Chapter 6
The Monetary Approach to the Balance of Payments*

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1 Introduction

We have seen that, in our basic monetary model of Chapter 5, both monetary policy (under flexible exchange rates) and exchange rate policy (under predetermined exchange rates) do not have real effects. We purposely chose such a set-up to focus exclusively on some important monetary phenomena without having to worry about possible links between the real and the monetary sectors. While that model is an important conceptual benchmark, it certainly does not provide us with tools to understand the possible real effects of monetary/exchange rate policy in an open economy. At its core, the main task of open economy monetary economics is to study departures from the world of Chapter 5.

The first such departure will be to abstract from interest-yielding assets. In other words, we will modify the model of Chapter 5 by assuming that money is the only asset available in the world economy. This will allow us

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to highlight a fundamental adjustment mechanism in economies operating under predetermined exchange rates: the so-called *monetary approach to the balance of payments*.[1] This approach emphasizes the idea that temporary "disequilibria" in the money market (in the sense of current money holdings that differ from the long-run demand for money) give rise to movements in international reserves (i.e., the balance of payments). More specifically, an excess demand for money will give rise to a trade surplus (and hence to a gain in international reserves) as the economy "imports" money from abroad to increase real money balances. Conversely, an excess supply of money will lead to a trade deficit (and thus to a loss of international reserves) as the economy gets rid of unwanted money balances. In this context, by increasing the price level, a devaluation reduces real money balances on impact and results in an excess money demand. This provokes a fall in consumption as households save in order to rebuild their cash balances. The resulting trade surplus leads to a gain in international reserves.[2] We can thus think of a devaluation as an *expenditure-reducing* mechanism. The reduction in expenditures translates into a trade surplus and reserve accumulation.

By introducing non-tradable goods and endogenous production into the picture, this chapter's model will also allow us to get a fuller picture of the anatomy of a devaluation. In addition to the expenditure-reducing effect isolated in the one-good model, we will encounter two additional effects: (i) an *expenditure-switching effect* whereby households switch consumption from tradable to non-tradable goods and (ii) a *production effect* whereby labor moves from the non-tradable to the tradable goods sector, thus increasing production of tradables at the expense of non-tradables. These two additional effects are induced by an increase in the relative price of tradable goods (i.e., a real depreciation of the currency). Intuitively, in response to a devaluation, households will want to reduce consumption of *both* tradable and non-tradable goods (the expenditure-reducing effect). This causes – at the initial relative price of non-tradable goods – an excess supply of non-

[1] Recall that the expression "balance of payments" – which reflects the common parlance of the 1960's and 1970's – refers to changes in international reserves at the Central Bank.

[2] In addition to being a fundamental adjustment mechanism in any open economy model, the monetary approach to the balance of payments has had a tremendous impact on policy. In fact, it has been at the core of IMF's policy recommendations for more than half a century. Typically, low international reserves at the central bank would prompt the IMF to call for a devaluation, on the rationale that such a measure would increase international reserves.

tradable goods. The resulting fall in the relative price of non-tradable goods induces households to consume more non-tradables (and less tradables) and firms to produce more tradables. This nicely illustrates the idea that a devaluation will improve the trade surplus by both reducing consumption of tradables and increasing production of tradable goods.

This two-good model will also allow us to make two additional important points. First, contrary to many economists' best instincts, a large comovement between nominal and real exchange rate is not *prima facie* evidence of sticky prices. In fact – and in spite of no nominal rigidities of any kind – the nominal price of non-tradable goods will respond very little to a devaluation if, as we expect in practice, there is little substitution in production. In fact, in the limiting case in which there is no production substitution at all, a back-of-the-envelope calculation reveals that a 10 percent devaluation results in an increase of only 2.7 percent in the price of non-tradable goods. Second, in this model a devaluation is *contractionary*, in the sense that it leads not only to a fall in consumption but also to a fall in production (in terms of tradable goods). This prediction is thus consistent with the idea that, particularly in developing countries, devaluations are often contractionary.[3]

Finally, we turn our attention to flexible exchange rates. We first show that, unlike the case of fixed/predetermined exchange rates studied so far in the chapter, the economy adjusts instantaneously to a change in the level or rate of growth of the money supply. Interestingly enough, therefore, removing interest-bearing bonds from the picture does not alter the results obtained in our pure monetary model of Chapter 5. The intuition is simply that the absence of interest-bearing assets does not affect the equilibrating mechanism under flexible exchange, which are changes in the nominal exchange rate. Hence, a doubling of the money supply will lead to a doubling of the nominal exchange rate with no real effects. We then introduce a celebrated twist into the model by assuming that domestic agents use two monies (i.e., there is currency substitution). The point is to show that, even under flexible exchange rates, the presence of currency substitution implies that the economy will adjust in much the same way as that emphasized by the monetary approach. Intuitively, adjustments in the stock of foreign currency can only take place through trade imbalances. Hence, a, say, increase in the rate of money growth, which induces households to increase their long-run holdings of foreign currency relative to domestic currency, will necessitate of

[3]See Chapter 8 for a review of the evidence on the output effects of devaluations.

a trade surplus to enable households to acquire more foreign currency.

The chapter proceeds as follows. Section 2 develops the main model of this chapter and analyzes several experiments that illustrate the adjustment mechanism highlighted by the monetary approach. Section 3 adds non-tradable goods and endogenous production to the model of Section 2 and focuses on the effects of a devaluation. Section 4 analyzes the model of Section 2 under flexible exchange rates. Section 5 introduces currency substitution into the picture and analyzes the response of the economy to an increase in the rate of growth of the money supply. Section 6 offers concluding remarks.

2 The monetary approach to the balance of payments

Building on Calvo (1981), we modify the model of Chapter 5 by assuming that money is the only asset available in the world economy. There is thus domestic money (used in the domestic economy for both transactions and store-of-value purposes) and foreign money (i.e., dollar bills) which is held as reserves by the Central Bank. There is a constant endowment of the only tradable (and non-storable) good, denoted by $\bar{y}$. The foreign price of the good is assumed to be constant and equal to one.

2.1 Consumer's problem

Let preferences be given by

$$\int_0^\infty [u(c_t) + v(m_t)]e^{-\beta t}dt, \tag{1}$$

where c_t denotes consumption and m are real money balances.

Since there are no bonds in this world, the consumer's flow constraint is given by

$$\dot{M}_t = P_t\bar{y} + P_t\tau_t - P_tc_t. \tag{2}$$

From the definition of real money balances ($m_t = M_t/E_t$), it follows that

$$\dot{m}_t = \frac{\dot{M}_t}{E_t} - \varepsilon_t m_t. \tag{3}$$

Dividing equation (2) by $P_t (= E_t)$ and using (3), we can rewrite it as

$$\dot{m}_t = \bar{y} + \tau_t - c_t - \varepsilon_t m_t. \tag{4}$$

The consumer maximizes (1) subject to the flow constraint (4). In terms of the current value Hamiltonian:[4]

$$H = u(c_t) + v(m_t) + \lambda_t \left(\bar{y} + \tau_t - c_t - \varepsilon_t m_t \right), \tag{5}$$

where c_t is the control variable, m_t is the state variable, and λ_t is the associated co-state variable. The co-state variable, λ, can be interpreted as the shadow value of an additional unit of real money balances in terms of time t utility. At each point in time, the consumer chooses consumption. The choice of consumption yields direct utility (as captured by the term $u(c_t)$) but also affects the stock of real money balances for the next instant through the flow budget constraint. The value of such a change is captured by the third term on the RHS of (5). Hence, for a given value of λ_t, the Hamiltonian captures the total contribution to utility of the choice of consumption.

Optimal conditions are given by

$$\frac{\partial H}{\partial c_t} = u'(c_t) - \lambda_t = 0, \tag{6}$$

$$\dot{\lambda}_t = \beta \lambda_t - \frac{\partial H}{\partial m_t} = (\beta + \varepsilon_t)\lambda_t - v'(m_t). \tag{7}$$

We can rewrite these optimality conditions as:

$$u'(c_t) = \lambda_t, \tag{8}$$

$$\underbrace{\frac{v'(m_t)}{\lambda_t} - \varepsilon_t}_{\text{"net dividend"}} + \underbrace{\frac{\dot{\lambda}_t}{\lambda_t}}_{\text{capital gain}} = \beta.$$

The first equation says that, at the margin, the consumer equates the benefits of an additional unit of consumption ($u'(c_t)$) to the shadow cost of that unit, λ. The second equation can be interpreted as an asset pricing equation. It

[4] Appendix 7.1 provides a basic introduction to optimal control techniques, which will be heavily used in the remainder of the book.

equates the total returns on real money balances – which consist of a "net dividend" and a capital gain – to the return on consumption, given by β.

For further reference, notice that by differentiating (8) with respect to time and using (7) we obtain:

$$\dot{c}_t = -\frac{1}{u''(c_t)}[v'(m_t) - (\beta + \varepsilon_t)u'(c_t)]. \tag{9}$$

2.2 Government

The government holds foreign currency as international reserves (still denoted by h in real terms). Since foreign currency does not pay interest, the government's flow budget constraint takes the form

$$\dot{h}_t = \frac{\dot{M}_t}{P_t} - \tau_t. \tag{10}$$

The rate of growth of domestic credit is given by

$$\frac{\dot{D}_t}{D_t} = \theta_t. \tag{11}$$

Differentiating with respect to time the Central Bank's balance sheet ($h_t + d_t = m_t$) and using the fact that $\dot{d}_t/d_t = \theta_t - \varepsilon_t$, we obtain:

$$\dot{h}_t = \dot{m}_t - d_t(\theta_t - \varepsilon_t).$$

As discussed in Chapter 5 – and to make the predetermined exchange rate system sustainable over time – we assume that $\theta_t = \varepsilon_t$. Hence,

$$\dot{h}_t = \dot{m}_t. \tag{12}$$

Conceptually, this is an extremely important equilibrium condition. It says that, under predetermined exchange rates, the change in international reserves will be determined by the change in real money demand. In other words, if, for some reason, real money demand is increasing (decreasing) over time, then the Central Bank will be gaining (losing) international reserves over time.

