rates but not under predetermined exchange rates.

Until this point in the chapter, money has been introduced by entering real money balances as an argument in the utility function. While this is a convenient shortcut, it does not make explicit the trading environment that may underlie the presence of money. A more explicit framework is analyzed in Section 6 where we switch from continuous to discrete time and introduce money via a *cash-in-advance constraint*. We adopt a formulation – originally due to Lucas (1982) – in which asset markets open before goods markets, which implies that money continues to be a veil.[1]

In sum, this chapter focuses on the simplest monetary model of a small open economy in which money is a veil in the sense that monetary/exchange

[1] As will become clear in Chapter 7, alternative timing assumptions in a discrete time environment (i.e., goods markets open before asset markets) or a continuous-time formulation of a cash-in-advance model imply that temporary changes in monetary/exchange rate policy will affect the real sector.

rate policy does not affect the real sector. Subsequent chapters will introduce various frictions into this benchmark model that will "remove the veil" and allow monetary variables to affect the real sector.

2 The basic monetary model

Consider a small open economy inhabited by a large number of identical, infinitely-lived consumers who are endowed with perfect foresight. The economy is perfectly integrated with the rest of the world in both goods and capital markets. There is only one (tradable and non-storable) good, whose price is given by the law of one price. The economy receives a flow endowment of the good (y_t). The international real interest rate (r) is given and constant over time.

2.1 Consumer's problem

2.1.1 Budget constraints

The consumer holds two assets: domestic money (M) and an internationally-traded bond denominated in the foreign currency (B^*). Nominal asset holdings are therefore:

$$A_t = M_t + E_t B_t^*, \tag{1}$$

where E is the nominal exchange rate (units of domestic currency per unit of foreign currency). By the law of one price,

$$P_t = E_t P_t^*, \tag{2}$$

where P_t is the domestic currency price of the good and P_t^* is the foreign currency price. By differentiating equation (2) with respect to time, we obtain

$$\pi_t = \varepsilon_t + \pi_t^*, \tag{3}$$

where $\pi_t(\equiv \dot{P}_t/P_t)$ is the rate of inflation, $\varepsilon_t(\equiv \dot{E}_t/E_t)$ is the rate of change of the exchange rate, and $\pi_t^*(\equiv \dot{P}_t^*/P_t)$ is the foreign inflation rate.[2]

The numeraire of this economy will be the tradable good. Hence, "real" variables will be defined in terms of tradable goods. Dividing (1) by P_t, we obtain

$$a_t = m_t + b_t, \tag{4}$$

where $a_t(\equiv A_t/P_t)$, $m_t(\equiv M_t/P_t)$, and $b_t(\equiv B_t^*/P_t^*)$ denote real financial assets, real money balances, and real foreign bonds, respectively.

The consumer's flow constraint in nominal terms is given by

$$\dot{A}_t = E_t i_t^* B_t^* + \dot{E}_t B_t^* + P_t y_t + P_t \tau_t - P_t c_t, \tag{5}$$

[2] As a matter of terminology, we will refer to ε as the rate of *devaluation* under predetermined exchange rates and as the rate of *depreciation* under flexible exchange rates.

where i_t^* denotes the foreign nominal interest rate, y_t is the endowment of the good, τ_t denotes real lump-sum transfers from the government and c_t denotes consumption. The term $E_t i_t^* B_t^*$ captures interest payments on the foreign bonds (in terms of domestic currency), while the term $\dot{E}_t B_t^*$ denotes capital gains on the foreign bonds.

To express the flow constraint in real terms, divide (5) by P_t (taking into account the law of on price) to obtain

$$\frac{\dot{A}_t}{P_t} = (i_t^* + \varepsilon_t) b_t + y_t + \tau_t - c_t, \tag{6}$$

where $\varepsilon (\equiv \dot{E}/E)$ denotes the rate of depreciation/devaluation. Given that, by definition, $a_t = A_t / E_t P_t^*$, it follows that

$$\dot{a}_t = \frac{\dot{A}_t}{P_t} - (\varepsilon_t + \pi_t^*) a_t. \tag{7}$$

Substituting (6) into (7) and rearranging terms:

$$\dot{a}_t = (i_t^* - \pi_t^*) a_t + y_t + \tau_t - c_t - (i_t^* + \varepsilon_t) m_t. \tag{8}$$

Assuming that the Fisher equation holds in the rest of the world (i.e., $i_t^* = r + \pi_t^*$) and taking into account that perfect capital mobility implies that interest parity will hold (i.e., $i_t = i_t^* + \varepsilon_t$), we can rewrite (8) as

$$\dot{a}_t = r a_t + y_t + \tau_t - c_t - i_t m_t. \tag{9}$$

Integrating forward equation (9) and imposing the transversality condition $\lim_{t \to \infty} a_t e^{-rt} = 0$ (for the reasons discussed in Chapter 1), we finally obtain

$$a_0 + \int_0^\infty (y_t + \tau_t) e^{-rt} dt = \int_0^\infty (c_t + i_t m_t) e^{-rt} dt. \tag{10}$$

This lifetime constraint makes perfect sense as it says that the present discounted value of "total expenditures" (given by the RHS, and which include the opportunity cost of holding real money balances) must equal the consumer's wealth (LHS), which comprises his/her initial real financial assets (a_0) and the present discounted value of the endowment and government transfers.

2.1.2 Utility maximization

The consumer's lifetime utility is given by

$$\int_0^\infty [u(c_t) + v(m_t)] e^{-\beta t} dt, \tag{11}$$

where $\beta (> 0)$ is the subjective discount rate, and the functions $u(c_t)$ and $v(m_t)$ are strictly increasing and strictly concave in their arguments:

$$
\begin{aligned}
u'(c_t) &> 0, \; v'(m_t) > 0, \\
u''(c_t) &< 0, \; v''(m_t) < 0.
\end{aligned}
$$

The rationale behind introducing money in the utility function is that real money balances provide liquidity services that can be thought of as proportional to the stock of real money balances. In other words, liquidity services are captured by ξm_t where, for simplicity, it is assumed that $\xi = 1$. Hence, the way to think of $v(m)$ is that consumers derive utility from the liquidity services provided by money.[3]

The consumer's problem consists in choosing $\{c_t, m_t\}$ for all $t \in [0, \infty)$ to maximize lifetime utility (11) subject to the lifetime constraint (10), for a given path of τ_t, i_t, and y_t and given values of r and a_0.

Assuming, as usual, that $\beta = r$, the first order conditions imply

$$
\begin{aligned}
u'(c_t) &= \lambda, && (12) \\
v'(m_t) &= \lambda i_t. && (13)
\end{aligned}
$$

Combining these two equations, we obtain

$$
u'(c_t) = \frac{v'(m_t)}{i_t},
$$

which implicitly defines a real money demand with standard properties:

$$
\begin{aligned}
m_t &= L(c_t, i_t), && (14) \\
\frac{\partial L}{\partial c_t} &= \frac{i_t u''(c_t)}{v''(m_t)} > 0, && (15) \\
\frac{\partial L}{\partial i_t} &= \frac{u'(c_t)}{v''(m_t)} < 0. && (16)
\end{aligned}
$$

Real money demand is thus increasing in consumption and decreasing in the nominal interest rate (the opportunity cost of holding money).[4]

2.2 Government

The government comprises the fiscal authority and the monetary authority (i.e., the Central Bank). Let H_t^* be the amount of net foreign bonds (measured in terms of foreign currency) that the government holds and $H_t (\equiv E_t H_t^*)$ denote

[3] There are two other popular ways of introducing money: via a cash-in-advance constraint (as analyzed later in this chapter and in Chapter 7) or a transactions costs technology (Chapter 7). All three ways have pros and cons that will be made clear as we proceed further.

[4] It is worth noting that, in any microfounded model of money, real money demand will depend on *consumption*, as opposed to income or output.

the domestic currency value of these bonds. The government's flow constraint in nominal terms is given by[5]

$$\dot{H}_t = \underbrace{i_t^* E_t H_t^*}_{\text{interest income}} + \underbrace{\dot{E}_t H_t^*}_{\text{capital gains}} + \underbrace{\dot{M}_t}_{\text{money printing}} - \underbrace{P_t \tau_t}_{\text{transfers}}. \qquad (17)$$

As indicated below the equation, the government has three sources of revenues: (i) interest income on its international reserves; (ii) capital gains on its international reserves; and (iii) money printing or revenues from money creation. The only government expenditure consists of lump-sum transfers.[6]

To obtain the government's flow constraint in real terms, we proceed in the same way as we did above for the consumer. Define the real value of international reserves (i.e., international reserves in terms of real dollars) as $h(\equiv H/P)$. It then follows that

$$\dot{h}_t = \frac{\dot{H}_t}{P_t} - (\varepsilon_t + \pi_t^*) h_t. \qquad (18)$$

Dividing (17) by P_t, substituting the resulting expression into (18) (and imposing the Fisher equation for the rest of the word), we obtain:

$$\dot{h}_t = r h_t + \frac{\dot{M}_t}{P_t} - \tau_t. \qquad (19)$$

It proves illuminating to write the real revenues from money creation, M_t/P_t, as

$$\frac{\dot{M}_t}{P_t} = \dot{m}_t + (\varepsilon_t + \pi_t^*) m_t. \qquad (20)$$

As indicated, real revenues from money creation can be divided into two components: (i) seigniorage (first term on the RHS), which refers to the revenues that may accrue to the government as the result of an increase in the public's *real* demand for money; and (ii) the inflation tax (second term on the RHS), which refers to the revenues that may accrue to the government as a result of the public's desire to replace the loss of value of real money balances due to a positive inflation rate.

Substituting equation (20) into equation (19), we obtain

$$\dot{h}_t = r h_t + \dot{m}_t + (\varepsilon_t + \pi_t^*) m_t - \tau_t. \qquad (21)$$

[5] Appendix 7.1 breaks down the government into the monetary and the fiscal authority and shows how equation (17) follows from consolidating their respective nominal budget constraints.

[6] At this point, we are, of course, abstracting from tax revenues from conventional taxes (i.e., taxes other than the inflation tax) and government spending.

Integrating forward equation (19) and imposing the transversality condition $\lim_{t\to\infty} h_t e^{-rt} = 0$, we obtain the government's intertemporal constraint:[7]

$$h_0 + \int_0^\infty \frac{\dot{M}_t}{P_t} e^{-rt} dt = \int_0^\infty \tau_t e^{-rt} dt. \tag{22}$$

In closing, notice that so far we have only looked at fiscal accounting and made no policy assumptions.

2.3 Equilibrium conditions

The assumption of perfect capital mobility implies that interest parity holds:

$$i_t = i_t^* + \varepsilon_t. \tag{23}$$

Let $k_t (\equiv b_t + h_t)$ denote the economy's stock of net foreign assets. Combining the consumer's flow constraint (equation (9)) with the government's (equation (21)) yields the economy's flow constraint:

$$\dot{k}_t = rk_t + y_t - c_t. \tag{24}$$

The economy accumulates net foreign assets (i.e., $\dot{k}_t > 0$) to the extent that the economy's income $(rk_t + y_t)$ exceeds the economy's consumption (c_t). To link the economy's flow constraint to standard balance of payments accounting, rewrite this equation as

$$\underbrace{\dot{h}_t}_{\Delta\text{h}} = \underbrace{-\dot{b}_t}_{\text{KA}} + \underbrace{\underbrace{r(b_t + h_t)}_{\text{IB}} + \underbrace{y_t - c_t}_{\text{TB}}}_{\text{CA}}, \tag{25}$$

where TB, IB, CA, KA, and Δh denote trade balance, income balance, current account, capital account, and increase in international reserves. As indicated, therefore, equation (25) constitutes the fundamental identity of balance of payment accounting which, in words, reads as[8]

Increase in international reserves = Capital account + Current account.

In the "real" world of Chapter 1, a current account deficit had to be financed by a capital account surplus (i.e., by borrowing from abroad). In contrast, a monetary economy can run a current account deficit *and* a capital account deficit (i.e., lending abroad) provided that the Central Bank is financing these two deficits by losing international reserves.

[7] Strictly speaking, the RHS of constraint (22) should include all future jumps in real money balances. For example, if such a jump occurred at time T, then we would include a term of the form $e^{-rT}\Delta m$.

[8] As a matter of terminology, notice that the change in international reserves is often referred to as the "balance of payments". If the country is gaining (losing) international reserves, the balance of payments is positive (negative).

Finally, notice that integrating forward the economy's flow constraint (equation (24)) and imposing the corresponding transversality condition yields the economy's resource constraint:

$$k_0 + \int_0^\infty y_t e^{-rt} dt = \int_0^\infty c_t e^{-rt} dt. \tag{26}$$

2.4 Perfect foresight equilibrium

We now characterize the perfect foresight equilibrium paths (PFEP) of consumption and real money balances. It follows from first-order condition (12) that consumption will be constant along a perfect foresight equilibrium. Then, from the economy's resource constraint (26),

$$\bar{c} = r\left(k_0 + \int_0^\infty y_t e^{-rt} dt\right). \tag{27}$$

Due to the separability of consumption and real money balances in the utility function, consumption is constant over time (and equal to permanent income) *regardless* of the path of the nominal variables. As in the endowment model of Chapter 1, consumption will be flat over time for any path of the endowment. In periods of low (high) endowment, the economy will run trade deficits (surpluses) to smooth consumption over time. Furthermore, consumption and the external accounts would respond to unanticipated endowment shocks in exactly the same way as they did in Chapter 1. Hence, in this basic monetary model, money is just a "veil" in the sense that the model exhibits the "dichotomy" between the real and monetary sectors emphasized in classical monetary economics.[9]

From (14), real money demand along a PFEP will be given by

$$m_t = L(\bar{c}, i_t). \tag{28}$$

Hence, real money demand will be constant if the nominal interest rate is constant over time. We now turn to the determination of the nominal interest rate and other nominal variables. (In what follows we will assume that the foreign inflation rate is constant at the level π^*.)

2.5 Nominal anchors

The determination of the nominal interest rate and the paths of both the nominal exchange rate and the nominal money supply will depend on the specific monetary regime adopted by the monetary authority. We will now study the determination of these variables under the two main nominal anchors: the exchange rate (predetermined exchange rates) and the money supply (flexible exchange rate).[10]

[9] It is worth noticing that this dichotomy refers to the fact that the monetary sector does not influence the real sector. But real shocks, of course, may affect monetary variables.

[10] In most modern countries, the central bank has a legal monopoly over the issuance of money. As with any monopolist, the central bank can therefore either set the price (and let

As a starting point, consider the Central Bank's balance sheet illustrated in Table 1.[11]

[Table 1]

From the balance sheet, it follows that

$$E_t H_t^* + D_t = M_t, \tag{29}$$

where D_t denotes the stock of nominal domestic credit.[12]

To organize our thoughts regarding the *mechanics* of nominal anchors and the determination of nominal magnitudes (we will deal with the economics below), it is helpful to rewrite equation (28) as

$$\frac{M_t}{E_t P_t^*} = L(\bar{c}, i_t), \tag{30}$$

where P_t^* is, of course, exogenously-given. As will be clear below, in a stationary equilibrium, the nominal interest rate, i_t, will be determined by the rate of growth of the nominal anchor. Hence, let us think of i_t as having been determined already for the purposes of thinking about the determination of nominal magnitudes. We can then viewed equations (29) and (30) as a two-equation system with four unknowns: E_t, H_t^*, D_t, and M_t. The monetary authority can then set two of these four variables and let the other two be determined endogenously. In addition, equation (30) tells us that either M_t or E_t must be set by the monetary authority (but not both because, in that case, equation (30) would be overdetermined).

In this context – and as Table 2 illustrates – we can define a predetermined exchange rate regime as one in which the monetary authority sets E_t and D_t and lets M_t and H_t^* be determined endogenously. Put differently, we can think of the monetary authority as setting E_t which, through equation (30), determines M_t. With E_t and M_t so determined and D_t being set by the monetary authority, the Central Bank's balance sheet (29) endogenously determines H_t^*.

[Table 2]

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the quantity be market-determined) or the quantity (and let the price be market-determined). The former is the case of predetermined exchange rates and the latter is the case of flexible exchange rates.

[11] Following common practice – and for simplicity – the balance sheet assumes that the central bank's capital is zero.

[12] In the context of this model – in which we do not explicitly model domestic bonds – the label "domestic credit" is somewhat of a misnomer. The label "domestic credit" is more natural when the monetary authority introduces money into the system via open-market operations whereby a purchase of domestic bonds (an increase in domestic credit) increases the money supply and a sale of domestic bonds (a reduction in domestic credit) reduces the money supply. In the current model, however, money should be thought of as being introduced into the economy through a "helicopter drop" (and being retired from the system with a giant vacuum cleaner!). Hence, "domestic credit" is essentially an accounting fiction that could, however, be interpreted as a non-interest bearing obligation of the private sector vis-a-vis the monetary authority.

To define a flexible exchange rate regime, we need to add an additional wrinkle. Under flexible exchange rates, the nominal exchange rate will be an endogenous variable. The Central Bank's balance sheet (29) would then tell us that, for given H_t^* and D_t, changes in E would change the money supply. For instance, an increase in the nominal exchange rate would lead the Central Bank to print more money. In practice, however, Central Banks typically do not "monetize" changes in the nominal exchange rate and simply credit/debit a non-monetary liability. To fix ideas, denote this non-monetary liability by NM and rewrite the balance sheet of the Central Bank as

$$E_t H_t^* + D_t = M_t + NM_t. \tag{31}$$

We can then view equations (30) and (31) as a system of two equations in 5 unknowns: E_t, H_t^*, D_t, M_t, and NM_t. The Central Bank can then set three of these five variables and let the other two be determined endogenously (see Table 2) . Under flexible exchange rates, the Central Bank sets D, M, and H_t^* and lets E_t and NM_t be determined endogenously. In other words, we can think of the Central Bank setting M_t and the money market equilibrium (30) determining E_t. With M_t and E_t so determined and the Central Bank setting H^*, equation (31) determines NM_t.

Having discussed the mechanics behind predetermined and flexible exchange rate regimes, we now turn to a more detailed discussion and solve the model under both regimes.

2.5.1 Predetermined exchange rates

Under predetermined exchange rates, the monetary authority sets the path of the nominal exchange rate (E_t) and the path of the nominal stock of domestic credit (D_t). The path of international reserves (H_t^*) and the path of the nominal supply of money (M_t) will be *endogenously* determined. The idea that the nominal money supply is *endogenous* under predetermined exchange rates is critical to the understanding of this regime.