Finally, to find out the transfer policy implied by the domestic credit rule (11) and the assumption $\theta_t = \varepsilon_t$, solve for τ_t from (10) and use (12) to obtain:

$$\tau_t = \varepsilon_t m_t. \tag{13}$$

2.3 Equilibrium conditions

Combining the consumer's flow constraint (given by equation (4)) with the government's (given by equation (10)), we obtain:[5]

$$\dot{h}_t = \bar{y} - c_t. \tag{14}$$

Since there are no bonds in this world, the capital account is zero by construction. Hence, (14) says that the increase in international reserves equals the current account. Using (12), we can rewrite equation (14) as

$$\dot{m}_t = \bar{y} - c_t. \tag{15}$$

2.4 Dynamic system

Equations (9) and (15) constitute a dynamic system in c_t and m_t for a constant value of ε_t, denoted by $\bar{\varepsilon}$. To characterize the steady-state of the system, set $\dot{c} = \dot{m} = 0$ to obtain:

$$c_{ss} = \bar{y}, \tag{16}$$

$$v'(m_{ss}) = u'(\bar{y})(\beta + \bar{\varepsilon}). \tag{17}$$

Since there are no interest-bearing assets in this world, steady-state consumption will always be equal to the constant endowment, as indicated by equation (16). Equation (17) implicitly defines a steady-state real money demand with standard properties. Solving for m_{ss}, we obtain:

$$m_{ss} = L(\bar{y}, \beta + \bar{\varepsilon}), \tag{18}$$

where

[5] To be sure, equation (14) can also be obtained by substituting (13) into (4). But the derivation in the text makes the point that equation (14) does not depend on the assumption that $\varepsilon_t = \theta_t$.

$$\begin{aligned}\frac{\partial L}{\partial \bar{y}} &= \frac{(\beta+\bar{\varepsilon})u''(\bar{y})}{v''(m_{ss})} > 0,\\ \frac{\partial L}{\partial(\beta+\bar{\varepsilon})} &= \frac{u'(\bar{y})}{v''(m_{ss})} < 0.\end{aligned}$$

We can think of $\beta+\bar{\varepsilon}$ as a "shadow nominal interest rate" since it captures the steady-state opportunity cost of holding money.

We proceed by linearizing this dynamic system around the steady state. The linear approximation of the dynamic system around the steady state is given by

$$\begin{bmatrix}\dot{c}_t\\ \dot{m}_t\end{bmatrix} = \begin{bmatrix}\beta+\bar{\varepsilon} & \frac{-v''(m_{ss})}{u''(\bar{y})}\\ -1 & 0\end{bmatrix}\begin{bmatrix}c_t-\bar{y}\\ m_t-m_{ss}\end{bmatrix}. \tag{19}$$

The determinant of the matrix associated with the linear approximation (denoted by Δ) is negative:

$$\Delta = \frac{-v''(m_{ss})}{u''(\bar{y})} < 0.$$

Since the determinant is the product of the two roots, a negative determinant implies that the system has one positive (i.e., unstable) and one negative (i.e., stable) root (see Appendix 7.2 for details). Given that there is one predetermined variable (i.e., m), the system exhibits saddle path stability: for a given value of m_0, c will adjust so as to position the system along the saddle path.[6]

We now proceed to fully characterize the qualitative behavior of the dynamic system in the (m,c) plane by resorting to the so-called *phase diagram* (Figure 1). To construct the phase diagram, we first draw the $\dot{c}_t = 0$ and the $\dot{m}_t = 0$ curves. To obtain these curves, set $\dot{c}_t = 0$ in equation (9) to obtain

[6]Notice that this is the first time in the book that we are using a model with a stable root. Up to now, all the models have had a zero root (or unit root in discrete time). The presence of a stable root means that the system will converge to the steady-state regardless of the initial conditions. (The reader is referred back to Box 4 in Chapter 2 for a discussion of roots in small open economy models.) Models with a stable root are often referred to as models with "intrinsic dynamics", as opposed to models with a unit root (or zero root) which only have "extrinsic" dynamics (i.e., they typically have dynamics only to the extent that exogenous processes follow non-constant paths).

$$(\beta + \bar{\varepsilon})u'(c) = v'(m). \tag{20}$$

These are the loci of points along which real money balances do not change. To figure out the slope of the $\dot{c}_t = 0$ locus, totally differentiate equation (20) to obtain

$$\left.\frac{dc}{dm}\right|_{\dot{c}_t=0} = \frac{v''(m)}{(\beta + \bar{\varepsilon})u''(c)} > 0.$$

Hence, the $\dot{c}_t = 0$ curve will slope up, as depicted in Figure 1.

[Figure 1]

To obtain the $\dot{m}_t = 0$ locus, set $\dot{m}_t = 0$ in equation (15) to obtain

$$c = \bar{y}.$$

Hence, the $\dot{m}_t = 0$ curve shows up in the phase diagram as a horizontal line.

The intersection of the $\dot{m}_t = 0$ curve and the $\dot{c}_t = 0$ curves (point A in Figure 1) characterizes the system's steady-state. If the system starts at that point, it will stay there. In addition, the $\dot{m}_t = 0$ and $\dot{c}_t = 0$ curves define four different regions, labeled I through IV in Figure 1. The next step is to determine in what direction the system will move in each of these four regions. To figure this out, we draw arrowheads in each of the four regions in the following way. Suppose that we are on a point along the $\dot{m}_t = 0$ curve in Figure 1 and that we increase c by a small amount. How will real money balances respond? To answer this question, differentiate equation (15) with respect to c to obtain:

$$\left.\frac{\partial \dot{m}_t}{\partial c_t}\right|_{\dot{m}_t=0} = -1 < 0.$$

This says that if c is increased by a small amount, $\dot{m}$ becomes negative; that is, real money balances fall. Graphically, this means that we can draw arrowheads pointing west above the $\dot{m}_t = 0$ curve (i.e., in regions I and II) and, conversely, arrowheads pointing east below the $\dot{m}_t = 0$ curve (i.e., in regions III and IV).

Suppose now that we are on a point along the $\dot{c}_t = 0$ curve in Figure 1 and we increase m by a small amount. How will consumption respond? This time differentiate the $\dot{c}_t = 0$ curve – given by (9) – with respect to m to obtain

$$\left.\frac{\partial \dot{c}_t}{\partial m_t}\right|_{\dot{c}_t=0} = \frac{-v''(m_{ss})}{u''(\bar{y})} < 0.$$

This implies that if m is raised by a small amount, consumption will fall. Conversely, if m is decreased a little bit, consumption will increase. Graphically, then, we can draw arrowheads pointing south to the right of the $\dot{c}_t = 0$ curve (i.e., in regions II and III) and pointing north to the left of $\dot{c}_t = 0$ curve (i.e., in regions I and IV).

The arrowheads that we have just drawn tell us in what direction the system will move in each of the four regions. Specifically, in region I the system will move in a northwestern direction; in region II in a southwestern direction, in region III in a southeastern direction and in region IV in a northeastern direction.

How do we determine graphically the only convergent equilibrium path (i.e., the saddle path)?[7] It should be clear that if the system starts in either region I or III, it will diverge over time. Hence, there are only two regions (regions II and IV) in which the system will in principle move in a direction that is consistent with convergence to the steady-state. Within these two regions, however, there is a unique path that will lead the system to the steady-state. To illustrate this, suppose that the initial level of real money balances at $t = 0$ is given by m_0 which, as drawn in Figure 1, is lower than m_{ss}. Since m is a predetermined variable but c is not, the system can position itself at any point along the vertical line corresponding to m_0. Suppose that c_0 is such that the system starts at point C. Then the system will travel in a northeastern direction, cross the $\dot{m}_t = 0$ locus with a vertical tangent and then begin to travel in a northwestern direction. Such a path clearly diverges. By the same token, suppose that c_0 is such that the system starts at a point like D. The system then will travel in a northeastern direction, cross the $\dot{c}_t = 0$ locus with a horizontal tangent and then head in a southeastern direction, clearly diverging. In fact, it is only if c_0 is such that the system starts at point B (i.e., on a point along the saddle path) that it will converge over time to the steady-state (i.e., point A).

[7] Appendix 7.2 provides an analytical derivation of the saddle path.

2.5 Initial steady state

Let us now fully characterize an initial steady-state for a constant rate of devaluation, $\bar{\varepsilon}$. The steady-state values of consumption and real money balances are given by equations (16) and (18), respectively. Clearly, the trade balance will be equal to zero:

$$TB_{ss} = \bar{y} - c_{ss} = 0.$$

Since the rate of growth domestic credit will be assumed constant (and equal to $\bar{\theta}$) and $\bar{\theta} = \bar{\varepsilon}$, the level of real domestic credit will be exogenously given at some value $\bar{d}$. Given the stock of real domestic credit, the steady-state level of international reserves follows from the Central Bank's balance sheet ($h_t + d_t = m_t$):

$$h_{ss} = m_{ss} - \bar{d}.$$

2.6 Effects of a devaluation

Suppose that the economy is initially in the steady-state given by point A in Figure 1.[8] There is then an unanticipated and permanent increase in the level of the exchange rate (i.e., a permanent devaluation) (Figure 2, Panel A). How will the economy respond?

[Figure 2]

Let us use the phase diagram to find out the behavior of consumption and real money balances. The first question to ask is the following: does this permanent devaluation change the steady-state? The answer is clearly no. As equations (16) and (17) make clear, the steady-state values of consumption and real money balances do not depend on the nominal exchange rate.

How does the system respond on impact? On impact, real money balances must fall. To see this, notice that the Central Bank's balance sheet at time 0 reads as

$$m_0 = h_0 + \frac{D_0}{E}.$$

[8] To fix ideas – and with no loss of generality – we assume that $\bar{\varepsilon} = \theta = 0$ so that the exchange rate is fixed.

International reserves cannot jump at time 0 and D_0 is a policy variable which, by assumption, does not change at time 0. Hence, the increase in E must lead to a fall in real money balances. In terms of the phase diagram depicted in Figure 1, real money balances jump from m_{ss} to a point such as m_0. What happens to c_0? Based on our previous logic, c_0 will have to adjust so that the system lies along the saddle path. Hence, on impact, the system jumps for point A to Point B in Figure 1. The system then travels back to the unchanged steady-state (point A). The corresponding time paths of consumption and real money balances are depicted in Figure 2, Panels B and C, respectively. Given that output is constant, the fall in consumption leads to a trade surplus at $t = 0$ (Figure 2, Panel F), which diminishes over time as consumption returns to its initial level.

How do international reserves respond? First, notice that real domestic credit falls at $t = 0$ and stays at that lower level thereafter (Figure 2, Panel E). It then follows from the Central Bank's balance sheet ($h_{ss} = m_{ss} - d_{ss}$) that international reserves will be higher in the new steady-state because steady-state real money balances do not change but real domestic credit is lower. Hence, in the steady-state, international reserves increase by precisely the amount that real domestic credit falls. On impact, international reserves do not change and then increase over time as equation (12) makes clear (see Figure 2, Panel D).

What is the intuition behind the adjustment of this economy to a devaluation? Since the devaluation reduces real money balances on impact but does *not* affect the desired long-run level of real money balances, the economy must rebuild real money balances over time. The only way for the economy to do so is to run a trade surplus (or, equivalently, a current account surplus) which increases international reserves over time and hence the nominal money supply. To run a trade surplus, the economy must reduce consumption in the short-run (which we will refer to as the *expenditure reducing effect* of a devaluation). Put differently, the excess demand for real balances at time 0 (relative to the long-run real money demand) forces this economy to "import" the desired real money balances from the rest of the world. The fact that the balance of payments (i.e., the change in international reserves) responds to a "disequilibrium" in the money market explains why we refer to this kind of model as the "monetary approach to the balance of payments".

The key conclusion of this experiment is that *a devaluation will lead to a gain in international reserves.* This result thus provides a rationale for the typical IMF recommendation to countries that are losing international reserves.

Two final observations are in order. First, the result that a devaluation leads to an increase in international reserves is, of course, the same conclusion that we reached in Chapter 5. The key difference is that in the model of Chapter 5 the gain in international reserves occurred instantaneously and therefore involved no adjustment in the real sector. In contrast, in this chapter's model, the gain in international reserves occurs gradually over time through a trade surplus. Since the real world is surely somewhere in between Chapter 5 (where there is perfect capital mobility) and Chapter 6 (no capital mobility), this type of model would predict that a devaluation should lead to both an increase in international reserves and a trade surplus.[9]

The second observation is that, in this model, a devaluation is contractionary in the sense that it leads to a fall in consumption. Output is, of course, exogenous so it does not change. What would happen if we endogeneized production in this one-good world? Exercise 1 at the end of this chapter introduces a labor/leisure choice and linear production into the model and shows that a devaluation would still lead to a trade surplus in order to replenish real money balances. The trade surplus, however, would come about through two different channels: lower consumption and higher output. Intuitively, the fall in consumption needed to "import money" must be accompanied by a fall in leisure because the relative price of consumption in terms of leisure is not affected. The fall in leisure means more labor and higher production. The devaluation is thus expansionary in terms of output although it still leads to lower consumption. The conventional wisdom in this area (as we will review in Chapter 8) is that devaluations are expansionary. The empirical evidence, however, suggests that, in developing countries, devaluations are often contractionary (see, for instance, Gupta, Mishra, and Sahay (2007)).

[9] Incidentally, we should note that we would not want to use this model to draw normative implications. The reason is that, unlike the model of Chapter 5, the private sector is not "consuming" the international reserves at the Central Bank. In Chapter 5, the consumer ends up consuming those reserves because the Central Bank transfers the interest on reserves at every instant and, hence, the present discounted value of such transfers is equal to the initial stock of international reserves.

2.7 Increase in domestic credit

Often times, developing countries finance government spending with domestic credit (i.e., the fiscal authority "borrows" from the monetary authority). In our model, such borrowing takes the form of the Central Bank printing money to buy (non-interest bearing) debt issued by the Finance Ministry.

Suppose, once again, that the economy is initially (i.e., just before $t = 0$) at the steady-state given by point A in Figure 1.[10] At $t = 0$, there is an unanticipated and permanent increase in the stock of domestic credit (Figure 3, Panel A). How does the economy respond?

[Figure 3]

In terms of Figure 1, notice that a permanent increase in the stock of domestic credit does not change the economy's steady-state, as equations (16) and (17) make clear. From the Central Bank's balance sheet, we infer that, on impact (i.e., at $t = 0$), real domestic credit increases and hence real money balances also increase. The system thus jumps from point A to a point like E in Figure 1 and then travels back to point A over time. The corresponding paths of consumption and real money balances are illustrated in Figure 3, Panels B and C. The trade balance thus goes into deficit at time 0 and gradually recovers over time (Panel F).

Since real domestic credit rises on impact and remains at that level thereafter (Panel E) and steady-state real money demand does not change, we infer that steady-state international reserves will fall. Hence, international reserves fall over time towards their lower steady-state value (Figure 3, Panel D).

Intuitively, on impact the increase in domestic credit puts more real money balances in the hands of the public. The public, however, does not wish to hold more real money balances. To get rid of these unwanted real money balances, the economy runs a trade deficit. The trade (and current) deficits leads to a persistent loss of international reserves that reduces the nominal money supply. In the new steady-state, the *level* of the nominal money supply will be the same as before the shock, but the *composition* will be different (i.e., international reserves are lower and real domestic credit is higher).

In conclusion, the model's key prediction is that an increase in domestic

[10] Again, suppose that $\bar{\varepsilon} = \bar{\theta} = 0$.

credit will lead to a loss of international reserves of the same order of magnitude. Since, in practice, many developing countries use Central Bank credit to finance government spending, this result provides a key link between loose fiscal policy and loss of international reserves (see Box 1). Based on this type of model, a typical component of an IMF policy package is to set a target for international reserves and, given an estimate of real money demand, set the growth of domestic credit in such a way as to meet that target (see Box 2 on IMF financial programming).

2.8 Increase in the rate of devaluation

Suppose that the initial steady-state (corresponding to the devaluation rate $\bar{\varepsilon}$) is given by point A in Figure 4. At $t = 0$, there is an unanticipated and permanent increase in the rate of devaluation from $\bar{\varepsilon}$ to ε^H (Figure 5, Panel A).[11]

[Figure 4]

How is the steady-state affected by an increase in the rate of devaluation? From (16) and (17), it follows that while steady-state consumption does not change, steady-state real money balances fall. Intuitively, since the opportunity cost of holding real money balances is higher, consumers reduce their demand for money. The new steady-state will be thus at a point like B in Figure 4.

[Figure 5]

How does the economy go from point A to point B? On impact, real money balances cannot jump. Hence, the economy must jump on impact from point A to point C to position itself along the saddle path. It then travels over time from point C towards point A. The corresponding paths of consumption and real money balances are illustrated in Figure 5, Panels B and C. The consumption paths implies that the economy runs a trade deficit over time (Figure 5, Panel F). Since real domestic credit remains unchanged (Panel

[11] To ensure that the new predetermined exchange rate regime will be sustainable over time, we assume that the rate of growth of domestic credit, $\bar{\theta}$, is increased by the same amount as $\bar{\varepsilon}$.

E), we know from the Central Bank's balance sheet that, in the new steady-state, international reserves will be lower. Hence, international reserves will fall over time (Panel D).

Intuitively, the increase in the rate of devaluation induces a fall in the steady-state real money demand. To get rid of these unwanted real money balances, the economy must run a trade (current account) deficit. In other words, the economy "exports" its unwanted real money balances.

Finally, notice that for this experiment the interest-rate elasticity of money demand plays a critical role. In other words, if real money demand were not interest rate elastic, consumers would not wish to reduce real money balances in the long-run and hence the monetary adjustment mechanism that we have just described would not take place. This is in sharp contrast to the previous two experiments – a permanent devaluation and a permanent increase in the stock of domestic credit – in which the interest-rate elasticity of money demand played no role since the shocks affected real money *supply*. It should then be clear that if we introduced money into the model via a cash-in-advance constraint (which implies a non interest-rate elastic money demand), the results of the first two experiments would go through but the results of this third experiment would not. (Exercise 2 at the end of this chapter asks you to verify this.) We thus conclude that, as long as the interest rate elasticity of money demand is not critical for the experiment at hand, nothing is lost by adopting a cash-in-advance specification that will in general simplify the model. If the interest rate elasticity is critical, then one should adopt a MIUF or transaction costs specification that gives rise to such a money demand since, in practice, money demand in developing countries is indeed interest rate elastic (recall Box 2 in Chapter 5).

3 Anatomy of a devaluation: Devaluation in a two-good world

The one-good model in the previous section has illustrated a key mechanism associated with a devaluation: the *expenditure-reducing effect*. As the model makes clear, this expenditure-reducing effect results from the contraction in real money balances brought about by the increase in the price level (i.e., the increase in the exchange rate), which induces households to reduce consumption in order to replenish money balances. There are, however, two other important effects of a devaluation that the model has abstracted from: (i) an *expenditure-switching effect*, and (ii) *a production effect*. These effects are critical in obtaining a full understanding of the effects of a devaluation in this type of model.

To capture these effects, this section adds two features to the previous model: a non-tradable good and endogenous production. Production will be endogeneized along the lines of Section 5 in Chapter 4. Tradables and non-tradables are produced with labor as the only input. Households are endowed with an exogenous amount of labor, which they supply inelastically to the labor market (i.e., there is no labor/leisure choice). For simplicity, we assume that households undertake the production themselves.

3.1 Household's problem

Preferences are now given by

$$\int_0^\infty [\log(c_t^T) + \log(c_t^N) + \log(z_t)]e^{-\beta t}dt, \tag{21}$$

where c_t^T and c_t^N denote consumption of tradable and non-tradable goods, respectively, and $z_t(\equiv M_t/P_t)$ denotes real money balances in terms of the price index P, given by[12]

$$P \equiv \sqrt{P^T P^N}. \tag{22}$$

Since money balances enter the utility function to capture the liquidity services provided by money, it seems natural to deflate nominal balances by a price index because the household consumes both goods.[13] Since, as easily verified, $z_t = m_t\sqrt{e}$ (where e_t denotes the relative price of tradable goods in terms of non-tradable goods), we can re-write the above preferences as

$$\int_0^\infty [\log(c_t^T) + \log(c_t^N) + \log(m_t) + \left(\frac{1}{2}\right)\log(e_t)]e^{-\beta t}dt, \tag{23}$$

[12]Notice that, to be consistent with our notation in this and the previous chapter, we will use z to denote real money balances in terms of the price index and continue to use m to denote real money balances in terms of tradable goods. (We will continue to use tradable goods as the numeraire.)