We use the term "predetermined exchange rates" rather than the more common label "fixed exchange rates" because, strictly speaking, the latter is just a particular case of the former. The key feature of a predetermined exchange rate regime is that, at any given point in time, the monetary authority stands ready to buy and sell foreign exchange at a given price (i.e., at a given exchange rate). In other words, the public can go to the Central Bank and sell foreign exchange (e.g., dollars) to the Central Bank in exchange for domestic currency and viceversa. If the price at which the Central Bank sells/buys foreign exchange is constant over time, we then talk of a fixed exchange rate regime. But if that price varies over time, then we need to use the more general term "predetermined exchange rate".[13]

Formally, setting the path of the nominal exchange rate implies setting the initial level, $\bar{E}_0$, and a constant rate of growth, $\bar{\varepsilon}$, of the nominal exchange

[13] Box 1 lists different kinds of predetermined exchange rate regimes.

rate.[14] Given $\bar{\varepsilon}$, the interest parity condition (23) determines a constant level of the nominal interest rate, $\bar{i}$:

$$\bar{i} = i^* + \bar{\varepsilon}. \tag{32}$$

The constancy of the nominal interest rate implies, from (28), that real money balances will be constant over time at a level given by:

$$\bar{m} = L(\bar{c}, \bar{i}) \tag{33}$$

Hence, $\dot{m}_t = 0$ for all $t \in [0, \infty)$. Since, by definition, $m = M/EP^*$, it follows that the rate of money growth will also be constant over time:

$$\bar{\mu} = \bar{\varepsilon} + \pi^*. \tag{34}$$

The only nominal variable yet to be determined is the initial level of the nominal money supply, M_0. Since equation (28) holds at time 0, we can write:

$$\frac{M_0}{P_0^* \bar{E}_0} = L(\bar{c}, \bar{i}).$$

Solving for M_0:

$$M_0 = P_0^* \bar{E}_0 L(\bar{c}, \bar{i}). \tag{35}$$

Interestingly enough, so far we have not made use of the path of nominal domestic credit. Hence, all the endogenous variables derived above are independent of the path of nominal domestic credit. The path of nominal domestic credit will matter though in determining the path of international reserves (h). As indicated above, the monetary authority sets the initial level, D_0, and the (constant) rate of growth (θ) of domestic credit. To determine the resulting path of international reserves, first express the Central Bank's balance sheet in real terms:

$$h_t + d_t = m_t.$$

Then differentiate this identity and solve for $\dot{h}_t$:

$$\dot{h}_t = \dot{m}_t - \dot{d}_t. \tag{36}$$

This equation says that, under predetermined exchange rates, any creation in real domestic credit in excess of changes in real money demand will result in a loss of international reserves. In other words, if $\dot{d}_t > \dot{m}_t$, then $\dot{h}_t < 0$. Intuitively, money printed by the Central Bank that is not demanded by the private sector will result in a loss of international reserves as the private sector gets rid of unwanted money balances by exchanging them for net foreign assets at the Central Bank. To gains additional insights, notice that, since $d \equiv D/EP^*$,

[14] Of course, the monetary authority could set a non-constant path of the devaluation rate. We will study some of those cases in later chapters.

$$\dot{d}_t = d_t(\theta - \bar{\varepsilon} - \pi^*). \tag{37}$$

Substituting (37) into (36), we obtain

$$\dot{h}_t = \dot{m}_t - d_t(\theta - \bar{\varepsilon} - \pi^*).$$

In this particular case, $\dot{m}_t = 0$ because, from (33), real money demand is constant. Hence, we can rewrite this equation as

$$\dot{h}_t = -d_t(\theta - \bar{\varepsilon} - \pi^*). \tag{38}$$

Given that real money demand is constant, if nominal domestic credit is growing faster than domestic inflation (i.e., if $\theta > \bar{\varepsilon} + \pi^*$), then the monetary authority will be losing international reserves (i.e., $\dot{h}_t < 0$). Needless to say, this has been a common situation in developing countries where the monetary authority is often coerced into printing money to finance the fiscal authority's spending. If this situation persists and there is some threshold below which international reserve cannot fall, the monetary authority will run out of international reserves and a balance of payment crisis will ensue (as analyzed in detail in Chapter 9). Conversely, a situation in which $\theta < \bar{\varepsilon} + \pi^*$would imply that the monetary authority accumulates international reserves without bound and can also be ruled out. Hence, for a predetermined exchange rate regime to be sustainable over time, the rate of domestic credit growth must equal the domestic inflation rate (i.e., $\theta = \bar{\varepsilon} + \pi^*$).[15] This will be, therefore, our maintained assumption.

Having established that the path of international reserves is constant over time, we need to establish their initial value, h_0. From the Central Bank's balance sheet at $t = 0$ and (33), it follows that

$$h_0 = L(\bar{c}, \bar{i}\,) - d_0. \tag{39}$$

In principle, h_0 could take any sign, since the Central Bank could have a negative asset position.

Finally, notice that the variable that adjusts to make the government constraint hold at any point in time is the level of transfers. From (19), and taking into account that h_t, ε_t, and m_t are all constant over time, it follows that

$$\tau_t = r\bar{h} + (\bar{\varepsilon} + \pi^*)\bar{m}. \tag{40}$$

To fix ideas, consider a fixed exchange rate regime ($\bar{\varepsilon} = 0$) and zero foreign inflation. Then, $\tau_t = r\bar{h}$. If $\bar{h} < 0$, then the government is financing the debt service by taxing the private sector.

[15] We should note that, strictly speaking, this is a sufficient, though not necessary, condition for a predetermined exchange rate regime to be sustainable (see Appendix 7.3).

2.5.2 Flexible exchange rates

Under flexible exchange rates, the monetary authority does not intervene in the foreign exchange market and allows the exchange rate to be determined by market forces. If the Central Bank does not intervene in the foreign exchange market, its international reserves will be constant over time.[16] For simplicity, it is typically assumed that this initial level of international reserves is zero.

If international reserves are zero, then the Central Bank's balance sheet reduces to:

$$D_t = M_t.$$

Hence, under flexible exchange rates, setting the path of nominal domestic credit is equivalent to setting the path of the nominal money supply.[17] In particular, the monetary authority sets the initial level, $\bar{M}_0$, and a constant rate of growth, $\bar{\mu}$, of the nominal money supply.

We first show that real money balances will be constant over time. To see this, notice that $\dot{m}_t/m_t = \bar{\mu} - \varepsilon_t - \pi^*$, and use (13) and (23) to obtain:

$$\dot{m}_t = m_t \left[r + \bar{\mu} - \frac{v'(m_t)}{\lambda} \right]. \tag{41}$$

Linearizing this equation around the stationary value for real money balances (given by $r + \bar{\mu} = v'(m_t)/\lambda$), we see that this is an unstable differential equation.[18] Formally,

$$\left. \frac{\partial \dot{m}_t}{\partial m_t} \right|_{ss} = -\frac{m_{ss} v''(m_{ss})}{\lambda} > 0.$$

This implies that unless m is already at its stationary value at $t = 0$, it will diverge over time. Intuitively, if m increases the nominal interest rate must fall to accommodate this increase. By interest parity, this implies a fall in the rate of depreciation (inflation), which in turn implies that real money supply will grow faster which requires a further fall in the nominal interest rate and so forth. Hence, the only convergent equilibrium path is for real money balances to be constant and (implicitly) given by

$$r + \bar{\mu} = v'(\bar{m})/\lambda,$$

for all $t \geq 0$. Given this value for real money balances, equation (28) determines a unique nominal interest rate, $\bar{i}$. Further, since $\dot{m}_t/m_t = \bar{\mu} - \varepsilon_t - \pi^* = 0$, the (constant) rate of depreciation will be given by:

[16] The central bank could, of course, choose a non-constant path of international reserves by appropriately intervening in the foreign exchange market. This would be the case of "dirty floating" analyzed in Exercise 1 at the end of this chapter.

[17] If the constant level of reserves had not been set equal to zero, then this statement would not be quite correct because changes in the nominal exchange rate would imply capital gains or losses, which would affect the nominal money supply. In practice, however, central banks do not "monetize" capital gains or losses but instead debit a non-monetary liability.

[18] As an example, notice that if $v(m) = \log(m)$, then equation (41) becomes a linear differential equation given by $\dot{m}_t = (r + \bar{\mu})m_t - 1/\lambda$ which is, of course, unstable.

$$\bar{\varepsilon} = \bar{\mu} - \pi^*. \tag{42}$$

From the interest parity condition (23) and (42) – and taking into account that $i^* = r + \pi^*$– it follows that the constant level of the nominal interest rate is given by:

$$\bar{i} = r + \bar{\mu}.$$

The only nominal variable yet to be determined is the initial level of the nominal exchange rate, E_0 (i.e., the initial price level). Since equation (28) holds at time 0, we can write:

$$\frac{\bar{M}_0}{P_0^* E_0} = L(\bar{c}, \bar{i}).$$

Solving for E_0:

$$E_0 = \frac{\bar{M}_0}{P_0^* L(\bar{c}, \bar{i})}.$$

We have thus shown that, as under predetermined exchange rates, the value of all nominal variables is perfectly well-defined under flexible exchange rates.

Finally, how does the level of transfers get determined? Since international reserves are equal to zero and $\dot{m}_t = 0$, then from (21):

$$\tau_t = (\bar{\varepsilon} + \pi^*)\bar{m}. \tag{43}$$

3 Neutrality and superneutrality results

We now proceed to show that, in our basic monetary model, monetary and exchange rate policy are neutral and superneutral. By "neutral" monetary policy, we mean that an (unanticipated) and permanent change in the *level* of the money supply has no real effects.[19] It simply leads to an equi-proportional change in the exchange rate. By the same token, a neutral exchange rate policy means that a permanent devaluation has no real effects and leads to an equiproportional change in the nominal money supply. By "superneutral" monetary or exchange rate policy, we mean that neither a change in the rate of change of the money supply or in the rate of devaluation has any real effects.

Since both monetary and exchange rate policy are neutral and superneutral, changes in monetary/exchange rate policy have no effects on the real economy. Conversely, real shocks will have the same real effects under either flexible or predetermined exchange rate regimes, as examined in Exercise 2 at the end of the chapter. Hence, money is a veil in this basic model in the sense that there is no interaction between real and monetary variables.

[19] In this context, it is always implicitly understood that "real effects" refers to real variables other than real monetary balances (which may change).

3.1 Exchange rate policy

We will now study the effects of (i) an unanticipated and permanent devaluation (i.e., an increase in the level of the exchange rate), and (ii) an unanticipated and permanent increase in the devaluation rate.

3.1.1 A permanent devaluation

Suppose that the economy is initially in the stationary perfect foresight equilibrium characterized above (for $\pi^* = 0$). For simplicity, assume that initially $\varepsilon = \theta = 0$, so that the exchange rate is initially fixed. At $t = 0$, there is an unanticipated and permanent increase in the level of the nominal exchange rate (i.e., a permanent devaluation); see Figure 1, Panel A. What will be the effects of this devaluation?

[Figure 1]

Clearly, the devaluation has no real effects since we have already shown that, along a PFEP, consumption is given by (27) regardless of the path of the exchange rate (Figure 1, Panel B).

From (32), we also see that the nominal interest rate will not change (Figure 1, Panel C). Hence, from (33), real money demand does not change either (Figure 1, Panel D). From (34), the same is true of the rate of money growth. From (35), we see that the initial level of nominal money balance will increase in the same proportion as the exchange rate.

The main action resulting from a permanent devaluation actually takes place in the Central Bank's balance sheet. Since the stock of nominal domestic credit is controlled by policymakers and hence is given at $t = 0$, the real stock of domestic credit falls at $t = 0$ and remains at that lower level thereafter (Figure 1, Panel E). We can then infer the path of international reserves from the Central Bank balance sheet ($h_t = \bar{m} - d_t$). It follows that international reserves jump up on impact and remain constant thereafter (Figure 1, Panel F). In fact – as follows from the Central Bank's balance sheet – the change in international reserves at $t = 0$ is exactly equal to the reduction in the real stock of domestic credit:

$$\Delta h_0 = -\Delta d_0 > 0.$$

We thus conclude that *a devaluation leads to a gain in international reserves.*

What is the intuition behind the increase in international reserves at the Central Bank? The key is that while the devaluation does not affect real money demand, it reduces real money supply for the initial nominal money supply. In other words, at the initial money supply, there is an incipient excess demand for money. To rebuild their money balances, consumers go to the Central Bank and exchange net foreign assets for nominal money balances.[20]

[20] An alternative interpretation, which focuses on Central Bank intervention, goes as follows. To get rebuild money balances, consumers sell foreign assets. This puts downward pressure

The idea of an "excess money demand" leading to a gain in international reserves – which occurs instantaneously in this model with no frictions – is one of the most fundamental monetary adjustment mechanisms under predetermined exchange rates. Introducing various frictions into this basic model (such as assuming that there are no interest bearing bonds as in Chapter 6 or assuming that there are capital controls) will force this adjustment to take place gradually over time but will not alter its fundamental nature.[21]

3.1.2 A permanent increase in the rate of devaluation

Suppose now that, starting from the same initial equilibrium (for $\pi^* = \varepsilon = \theta = 0$), there is an unanticipated and permanent increase in the devaluation rate (Figure 2, Panel A).[22] Once again, consumption remains constant (Figure 2, Panel B). From (32), we infer that the nominal interest rate increases *pari passu* with the devaluation rate (Figure 2, Panel C). Since the opportunity cost of holding money increases, real money demand falls at $t = 0$, as follows from (33) (Figure 2, Panel D). The rate of money growth increases (from (34)). From (35), the initial level of the money supply also falls. The path of real domestic credit remains unchanged (Figure 2, Panel E). Finally, since real money demand falls, we infer from (39) that international reserves fall at $t = 0$.

[Figure 2]

How does this adjustment take place? In response to the increase in the opportunity cost of holding money, the consumer wants to reduce his/her real money holdings. To do so, he/she goes to the Central Bank and exchanges domestic money for foreign assets (i.e., sells domestic currency and buys foreign assets). As a result, international reserves at the Central Bank fall whereas private holdings of net foreign assets go up. For the economy as a whole, net foreign assets do not change.

It is interesting to note that while a permanent devaluation leads to an *increase* in international reserves, a permanent increase in the devaluation rate results in a *fall* in international reserves. The difference is due to the different way in which equilibrium in the money market is affected. In the first case – and for the initial nominal money supply – the devaluation leads to an incipient excess real money demand which requires an upward adjustment in the nominal

on the nominal exchange rate (the nominal price of foreign bonds). To prevent the domestic currency from appreciating, the central bank must step in and buy the foreign assets offered by the private sector. By so doing, the central bank increases the money supply until the money market is in equilibrium, at which point there are no further pressures on the nominal exchange rate.

[21] By the same token, a revaluation (i.e., a fall in E) would lead to a loss in international reserves because, at the initial nominal money supply, there would be an excess supply of money.

[22] To ensure that the predetermined exchange rate regime continues to be sustainable, we assume that θ increases by the same amount.

money supply.[23] In the second case – and for the initial nominal money supply – the increase in the rate of devaluation leads to an incipient excess real money supply which requires a fall in the nominal money supply.

3.2 Monetary policy

We now turn to flexible exchange rates and examine the effects of (i) an unanticipated and permanent increase in the stock of nominal money and (ii) an unanticipated and permanent increase in the rate of money growth.

Permanent increase in money supply Suppose that, starting from the initial equilibrium described above (with $\bar{\mu} = \pi^* = 0$), there is an unanticipated and permanent increase in the nominal money supply (Figure 3, Panel A). Consumption, of course, remains constant (Figure 3, Panel B). In the new equilibrium, real money balances must remain constant for, if they did not, the path of real money balances would diverge over time (Figure 3, Panel C). The constancy of real money balances implies that, on impact, the nominal exchange rate will increase by the same proportion as the nominal money supply and remain at that level thereafter (Figure 3, Panel D). The fact that $\dot{m}_t = 0$ for all t implies that the rate of depreciation continues to be equal to 0 (Figure 3, Panel E). This implies that the nominal interest rate also remains constant (Figure 3, Panel F).

[Figure 3]

We conclude that a permanent increase in the nominal money supply leads to an equi-proportional increase in the nominal exchange rate, leaving unchanged all other variables.

Permanent increase in the rate of money growth Suppose that, starting from the initial equilibrium described above (with $\bar{\mu} = \pi^* = 0$), there is an unanticipated and permanent increase in the rate of money growth (Figure 4, Panel A). Consumption, of course, remains unchanged (Figure 4, Panel B). Since real money balances are governed by an unstable differential equation, they must adjust immediately to their new and lower stationary value (Figure 4, Panel C). Otherwise, the path of real money balances would diverge over time. Since the nominal money supply is given at $t = 0$, we infer that the nominal exchange rate increases on impact (Figure 4, Panel E). The fact that $\dot{m}_t = 0$ for all t implies that the rate of depreciation increases on impact and remains at that level thereafter. This, in turn, implies that the nominal exchange rate increases at the rate of money growth from $t = 0$ onwards (Figure 4, Panel D). By interest parity, the nominal interest rate increases on impact and stays constant thereafter (Figure 4, Panel F).

[23] In contrast, a permanent increase in domestic credit (i.e., an increase in D_t) would lead to an incipient excess suppy of money and hence a loss of international reserves, as analyzed in Exercise 3 at the end of the chapter.

[Figure 4]

We conclude that a permanent increase in the rate of money growth leads to an increase in the exchange rate (a nominal depreciation) and a corresponding increase in the rate of devaluation and the nominal interest rate.

4 Equivalence results

We will now show that there is a fundamental equivalence between predetermined and flexible exchange rates. Consider an economy operating under predetermined exchange rates. (For simplicity, assume foreign inflation is zero.) For any given path of the exchange rate set by the monetary authority, there will be a corresponding path of the nominal money supply. If, under flexible rates, the monetary authority were to set exogenously that path of the nominal money supply, the same equilibrium paths would obtain. In fact – and unless you were able to see the movements in international reserves at the Central Bank – you would be unable to tell the two regimes apart.