[13]This price index corresponds to the minimum nominal expenditure required to achieve a certain level of utility (see Appendix 7.3 for the derivation).

which makes clear that, in the logarithmic case, the price deflator is irrelevant. In other words, we will get exactly the same results as if we had assumed that m, instead of z, yields liquidity services.

The flow budget constraint is given by

$$\dot{m}_t = y_t^T + \frac{y_t^N}{e_t} + \tau_t - c_t^T - \frac{c_t^N}{e_t} - \varepsilon_t m_t, \tag{24}$$

where y^T and y^N denote production of tradable and non-tradable goods, respectively. Production is given by

$$y_t^T = Z^T \left(n_t^T\right)^{\alpha}, \tag{25}$$

$$y_t^N = Z^N n_t^N, \tag{26}$$

where Z^T and Z^N are positive productivity parameters and $0 < \alpha < 1$.[14] As in Chapter 4, the non-tradable sector is assumed to be more labor intensive than the tradable sector.

The labor supply constraint is given by

$$\bar{n} = n_t^T + n_t^N, \tag{27}$$

where $\bar{n}$ is the exogenous labor endowment.

Substituting (25), (26) and (27) into the flow constraint (24), we can set up the Hamiltonian:

$$\begin{aligned} H &= \log(c_t^T) + \log(c_t^N) + \log(m_t\sqrt{e}) \\ &\quad + \lambda_t \left[Z^T \left(n_t^T\right)^{\alpha} + \frac{Z^N\left(\bar{n} - n_t^T\right)}{e_t} + \tau_t - c_t^T - \frac{c_t^N}{e_t} - \varepsilon_t m_t \right]. \end{aligned}$$

The control variables are c_t^T, c_t^N, and n_t^T, the state variable is m_t, and the co-state variable is λ_t. The first-order conditions are given by

[14] The case $\alpha = 1$ is also well-defined and corresponds to linear production in both sectors (Exercise 3 at the end of this chapter asks you to solve for this linear case). This extreme case proves quite useful as a a benchmark for some questions posed below. The case $\alpha = 0$, on the other hand, is not formally well-defined but, conceptually, corresponds to the endowment case discussed below.

$$\begin{aligned}
\frac{1}{c_t^T} &= \lambda_t, && (28)\\
\frac{1}{c_t^N} &= \frac{\lambda_t}{e_t}, && (29)\\
\alpha Z^T \left(n_t^T\right)^{\alpha-1} &= \frac{Z^N}{e_t}, && (30)\\
\dot{\lambda}_t &= \left(\beta+\varepsilon_t\right)\lambda - \frac{1}{m_t}. && (31)
\end{aligned}$$

Relative to the previous model in this chapter, the only new optimality condition is equation (30) which, nonetheless, should be familiar from Chapter 4. This condition, which captures efficiency in production, requires that the value of the marginal productivity of labor be equated across sectors. If that were not the case, total production could be increased by shifting labor away from the low marginal productivity sector to the high marginal productivity sector.

Combining conditions (28) and (29), we get the familiar consumption-efficiency condition whereby the marginal rate of substitution between tradables and non-tradables is equal to the relative price of tradables:

$$\frac{c_t^N}{c_t^T} = e_t. \tag{32}$$

Further, combining (30) and (32), we see that, an optimum, the marginal rate of substitution in consumption and in production are equalized:

$$\frac{c_t^T}{c_t^N} = \frac{\alpha Z^T \left(n_t^T\right)^{\alpha-1}}{Z^N}. \tag{33}$$

3.2 Government

The government sector remains unchanged. Constraints (10), (12), and (13) therefore remain valid.

3.3 Equilibrium conditions

Equilibrium in the non-tradable goods market requires that

$$c_t^N = y_t^N. \tag{34}$$

Substituting the government's flow constraint (10) into the households' flow constraint (24) and imposing equilibrium in the non-tradable goods market, given by (34), yields:

$$\dot{h}_t = y_t^T - c_t^T.$$

Substituting (12) into the last equation:

$$\dot{m}_t = y_t^T - c_t^T. \tag{35}$$

3.4 Dynamic system

As before, we will set up a dynamic system in c^T and m. The first dynamic equation follows from time-differentiating (28) and using (31) to obtain:

$$\dot{c}_t^T = c_t^T \left(\frac{c^T}{m_t} - \beta - \varepsilon_t \right). \tag{36}$$

To derive the second dynamic equation, substitute (25) into (35) to obtain:

$$\dot{m}_t = Z^T \left(n_t^T\right)^{\alpha} - c_t^T. \tag{37}$$

We will now express n_t^T as a function of c_t^T. Using (26), (27), (28), (29), (30), and (34), it follows that

$$c_t^T = \frac{\alpha Z^T (\bar{n} - n_t^T)}{\left(n_t^T\right)^{1-\alpha}}, \tag{38}$$

which implicitly defines n^T as a decreasing function of c^T:

$$n_t^T = \Psi(c_t^T), \tag{39}$$

where

$$\Psi'(c_t^T) = -\frac{1}{\alpha Z^T \left[\left(n_t^T\right)^{\alpha-1} + \frac{(1-\alpha)(\bar{n}-n_t^T)}{\left(n_t^T\right)^{2-\alpha}} \right]} < 0$$

Substituting (39) into (37) yields our second differential equation:

$$\dot{m}_t = Z^T \left[\Psi(c_t^T)\right]^{\alpha} - c_t^T. \tag{40}$$

Equations (36) and (40) constitute a dynamic system in c^T and m, for a given and constant value of the rate of devaluation, $\bar{\varepsilon}$.

The system's steady-state is implicitly given by

$$m_{ss} = \frac{c_{ss}^T}{\beta + \bar{\varepsilon}}, \tag{41}$$

$$c_{ss}^T = Z^T \left[\Psi(c_{ss}^T)\right]^{\alpha}. \tag{42}$$

Notice that equation (42) defines a unique value of c_{ss}^T because the LHS is an increasing function of c_{ss}^T whereas the RHS is a decreasing function of c_{ss}^T.

Linearizing the system and using (41) and (42), we obtain:

$$\begin{bmatrix} \dot{c}_t^T \\ \dot{m}_t \end{bmatrix} = \begin{bmatrix} \beta + \bar{\varepsilon} & -(\beta + \bar{\varepsilon})^2 \\ \alpha Z^T \left(n_{ss}^T\right)^{\alpha - 1} \Psi' - 1 & 0 \end{bmatrix} \begin{bmatrix} c_t - c_{ss}^T \\ m_t - m_{ss} \end{bmatrix}.$$

The determinant of the matrix associated with the linear approximation is given by

$$\Delta = (\beta + \bar{\varepsilon})^2 \left[\alpha Z^T \left(n^T\right)^{\alpha - 1} \Psi' - 1\right] < 0,$$

which indicates that the system has one negative and one positive root and is thus saddle-path stable.

To construct the phase diagram, we proceed to find out the slope of the $\dot{c}^T = 0$ and $\dot{m} = 0$ loci. From (36), it follows that

$$\left.\frac{dc^T}{dm}\right|_{\dot{c}_t^T = 0} = \beta + \bar{\varepsilon} > 0.$$

Clearly, from (40), the $\dot{m} = 0$ schedule is a constant value of c^T. Qualitatively, therefore, we have the same phase diagram as before (Figure 1).

3.5 Initial steady-state

We will consider an initial steady-state in which the exchange rate is fixed (i.e., the rate of devaluation is zero). Then, from (41) and (42), it follows that

$$
\begin{aligned}
m_{ss} &= \frac{c^T_{ss}}{\beta}, \\
c^T_{ss} &= Z^T \left[\Psi(c^T_{ss})\right]^{\alpha}.
\end{aligned}
$$

Given c^T_{ss}, the steady-state value of n^T is implicitly determined by (recall equation (38))

$$
c^T_{ss} = \frac{\alpha Z^T(\bar{n} - n^T_{ss})}{(n^T_{ss})^{1-\alpha}}.
$$

Given n^T_{ss}, the labor supply constraint (27) determines n^N_{ss}:

$$
n^N_{ss} = \bar{n} - n^T_{ss}.
$$

Production of tradables and non-tradables goods is then read off the production functions:

$$
\begin{aligned}
y^T_{ss} &= Z^T \left(n^T_{ss}\right)^{\alpha}, \\
y^N_{ss} &= Z^N n^N_{ss}.
\end{aligned}
$$

Equilibrium in the non-tradable goods market determines consumption of non-tradable goods:

$$
c^N_{ss} = Z^N n^N_{ss}
$$

The real exchange rate follows from condition (33):

$$
e_{ss} = \frac{c^N_{ss}}{c^T_{ss}}.
$$

Finally, notice that the steady-state trade balance is zero:

$$
TB_{ss} \equiv y^T_{ss} - c^T_{ss} = 0.
$$

As should be clear, the steady-state conditions do no depend on the *level* of the nominal exchange rate. Hence, as in the one-good model, a devaluation is neutral in the long-run.[15]

[15] Only steady-state real money balances would be affected by a change in the rate of devaluation, so exchange rate policy is also super-neutral in the long run.

Figure 6 offers a graphical representation of the initial steady-state. It depicts the production possibility frontier (henceforth PPF), which should be thought of as the locus of production points that are attainable by this economy (if, as is always the case in the model, the labor supply $\bar{n}$ is fully employed). To check that the PPF is negatively sloped, solve for n^T and n^N from (25) and (26), respectively, and substitute into the labor constraint (27) to obtain:

$$\bar{n} = \left(\frac{y_t^T}{Z^T} \right)^{\frac{1}{\alpha}} + \frac{y_t^N}{Z^N}.$$

[Figure 6]

Totally differentiate to obtain:

$$\frac{dy_t^T}{dy_t^N} = -\alpha \frac{Z^T}{Z^N} \left(\frac{y_t^T}{Z^T} \right)^{1-\frac{1}{\alpha}} < 0. \tag{43}$$

It is easy to check that $d^2 y_t^T / \left(dy_t^N \right)^2 < 0$, which confirms that the PPF is concave.

At point A, the PPF and the indifference curve between tradables and non-tradables are tangent to each other and to the relative price of non-tradable goods, $1/e_{ss}$. Production of tradables, y_{ss}^T, can be read off the vertical axis while production of non-tradable goods, y_{ss}^N, is read off the horizontal axis. Consumption of tradables equals production so that the trade balance is zero.

3.6 Devaluation

Suppose that the economy is initially in the steady-state equilibrium just described. At time 0, there is an unanticipated and permanent devaluation (Figure 7, Panel A). How does the economy react?

[Figure 7]

3.6.1 Dynamic response

Such increase in the nominal exchange rate reduces on impact real money balances, which takes the dynamic system from point A to a point such as B in Figure 1. The dynamic system then travels back to the unchanged steady-state along the saddle path. The path of c^T is depicted in Figure 7, Panel B.

Since c^T and n^T are negatively related at all points in time (recall equation (39)), the path of n^T (and hence of production of tradable goods) will be the mirror image of that of c^T, as illustrated in Figure 7, Panel D. Production of tradable goods increases on impact and then falls gradually over time. Since on impact, production of tradables increase while consumption of tradable falls, the trade balance improves on impact (Figure 7, Panel E). As time goes by, the increase in consumption of tradables and the fall in production of tradables implies that the trade balance falls over time.

Given the behavior of n^T, the path of n^N follows from the labor supply constraint (27). Production – and thus consumption – of non-tradables falls on impact and then increases over time towards its unchanged steady-state (Figure 7, Panel C). Through the production efficiency condition, the path of n^T also allows us to infer the path of the real exchange rate. The real exchange rate increases on impact (real depreciation) and then falls over time (Figure 7, Panel F).

In sum, a devaluation leads to a fall in consumption of both goods, a switch in production from the non-tradable sector to the tradable goods sector, a trade surplus, and real depreciation. What is the intuition behind these results? While the devaluation does not affect steady-state real money demand, it reduces real money balances on impact by increasing the price level. To rebuild money balances over time – and for a given real exchange rate – households wish to reduce consumption of both goods. This results in excess supply of non-tradable goods, which must cause a fall in their relative price (i.e., a rise in e). The fall in the relative price of non-tradable goods reduces profitability in the non-tradable goods sector and induces labor to shift from this sector to the tradable good sector.

We now look in more detail at some key features related to the impact effect of a devaluation on expenditure, GDP, and prices.

3.6.2 Expenditure reducing versus expenditure switching

Figure 6 will help us analyze the expenditure-reducing versus expenditure-switching effects. As described above, initial production and consumption are given by point A. Immediately after the devaluation (i.e., at time 0), consumption shifts to point B – where the indifference curve is tangent to $1/e_0$ – and production to point C –where the PPF is tangent to $1/e_0$. The picture thus shows the fall in both consumption of tradable and non-tradable goods and the increase in production of tradable goods at the expense of non-tradable goods. On impact, the trade balance is given by the vertical distance between y_0^T and c_0^T.

While consumption jumps immediately from point A to point B, from a conceptual point of view we can decompose this adjustment in consumption into two effects:

- An *expenditure-reducing* effect (from point A to point D). This effect captures the fall in consumption that takes place for an unchanged relative price. This contraction in consumption is due to the households' desire to save in order to rebuild real money balances. This is, of course, the only effect present in the one-good model studied earlier in this chapter.[16]

- An *expenditure-switching* effect (from point D to Point B). This effect captures the substitution in consumption that takes place as a result of the fall in the relative price of non-tradable goods from $1/e_{ss}$ to $1/e_0$. As non-tradable goods become relatively cheaper, households consume more non-tradable goods and less tradable goods relative to point D. At point B, therefore, the ratio c^N/c^T is higher than at point D. In absolute terms, as well, consumption of tradables is lower and consumption of non-tradables is higher.[17]

It is worth noting that, when it comes to consumption of tradable goods, both effects reinforce each other. In contrast, for consumption of non-

[16] It is also the only effect in the case in which both production functions are linear, as Exercise 3 at the end of this chapter makes clear. In this case, the relative price of non-tradable goods is completely determined by the technology.

[17] Although not relevant for our argument, notice that consumption expenditure falls as we move from point D to point B due to the increase in e. To see this, use condition (32) to rewrite consumption expenditure ($c^T + c^N/e$) as $2c^T$. Since c^T falls, consumption expenditure falls as well.

tradable goods, the two effects go in opposite direction. We have already established, however, that the expenditure-reducing effect dominates and consumption of non-tradable goods falls.

Finally, notice that the change in production from point A to point C captures the *production effect*. At the pre-shock labor allocation, the increase in the relative price of tradable goods increases the value of the marginal productivity of labor in the tradable goods sector relative to that in the non-tradable goods sector. As a result, labor switches from the non-tradable to the tradable sector. Hence, all three effects – expenditure-reducing, expenditure switching, and production – reinforce each other in bringing about a trade surplus on impact: the first two by reducing consumption of tradable goods and the third one by increasing production.

3.6.3 Impact effect on GDP

We have shown that on impact – and as a result of the reallocation of labor across sectors – production of tradable goods increases while production of non-tradable goods falls. But what happens to total production? In other words, does a devaluation lead to an overall reduction in GDP? To answer this question, define total production (in terms of tradable goods) as:

$$y_t \equiv y_t^T + \frac{y_t^N}{e_t}. \tag{44}$$

This definition of total production corresponds to GDP (in terms of tradable goods).[18] In terms of Figure 6, this corresponds to output measured along the vertical axis. Thus, y_{ss} indicates the pre-shock level of output in terms of tradable goods. To find out how GDP responds to a (small) devaluation, differentiate (44) with respect to E, using (25) and (26), to obtain:

$$\left.\frac{dy_t}{dE_t}\right|_{t=0} = \left[\frac{Z^T\alpha}{\left(n_t^T\right)^{1-\alpha}} - \frac{Z^N}{e_t}\right]\frac{dn_t^T}{dE_t} - \frac{Z^N n_t^N}{e_t}\frac{de_t}{dE_t}$$

But notice that, in light of the production efficiency condition (30), the term in square brackets on the RHS is zero. Hence, the above equation reduces to

$$\left.\frac{dy_t}{dE_t}\right|_{t=0} = -\frac{Z^N n_t^N}{e_t}\frac{de_t}{dE_t} < 0,$$

[18] In this particular model, it also corresponds to GNP because there is no debt service.

since we know that, on impact, $de_t/dE_t > 0$ (Figure 7, Panel F). Intuitively, since production efficiency always holds at an optimum, the net effect of the labor reallocation on the value of production is zero. Hence, the only effect on total production comes from a valuation effect: the initial production of non-tradable goods (in terms of tradable goods) falls as a result of the increase in the relative price of tradable goods. We thus conclude that a devaluation will reduce GDP in terms of tradable goods.[19,20]

What would happen for a "larger" devaluation? To answer this question, it proves useful to write the change in output on impact as:

$$y_0 - y_{ss} = \underbrace{y^T_{ss} + \frac{y^N_{ss}}{e_0} - \left(y^T_{ss} + \frac{y^N_{ss}}{e_{ss}}\right)}_{\text{valuation effect (-)}} + \underbrace{\left(y^T_0 + \frac{y^N_0}{e_0}\right) - \left(y^T_{ss} + \frac{y^N_{ss}}{e_0}\right)}_{\text{reallocation effect (+)}}.$$

As indicated below the equation, we can decompose the change in output into two effects: a valuation effect and a reallocation effect. Figure 8 illustrates these two effects as well as the total effect. The valuation effect is the one that we just isolated when looking at a small devaluation and, graphically, is represented by the distance along the vertical axis between points y_{ss} and and $(y_{ss})_{e_0}$. This effect reduces GDP because the initial production point (point A) evaluated at the new relative price ($(y_{ss})_{e_0}$) is lower than evaluated at the initial relative price (y_{ss}). The second effect, the reallocation effect, is captured graphically by the distance between y_0 and $(y_{ss})_{e_0}$. This effect captures the labor reallocation from the non-tradable goods sector to the tradable goods sector. This effect increases GDP because, at the new relative price, the marginal productivity in the tradable goods sector is larger than in the non-tradable goods sector. Hence, the output value of a unit of labor

[19] Notice that the theoretical concept of GDP in terms of tradable goods corresponds to what in practice is often referred to as "GDP in dollars". It is a well-known fact that, in developing countries, the GDP in dollars plummets in response to a devaluation. Just to give one example, in Uruguay, after an exchange rate band was abandoned in mid-2001 (as a result of the Argentinean crisis), in 2002 the nominal exchange rate increased by roughly 94 percent, the CPI by 26 percent (which yields a real devaluation of 54 percent) and GDP in dollars fell by 34 percent.

[20] On the other hand – and as can be readily verified – GDP in terms of non-tradable goods actually increases due to the fact that, in terms of non-tradable goods, production of tradable goods is higher. The more relevant measure of GDP, however, is in terms of tradables since this determines the country's flow resources relative to the rest of the world.

– except for the marginal one – will be higher in the tradable than in the non-tradable goods sector. Since the reallocation effect is of second-order, the valuation effect will dominate. This is clear from Figure 8, where we see that $y_0 < y_{ss}$.

This initial fall in GDP is often mentioned as a "cost" of a devaluation. In fact, it is often argued that the price to be paid for the improvement in the trade balance is that the economy becomes "poorer". This model nicely illustrates this idea.

[Figure 8]

3.6.4 How do prices respond to a devaluation?

This section's model can also shed light on a highly relevant question: how do prices respond to a devaluation? In the context of our model, we take the question to be: what will be the effect of a rise in E on the price of non-tradable goods, P^N? (Of course, the response of the price index, P, will simply be a geometric average of the rise in E and P^N.)