We will illustrate this point by looking at two examples. First, we will assume that the economy is operating under a predetermined exchange rate regime with a non-constant rate of devaluation and study the implications for the path of the nominal money supply. We will then examine the opposite case: we will assume that the economy is operating under flexible exchange rates with a non-constant path of the money growth rate and examine the implications for the path of the nominal exchange rate.[24]

4.1 A non-constant rate of devaluation

Suppose that the economy is operating under predetermined exchange rates. Consider the perfect foresight equilibrium corresponding to the path of the devaluation rate illustrated in Figure 5, Panel A. The rate of devaluation is constant until time T (at the level ε^H) at which time it falls to a lower level (ε^L) and stays there afterwards. The corresponding path of the nominal exchange rate is illustrated in Figure 5, Panel B. By the interest parity condition (equation (23)), the nominal interest rate is high and then low (Figure 5, Panel C). From (28), real money demand will be low until time T and then jump to a higher level at time T (Figure 5, Panel D). (Consumption is of course constant and independent of the path of the nominal exchange rate.) The path of the nominal money supply (Figure 5, Panel E) follows from the path of real money balances and that of the nominal exchange rate. Since real money balances are constant until time T, the nominal money stock must be growing at the same rate as the nominal exchange rate (ε^H). At time T, the nominal money stock increases discretely as the public exchanges foreign assets for domestic money at the

[24] Exercise 4 at the end of the chapter looks at a third example (an anticipated increase in the money supply under flexible rates).

Central Bank. After time T, the nominal money supply increases at the lower rate of devaluation (ε^L). Finally, since $\dot{m}_t = 0$ for all $t \geq 0$, then $\mu_t = \varepsilon$. Hence, μ is first high and then falls at time T (Figure 5, Panel F).

[Figure 5]

Consider now this same economy operating under flexible exchange rates. Further, suppose that the path of the nominal money supply set by the Central Bank is described by Figure 5, Panel E. In other words, the Central Bank has announced that the nominal money supply will grow at the rate ε^H until time T, increase discretely at time T, and then grow at the lower rate ε^L. We will now check that – as we should expect – the corresponding paths of the rate of depreciation, the nominal exchange rate, the nominal interest rate, and real money balances are as illustrated in Figure 5.

Recall that, under flexible exchange rates, real money balances are governed by the unstable differential equation given by (41). Between 0 and T, the dynamics of real money balances will be governed by the laws of motion corresponding to the stationary equilibrium given implicitly by $r + \varepsilon^H = v'(m)/\lambda$. At time T, real money balances will jump since nominal money supply goes up but the nominal exchange rate does not change. By construction (recall Figure 5, Panel D), the jump in nominal money at time T is exactly the increase needed to take real money balances from their level between $[0, T)$ to their new and higher level. Hence, real money balances will stay at their stationary value until time T and at that point jump to their new value. This is, of course, the same path described by Figure 5, Panel D. Having established that $\dot{m}_t = 0$ for all $t \geq 0$, it then follows that the rate of depreciation is given by the rate of money growth at all points in time and thus follows the path illustrated in Figure 5, Panel A. The corresponding path of the nominal exchange rate is given by Figure 5, Panel B.

In sum, if under flexible rates the monetary authority sets the path of the nominal supply given by Figure 5, Panel E, the equilibrium paths of the rate of depreciation, the nominal exchange rate, the nominal interest rate, and real money balances would be exactly the same as under a predetermined exchange rates system in which the Central Bank sets the path of devaluation given by Figure 5, Panel A. An outside observer who can only see the six variables illustrated in Figure 5 would *not* be able to tell if the economy is operating under predetermined exchange rates or flexible exchange rates. But, of course, if the observer could see the Central Bank's balance sheet, he/she would be able to tell by the changes in the balance sheet at time T. Under predetermined exchange rates, he/she would see the increase in the monetary base accompanied by an increase in international reserves whereas under flexible exchange rates, he/she would see an increase in nominal domestic credit.[25]

[25] Of course, if under predetermined exchange rates, the central bank increased domestic credit at T by precisely the amount needed to meet the additional real money demand, our observer would not be able to tell which regime the economy is operating under even by looking at the central bank's balance sheet!

4.2 A non-constant path of the money growth rate

Suppose now that the economy is operating under flexible exchange rates and the Central Bank sets a non-constant rate of money growth, as illustrated in Figure 6, Panel A. The corresponding path of the nominal money supply is depicted in Figure 6, Panel B.

[Figure 6]

To solve for the perfect foresight path of real money demand, we need to make the critical observation that real money balances cannot jump at T. To see this, recall that, by definition, $m = M/EP^*$. The path of the nominal money supply is, by assumption, continuous at time T. The same is true of P^*. Furthermore, in equilibrium, the nominal exchange rate cannot jump at T (i.e., the nominal exchange rate cannot jump in an anticipated fashion). If it did, there would be infinite arbitrage opportunities which would be inconsistent with equilibrium. For example, suppose that the exchange rate were expected to increase at time T. Since the consumer knows this with certainty, he/she would get rid of all money balances just an instant before the increase in the exchange rate. Real money demand would thus fall to zero an instant before T, which is inconsistent with equilibrium (as it would lead to an infinite price level or exchange rate). Conversely, if the exchange rate were expected to fall at T, consumers would want to get rid of all net foreign assets and switch into domestic money in anticipation of infinite returns at time T. Money demand just an instant before T would be infinite which, again, is inconsistent with equilibrium. We conclude that real money balances must be continuous at time T.

Given that real money balances cannot jump at T, we infer that m will have to converge in a continuous fashion to its higher stationary equilibrium. Hence, m_0 will need to be above the level corresponding to the stationary equilibrium implicitly defined by $r + \mu^H = v'(m)/\lambda$ so that it increases during the transition period (Figure 6, Panel C). Since consumption is always equal to permanent income in this model, we can infer the behavior of the nominal interest rate from the real money demand equation (28). The nominal interest rate will thus fall over time towards its stationary value (Figure 6, Panel D).

As for the path of the rate of depreciation, notice that

$$\frac{\dot{m}_t}{m_t} = \mu^H - \varepsilon_t > 0, \qquad t \in [0, T).$$

We infer that during the transition the rate of depreciation will be below μ^H. From the interest parity condition, we also know that the rate of depreciation will be falling over time and be continuous at T (see Figure 6, Panel E). The corresponding path of the nominal exchange rate is depicted in Figure 6, Panel F.

Suppose now that the economy were operating under predetermined exchange rates and that the Central Bank set the path for the nominal exchange rate depicted in Figure 6, Panel F. In other words, the Central Bank would set a path that would involve a declining rate of devaluation over time until it converges to μ^L. From the interest parity condition (23) , the nominal interest rate would follow the path illustrated in Figure 6, Panel D. From the real money demand equation and the fact that consumption is constant throughout, the path of real money balances would be as illustrated in Figure 6, Panel C. The change in real money balances over time combined with the rate of devaluation would yield, by construction, the path for the rate of money growth illustrated in Figure 6, Panel A.

Once again, if an outside observer saw the behavior of the six variables in Figure 6, he/she would be unable to tell which exchange rate regime the economy is operating under. The observer would need to see the Central Bank's balance sheet to find out the exchange rate regime. Under predetermined exchange rates, the Central Bank is gaining international reserves between time 0 and time T whereas, under flexible rates, domestic credit is increasing.[26]

4.3 On the non-equivalence in practice

Even though, as we just saw, there is a fundamental theoretical equivalence between predetermined and flexible exchange rates, it does *not* follow that exchange rate regimes are irrelevant. Far from it – and as we will see in this and subsequent chapters – the exchange rate regime is often crucial in determining the economy's response to various shocks. The reason is that, in practice, policymakers set relatively simple paths for the exchange rate (under predetermined exchange rates) or for the money supply (under flexible exchange rates). Given these simple paths, and the fact that economies are subject to a multitude of both real and monetary shocks, the nominal money supply (under predetermined exchange rates) or the nominal exchange rate (under flexible exchange rates) will follow very volatile paths. For exchange rate regimes to be equivalent in practice, we would need to observe policymakers setting, say, some simple path for the nominal exchange rate (under predetermined exchange rates) and then some other policymakers setting the corresponding, and very volatile, paths for the nominal money supply (under flexible rates). Or viceversa, we would need to observe some policymakers setting simple paths for the money supply and others setting the equivalent (and possibly highly volatile) paths for the nominal exchange rate. This is not how policymakers operate in practice. In the actual world, policymakers operating under predetermined exchange rates will set some simple paths for the nominal exchange rates and policymakers operating under flexible exchange rates will set simple paths for the nominal money supply.[27] After all, nominal exchange rate rules or money supply rules are credible and provide an anchor to inflationary expectations provided that

[26] If, under predetermined exchange rates, the Central Bank were increasing domestic credit so as to satisfy the increasing money demand, then the two systems would behave identically in all dimensions.

[27] We will deal with nominal interest rate rules in Chapter 9 but the same argument would apply. A given nominal interest rate rule would correspond to very volatile paths of the nominal money supply and nominal exchange rate, depending on what shocks are hitting the economy.

they are simple and easily understandable by the public. As result, exchange rate regimes are not equivalent in practice. The next section provides an illustration of this by studying the economy's response to the same shock under predetermined and then flexible exchange rates. We will see that, as we would expect, the response is quite different.

5 Temporary money shocks

This section illustrates the practical "non-equivalence" of exchange rate regimes by analyzing how an anticipated negative shock to money demand will lead to quite a different dynamic response under predetermined and flexible exchange rates. Under predetermined, the inflation rate (i.e., the rate of devaluation) is controlled by policymakers and the negative demand shock will show up as a loss of reserves. In sharp contrast, under flexible rates, inflation (i.e., the rate of devaluation) will increase in *anticipation* of the shock and reach its maximum just before the shock actually happens (which is reminiscent of Sargent and Wallace's (1981) unpleasant monetarist arithmetic).

To tackle money demand shocks in our framework, we modify preferences in the following way:

$$\int_0^\infty [u(c_t) + \gamma_t v(m_t)] e^{-\beta t} dt, \tag{44}$$

where $\gamma_t (> 0)$ is a shock to the liquidity services provided by real money balances. The rest of the model remains unchanged.

While the first-order condition for consumption remains given by (12), the first-order condition for real money balances now reads as:

$$\gamma_t v'(m_t) = \lambda i_t. \tag{45}$$

Combining (12) and (45) implicitly defines a real money demand of the form:

$$\begin{aligned} m_t &= L(c_t, i_t, \gamma_t), \\ \frac{\partial L}{\partial c_t} &= \frac{i_t u''(c)}{v''(m)\gamma_t} > 0, \\ \frac{\partial L}{\partial i_t} &= \frac{u'(c)}{\gamma_t v''(m)} < 0, \\ \frac{\partial L}{\partial \gamma_t} &= -\frac{v'(m)}{\gamma_t v''(m)} > 0. \end{aligned} \tag{46}$$

As we should have expected, an increase in γ_t raises real money demand.

We will now consider a perfect foresight equilibrium path for a *non-constant* path of the money shock parameter, γ_t. (We assume that foreign inflation is constant over time and equal to $\overline{\pi^*}$.) Specifically, suppose that γ_t is constant until T at which point it falls. In other words, there is an anticipated negative money demand shock. Formally:

$$\gamma_t = \begin{cases} \gamma^H & 0 \leq t < T, \\ \gamma^L & t \geq T, \end{cases} \tag{47}$$

where $\gamma^H > \gamma^L$.

We will now characterize the corresponding PFEP under both predetermined exchange rates and flexible exchange rates.

5.1 Predetermined exchange rates

Suppose that the economy is operating under a predetermined exchange rate regime with a constant rate of devaluation given by $\bar{\varepsilon}$. The path of the money shock parameter is given by (47) (Figure 7, Panel A). How will the economy behave under predetermined exchange rates?

[Figure 7]

We know, of course, that consumption is not affected (Figure 7, Panel B). The path of the nominal interest rate which, given the interest parity condition (23), is determined by the constant rate of devaluation, is also constant over time (Figure 7, Panel F). Real money balances, however, will fall at time T in response to the negative money demand shock, as follows from the real money demand (46) (Figure 7, Panel C). Since the real stock of domestic credit remains constant throughout, we know that the nominal money supply falls at T. This, in turn, corresponds to a fall in international reserves at T (Figure 7, Panel D).

5.2 Flexible exchange rates

Suppose now that the economy is operating under a flexible exchange rate regime with a constant rate of money growth given by $\bar{\mu}$. Again, the path of the money shock parameter is given by (47) (Figure 8, Panel A). How will this economy behave over time?

[Figure 8]

Under flexible exchange rates, the path of real money demand will be governed by an unstable differential equation along the lines of (41). Given that we now have a money demand shock in the model, it is straightforward to verify that the corresponding differential equation for real money balances is given by

$$\dot{m}_t = m_t \left[r + \bar{\mu} - \frac{\gamma_t v'(m_t)}{\lambda} \right].$$

The stationary value of real money demand is therefore implicitly given by:

$$r + \bar{\mu} = \frac{\gamma_t v'(m_t)}{\lambda}. \tag{48}$$

It follows from (48) that at time T, the stationary value of real money balances falls (recall that λ is constant along a perfect foresight path). Further, for the same reasons argued above (i.e., both M and E are continuous functions of time at T), m will be continuous at time T. For the path of m to be continuous at time T, m needs to start below the stationary equilibrium corresponding to γ^H, fall over time, and reach the stationary equilibrium corresponding to γ^L at precisely time T. The path of real money balances is illustrated in Figure 8, Panel C.

Given the path of real money balances depicted in Figure 8, Panel C, we can derive the path of the rate of depreciation and hence of the nominal interest rate. Since $\dot{m}_t/m_t = \bar{\mu} - \varepsilon_t - \pi^*$, it follows that

$$\varepsilon_t = \bar{\mu} - \pi^* - \frac{\dot{m}_t}{m_t}. \tag{49}$$

Equation (49) tells us four critical things. First, for $t \geq T$, $\dot{m}_t = 0$ and therefore ε_t will be constant and equal to $\bar{\mu} - \pi^*$. Second, for $t \in [0, T)$, the fact that $\dot{m}_t < 0$ implies that $\varepsilon_t > \bar{\mu} - \pi^*$. Third, at time T ε_t will jump down because, as Figure 8, Panel C makes clear, $\dot{m}_t$ is negative until just before T and then becomes zero at T. Fourth, to find out how ε_t behaves for $t \in [0, T)$, differentiate (49) with respect to time to obtain:

$$\dot{\varepsilon}_t = -\frac{1}{m_t^2}(\ddot{m}_t m_t - \dot{m}_t^2) > 0,$$

where the sign follows from the fact that $\ddot{m} < 0$. Putting together all these pieces of information, we conclude that ε_t starts above its stationary equilibrium, increases over time, and then falls at time T (Figure 8, Panel D). Given interest parity, the path of the nominal interest rate will follow exactly the same pattern (Figure 8, Panel F).

What about the nominal exchange rate? We already know that the nominal exchange rate cannot jump at T. Hence, given that the rate of depreciation itself increases over time, the logarithm of the nominal exchange rate increases over time at an increasing rate and then becomes constant at time T (Figure 8, Panel E.)

The remarkable feature of this case is that the inflation rate is high (and increasing over time) *before* the negative money demand shock actually occurs. In fact, an outside observer who is not aware of the forthcoming shock to money demand would be quite dumbfounded by the emergence of inflation when the money supply has not changed at all!

In sum – and comparing Figures 7 and 8 – we can see the very different response of this economy to an anticipated fall in money demand. Under predetermined exchange rates, this shock has no effect whatsoever on the inflation rate (i.e., on the rate of devaluation) or the nominal interest rate. In sharp contrast, under flexible exchange rates, the rate of inflation (i.e., depreciation) rises in anticipation of the shock, as does the nominal interest rate.[28,29]

6 Money as a veil in a cash-in-advance model

So far in this chapter we have examined monetary phenomena in a money-in-the-utility function model in which money is a veil in the sense that the real equilibrium is independent of monetary/exchange rate policy. Another popular way of introducing money into the endowment model of Chapter 1 is through a cash-in-advance constraint.[30] We will now study a discrete-time formulation of a cash-in-advance model in which money is a veil and analyze how the monetary equilibrium is determined under both predetermined and flexible exchange rates. In so doing, we will also have a chance to go over some important accounting in discrete-time terms.

6.1 Households

6.1.1 Budget constraints

We first need to be specific about the economic environment in which households operate. Households enter period t with a certain amount of nominal cash balances, M_{t-1}, and a certain amount of nominal foreign bonds, B^*_{t-1}. Periods are divided into two sub-periods. In the first sub-period, asset markets open. In the asset markets, agents receive/pay interest on the net foreign bonds they carried over from last period, buy or sell bonds in exchange for money (at the Central Bank under predetermined exchange rates or in the foreign exchange market under flexible exchange rates) and receive nominal transfers from the government, $P_t\tau_t$. Households exit the asset market with a quantity M^p_t of nominal cash balances and B^*_t of nominal bonds. Formally:

$$M^p_t + E_tB^*_t = E_t(1+i^*_{t-1})B^*_{t-1} + M_{t-1} + P_t\tau_t. \qquad (50)$$

In the second sub-period, goods market open. Think of households as composed of two individuals: a shopper and a seller. The shopper and seller part at the beginning of the goods market sub-period and do not meet again until goods markets close.[31] Think of the seller as staying in the store selling the endowment of the good to other households' shoppers. The shopper leaves the store with nominal money balances of M^p_t and uses part or all of this money to buy goods from other stores. Since, by assumption, the shopper needs to use

[28] Drazen and Helpman (1990) study the inflationary consequences of anticipated policies in the context of a closed economy.

[29] Exercise 5 at the end of this chapter provides yet another illustration of how the economy responds differently depending on the exchange rate regime by studying the consequences of an anticipated increase in the rate of growth of the nominal anchor.

[30] See Helpman (1981) and Lucas (1982).