When faced with an episode in which a devaluation has been followed by a less than proportional increase in P^N (and hence in P), economists often ask: why? This is a good, but potentially misleading, question. It is potentially misleading because it seems to presume that, even in the short-run, a devaluation of x percent should be followed by an x percent rise in P^N. In fact, this presumption is often so entrenched in some economists' minds that when the data do not show an equi-proportional increase in P^N, they infer that it must be due to sticky prices! This inference is, however, incorrect. Of course, the less-than-proportional increase in prices *may* be due to sticky prices (as we will see in Chapter 8), but that is not *necessarily* the case. This section's model will enable us to illustrate this point and thus dispel the notion that a less than proportional increase in P^N is somehow evidence of sticky prices.

Let us turn to the derivation of the path of P^N, illustrated in Figure 7, Panel A. In our model, the path of P^N will be determined by the need to accommodate the path of the real exchange rate illustrated in Figure 7, Panel F. Recall that, by definition, $e = E/P^N$. Since the real exchange rate does not change across steady-states, it follows that in the long run P^N will rise by the same proportion as E. Given that that e increases on impact, we infer that P^N rises by less than the nominal exchange rate. Over time P^N

rises. We thus conclude that, in spite of no nominal rigidities of any kind, the price of non-tradable goods rises on impact by less than the nominal exchange rate.

What determines in the model the magnitude of the impact response of P^N? The key parameter is α, which is a measure of the substitution in production between tradables and non-tradables.[21] Table 1 indicates the impact response of the real exchange (e), the price of non-tradable goods (P^N), and the price index (P) to a devaluation of 10 percent for different values of α.[22] For instance, for $\alpha = 0.4$, a devaluation of 10 percent leads to an increase in the real exchange rate of 3.8 percent, a rise in P^N of 6.0 percent, and a rise in P of 8 percent. As the table makes clear, as α falls, the increase in the real exchange rate is larger and hence the rise in P^N is smaller. In particular, for $\alpha = 1$ the nominal price of non-tradables increases by the same proportion as the nominal exchange rate, whereas in the endowment case (no substitution in production), the rise in P^N is only 2.7 percent.

[Table 1]

Intuitively, let us go back to Figure 6 and notice that at point D (which indicates what consumption would be at the pre-shock) there is an incipient excess supply of non-tradable goods. The adjustment to this incipient excess supply will take the form of some combination of lower production of non-tradables (relative to point A) and higher consumption of non-tradables (relative to point D). The parameter α controls how much of this adjustment will be reflected in lower production and how much will be reflected in higher consumption. To fix ideas, consider the two extreme cases: the linear case ($\alpha = 1$) and the endowment case (which corresponds, conceptually, to the $\alpha = 0$ case). The linear case is illustrated in Figure 9, Panel A. As you may recall from Chapter 4, in this case the real exchange rate is fully determined by the technology (i.e., $e_t = Z^N/Z^T$). Since the relative price cannot adjust to clear the non-tradable goods market, the entire adjustment must come through a shift in production. In terms of Figure 9, Panel A, the incipient

[21] If we had CES preferences, substitution in consumption would also matter. The more substitutable are tradables and non-tradables, the smaller the required relative price adjustment and hence the larger the initial increase in P^N. If both goods were perfect substitutes, then the relative price would not change and P^N would rise by the same proportion as E.

[22] The computations underlying Table 1 are presented in Appendix 7.4. The Mathematica program used for the computations is available upon request.

excess supply of non-tradable goods at point B (given by $y_{ss}^N - y_0^N$) will be taken care of by a shift in production from point A to point C. Hence, as Table 2 makes clear, P^N increases by the same proportion as the nominal exchange rate (10 percent).

[Figure 9]

Consider now the other extreme case in which the supply of both goods is fixed (referred to as the endowment case in Table 2), illustrated in Figure 9, Panel B. The fixed supply of goods is denoted by $\overline{y^T}$ and $\overline{y^N}$, respectively. The initial steady-state is at point A. After the devaluation –and if the relative price did not change – consumption would take place at a point like C, with an incipient excess supply of non-tradable goods. Since the supply of non-tradables is fixed and cannot adjust, all the burden of the adjustment must be borne by the relative price of non-tradable goods, which must fall until households are willing to consume the available supply of non-tradable goods (point B). This case thus captures the largest adjustment in the relative price and, hence, the smallest increase in P^N. As Table 1 indicates, the real exchange rate increases by 7.1 percent, which implies that P^N goes up by only 2.7 percent.

All other cases (i.e., for $1 < \alpha < 0$) fall in between these two extreme cases just considered. In these intermediates cases – and as illustrated in Figure 6 – the adjustment takes the form of both a fall in the relative price of non-tradable goods and a switch in resources from the non-tradable to the tradable goods sector. The lower is α, the lower is the substitution between the two sectors and thus the larger the increase in the real exchange rate.

We thus conclude that, in spite of no nominal rigidities, the price of non-tradable goods will rise by less than the nominal exchange rate. In fact, since in practice we expect substitutability between the two sectors of productions to be rather low in the short-run (which would correspond to a low value of α in Table 2), this model would predict a small impact response of prices of non-tradable goods. This prediction is in line with the stylized facts associated with large recent devaluations (see Box 3).

4 Flexible exchange rates

Consider once again the monetary model with no bonds developed in Section 2 but suppose now that the economy is operating under flexible exchange rates. Interestingly enough, we will show that an unanticipated and permanent increase in either the level or the rate of growth of the money supply has no real effects. In other words, money is neutral and superneutral.

Let us first solve the model for a constant value of the money growth rate, $\bar{\mu}$. Under flexible exchange rates, the change in international reserves is, by definition, zero. Hence, equation (14) implies that $c_t = \bar{y}$ for all t. In this light, equation (9) reduces to

$$v'(m_t) = (\beta + \varepsilon_t)u'(\bar{y}). \tag{45}$$

This is just an equilibrium condition because both m_t and ε_t are endogenous variables under flexible exchange rates. From the definition of real money balances,

$$\frac{\dot{m}_t}{m_t} = \bar{\mu} - \varepsilon_t. \tag{46}$$

Solving for ε_t from (45) and substituting into (46), we obtain the following differential equation in m_t:

$$\dot{m}_t = m_t \left[\bar{\mu} + \beta - \frac{v'(m_t)}{u'(\bar{y})} \right]. \tag{47}$$

It is easy to check that this is an unstable differential equation. Hence, a convergent equilibrium path requires that m_t be constant over time and implicitly defined by:

$$\frac{v'(m_t)}{u'(\bar{y})} = \bar{\mu} + \beta, \tag{48}$$

which is of course a money demand-type equation.

Suppose now that there is an unanticipated and permanent increase in the level of the money supply. Clearly, the stationary value of m_t implicitly defined by condition (48) is not affected. We thus infer that the nominal exchange rate changes in the same proportion as M so as to leave real money balances unchanged. In other words, monetary policy is neutral.

Suppose now that there is an unanticipated and permanent increase in the rate of growth of the money supply. From (48), we infer that real money balances must fall. Intuitively, since the opportunity cost of holding money has increased, money demand falls. But since m_t is governed by the unstable differential equation (47), m needs to jump immediately to its new and lower value. This fall will be effected through a rise in the nominal exchange rate. There are thus no real effects: monetary policy is also superneutral.

Intuitively, the reason why money is neutral/superneutral is that the absence of capital mobility does not affect the economy's adjustment mechanism to changes in the money supply which, as we saw in Chapter 5, are changes in the exchange rate (price level). As a result, the absence of interest-bearing bonds is inconsequential for the case of flexible exchange rates.

5 A currency substitution model

Until August 15, 1971, when Richard Nixon suspended convertibility of dollars into gold, most of the world operated under fixed exchange rates. Furthermore, capital mobility was limited by modern standards. It is thus not surprising that most theoretical analyses in open economy macroeconomics were cast in a framework similar to the model studied above in Section 2.[23] When the world began to switch to more flexible exchange rate arrangements, this paradigm became less relevant to explain the real world. Moreover – and as shown in Section 4 – the fact that under flexible exchange rates, money is neutral and superneutral prevented researchers from using it to explain real-world phenomena. For instance, suppose one wanted to ask the question: what would be the real effects of an increase in the in the rate of money growth? None would be the answer of that model. Given this state of affairs, Calvo and Rodriguez (1977) came up with the idea of introducing currency substitution as a useful and certainly relevant friction in the model. Interestingly enough, even though the model is one of flexible exchange rates, the adjustment mechanisms involved are very much reminiscent of the monetary approach.

5.1 The model

Preferences now take the form:

$$\int_0^\infty [\log(c_t) + \log(x_t)]e^{-\beta t}dt, \tag{49}$$

where c_t denotes consumption of the only (tradable) good and x is a liquidity variable such that

[23] Needless to say, by providing explicit microfoundations and assuming rational expectations (i.e., perfect foresight in this case), we have presented a modern version of this old paradigm.

$$
\begin{aligned}
qx_t &= f_t, && (50) \\
(1-q)x_t &= m_t, && (51)
\end{aligned}
$$

where $0 \leq q < 1$ and m and f denote real domestic and foreign currency, respectively. (All real variables are defined in terms of tradable goods.) One can therefore think of x as a composite of domestic and foreign currency. Notice that if $q = 0$ the model reduces to the one in Section 2.

Let a denote real financial wealth, defined as

$$
a_t \equiv m_t + f_t. \tag{52}
$$

The consumer's flow constraint is now given by

$$
\dot{a}_t = \bar{y} + \tau_t - c_t - \varepsilon_t m_t, \tag{53}
$$

where y^T denotes the constant endowments of tradable goods.

Before proceeding to the consumer's maximization, it will prove convenient to express the flow constraint in terms of x. To this end, use (50), (51), and (52) to rewrite the flow constraint (53) as

$$
\dot{x}_t = \bar{y} + \tau_t - c_t - (1-q)\varepsilon_t x_t. \tag{54}
$$

Intuitively, notice that the opportunity cost of holding the composite currency, x, is $(1-q)\varepsilon_t$ as we should have expected.

We can now write the current value Hamiltonian as

$$
H \equiv \log(c_t) + \log(x_t) + \lambda_t \left[\bar{y} + \tau_t - c_t - (1-q)\varepsilon_t x_t\right].
$$

The optimality conditions are given by

$$
\begin{aligned}
\frac{1}{c_t} &= \lambda_t, && (55) \\
\dot{\lambda}_t &= \beta\lambda_t - \frac{\partial H}{\partial x_t} = \lambda_t\left[\beta + (1-q)\varepsilon_t\right] - \frac{1}{x_t}. && (56)
\end{aligned}
$$

5.1.1 Government

We will assume, as usual, that the government holds no international reserves. The government's flow constraint thus becomes

$$\tau_t = \frac{\dot{M}_t}{E_t}. \tag{57}$$

Since the economy is operating under flexible exchange rates, the monetary authority will set the path of the nominal money supply. Let $\bar{\mu}$ denote the constant rate of growth set by the monetary authority.

5.1.2 Equilibrium conditions

Substituting the government's flow constraint (57) into the consumer's flow constraint (53), we obtain the economy's flow constraint:

$$\dot{f}_t = \bar{y} - c_t. \tag{58}$$

This equation says that in order for the private sector to accumulate foreign currency, it must run a trade surplus.

5.1.3 Dynamic system

To solve the model, we will set up a dynamic system in c and x. To this end, differentiate first-order condition (55) and use (56) and (55) to obtain:

$$\dot{c}_t = c_t \left[\frac{c_t}{x_t} - \beta - (1-q)\varepsilon_t \right]. \tag{59}$$

Since $(1-q)x_t = m_t$ and $\dot{m}_t/m_t = \bar{\mu} - \varepsilon_t$, it follows that

$$\varepsilon_t = \bar{\mu} - \frac{\dot{x}_t}{x_t}.$$

Using equation (58) and the fact that $qx_t = f_t$ we can rewrite this last equation as

$$\varepsilon_t = \bar{\mu} - \frac{\bar{y} - c_t}{qx_t}.$$

Substituting this equation into (59), we obtain:

$$\dot{c}_t = c_t \left[\frac{(2q-1)\, c_t}{q x_t} + \frac{(1-q)y}{q x_t} - \beta - (1-q)\bar{\mu} \right]. \tag{60}$$

Using (50), we can rewrite (58) as

$$\dot{x}_t = \frac{1}{q}(\bar{y} - c_t). \tag{61}$$

Equations (60) and (61) constitute a dynamic system in c_t and x_t, for a given value of $\bar{\mu}$. The steady state is given by

$$\begin{aligned} c_{ss} &= \bar{y}, & (62) \\ x_{ss} &= \frac{\bar{y}}{\beta + (1-q)\bar{\mu}}. & (63) \end{aligned}$$

The linear approximation of the dynamic system around the steady state is

$$\begin{bmatrix} \dot{c}_t \\ \dot{x}_t \end{bmatrix} = \begin{bmatrix} \left(\frac{2q-1}{q}\right)[\beta + (1-q)\mu] & -[\beta + (1-q)\mu]^2 \\ -\frac{1}{q} & 0 \end{bmatrix} \begin{bmatrix} c_t - \bar{y} \\ x_t - x_{ss} \end{bmatrix}.$$

The determinant of the matrix associated with the linear approximation is thus with the linear approximation is

$$\Delta = -\frac{[\beta + (1-q)\mu]^2}{q} < 0,$$

that the system is saddle-path stable.

Graphically, we can draw the phase diagram depicted in Figure 10. To draw the $\dot{c} = 0$ and $\dot{x} = 0$ loci, set equations (60) and (61) to zero to obtain, respectively:

$$\begin{aligned} x_t &= \frac{\bar{y}}{\beta + (1-q)\bar{\mu}}, \\ c &= \bar{y}. \end{aligned}$$

It follows that the $\dot{c} = 0$ locus is a vertical line while the $\dot{x} = 0$ locus is a horizontal line. These two loci define four regions. Proceeding as before, we can establish how the system will move in each of these regions by drawing the arrowheads as indicated. We thus conclude that, as shown in Figure 10, the saddle-path is positively sloped.

[Figure 10]

5.1.4 Permanent increase in rate of money growth

Suppose that the economy is initially at the steady-state given by point A in Figure 10. At $t = 0$, there is an unanticipated and permanent increase in the rate of money growth from $\bar{\mu}$ to $\bar{\mu}^H$ (Figure 11, Panel A). How will the economy react?

[Figure 11]

In the new steady-state – given by point B in Figure 10 – c is unchanged but x has fallen, as follows from (62) and (63). The system will therefore jump on impact from point A to point C and then proceed along the saddle path towards point B. The corresponding paths of c and x are depicted in Figure 11, Panel B and C, respectively. Naturally, since m and f are a fixed proportion of x, they will also follow a path qualitatively identical to that of x (as illustrated in Panel D for foreign currency). Since consumption rises on impact, the economy runs a trade deficit throughout the transition (Figure 11, Panel F).

What will happen to the rate of depreciation? Notice that we can write

$$\varepsilon_t = \bar{\mu}^H - \frac{\dot{x}_t}{x_t} > \bar{\mu}^H,$$

since $\dot{x}_t < 0$. Furthermore, the rate of depreciation will be higher in the new steady-state. Differentiating this last equation with respect to time (and noticing that $\ddot{x}_t > 0$), we can check that $\dot{\varepsilon}_t < 0$. The rate of depreciation thus follows the path illustrated in Figure 11, Panel E. We also know that the nominal exchange rate rate will not jump on impact since x and hence m do not jump on impact.

What is the intuition behind our results? The increase in the rate of money growth implies a higher opportunity cost of the composite currency (x). As a result, the public wishes to reduce its holdings of this composite money in the long run. Since this composite money consists of domestic and foreign currency in fixed proportions, foreign currency holdings must also fall in the long run. To achieve this goal, the economy must run a trade deficit which requires an increase in consumption.

Interestingly enough – and even though the economy is operating under flexible exchange rates – the adjustment mechanism is analogous to that emphasized by the monetary approach to the balance of payments. While, under flexible exchange rates, the nominal exchange rate can adjust to bring about the desired real holdings of *domestic* currency, it is obviously powerless when it comes to changing holdings of *foreign* currency. Hence, changes in foreign currency holdings must take place through trade imbalances, as emphasized by the monetary approach.

6 Concluding remarks

This chapter departed from the frictionless monetary world of Chapter 5 by assuming away the existence of interest-bearing bonds. In such a world, which can be taken as a proxy for situations of low capital mobility, exchange rate policy ceases to be neutral or superneutral. Indeed, a devaluation leads to a trade surplus and an increase in the Central Bank's international reserves, whereas an increase in the rate of devaluation results in a trade deficit and a loss of international reserves. These experiments illustrate the so-called monetary approach to the balance of payments, which emphasizes the idea that changes in international reserves are caused by "disequilibria" in the money markets (in the sense of the current stock of real money balances differing from its long-run equilibrium). In sharp contrast, monetary policy under flexible exchange rates continues to be neutral and superneutral because the absence of interest-bearing bonds does not affect the adjustment channel (changes in the nominal exchange rate) operating in the frictionless world of Chapter 5.

The next two chapters will introduce alternative frictions in the world of Chapter 5 that will also imply that monetary/exchange rate policy and the real economy.

7 Appendices

7.1 Dynamic optimization in continuous time

For those of you who are not familiar with optimal control techniques, this appendix provides a "cookbook" for deriving optimality conditions in continuous-time dynamic problems and then illustrates this method with the familiar one-sector growth model.[24] This is all that you will need for the purposes of this book. For formal proofs of these techniques, see Kamien and Schwartz (1981) or Chiang (1992). The second part of either book deals with optimal control techniques and contains formal proofs and examples.

7.1.1 The general problem

Consider the following standard problem in many areas of economics. Maximize

$$\int_0^\infty f(x_t, u_t)e^{-\beta t}dt, \tag{64}$$

subject to

$$\dot{x}_t = g(x_t, u_t), \tag{65}$$

$$x_0 \text{ given.} \tag{66}$$

In optimal control problems, variables are divided into *state* variables and *control* variables. In the problem above, x is the state variable and u is the control variable. (The extension to several control and state variables is straightforward.) The movement of state variables is governed by first-order differential equations such as (65). Note that the state variable may or may not enter the objective function $f(.)$: in the one-sector growth model below, the state variable does not enter the objective function, in the model in the text it does.

Faced with this maximization problem, the first step is to set up the *current value Hamiltonian*, denoted by $H(.)$:

[24] For extensive use of optimal control techniques in growth models, see Barro and Sala-i-Martin (1995).

$$H(x_t, u_t, \lambda_t) \equiv f(x_t, u_t) + \lambda_t g(x_t, u_t), \tag{67}$$

where λ_t is an auxiliary variable (analogous to a Lagrange multiplier) which is referred to as the *co-state* variable. This variable λ_t can be interpreted as the marginal valuation at time t of the associated state variable.

It can be shown (see Kamien and Schwartz (1981) or Chiang (1992)) that, at an optimum, the following necessary conditions must be satisfied:

$$\frac{\partial H}{\partial u_t} = f_u(x_t, u_t) + \lambda_t g_u(x_t, u_t) = 0 \tag{68}$$

$$\dot{\lambda}_t = \beta\lambda_t - \frac{\partial H}{\partial x_t} = \beta\lambda_t - f_x(x, u) - \lambda_t g_x(x_t, u_t) \tag{69}$$

Condition (69) gives the law of motion for the co-state variable. If $f(x_t, u_t)$ and $g(x_t, u_t)$ are concave in both arguments, the necessary conditions are also sufficient for an optimum.

In the typical optimal control problem, one would solve for u as a function of λ and x from (68) (i.e., $u = \tilde{u}(x, \lambda)$) and substitute this into (65) and (69) to obtain:

$$\begin{aligned} \dot{x}_t &= g[x_t, \tilde{u}(x_t, \lambda_t)], \\ \dot{\lambda}_t &= \beta\lambda_t - f_x[x_t, \tilde{u}(x_t, \lambda_t)] - \lambda_t g_x[x_t, \tilde{u}(x_t, \lambda_t)]. \end{aligned}$$

This is a differential equation system in x and λ, which can be solved using standard phase-diagram techniques (an excellent discussion is contained in Kamien and Schwartz (1991) in the chapter "Equilibria in infinite horizon autonomous problems.")