[31] If it helps you, think of a "period" as a day. Assets markets open in the morning and close at noon. Goods markets open at noon and close at 5 pm. The shoppper and the seller say good-bye at noon and do not see each other until after 5 pm.

money acquired in the asset markets to buy goods in the goods markets, we refer to this as a cash-in-advance constraint.[32] Formally:

$$M_t^p \geq P_t c_t. \tag{51}$$

What are the households's money balances at the end of period t (denoted by M_t)? Households will have the money obtained from selling goods at the store ($P_t y_t$) and the money brought from the asset markets that was not spent on purchasing goods ($M_t^p - P_t c_t$). Formally,

$$M_t = M_t^p - P_t c_t + P_t y_t. \tag{52}$$

By substituting (52) into (50), we obtain the households' flow constraint for period t as a whole:

$$M_t + E_t B_t^* = E_t(1 + i_{t-1}^*)B_{t-1}^* + M_{t-1} + P_t \tau_t + P_t y_t - P_t c_t. \tag{53}$$

For the sake of comparison with the continuous-time case, we can define nominal assets as $A_t \equiv M_t + E_t B_t^*$ and, by adding and subtracting $E_{t-1}B_{t-1}^*$ in the LHS of equation (53), rewrite it as:

$$A_t - A_{t-1} = \underbrace{E_t i_{t-1}^* B_{t-1}^*}_{\text{interest payments}} + \underbrace{(E_t - E_{t-1})B_{t-1}^*}_{\text{capital gains}} + P_t \tau_t + P_t y_t - P_t c_t,$$

which is the discrete-time counterpart of flow constraint (5) in the continuous-time case.

To express the flow constraint in real terms, divide both sides of (53) by P_t and manipulate terms to obtain:

$$m_t + b_t = \frac{E_t}{P_t} P_{t-1}^*(1 + i_{t-1}^*)b_{t-1} + \frac{P_{t-1}}{P_t} m_{t-1} + \tau_t + y_t - c_t,$$

where, by definition, $m_t \equiv M_t/P_t$ and $b_t \equiv E_t B_t^*/P_t = B_t^*/P_t^*$ denote real money balances and real bond holdings. Defining the inflation rate in period t as $1 + \pi_t = P_t/P_{t-1}$, assuming – as usual – that the Fisher equation holds in the rest of the world (i.e., $1 + i_{t-1}^* = (1 + r)(P_t^*/P_{t-1}^*)$), and using the law of one price (i.e., $P_t = EP_t^*$), we can rewrite the last equation as

$$b_t + m_t = (1 + r)b_{t-1} + \frac{m_{t-1}}{1 + \pi_t} + \tau_t + y_t - c_t. \tag{54}$$

Adding and subtracting $(1 + r)m_{t-1}$ from the RHS of the last equation and taking into account that, by definition, $a_t = m_t + b_t$, we obtain:

$$a_t = (1 + r)a_{t-1} + \tau_t + y_t - c_t - \frac{i_{t-1}}{1 + \pi_t} m_{t-1}, \tag{55}$$

[32] The implicit assumption is that even though all households are identical, they do not consume their own endowment. If it helps, think of each households as being endowed with the same good (say, candy) but that candy comes in different colors. Households do not like the color of their own candy and wish to buy other household's candies.

which is the discrete-time counterpart of equation (9). To understand intuitively the opportunity cost term (i.e., the last term on the RHS), notice that by holding nominal money balances given by M_{t-1} from period $t-1$ to period t, the household foregoes interest payments in the amount of $i_{t-1}M_{t-1}$, the real value of which is $i_{t-1}M_{t-1}/P_t$ in period t. By multiplying and dividing by P_{t-1}, this term can be expressed as $i_{t-1}m_{t-1}/(1+\pi_t)$.

6.1.2 Utility maximization

We can set up the household's maximization problem in various ways depending on which constraints we use. It proves convenient to substitute equation (50) into the cash-in-advance constraint (51) to obtain:

$$E_t(1+i^*_{t-1})B^*_{t-1} + M_{t-1} + P_t\tau_t - E_tB^*_t \geq P_tc_t. \tag{56}$$

Expressing this equation in real terms, we obtain:

$$(1+r)b_{t-1} + \frac{m_{t-1}}{1+\pi_t} + \tau_t - b_t \geq c_t. \tag{57}$$

Households then choose $\{c_t, b_t, m_t\}_{t=0}^{\infty}$ to maximize lifetime utility subject to a sequence of flow budget constraints given by (54) and a sequence of *inequality* constraints given by the cash-in-advance constraints (57). In terms of the Lagrangian,

$$\begin{aligned}\mathcal{L} = \quad & \sum_{t=0}^{\infty}\beta^t u(c_t) \\ & + \sum_{t=0}^{\infty}\beta^t\lambda_t\left[(1+r)b_{t-1} + \frac{m_{t-1}}{1+\pi_t} + \tau_t + y_t - c_t - b_t - m_t\right] \\ & + \sum_{t=0}^{\infty}\beta^t\Psi_t\left[(1+r)b_{t-1} + \frac{m_{t-1}}{1+\pi_t} + \tau_t - b_t - c_t\right].\end{aligned}$$

The first-order conditions for c_t, m_t, and b_t are given, respectively, by

$$u'(c_t) = \lambda_t + \Psi_t, \tag{58}$$

$$-\lambda_t + \frac{\beta\lambda_{t+1}}{1+\pi_{t+1}} + \frac{\beta\Psi_{t+1}}{1+\pi_{t+1}} = 0, \tag{59}$$

$$\beta(1+r)(\lambda_{t+1} + \Psi_{t+1}) = \lambda_t + \Psi_t. \tag{60}$$

The first-order condition for λ_t recovers equation (54). The first-order condition for Ψ_t takes the form of a Kuhn-Tucker condition:

$$(1+r)b_{t-1} + \frac{m_{t-1}}{1+\pi_t} + \tau_t - b_t \geq c_t \qquad \Psi_t \geq 0, \qquad \left[(1+r)b_{t-1} + \frac{m_{t-1}}{1+\pi_t} + \tau_t - b_t - c_t\right]\Psi_t = 0. \tag{61}$$

In words, if the cash-in-advance holds as an inequality, the associated multiplier will be zero.

Given the assumption that $\beta(1+r)=1$, it follows from condition (60) that $\lambda_{t+1}+\Psi_{t+1}=\lambda_t+\Psi_t$ and hence, from condition (58), that

$$u'(c_{t+1})=u'(c_t). \tag{62}$$

As in the endowment economy of Chapter 1, consumption will be constant along any perfect foresight path. Moreover, since we have yet to say anything about monetary/exchange rate policy, full consumption smoothing will hold irrespective of the exchange rate regime in place. This proves that, in this version of the cash-in-advance model, money is a veil in the sense that the real equilibrium will not depend on the path of monetary variables. Hence, in respond to fluctuations in the endowment, the economy will use the trade balance as a shock absorber and keep consumption constant by borrowing in bad times (i.e., when the endowment is low) and repaying in good times (i.e., when the endowment is high).

We will now show that if the nominal interest rate is positive, the cash-in-advance constraint will bind. Taking into account that $\beta=1/(1+r)$, condition (59) becomes

$$\frac{\lambda_{t+1}+\Psi_{t+1}}{1+i_t}=\lambda_t.$$

Since $\lambda_{t+1}+\Psi_{t+1}=\lambda_t+\Psi_t$, we can rewrite this last condition as:

$$\lambda_t i_t=\Psi_t.$$

Since $\lambda_t>0$, it follows that if $i_t>0$, then $\Psi_t>0$ which implies, from the slackness condition, that the cash-in-advance binds. This is, of course, very intuitive. If the nominal interest rate is positive, it makes no sense for households to leave the asset markets with more money than they need to purchase goods when goods markets open because they could have always used that money to buy bonds and receive interest rate payments at the beginning of the following period.

On the other hand, if $i_t=0$, $\Psi_t=0$ and the cash-in-advance constraint is non-binding in the sense that it becomes irrelevant for the consumer's choice. In this case, households will be indifferent between holding money or bonds and hence the choice of money balances is indeterminate.[33]

6.2 Government

As before, let H^* denote the foreign currency value of international reserves. The government's flow budget constraint in nominal terms is thus given by

[33] Of course, from a purely mathematical point of view, the cash-in-advance constraint could still hold as an equality even if $\Psi_t=0$. But, from an economic point of view, this is irrelevant since money and bonds are perfect substitutes and the cash-in-advance does not constrain consumers' choices.

$$E_t H_t^* = E_t(1+i_{t-1}^*)H_{t-1}^* + M_t - M_{t-1} - P_t\tau_t.$$

To express it in real terms, divide both sides of this equation by P_t to obtain

$$h_t = (1+r)h_{t-1} + \frac{M_t - M_{t-1}}{P_t} - \tau_t, \tag{63}$$

where we have used the fact that, by definition, $h_t \equiv H_t^*/P_t^*$, from the law of one price, $P_t = E_t P_t^*$, and, from the Fisher equation in the rest of the world, $1+i_{t-1}^* = (1+r)(P_t^*/P_{t-1}^*)$.

Finally, and for further reference, notice that revenues from money creation may be expressed as follows:

$$\frac{M_t - M_{t-1}}{P_t} = m_t - m_{t-1} + \frac{\pi_t}{1+\pi_t} m_{t-1}. \tag{64}$$

6.3 Equilibrium conditions

Perfect capital mobility implies that interest parity holds:

$$1 + i_t = (1+i_t^*)\frac{E_{t+1}}{E_t}. \tag{65}$$

To obtain the economy's flow constraint, combine the consumer's flow constraint (given by equation (54)) with the government's (given by equation (63)) – taking into account (64) – to obtain:

$$b_t + h_t = (1+r)(b_{t-1} + h_{t-1}) + y_t - c_t. \tag{66}$$

Let $k(\equiv b + h)$ denote the economy's total net foreign assets. Iterating forward and imposing the transversality condition $\underset{t\to 0}{lim} \frac{k_t}{(1+r)^t} = 0$ yields:

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = (1+r)(b_{-1} + h_{-1}) + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}. \tag{67}$$

6.3.1 Perfect foresight equilibrium

We have established from (62) that consumption is constant over time. From the resource constraint (67), it then follows that:

$$\bar{c} = \frac{r}{1+r}\left[(1+r)(b_{-1} + h_{-1}) + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}\right]. \tag{68}$$

As in the continuous-time case, the constant level of consumption is equal to permanent income.

Taking into account (51) and the fact that we have established that the CIA constraint will bind, the constant value of consumption will determine the value of real money balances taken into the goods markets:

$$\frac{M_t^p}{P_t} = \bar{c}. \tag{69}$$

Further, notice that substituting (69) into (52), it follows that

$$M_t = P_t y_t, \tag{70}$$

which you will recognize as a quantity theory equation with unitary velocity.

6.4 Predetermined exchange rates

Under predetermined exchange rates, the Central Bank sets the path of the nominal exchange rate (i.e., E_0, E_1, ...) and the path of the stock of nominal domestic credit (D_0, D_1, ...). Specifically – and as in the continuous-time case – suppose that the monetary authority sets E_0 and then a constant rate of devaluation. Formally,

$$\frac{E_{t+1}}{E_t} = 1 + \bar{\varepsilon}, \qquad t = 0, 1, 2, \ldots.$$

Given this constant rate of devaluation, the nominal interest rate follows from the interest parity condition (65):

$$1 + i_t = (1 + i^*)(1 + \bar{\varepsilon}). \tag{71}$$

Let us now determine the path of the price level. Let the rate of foreign inflation be constant; that is: $P_{t+1}^*/P_t^* = 1 + \overline{\pi^*}$. Then, given the law of one price ($P = EP^*$), the initial price level is given by $P_0 = E_0 P_0^*$. The law of one price also determines a constant level of inflation:

$$1 + \bar{\pi} = (1 + \bar{\varepsilon})\left(1 + \overline{\pi^*}\right),$$

where $1 + \bar{\pi} = P_{t+1}/P_t$.

We now turn to the path of the nominal money supply. Given the quantity theory equation (70), $M_0 = P_0 y_0$. The rate of growth of the nominal money supply also follows from the quantity theory equation (70):

$$1 + \mu_{t+1} = (1 + \bar{\pi})\,\frac{y_{t+1}}{y_t},$$

where $1 + \mu_{t+1} \equiv M_{t+1}/M_t$. Money supply growth will be higher in good times and lower in bad times because a higher (lower) endowment implies higher (lower) sales during the goods market period and hence more (less) nominal money balances carried over to next period.

To derive the path of international reserves, start from the Central Bank's balance sheet, given by

$$h_t + \frac{D_t}{E_t P_t^*} = \frac{M_t}{E_t P_t^*}.$$

Considering the same identity for $t+1$ and subtracting, we get:

$$h_{t+1} - h_t = \frac{M_{t+1}}{E_{t+1}P^*_{t+1}} - \frac{M_t}{E_t P^*_t} - \frac{D_t}{E_t P^*_t}\left[\frac{(1+\bar{\theta}) - (1+\bar{\varepsilon})(1+\overline{\pi^*})}{(1+\bar{\varepsilon})(1+\overline{\pi^*})}\right],$$

where $\bar{\theta}$ is the constant rate of domestic credit set by the Central Bank. For the same reasons discussed for the continuous-time case, we will assume that $1+\bar{\theta} = (1+\bar{\varepsilon})(1+\pi^*)$ to ensure that the predetermined exchange rate system is sustainable over time. Imposing that assumption, we can rewrite this last expression as

$$h_{t+1} - h_t = \frac{M_{t+1}}{E_{t+1}P^*_{t+1}} - \frac{M_t}{E_t P^*_t},$$

which is, of course, the counterpart to equation (38) in the continuous-time case. Changes in international reserves will solely reflect changes in real money balances. Using the quantity theory equation (70), we can rewrite this last equation as

$$h_{t+1} - h_t = y_{t+1} - y_t.$$

Increases (decreases) in the endowment will be reflected in higher (lower) reserves through their effect on real money demand. This is a feature that is not present in the continuous-time case because in that case the relevant variable for price level determination is consumption and not output.

This characterization of predetermined exchange rates makes clear the dichotomy between the real and monetary sectors of the economy. Specifically – and since we made no assumption on exchange/monetary policy in deriving the constant level of consumption (68) – exchange rate policy is both neutral and superneutral.

6.5 Flexible exchange rates

Under flexible exchange rates, the Central Bank controls the path of the money supply by setting M_t, $t \geq 0$. Specifically, we assume that the monetary authority sets M_0 and then sets a constant rate of money growth:

$$M_t = (1+\bar{\mu})M_{t-1}, \qquad t \geq 1.$$

The rate of inflation follows from the quantity theory equation, given by expression (70):

$$\frac{P_{t+1}}{P_t} = \frac{1+\bar{\mu}}{y_{t+1}/y_t}, \qquad t \geq 0. \tag{72}$$

Since the quantity theory equation holds, of course, for $t=0$, the initial price level is determined by

$$P_0 = \frac{M_0}{y_0}.$$

The nominal exchange rate is determined by the law of one price:

$$E_t = \frac{P_t}{P_t^*}. \tag{73}$$

The rate of depreciation is thus (once again, assume a constant foreign inflation rate)

$$\frac{E_{t+1}}{E_t} = \frac{P_{t+1}/P_t}{1+\overline{\pi^*}}. \tag{74}$$

From (65), (72), and (74), the nominal interest rate is thus given by

$$1 + i_t = \left(\frac{1+i^*}{1+\overline{\pi^*}}\right)\left(\frac{1+\bar{\mu}}{y_{t+1}/y_t}\right).$$

Under flexible exchange rates, fluctuations in the endowment will be reflected in fluctuations in inflation, the rate of depreciation, and the nominal interest rate. Under predetermined exchange rates, the same fluctuations would be reflected in movements in international reserves.

Once again, it is easy to see that monetary policy would be both neutral and superneutral.

6.6 Concluding remarks

As discussed before, in this basic monetary model, money is a "veil" in the sense that changes in monetary/exchange rate policy do not affect the path of real variables. While this model provides the natural conceptual benchmark for studying monetary economics in the open economy – and could even be a good description of the actual world in the long run or under extreme hyperinflationary conditions – it certainly does not provide us with tools to understand the possible real effects of monetary/exchange rate policy in an open economy. In essence, the main task of monetary economics in the open economy is to study departures from this benchmark in which monetary and exchange rate policy have real effects.

Subsequent chapters will thus introduce various frictions that will remove the veil and allow monetary/exchange rate policy to affect the real sector. Specifically, Chapter 6 will assume that there are no interest-bearing bonds in the economy; Chapter 7 will introduce money through a continuous-time cash-in-advance constraint that will establish a link between nominal interest rates and consumption; and Chapter 8 will introduce sticky prices.

7 Appendices

7.1 Breaking down the government into the monetary and the fiscal authority

To simplify the presentation of the basic monetary model, we considered in the text the government as a whole and did not break it down into its separate entities (i.e., the monetary and the fiscal authority). It proves illuminating, however, to consider each entity separately and see how aggregating them leads to equation (19) in the text.

7.1.1 The monetary authority

The Central Bank holds international reserves, prints money, lends to the government by issuing domestic credit (think of domestic credit as loans to the government), and gives transfers to the fiscal authority. As in the text, let H_t^* denote the net foreign assets (measured in the foreign currency) held by the monetary authority and H_t denote the domestic currency value of these international reserves (i.e., $H_t \equiv E_t H_t^*$). The Central Bank's flow budget constraint in domestic currency terms is then given by

$$\dot{H}_t = \underbrace{i_t^* E_t H_t^* + \dot{E}_t H_t^* + \ i_t D_t + \dot{M}_t}_{\text{revenues}} - \underbrace{(\dot{D} + P_t \tau_t^g)}_{\text{expenditures}}. \tag{75}$$

The first four terms on the RHS of equation (17) represent sources of revenues for the Central Bank. Specifically, the first term ($i_t^* E_t H_t^*$) captures the domestic-currency value of the interest proceeds on the stock of international reserves; the second term ($\dot{E}_t H_t^*$) denotes the capital gains on the existing stock of international reserves; the third term ($i_t D_t$) captures the interest income on the stock of nominal domestic credit; and the fourth term ($\dot{M}_t$) indicates that money printing is a source of revenues for the Central Bank. The last two terms on the RHS of equation (17) capture the Central Bank's expenditures. The Central Bank buys domestic bonds issued by the fiscal authority ($\dot{D}_t$) and transfers to the fiscal authority whatever profits it makes ($P_t \tau_t^g$).[34]

Dividing (75) by P_t – and recalling that $P_t = E_t P_t^*$– we obtain

$$\frac{\dot{H}_t}{P_t} = \ i_t^* h_t + \varepsilon_t h_t + \ i_t d_t + \frac{\dot{M}_t}{P_t} - \frac{\dot{D}_t}{P_t} - \tau_t^g, \tag{76}$$

where $h_t \equiv H_t^*/P_t^*$.

Recall that

$$\dot{h}_t = \frac{\dot{H}_t}{P_t} - (\varepsilon_t + \pi_t^*)h_t. \tag{77}$$

[34] As will become clear below, the standard convention is to assume that the Central Bank does not accumulate/decumulate wealth (i.e., it keeps its net worth constant over time).

Substituting (76) into (77) (and imposing the Fisher equation for the rest of the world), we obtain:

$$\dot{h}_t = rh_t + i_t d_t + \frac{\dot{M}_t}{P_t} - \frac{\dot{D}_t}{P_t} - \tau_t^g. \tag{78}$$

We can rewrite this expression as:

$$\dot{h}_t + \dot{d}_t - \dot{m}_t = rh_t + i_t d_t + (\varepsilon_t + \pi_t^*)m_t - (\varepsilon_t + \pi_t^*)d_t - \tau_t^g. \tag{79}$$

By convention, we assume that the Central Bank's net worth (i.e., its capital) remains constant over time. In other words, $\dot{h}_t + \dot{d}_t - \dot{m}_t = 0$. Imposing this condition in the last expression and rearranging terms, we obtain

$$\tau_t^g = i_t m_t. \tag{80}$$

Intuitively, the Central Bank transfers to the fiscal authority its "profits" so as to keep its net worth constant. The Central Bank's profits derive from the fact that while its assets bear market interest rates (remember that both the international reserves and the stock of domestic credit bear interest), its liabilities (the money supply) bear no interest. Alternatively, using the Central Bank's balance sheet ($m_t = h_t + d_t$) and the interest parity condition ($i_t = r + \pi_t^* + \varepsilon_t$), the last expression can be rewritten as

$$\tau_t^g = rh_t + rd_t + (\varepsilon_t + \pi_t^*)m_t.$$

In this interpretation, the Central Bank's profits consist of the real interest rate on reserves and domestic credit and the inflation tax on the real money supply.

7.1.2 The fiscal authority

The fiscal authority (in practice the Finance Ministry) borrows from the Central Bank (i.e., sells bonds to the Central Bank), pays interest on these bonds (i.e., on the stock of domestic credit), receives a transfer of τ_t^g from the Central Bank, and makes lump-sum transfers to the private sector.[35] The fiscal authority's flow constraint in nominal terms is thus

$$\dot{D}_t = \underbrace{i_t D_t + P_t \tau_t}_{\text{expenditures}} - \underbrace{P_t \tau_t^g}_{\text{revenues}}. \tag{81}$$

Dividing this expression by P_t, we get

$$\frac{\dot{D}_t}{P_t} = i_t d_t + \tau_t - \tau_t^g. \tag{82}$$

Using (80) and rearranging terms, we obtain

[35] Needless to say, the fiscal authority would typically spend on goods (as in chapter 4), a feature that we are abstracting from in this chapter and to which we will return in later chapters.

$$\tau_t = \dot{d}_t + (\varepsilon_t + \pi_t^*)m_t + rh_t.$$

We see that if $\dot{d}_t = 0$ (the usual assumption), then $\tau_t = (\varepsilon_t + \pi_t^*)m_t + rh_t$, which coincides of course with what we derived in the text (equation (40) in the case of predetermined exchange rates and equation (43) in the case of flexible rates, in which case $h_t = 0$ for all t)).

7.1.3 Aggregating the monetary and the fiscal authority

Combining the monetary's authority nominal budget constraint – given by equation (75) – and the fiscal authority's nominal budget constraint – given by equation (5) – we obtain

$$\dot{H}_t = i_t^* E_t H_t^* + \dot{E}_t H_t^* + \dot{M}_t - P_t \tau_t, \tag{83}$$

which, of course, coincides with equation (17) in the text.

By the same token, by combining the monetary authority's flow constraint – given by (78) – and the fiscal authority's flow constraint – given by (82) – we obtain:

$$\dot{h}_t = rh_t + \frac{\dot{M}_t}{P_t} - \tau_t, \tag{84}$$

which coincides with constraint (19) in the text. As a final remark, notice that we have just gone over a purely accounting exercise so that all the derivations in this appendix hold for any exchange rate regime.

7.2 Accounting in basic monetary model with jumps in real money balances

We mentioned in the text that, strictly speaking, both the household's and the government's budget constraints should take into account the possibility of jumps in real money balances (see, for example, Drazen and Helpman (1987)). We develop such a formulation in this appendix.

7.2.1 Consumer

The consumer's flow constraint takes the form

$$\begin{array}{ll} b_t - b_{t^-} = -(M_t - M_{t-})/E_t, & \text{if } t \in J, \\ \dot{b}_t = rb_t + y_t + \tau_t - c_t - \dot{m}_t - \varepsilon_t m_t, & \text{if } t \notin J. \end{array} \tag{85}$$

These flow constraints allow for the possibility of discrete changes in b_t and m_t at a finite set of points (which may include $t = 0$) belonging to the set J.[36] The precise points that belong to set J will naturally depend on the specific

[36] A jump at $t = 0$ may occur in the case of an unanticipated shock at t $t = 0$. If we are analyzing a PFEP, then, by construction, there will be no jumps at $t = 0$.

problem at hand. For example, when we analyze the effects of an unanticipated and permanent shock under predetermined exchange rate, the set J will typically include two points: $t = 0$ and $t = T$. Notice also that, along a perfect foresight path, the total level of financial assets, a_t, cannot change discretely at any point in time.

Integrating forward and imposing the condition $\lim_{t\to\infty} e^{-rt}b_t = 0$, we obtain the following consumer's lifetime budget constraint:

$$b_{0^-} + \int_0^\infty (y_t + \tau_t)\, e^{-rt}dt = \int_0^\infty (c_t + \dot{m}_t + \varepsilon_t m_t)e^{-rt}dt + \sum_{j\in J} \frac{M_t - M_{t-}}{E_t} e^{-rT_j}, \tag{86}$$

where b_{0^-} denote net foreign assets an instant before $t = 0$ and T_j are values of t that belong to set J.

This intertemporal constraint can be further simplified if we impose the condition that $\lim_{t\to\infty} e^{-rt}m_t = 0$ and use the interest parity condition ($i_t = r + \varepsilon_t$) to obtain:[37]

$$b_{0^-} + \frac{M_{0^-}}{E_0} + \int_0^\infty (y_t + \tau_t)\, e^{-rt}dt = \int_0^\infty (c_t + i_t m_t)e^{-rt}dt. \tag{87}$$

Unless there is a discrete change in the consumer's real financial assets at $t = 0$, then $b_{0^-} + M_{0^-}/E_0 = a_0$ and this expression coincides with constraint (10) in the text. Even if there is a discrete change in the consumer's real financial assets at $t = 0$ – which would be the case if an unanticipated devaluation occur – our assumption that the government transfers back to the consumer all proceeds ensures that the solution would be the same as in the text. Of course, if the government used the proceeds of a discrete devaluation to, say, increase spending, we would need to modify the analysis.

7.2.2 Government

The government's flow constraint should read as

$$\begin{aligned} h_t - h_{t^-} &= (M_t - M_{t-})/E_t, \quad \text{if } t \in J, \\ \dot{h}_t &= rh_t + \dot{m}_t + \varepsilon_t m_t - \tau_t. \quad \text{if } t \notin J, \end{aligned} \tag{88}$$

where the set J has been defined above. Under predetermined exchange rates, these discrete changes will take place when consumers decide to trade net foreign assets for domestic money (or viceversa) at the Central Bank.

If we integrate forward equation (88), imposing the condition that $\lim_{t\to\infty} e^{-rt}h_t = 0$, we obtain the following intertemporal constraint for the government:

$$\int_0^\infty \tau_t e^{-rt}dt = h_{0^-} + \int_0^\infty (\dot{m}_t + \varepsilon_t m_t)e^{-rt}dt + \sum_{j\in J} \frac{M_t - M_{t-}}{E_t} e^{-rT_j}, \tag{89}$$

[37] This condition will hold in equilibrium because, at an optimum, the choice of m_t will be finite.

where h_{0^-} denotes the level of international reserves an instant before $t = 0$. Notice that a, say, discrete fall in real money balances implies a loss of revenues for the government. This intertemporal constraint simply says the present discounted value of transfers must be financed with the initial stock of international reserves plus the present discounted value of revenue from money creation (including discrete jumps).

This intertemporal constraint can be further simplified if we impose the condition that $\lim_{t\to\infty} e^{-rt}m_t = 0$ and use the interest parity condition ($i_t = r + \varepsilon_t$) to obtain:[38]

$$\int_0^\infty \tau_t e^{-rt} dt = h_{0^-} - \frac{M_{0^-}}{E_0} + \int_0^\infty i_t m_t e^{-rt} dt, \tag{90}$$

7.2.3 Aggregation

Combining the consumer's intertemporal constraint, given by equation (87), with the government's, given by equation (90), we obtain

$$b_{0^-} + h_{0^-} + \int_0^\infty y_t e^{-rt} dt = \int_0^\infty c_t e^{-rt} dt.$$

Unless there is a jump in the economy's net foreign assets at $t = 0$, then $b_{0^-} + h_{0^-} = k_0$ and this intertemporal constraint coincides with expression (26) in the text.

7.3 Restrictions on the rate of growth of domestic credit

In deriving the government's intertemporal constraint – given by equation (22) – we imposed the condition

$$\lim_{t\to\infty} h_t e^{-rt} = 0. \tag{91}$$

Similarly, in deriving the economy's resource constraint – given by equation (26) – we imposed the condition

$$\lim_{t\to\infty} k_t e^{-rt} = 0. \tag{92}$$

Since $k_t = h_t + b_t$, conditions (91) and (92) imply that

$$\lim_{t\to\infty} b_t e^{-rt} = 0. \tag{93}$$

Further, in deriving the consumer's intertemporal constraint – given by equation (10) – we imposed the condition

$$\lim_{t\to\infty} (b_t + m_t) e^{-rt} = 0. \tag{94}$$

[38] This condition will hold in equilibrium because, at an optimum, the choice of m_t will be finite.

Conditions (93) and (95) imply that

$$\lim_{t\to\infty} m_t e^{-rt} = 0. \tag{95}$$

Since, from the Central Bank's balance sheet, $d_t = m_t - h_t$, conditions (91) and (95) imply

$$\lim_{t\to\infty} d_t e^{-rt} = 0. \tag{96}$$

By definition $d_t = D_t/E_t P_t^*$. Hence,

$$d_t = d_0 e^{\int_0^t (\theta_s - \varepsilon_s - \pi_s^*) d_s}.$$

Using the latter equation, we can rewrite (96) as

$$\lim_{t\to\infty} d_0 e^{\int_0^t (\theta_s - \varepsilon_s - \pi_s^* - r) d_s} = 0. \tag{97}$$

This condition provides an upper bound on the rate of expansion of domestic credit which is consistent with the maintenance of a predetermined exchange rate regime. In the case in which θ_s, ε_s, and π_s^* are constant over time and equal to θ, ε, and π^*, respectively, then the condition $\theta = \varepsilon + \pi^*$, as assumed in the text, is sufficient to ensure that (97) holds. In general, however, all that we need for condition (97) to hold is that, "on average", $\theta_s - \varepsilon_s - \pi_s^* - r < 0$. In the case in which the exchange rate is fixed ($\varepsilon_t = 0$), foreign inflation is zero, and the rate of growth of domestic credit is constant at θ, then condition (97) holds as long as $\theta < r$.[39]. Intuitively, under a fixed exchange rate, as long as $\theta > 0$, the Central Bank is increasing its external debt (i.e., international reserves become increasingly negative) but can finance the debt service by lump-sum taxing the private sector. If, however, $\theta > r$, it is no longer possible to service the debt because the rate of growth of government's debt exceeds the real interest rate. As Obsfeld (1986) points out, if lump-sum taxation were not available, then any $\theta > 0$ is no longer feasible because it would imply ever-growing revenue needs that can only be raised by distortionary taxation.

7.4 MIUF model in discrete time

This appendix develops a discrete-time version of the continuous-time MIUF model developed in the text. This will prove useful for subsequent chapters in the book. Unlike real models in which the switch from continuous to discrete time does not pose any particular methodological problem, in monetary models this switch is non-trivial because – as will become clear below – timing issues play a critical role.

[39] This is one of Obstfeld's (1984) points: if there are no constraints on the government's external borrowing, then a fixed exchange rate regime is sustainable as long as the rate of growth of domestic credit does not exceed the world real interest rate.

7.4.1 Households' maximization

Suppose that preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v\left(\frac{M_t}{P_t}\right)\right]. \tag{98}$$

A critical issue in discrete-time monetary models is related to timing issues. The more common assumption in the literature for MIUF models – which is captured in the above preferences – is that *end-of-period* money balances enter the utility function. While this assumption may look somewhat odd, it is the most common assumption in the literature (see, for example, Calvo and Leiderman (1992)).[40] The intertemporal budget constraint remains given by (55).

Households thus choose $\{c_t, m_t, a_t\}_{t=0}^{\infty}$ to maximize lifetime utility – given by (98) – subject to a sequence of flow budget constraints given by (55). In terms of the Lagrangian:

$$\begin{aligned}\mathcal{L} = \quad & \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v\left(m_t\right)\right] \\ & + \sum_{t=0}^{\infty} \beta^t \lambda_t \left[(1+r)a_{t-1} + \tau_t + y_t - c_t - \frac{i_{t-1}}{1+\pi_t} m_{t-1} - a_t \right].\end{aligned}$$

The first-order conditions with respect to c_t, m_t, and a_t are given by, respectively,

$$u'(c_t) - \lambda_t = 0, \tag{99}$$

$$v'\left(m_t\right) - \lambda_{t+1}\beta \frac{i_t}{1+\pi_{t+1}} = 0, \tag{100}$$

$$-\lambda_t + \beta(1+r)\lambda_{t+1} = 0, \tag{101}$$

Notice that, under our usual assumption that $\beta(1+r) = 1$, the last condition reduces to:

$$\lambda_{t+1} = \lambda_t.$$

The Lagrange multiplier is constant over time. Denote the constant value of the multiplier by λ (i.e., $\lambda_{t+1} = \lambda_t = \lambda$). Hence, from (99), it follows that

$$u'(c_t) = \lambda. \tag{102}$$

As expected, consumption is constant over time.

[40] As discussed by Calstrom and Fuerst (2001), the assumption is rather odd because it amounts to assuming that what matters for liquidity purposes is the money balances that the consumer *leaves* the grocery store with, rather than the money that he/she *enters* the store with! The more natural assumption would be that *beginning-of-period money* balances provide liquidity services, a specification analyzed below.

Taking into account that (i) the Lagrange multiplier is constant; (ii) $\beta(1+r)=1$ and (iii) by definition, $1+i_t=(1+r)(1+\pi_{t+1})$, we can rewrite first-order condition (100) as

$$v'\left(m_t\right)=\lambda\frac{i_t}{1+i_t}. \tag{103}$$

This is the condition analogous to equation (13) in the continuous-time version. As equation (103) makes clear, the opportunity cost of holding real money balances is $i_t/(1+i_t)$. Intuitively, in period t the household uses M_t to acquire money balances (instead of purchasing interest-bearing bonds) and hence foregoes i_tM_t in interest payments at the beginning of period $t+1$. The real value of these interest payments in $t+1$ is i_tM_t/P_{t+1}, which discounted to time t amounts to $i_t(M_t/P_{t+1})/(1+r)$. Multiplying and dividing by P_t and recalling that, by definition, $1+i_t=(1+r)(P_{t+1}/P_t)$, this expression can be written as $[i_t/(1+i_t)]m_t$.

Substituting (102) into (103), we obtain

$$v'\left(m_t\right)=u'(c_t)\frac{i_t}{1+i_t}, \tag{104}$$

which implicitly defines a real money demand with standard properties.

7.4.2 Perfect foresight equilibrium

The aggregate conditions (66) and (67) still hold in this model. Equation (102) tells us that consumption is constant over time. Hence, from the resource constraint,

$$\bar{c}=\frac{r}{1+r}\left[(1+r)(b_{-1}+h_{-1})+\sum_{t=0}^{\infty}\frac{y_t}{(1+r)^t}\right]. \tag{105}$$

7.4.3 Predetermined exchange rates

We now solve the model under predetermined exchange rates. Needless to say, the same logic that applied to the continuous-time case remains valid. The monetary authority sets the path of both the nominal exchange rate and the stock of domestic credit. Formally, the monetary authority sets E_t, $t=0,1,...$and $D_t, t=0,1,...$Suppose, for simplicity, that the monetary authority sets a constant rate of devaluation. Formally,

$$\frac{E_{t+1}}{E_t}=1+\bar{\varepsilon}.$$

Through the interest parity condition, the constant rate of devaluation determines a constant nominal interest rate (assuming, of course, a constant foreign inflation rate):

$$1+\bar{i}=(1+i^*)(1+\bar{\varepsilon}).$$

Given constant consumption and a constant nominal interest rate, real money demand will also be constant. Given a constant real money demand, the path of international reserves will be given by

$$h_{t+1} - h_t = -\frac{D_t}{E_t P_t^*}\left[\frac{(1+\bar{\theta}) - (1+\bar{\varepsilon})(1+\overline{\pi^*})}{(1+\bar{\varepsilon})(1+\overline{\pi^*})}\right]$$

Imposing the condition that the predetermined exchange rate regime be sustainable over time (i.e., $1+\bar{\theta} = (1+\bar{\varepsilon})(1+\pi^*)$), we obtain

$$h_{t+1} = h_t.$$

International reserves will be constant over time.

7.4.4 Flexible exchange rates

We now solve the model under flexible exchange rates. Assume, for simplicity, that the monetary authority sets an initial level of the nominal money supply, M_0, and a constant rate of money growth:

$$\frac{M_{t+1}}{M_t} = 1 + \bar{\mu}, \qquad t = 0, 1, ... \tag{106}$$

We will proceed in an analogous way to the continuous-time case and derive an unstable difference equation for real money balances. To this effect, multiply and divide by P_{t+1} and P_t on the LHS of equation (106) to obtain:

$$\frac{m_{t+1}}{m_t} = \frac{1}{\frac{P_{t+1}}{P_t}}\left(1+\bar{\mu}\right). \tag{107}$$

Using the interest parity condition and the law of one price, we can rewrite this equation as

$$\frac{m_{t+1}}{m_t} = \frac{\left(1+\bar{\mu}\right)\left(1+r\right)}{1+i_t}. \tag{108}$$

Solving for i_t from (103) and substituting the resulting expression into the last equation, we obtain a difference equation in m_t,

$$m_{t+1} = m_t\left(1+\bar{\mu}\right)\left(1+r\right)\left[1 - \frac{v'\left(m_t\right)}{\lambda}\right]. \tag{109}$$

This difference equation is the analog to the differential equation (41) in the continuous-time case.

The stationary state corresponding to this equation is implicitly given by (as follows from setting $m_{t+1} = m_t = \bar{m}$)

$$v'\left(\bar{m}\right) = \lambda\frac{\left(1+\bar{\mu}\right)\left(1+r\right) - 1}{\left(1+\bar{\mu}\right)\left(1+r\right)}.$$

By linearizing this equation around the stationary state, we can check that this is an unstable differential equation:[41]

$$\left.\frac{\partial m_{t+1}}{\partial m_t}\right|_{ss} = 1 - \bar{m}\frac{(1+\mu)(1+r)}{\lambda}v''(\bar{m}) > 1.$$

Given the instability of (109), m_t will need to be at its stationary state from $t = 0$ onwards. Otherwise, it would diverge over time. In other words, the constant value of m_t, $t = 0, 1, ...$ will be given by

$$v'(\bar{m}) = u(\bar{c})\frac{(1+\bar{\mu})(1+r) - 1}{(1+\bar{\mu})(1+r)}.$$

It then follows from (108) that the nominal interest rate will also be constant over time and given by

$$i_t = (1+\bar{\mu})(1+r) - 1.$$

Through interest parity, it then follows that the rate of depreciation will also be constant.

7.5 Beginning-of-period money balances

While it is, by far, the most popular MIUF specification in the literature, the end-of-period money balances specification is, to say the least, conceptually problematic. A more natural formulation would have beginning-of-period money balances provide liquidity services during the current period. In this case, preferences would be given by

$$\sum_{t=0}^{\infty}\beta^t\left[u(c_t) + v\left(\frac{M_{t-1}}{P_t}\right)\right]. \tag{110}$$

(Below it will prove convenient to express M_{t-1}/P_t as $m_{t-1}(P_{t-1}/P_t)$.) The budget constraint remains unchanged. Households thus choose $\{c_t, m_t, a_t\}_{t=0}^{\infty}$ to maximize lifetime utility – given by (110) – subject to a sequence of flow budget constraints given by (55). In terms of the Lagrangian:

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty}\beta^t\left[u(c_t) + v\left(m_{t-1}\frac{P_{t-1}}{P_t}\right)\right] \\ &\quad + \sum_{t=0}^{\infty}\beta^t\lambda_t\left[(1+r)a_{t-1} + \tau_t + y_t - c_t - \frac{i_{t-1}}{1+\pi_t}m_{t-1} - a_t\right].\end{aligned}$$

The first-order conditions with respect to $c_t, m_t,$and a_t are given by, respectively (note that, relative to the end-of-period case, the only first-order condition that

[41] As in the continous case, if $v(m) = \log(m)$, this difference equation becomes a linear difference equation given by $m_{t+1} = (1+\bar{\mu})(1+r)(m_t - 1/\lambda)$.

changes is the one for real money balances):

$$\begin{aligned} u'(c_t) - \lambda_t &= 0, \quad (111) \\ v'\left(m_t \frac{P_t}{P_{t+1}}\right) \frac{P_t}{P_{t+1}} - \lambda_{t+1} \frac{i_t}{1+\pi_{t+1}} &= 0 \\ -\lambda_t + \beta(1+r)\lambda_{t+1} &= 0, \end{aligned}$$

As in the previous case, the last equation implies that $\lambda_{t+1} = \lambda_t$ and hence, from (111), that consumption will be constant over time. In turn, the first-order condition for real money balances simplifies to:

$$v'\left(\frac{M_{t-1}}{P_t}\right) = \lambda_t i_{t-1}, \tag{112}$$

which makes clear that the opportunity cost of the relevant measure of real money balances is simply i_{t-1}. Intuitively, if last period the consumer had bought bonds – instead of holding money – he/she would have received $i_{t-1}M_{t-1}$ at the beginning of period t, the real value of which is $i_{t-1}M_{t-1}/P_t$.

It would be straightforward to solve the model under predetermined exchange rates. We solve instead the flexible exchange rates case. Once again, we will proceed by deriving an unstable difference equation. To this effect, it proves convenient to define a new measure of real money balances, denoted by n_t, as

$$n_t \equiv \frac{M_{t-1}}{P_t}.$$

Notice that the variable n_t captures the measure of real money balances that is relevant for period t purposes. In light of this definition, we can rewrite condition (112) as

$$v'(n_t) = \lambda_t i_{t-1}. \tag{113}$$

To derive the difference equation, start from the fact that $M_t/M_{t-1} = 1+\bar{\mu}$, multiply and divide on the LHS by P_t and P_{t+1}, and use the interest parity condition to obtain:

$$\frac{n_{t+1}}{n_t} = \frac{(1+\bar{\mu})(1+r)}{1+i_t}.$$

Solve for i_t from (113) (iterated forward one period) and substitute the resulting expression into the last equation to obtain, after rearranging terms,

$$\frac{n_{t+1}}{n_t} = \frac{(1+\bar{\mu})(1+r)}{1+\frac{v'(n_{t+1})}{\lambda}},$$

which implicitly defines a difference equation in n_t. Moreover, since

$$\frac{dn_{t+1}}{dn_t} = \frac{1}{1+\bar{n}\frac{(1+\mu)(1+r)}{\left[1+\frac{v'(\bar{n})}{\lambda}\right]^2}\frac{v''(\bar{n})}{\lambda}} > 1,$$

this difference equation is unstable. We thus conclude that the path of n_t will be constant over time. Proceeding as before, we can easily characterize the rest of the system.

Exercises[42]

1. Dirty floating

This exercise illustrates how one would think about "dirty floating" in the monetary model analyzed in the main section. Specifically, we analyze how the economy would respond to positive monetary shock that would lead to an appreciation of the domestic currency and how the monetary authority (MA) might intervene to partly offset such an appreciation (perhaps because, for reasons left out of the model, the MA fears that a large appreciation might worsen the trade balance).

Consider the model of Section 2 with the only modification that preferences are now given by:

$$\int_0^\infty [u(c_t) + \alpha_t v(m_t)] e^{-\beta t} dt, \tag{114}$$

where α should be thought of as a money demand shock. In the context of this model:

(a) Consider the case of flexible exchange rates (with $\mu = 0$). Suppose that just before $t = 0$, the economy is in a stationary equilibrium with a constant α. At $t = 0$, there is unanticipated and permanent increase in α. Solve for the non-intervention case (i.e., a "pure floating"). Explain the intuition behind the results.

(b) Solve for the extreme case of "full intervention" (i.e., the MA reacts in such a way that it does not let the nominal exchange rate change). Explain intuitively how this policy works.

(c) Consider an "intermediate case" in which the MA chooses to intervene in the foreign exchange market (but allows some of the adjustment to take place through the nominal exchange rate). In particular, derive a "policy reaction function" that would tell the MA how much to intervene as a function of the change in real money demand (which the MA must take as given) and the targeted change in the nominal exchange rate. [HINT: a) Think of small changes so that you can use differentiation to compute changes at $t = 0$. b) Think of the MA as having an initial positive stock of international reserves and that capital gains/losses reserves are not monetized; that is, there is some non-monetary liability (call it NM) that is adjusted.]

2. Demand shocks

This exercise shows that, as one would expect, the dichotomy between the real and the monetary sectors is still valid when the path of consumption

[42] An answer key is available from the author upon request.

is not constant over time. To this effect, consider the following variation of the model in the text. Preferences are given by:

$$\int_0^\infty [\alpha_t u(c_t) + v(m_t)] e^{-\beta t} dt,$$

where α_t is a preference shock. The rest of the model is unchanged. The parameter α_t can be viewed as a demand shock. Suppose that the path of α_t is as follows:

$$\alpha_t = \begin{cases} \alpha^H, & 0 \leq t < T, \\ \alpha^L & t \geq T, \end{cases}$$

where $\alpha^H > \alpha^L$. In this context:

(a) Solve for the perfect foresight equilibrium path corresponding to predetermined exchange rates.

(b) Solve for the perfect foresight equilibrium path corresponding to flexible exchange rates and show that it coincides with the one you just derived for predetermined exchange rates.

3. Increase in domestic credit

Consider the economy analyzed in the text and operating under predetermined exchange rates. In this context, analyze the effects of an unanticipated and permanent increase in the stock of domestic credit at time 0. Explain the intuition behind the results.

4. Equivalence between predetermined and flexible rates illustrated with an anticipated increase in the level of the money supply

Consider the economy analyzed in the text operating under flexible exchange rates. Suppose that the rate of money growth is zero (i.e., $\mu = 0$) and that the level of the money supply follows the path given by:

$$M_t = \begin{cases} M^L, & 0 \leq t < T, \\ M^H & t \geq T, \end{cases}$$

where $M^L < M^H$.

Suppose that the money supply is constant up to time T, increases at T, and remains unchanged thereafter. In this context:

(a) Solve for the perfect foresight path of all relevant variables.

(b) Show that if the economy were operating under predetermined exchange rates and the Central Bank set the path of the nominal exchange rate that you obtained in (a) above, the same equilibrium would obtain.

5. Inflationary consequences of anticipated changes in policy

This exercise explores yet another important distinction between a predetermined and flexible exchange rate systems: the behavior of the inflation rate in response to an anticipated changes in policy.

Consider the model of Section 2. Characterize the perfect foresight equilibrium paths corresponding to the following cases:

(a) Under predetermined exchange rates, suppose that the rate of devaluation is zero between 0 and T and increases to $\bar{\varepsilon} > 0$ at $t = T$. Solve for the path of all relevant variables.

(b) Under flexible exchange rates, suppose that the rate of money growth is zero between 0 and T and increases to $\bar{\mu} > 0$ at time T. Solve for the path of all relevant variables.

(c) How does the behavior of inflation differ? What is the intuition behind the results?

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Box 1. Exchange rate regimes in practice

Section 2.5 analyzes the main characteristics of predetermined exchange rates and flexible exchange rates regimes. For conceptual clarity, we studied those regimes in their "pure" form: under predetermined exchange rates, the monetary authority controls the path of the nominal exchange rate and, under flexible exchange rates, the monetary authority does not intervene at all in the exchange market. In real life, however, such "pure" forms are hard to find (particularly pure floaters) and, in fact, we observe a whole continuum of exchange rate regimes in between these two extremes. In addition, there are exchange rate regimes that in fact go beyond predetermined exchange rates, such as "full dollarization" (i.e., no domestic currency). What follows is an exchange rate taxonomy – based on the degree of exchange rate flexibility and the existence of formal or informal commitments to exchange rate paths – reported in the 2008 IMF report on "De Facto Classification of Exchange Rate Regimes and Monetary Policy Framework."[43]

No separate legal tender The currency of another country circulates as the sole legal tender or the member belongs to a monetary or currency union in which the same legal tender is shared by the members of the union. Adopting such regimes implies the complete surrender of the monetary authorities' control over domestic monetary policy. Examples: Ecuador, Panama, and El Salvador.

Currency board arrangements A monetary regime based on an explicit legislative commitment to exchange domestic currency for a specified foreign currency at a fixed exchange rate, combined with restrictions on the issuing authority to ensure the fulfillment of its legal obligation. This implies that domestic currency will be issued only against foreign exchange and that it remains fully backed by foreign assets, leaving little scope for discretionary monetary policy and eliminating traditional Central Bank functions, such as monetary control and lender-of-last-resort. Some flexibility may still be afforded, depending on how strict the banking rules of the currency board arrangement are. Hong Kong, for instance, has had a currency board (with the exchange rate fixed at 7.8 Honk Kong dollars per one U.S. dollars) since October 17, 1983. Other examples: Estonia, Lithuania, and Djibouti.[44]

Conventional fixed peg arrangements The country pegs its currency within margins of ± 1 percent or less vis-à-vis another currency; a cooperative

[43] See http://www.imf.org/external/np/mfd/er/2008/eng/0408.htm. The description of the different regimes is taken from the IMF report. Examples given are current as of April 31, 2008. This classification system is based on members' actual, de facto, arrangements as identified by IMF staff, which may differ from their officially announced arrangements (see the discussion below on de jure versus de facto regimes).

[44] Argentina's Converbitility Plan (1991-2001) would be another recent and highly publicized example of a currency board.

arrangement, such as the ERM II; or a basket of currencies, where the basket is formed from the currencies of major trading or financial partners and weights reflect the geographical distribution of trade, services, or capital flows. The currency composites can also be standardized, as in the case of the SDR. There is no commitment to keep the parity irrevocably. The exchange rate may fluctuate within narrow margins of less than ±1 percent around a central rate – or the maximum and minimum value of the exchange rate may remain within a narrow margin of 2 percent – for at least three months. The monetary authority maintains the fixed parity through direct intervention (i.e., via sale/purchase of foreign exchange in the market) or indirect intervention (e.g., via the use of interest rate policy, imposition of foreign exchange regulations, exercise of moral suasion that constrains foreign exchange activity, or through intervention by other public institutions). Flexibility of monetary policy, though limited, is greater than in the case of exchange arrangements with no separate legal tender and currency boards because traditional Central Banking functions are still possible, and the monetary authority can adjust the level of the exchange rate, although relatively infrequently. Examples: Venezuela, Honduras and Argentina.

Pegged exchange rates within horizontal bands The value of the currency is maintained within certain margins of fluctuation of more than ±1 percent around a fixed central rate or the margin between the maximum and minimum value of the exchange rate exceeds 2 percent. As in the case of conventional fixed pegs, reference may be made to a single currency, a cooperative arrangement, or a currency composite. There is a limited degree of monetary policy discretion, depending on the band width. Examples: Slovak Republic, Syria, and Tonga.

Crawling pegs The currency is adjusted periodically in small amounts at a fixed rate or in response to changes in selective quantitative indicators, such as past inflation differentials vis-à-vis major trading partners, differentials between the inflation target and expected inflation in major trading partners. The rate of crawl can be set to adjust for measured inflation or other indicators (backward looking), or set at a preannounced fixed rate and/or below the projected inflation differentials (forward looking). Maintaining a crawling peg imposes constraints on monetary policy in a manner similar to a fixed peg system. Examples: Bolivia, Botswana and China.

Managed floating with no predetermined path for the exchange rate The monetary authority attempts to influence the exchange rate without having a specific exchange rate path or target. Indicators for managing the rate are broadly judgmental (e.g., balance of payments position, international reserves, parallel market developments), and adjustments may not be automatic. Intervention may be direct or indirect. Examples: Algeria, Ukraine and Colombia.

Independently floating The exchange rate is market-determined, with any official foreign exchange market intervention aimed at moderating the rate of change and preventing undue fluctuations in the exchange rate, rather than at establishing a level for it. Examples: United States, Japan, Mexico and Euro area.

When it comes to quantifying the effects of different exchange rate regimes on, among other things, growth and inflation, a major issue confronting empirical researchers is whether to use de jure or de facto classifications of exchange rate regimes. Until recently, most papers followed de jure classifications, typically based on the approach that the IMF followed until 1997 of asking member countries to self-declare their arrangements. More recently, however, Levy-Yeyati and Sturzenegger (2002) and Reinhart and Rogoff (2004) have provided de facto classifications (like the one above by the IMF). These authors argue that de jure classifications usually fail to describe actual exchange rate regimes and that the gap between de facto (what countries actually do) and de jure (what countries say they do) classifications may be critical for understanding/quantifying a host of important issues. An example of this gap is the so-called "fear of floating" phenomenon analyzed in Calvo and Reinhart (2000), who argue that, while many emerging markets claim to pursue flexible exchange rate arrangements, many in fact intervene heavily to keep the nominal exchange rate within certain ranges. Table 3 illustrates the gap between de jure and de facto regimes by comparing the traditional de jure IMF classification with the de facto classification provided by Reinhart and Rogoff (2004). According to this table, 487 annual observations with a de jure flexible exchange rate are reclassified by Reinhart and Rogoff (2004) as a fixed exchange rate regime. Two interesting cases are China, 1994-1997, and Mexico 1992-1994, both with a de jure flexible exchange rate regime but classified as fixed by Reinhart and Rogoff (2004).

Box 2. What is the interest rate elasticity of money demand?

As shown in equation (14), the theoretical framework derived in this chapter models money demand as a positive function of consumption and a negative function of the nominal interest rate. The interest-rate elasticity of money demand is a particularly important parameter for both researchers and policymakers alike because it tells us the extent to which money demand will be affected by changes interest rates and inflation.

The estimation procedure typically used in finding the empirical counterpart of expression (14) starts by developing an equilibrium model in discrete time similar to the one outlined in Appendix 7.4. Given the timing assumptions of most models, the interest rate enters the money demand equation through the expression $i/(1+i)$, as is the case in the end-of-period specification captured in equation (104). Arrau *et al* (1995) introduce money in a general equilibrium model through a transaction cost technology that takes the form

$$H(m_t, \theta_t, c_t) = \frac{1}{c_t^{1-\alpha}} h\left(\frac{m_t}{c_t^{\alpha}}, \theta_t\right),$$

where $H(.)$ represents transactions costs per unit of consumption (in other words, total transactions costs are $c_t H(.)$) and $h(.)$ is given by

$$h\left(\frac{m_t}{c_t^{\alpha}}, \theta_t\right) = K\theta_t + \frac{1}{\beta}\left[\frac{m_t}{c_t^{\alpha}} log\left(\frac{m_t}{c_t^{\alpha}\theta_t}\right) - \frac{m_t}{c_t^{\alpha}}\right],$$

where θ_t is a technological parameter that captures financial innovation and ϕ is a parameter that represents the degree of scale economies in transaction (when $\phi = 1$, H is a function of the ratio m/c, which is the most common theoretical specification).

Under this functional form, it is easy to check that the money demand equation can be expressed as:

$$log(m_t) = log(\theta_t) + \alpha log(c_t) - \beta\left(\frac{i_t}{1+i_t}\right). \tag{115}$$

The parameter β is usually referred to as he interest rate semi-elasticity of money demand or, more generally, the opportunity cost semi-elasticity of money. Arrau *et al* (1995) estimate the money demand function for a sample of ten developing countries using quarterly time series from the mid-70's to the early 80's and using M1 as their measure of money balances. Since their paper focuses on the role of financial innovation in money demand, they allow for a time varying θ_t in equation (115). They also run regressions using i instead of $i/(1+i)$ as the opportunity cost of holding money . Panel A in Table 4 presents their estimates for β for the five countries in which they find a long-run (cointegrating) relation between the dependent and independent variables. Their estimates vary between -0.5 and -3 and are, for the most part, significantly different from zero.

Building up on the importance of financial innovation when modeling money demand, Reinhart and Vegh (1995) develop a general equilibrium framework in which money is used because of its services in reducing transaction costs. Under certain functional forms, they derive the following money demand equation:

$$log(m_t) = \delta + \varphi log(\theta_t) + \alpha log(c_t) + \beta log\left(\frac{i_t}{1+i_t}\right) \tag{116}$$

where θ_t is a proxy for financial innovation. The authors simultaneously estimated equation (116) and the model's intertemporal optimality condition (the standard Euler equation) applying Hansen's (1982) generalized method of moments (GMM). Their database consists of quarterly time series for the 70's and 80's for Argentina, Chile and Uruguay. Panel B in Table 4 presents their estimates for β. Notice that in this case β corresponds to the interest rate elasticity of money and not to a semi-elasticity as in equation (115).[45] Their estimates are significantly differently from zero for all countries and not inconsistent with the findings of Arrau *et al* (1995).

Some researchers have argued that many developing countries, particularly high-inflation ones, have gone through periods of interest rate controls, which limits the use of the interest rate when measuring the opportunity cost of holding money. Easterly *et al* (1995), for example, use the inflation rate, π_t, instead of the interest rate when constructing their opportunity cost variable, estimating an equation of the form

$$log\left(\frac{m_t}{y_t}\right) = \delta + \beta\left(\frac{\pi_t}{1+\pi_t}\right)^{\gamma}, \tag{117}$$

where y_t is output and γ is a parameter that captures the possibility of a nonlinear relationship between money balances and the corresponding opportunity cost. The authors use output instead of consumption as their scale variable because this aggregate is less subject to measurement error in their particular sample of countries. Panel C presents the results of their estimations for a panel of eleven high inflation countries using annual data for the 1960-1990 period, and using M1 as their money balances measure. The first two rows correspond to equation (117) estimated in levels, while the second two rows correspond to equation (117) estimated in first differences due to the difficulty of finding a cointegrating relationship in the levels specification. The authors find that the γ coefficient is positive and statistically significant in the nonlinear versions of both specifications, and therefore argue that in high-inflation countries the opportunity cost semi-elasticity of money is not constant but rather increases with inflation.

How large is the interest rate elasticity of money in the United States? Goldfeld and Sichel (1990) explain that until de mid-1970s the behavior of money demand in the U. S. was well explained by a simple partial adjustment model of the following form:

$$log(m_t) = a_0 + a_1 log(y_t) + a_2 log(m_{t-1}) + a_3 log(\pi_t) + \beta log\left(i_t\right) + error_t \tag{118}$$

[45] It can be easily checked that the interest-rate elasticity equals i times the semi-elasticity.

Panel D shows the results of estimating (118) applying a Cochrane-Orcutt procedure to quarterly data for the 1952-1986 period, taking M1 as the relevant measure of money holdings and using two rates to represent i: the commercial paper rate (RCP) and the commercial bank passbook rate (RCBP). In this case, the value of the interest rate elasticity is much smaller than the semi-elasticity estimates for developing countries presented in panels A though C. The authors also run regressions for different time periods and different variants of equation (118), obtaining low interest rate elasticity estimates in almost every case. Estimates for other G-7 countries summarized in Goldfeld and Sichel (1990) also point out to small numbers.

Taking into account the overall perform of their estimations, Goldfeld and Sichel (1990) suggested the need for rethinking the conventional specification, and experimented with different alternatives like estimating the money demand equation by maximum likelihood, using last day of quarter flow of funds data for M1, and amending the initial model by a buffer-stock component in the partial adjustment equation. In every instance, the estimated elasticity is below 0.04. We thus conclude that the low value of the estimated interest rate elasticity of money in the U.S. provides some empirical support for modeling money through a cash in advance constraint. Indeed, this has been the route followed by Cooley and Hansen (1989, 1995) in their seminal work introducing money in a real business cycles model.

Figure 1. Permanent devaluation

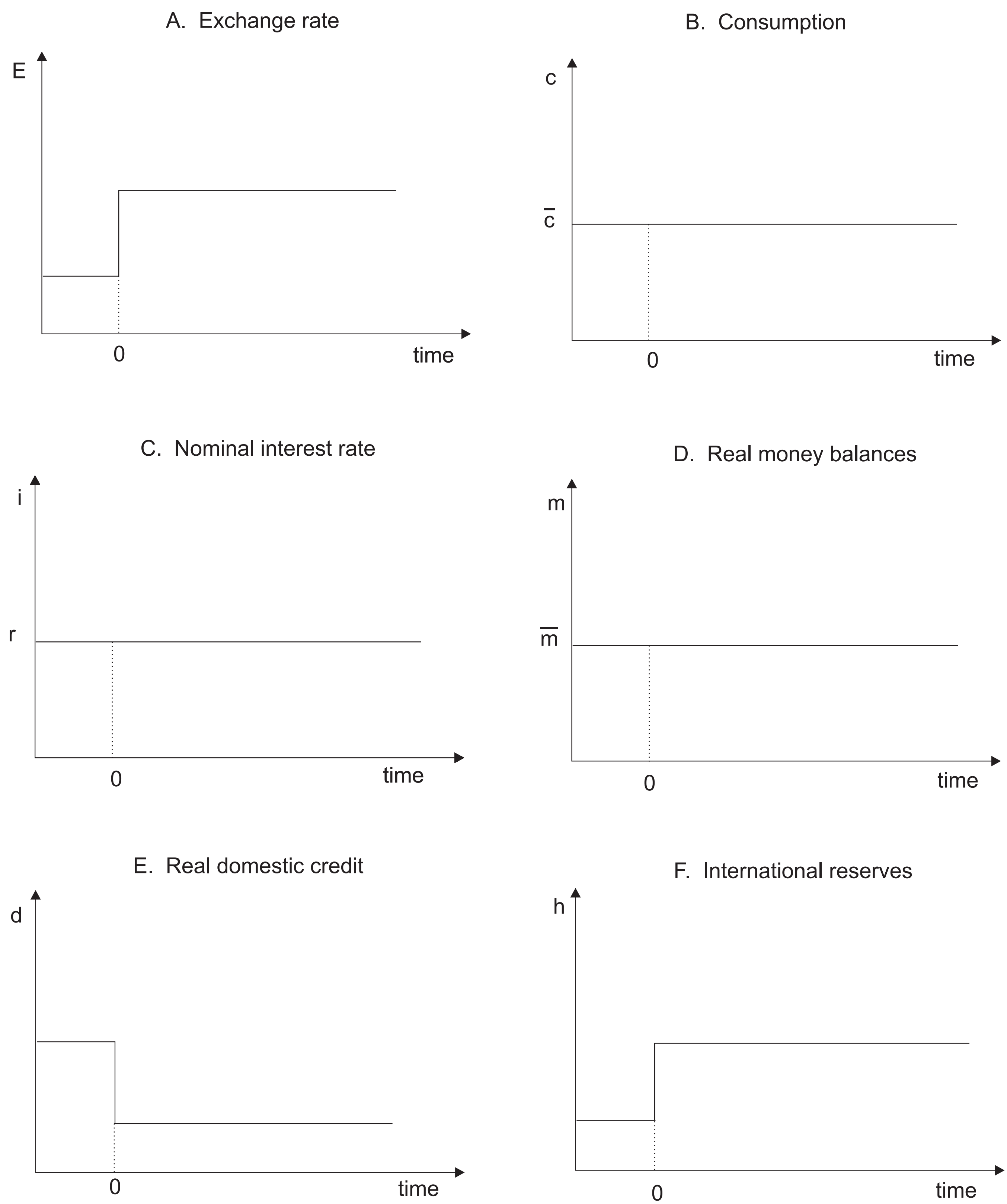

Figure 2. Permanent increase in devaluation rate

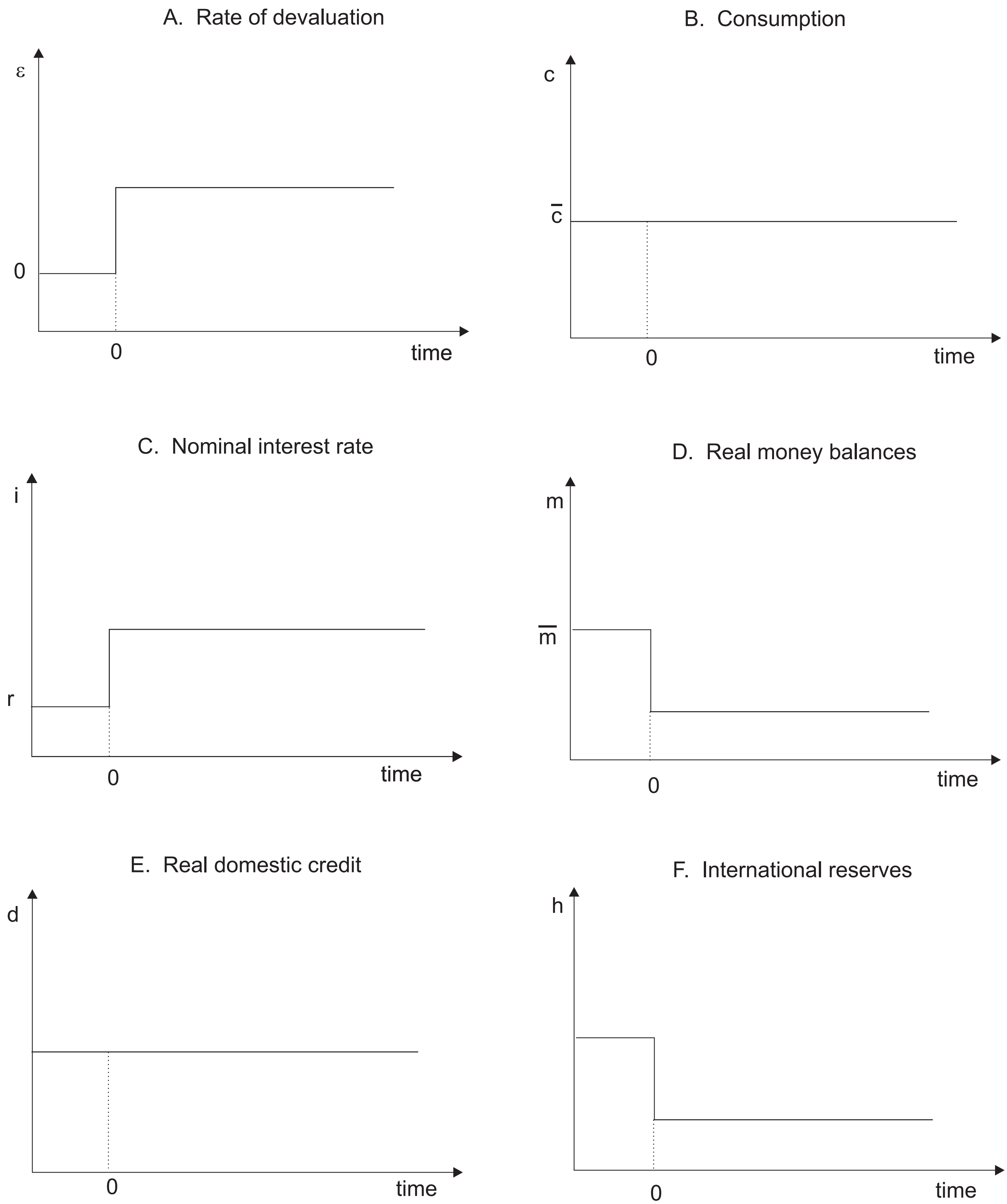

Figure 3. Permanent increase in domestic credit

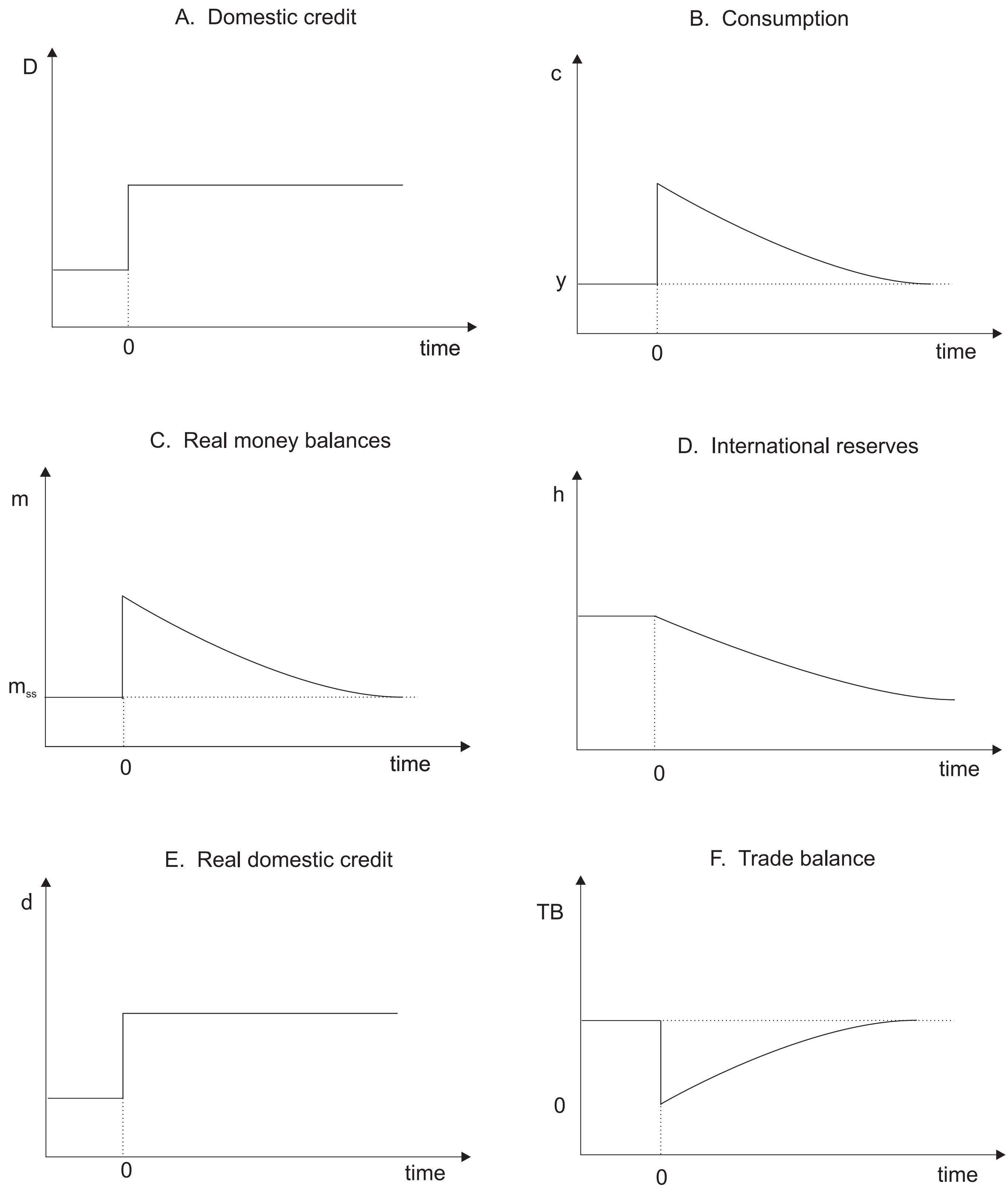

Figure 4. Permanent increase in rate of money growth

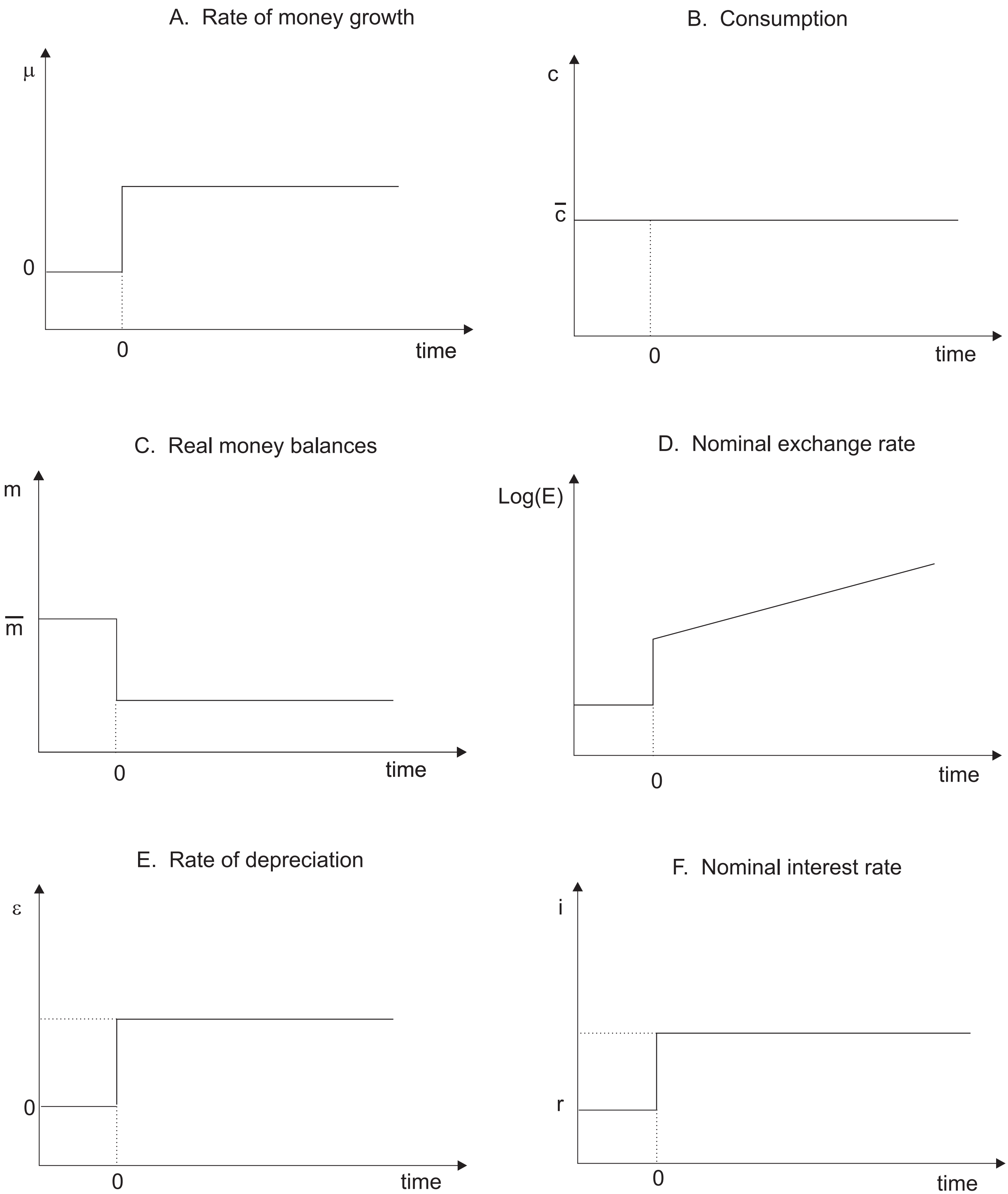

Figure 5. Equivalence between predetermined and flexible exchange rates I

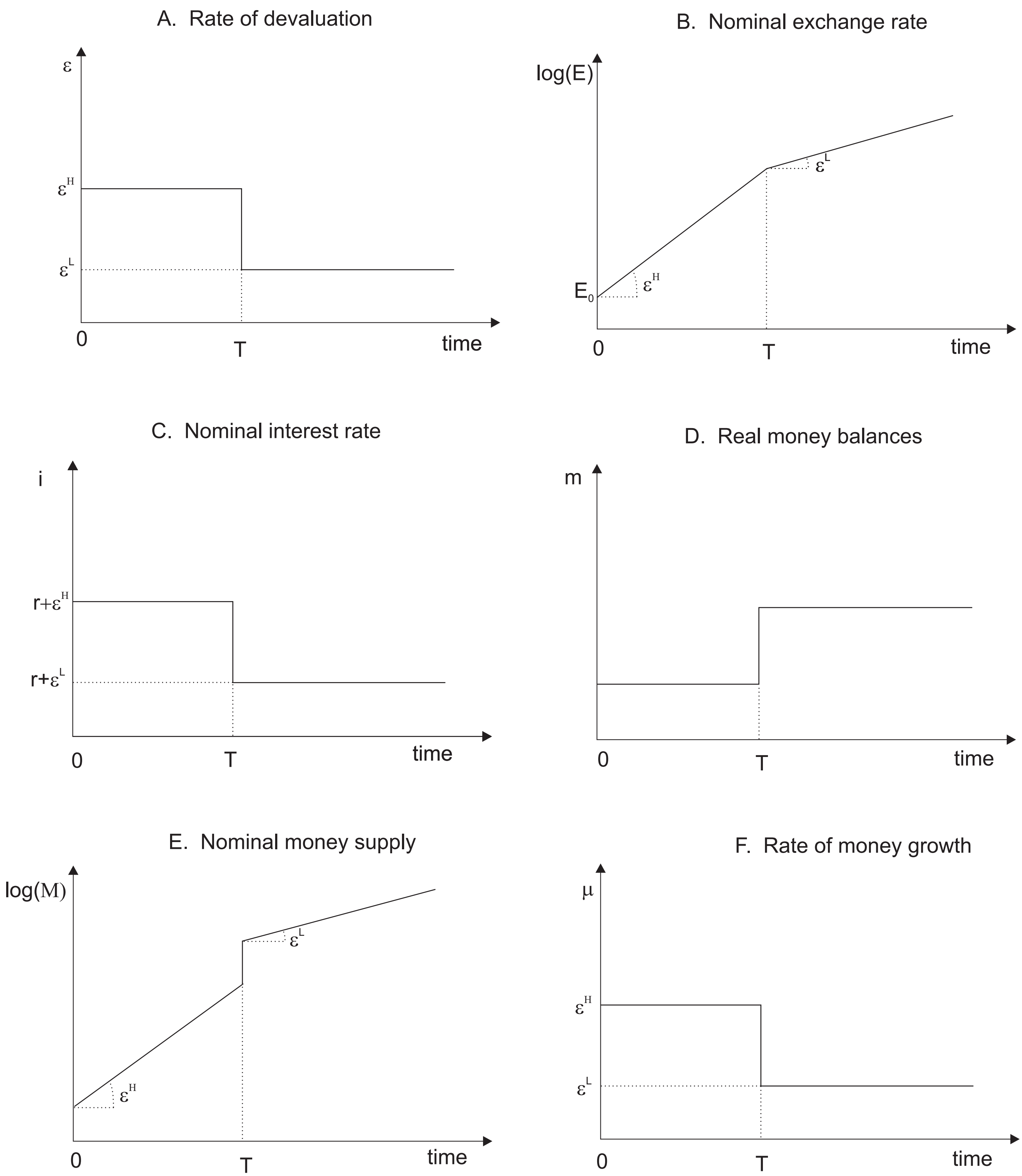

Figure 6. Equivalence between predetermined and flexible exchange rates II

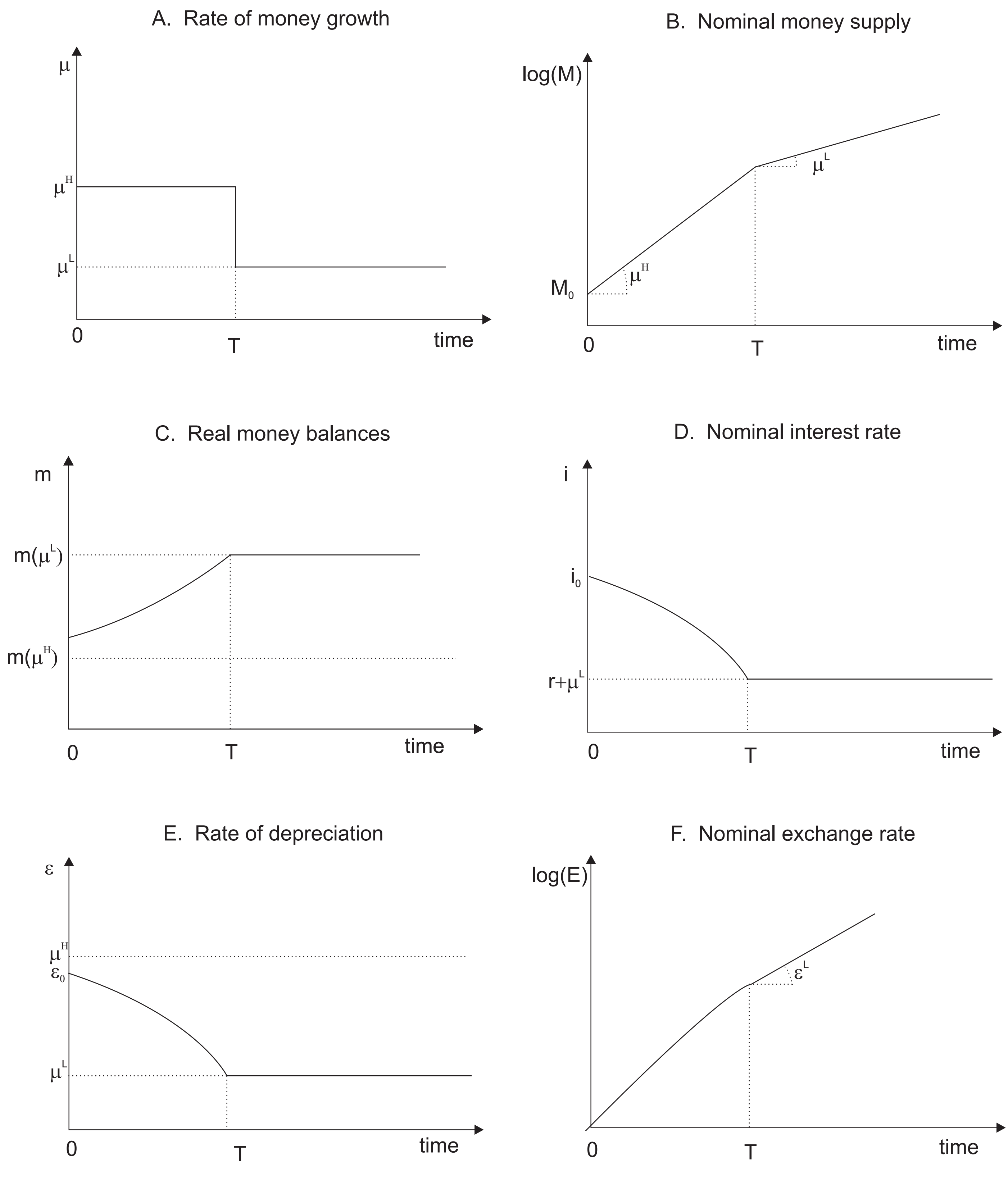

Figure 7. Negative money demand shock under predetermined exchange rates

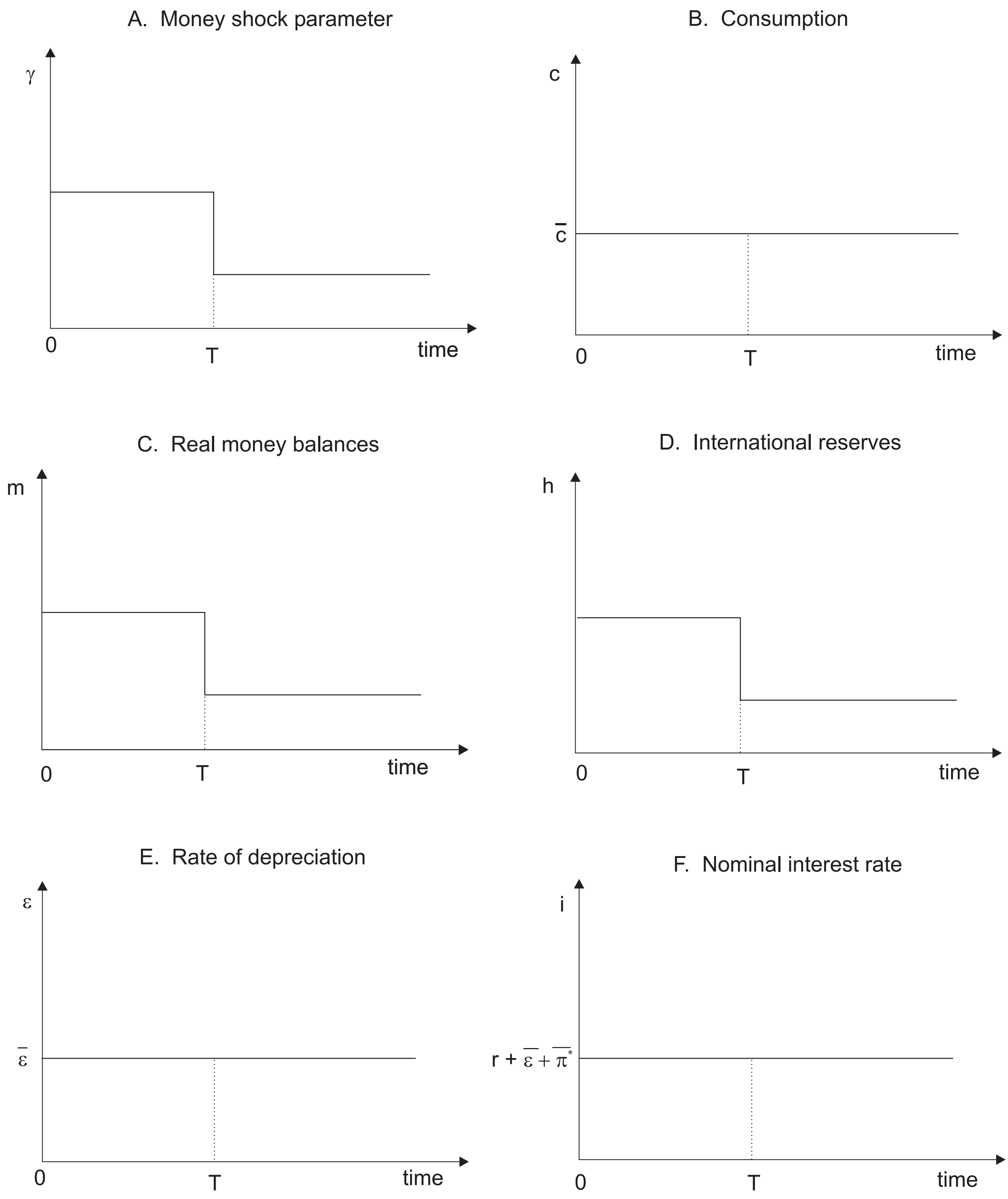

Figure 8. Negative money demand shock under flexible exchange rates

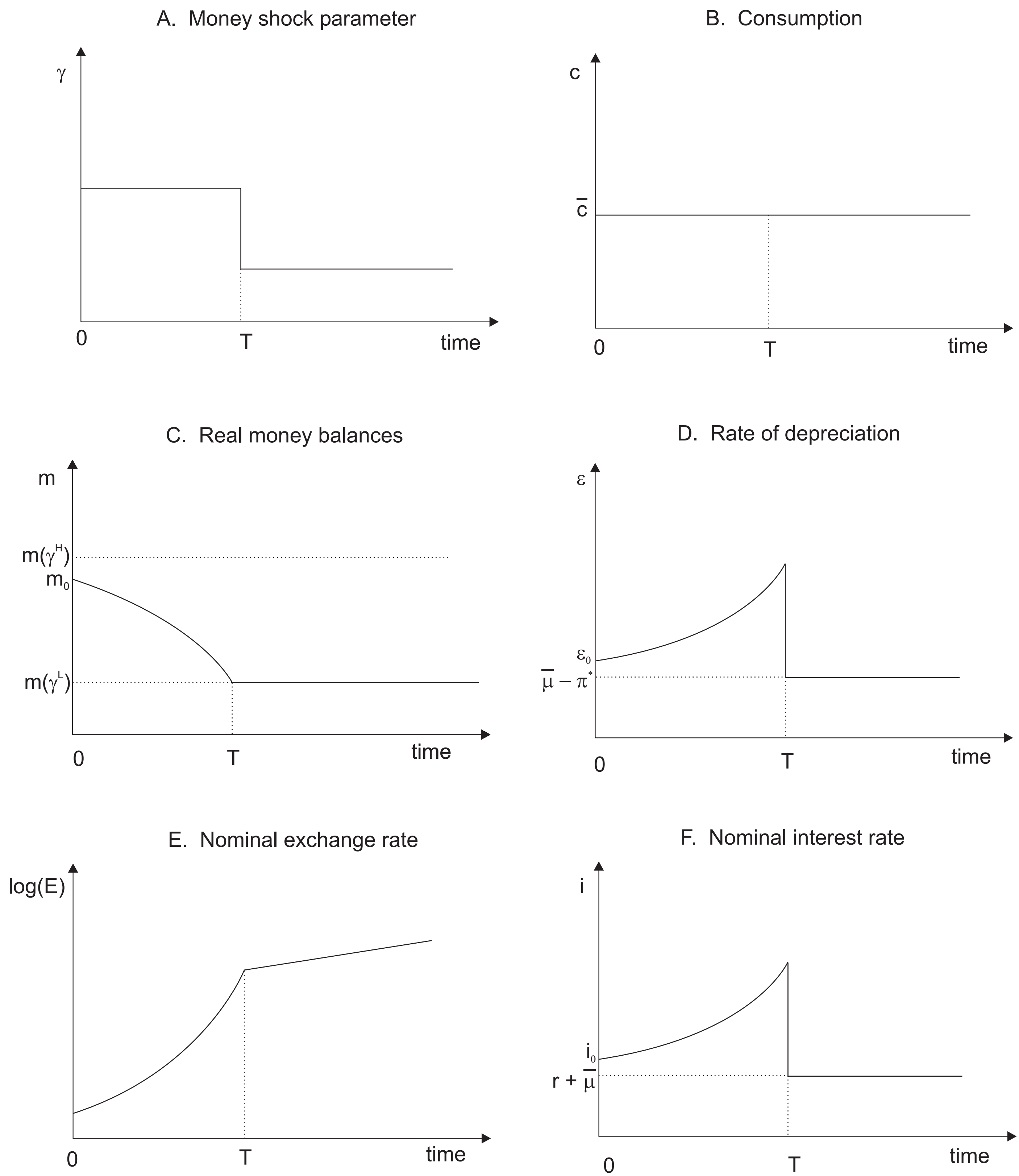

Table 1

Balance Sheet

$E_t H_t^*$	M_t
D_t	

Table 2. Nominal anchors

	Flexible	Predetermined
E	Set by policy	Endogenous
H*	Endogenous	Set by policy
D	Set by policy	Set by policy
M	Endogenous	Set by policy
NM	n/a	Endogenous

Note: n/a means "not applicable"

Table 3. *De jure* versus *de facto* exchange rate regimes

		RR de facto classification	
		Fixed	Flexible
IMF de jure classification	Fixed	1363	366
	Flexible	487	303

Source: IMF and Reinhart and Rogoff (2002). Data points are annual observations for 132 developing countries for the period 1970-2001

Table 4. Empirical estimates of the *opportunity cost semi-elasticity of money*

A.	Arrau *et al* (1995). Country-specific regressions*	
	Estimated β when the opportunity cost measure is:	
Country	*i*	*i/(1+i)*
Argentina		-0.47
		(-2.14)
Brazil		-2.17
		(-3.50)
India	-2.83	
	(-1.98)	
Israel		-2.97
		(-13.70)
Korea	-2.97	
	(-1.14)	
B.	Reinhart and Vegh (1995). Country-specific regressions*	
Country	*Estimated β***	
Argentina	-0.10	
	(-5.00)	
Chile	-0.09	
	(-2.25)	
Uruguay	-0.22	
	(-2.20)	
C.	Easterly et. al. (1995). Panel regressions*	
Specification	*Estimated β*	*Estimated γ*
Levels – linear (γ=1)	-1.42	
	(-11.46)	
Levels – Non-linear	-1.53	1.59
	(-10.04)	(6.78)
First-differences – linear (γ=1)	-0.74	
	(-6.53)	
First-differences – Non-linear	-0.92	2.20
	(-4.15)	(4.06)
D.	Goldfled and Sichel (1990). Money Demand in the US*	
Interest Rate	Estimated β**	
RCP	-0.013	
	(5.2)	
RCBP	-0.003	
	(0.9)	

* t-values in parenthesis

**Estimates correspond to the interest rate elasticity of money