7.1.2 An example: the one-sector growth model

Consider the following one-sector growth model. Maximize

$$\int_0^\infty \log(c_t) e^{-\beta t} dt, \tag{70}$$

subject to

$$\dot{k}_t = f(k_t) - c_t - \delta k_t, \tag{71}$$
$$k_0 \text{ given}, \tag{72}$$

where c is consumption, k is the capital stock, δ is the rate of depreciation and $f(k)$ is a strictly increasing and strictly concave production function. In this case, c is the control variable and k is the state variable.

The current value Hamiltonian is given by

$$H(c_t, k_t) \equiv \log(c_t) + \lambda_t[f(k_t) - c_t - \delta k_t]. \tag{73}$$

The variable λ_t can be interpreted as the price or value of an extra unit of capital at time t in terms of utility at time t. The Hamiltonian can be interpreted as follows. At each instant in time, the agent consumes c_t and owns a capital stock k_t. These two variables affect utility through two channels. First, the direct contribution of consumption to utility is captured by the first term, $\log(c_t)$. Second, the choice of consumption affects the change in the capital stock through (71). The *value* of this change (in utility terms) is given by the second term on the RHS of (73). Hence, for a given value of the shadow price λ_t, the Hamiltonian captures the total contribution to utility of the choice of c_t.

Using (68) and (69), the optimality conditions are:

$$\frac{\partial H}{\partial c_t} = \frac{1}{c_t} - \lambda_t = 0 \tag{74}$$
$$\dot{\lambda}_t = \beta\lambda_t - \frac{\partial H}{\partial k_t} = \beta\lambda_t - \lambda_t[f'(k_t) - \delta] = \lambda_t[\beta + \delta - f'(k_t)]. \tag{75}$$

Notice that one can rewrite (75) as

$$\frac{1}{\lambda_t}\frac{\partial H}{\partial k_t} + \frac{\dot{\lambda}_t}{\lambda_t} = \beta,$$

and interpret it as an asset pricing equation. The term $\frac{1}{\lambda_t}\frac{\partial H}{\partial k_t}$ is the "dividend" rate received by the agent (i.e., the marginal contribution of capital to utility divided by the price of the asset), $\frac{\dot{\lambda}_t}{\lambda_t}$ is the rate of capital gain (i.e., the rate of change in the price of the asset) and β is the rate of return of an alternative asset (consumption). Hence, at an optimum, the agent is indifferent between the two types of return because the overall return to investment, $\frac{1}{\lambda_t}\frac{\partial H}{\partial k_t} + \frac{\dot{\lambda}_t}{\lambda_t}$, equals the return to consumption, β.

Solving for c_t from (74) and substituting it into (71) and (75) yields a dynamic system in k and λ:

$$\dot{k}_t = f(k_t) - \frac{1}{\lambda_t} - \delta k_t, \tag{76}$$

$$\dot{\lambda}_t = \lambda_t[\beta + \delta - f'(k_t)]. \tag{77}$$

The steady state is given by:

$$f'(k_{ss}) = \beta + \delta,$$
$$\frac{1}{\lambda_{ss}} = f(k_{ss}) - \delta k_{ss}.$$

By linearizing the system around the steady-state, it is easy to check that this dynamic system is saddle-path stable (you may want to check this as an exercise). (Chiang (1992), Chapter 9, contains a detailed discussion of the one-sector growth model.) Hence, for a given value of k_0, the value of λ_0 will be endogenously determined so as to place the dynamic system on the saddle path. (Alternatively, you could differentiate (74) with respect to time and use (75) to set up a dynamic system in k and c; this is what Chiang (1992) does.)

Finally, note that if the production function were linear (i.e., $f(k) = rk$, $r > 0$) and there were no depreciation (i.e., $\delta = 0$), then it is clear from (77) that we would need to assume that $\beta = r$ for a stationary equilibrium to exist. This is exactly the reason why we assume that $\beta = r$ in small open economy models. (Intuitively, for a small open economy to have access to perfect international capital markets is like having access to a linear technology with marginal productivity of capital equal to r.) In that case, $\dot{\lambda}_t = 0$ for all t and therefore λ would be constant along a perfect foresight equilibrium path.

7.2 Analytical solution for the saddle path

In the text, we resorted to the phase diagram to graphically derive the saddle path . It will prove useful to solve for the saddle path analytically.[25] In the

[25] A classic reference on differential equations systems is Hirsh and Smale (1974).

process, we will also review how to solve a two-equation differential equation system.

For convenience, we restate the linear version of the model, given by (19), as

$$\begin{bmatrix} \dot{c}_t \\ \dot{m}_t \end{bmatrix} = A \begin{bmatrix} c_t - \bar{y} \\ m_t - m_{ss} \end{bmatrix}, \tag{78}$$

where

$$A \equiv \begin{bmatrix} \beta + \varepsilon & \frac{-v''(m_{ss})}{u''(\bar{y})} \\ -1 & 0 \end{bmatrix}$$

is the matrix associated with the linear approximation of the system.

To find the eigenvalues of the system, we need to solve for the characteristic polinomial of matrix A (recall that the eigenvalues are the roots of the characteristic polinomial of matrix A). Formally, we need to solve for

$$|A - \delta I| = 0,$$

where I is the identity matrix. In other words, we need to subtract δ from the diagonal elements of matrix A, set the determinant of this matrix equal to zero, and solve for δ. Proceeding in this way yields:

$$\begin{vmatrix} \beta + \varepsilon - \delta & \frac{-v''(m_{ss})}{u''(\bar{y})} \\ -1 & -\delta \end{vmatrix} = \delta^2 - (\beta + \varepsilon)\delta - \frac{v''(m_{ss})}{u''(\bar{y})} = 0.$$

The characteristic polynomial is a quadratic equation in δ that can be solved to yield:

$$\delta_{1,2} = \frac{\beta + \varepsilon \pm \sqrt{(\beta + \varepsilon)^2 + 4\frac{v''(m_{ss})}{u''(\bar{y})}}}{2}. \tag{79}$$

Since $(\beta + \varepsilon)^2 + 4\frac{v''(m_{ss})}{u''(\bar{y})} > 0$, the two roots will be real and distinct. Hence the characteristic roots are given by:

$$\delta_1 = \frac{\beta + \varepsilon - \sqrt{(\beta + \varepsilon)^2 + 4\frac{v''(m_{ss})}{u''(\bar{y})}}}{2} < 0, \tag{80}$$

$$\delta_2 = \frac{\beta + \varepsilon + \sqrt{(\beta + \varepsilon)^2 + 4\frac{v''(m_{ss})}{u''(\bar{y})}}}{2} > 0. \tag{81}$$

Using (80) and (81), we can verify that the product of the characteristic roots is given by the determinant of the matrix A:

$$\delta_1\delta_2 = -\frac{v''(m_{ss})}{u''(\bar{y})} = |A|.$$

For future reference also note that the trace of matrix A (the trace is the sum of the diagonal elements of a square matrix) corresponds to the sum of the characteristic roots:

$$\delta_1 + \delta_2 = \beta + \varepsilon = \mathrm{Tr}(A).$$

It proves useful to notice that we can write (79) as a

$$\delta_{1,2} = \frac{\mathrm{Tr}(A) \pm \sqrt{[\mathrm{Tr}(A)]^2 - 4|A|}}{2}.$$

The roots are therefore real and distinct if $[\mathrm{Tr}(A)]^2 - 4|A| > 0$, they are non-real complex conjugates if $[\mathrm{Tr}(A)]^2 - 4|A| < 0$, and there is only root – necessarily real – if $[\mathrm{Tr}(A)]^2 = 4|A|$.

Since A is a 2x2 matrix with two distinct real eigenvalues, then every solution to the differential system (78) is of the form

$$\begin{aligned} c_t - \bar{y} &= \omega_1 h_{11} e^{\delta_1 t} + \omega_2 h_{21} e^{\delta_2 t}, \\ m_t - m_{ss} &= \omega_1 h_{12} e^{\delta_1 t} + \omega_2 h_{22} e^{\delta_2 t}, \end{aligned}$$

where ω_1 and ω_2 are arbitrary constants and h_{i1} and h_{i2} are the eigenvectors corresponding to the eigenvalues δ_i, $i = 1, 2$. Recall that there is a stable root ($\delta_1 < 0$) and an unstable root ($\delta_2 > 0$). Clearly, if $\omega_2 \neq 0$, as $t \to \infty$, the system will diverge (i.e., the limit of $c_t - \bar{y}$ and $m_t - m_{ss}$ is $\pm\infty$). Therefore, to ensure convergence along a perfect foresight path, we set to zero the constant corresponding to the unstable root (i.e., we set $\omega_2 = 0$). By doing so, the system reduces to

$$\begin{aligned} c_t - \bar{y} &= \omega_1 h_{11} e^{\delta_1 t}, \\ m_t - m_{ss} &= \omega_1 h_{12} e^{\delta_1 t}. \end{aligned} \tag{82}$$

To find the eigenvector corresponding to the eigenvalue δ_1, we must solve the vector equation:

$$(A - \delta_1 I)h_1 = 0,$$

where h_1 is the eigenvector. Then,

$$\begin{bmatrix} \beta + \varepsilon - \delta_1 & \frac{-v''(m_{ss})}{u''(\bar{y})} \\ -1 & -\delta_1 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Since an eigenvector is determined up to a factor of proportionality, we need only solve for the second equation:

$$-h_{11} = \delta_1 h_{12}$$

As a normalization, set $h_{12} = 1$. (It is always convenient to normalize to one the element of the eigenvector corresponding to the predetermined variable.) Then,

$$h_{11} = -\delta_1 > 0. \tag{83}$$

The last step in this derivation is to determine the constant ω. To do, evaluate equation (82) at $t = 0$, to obtain

$$m_0 - m_{ss} = \omega_1. \tag{84}$$

Using (83) and (84), we can write the solution as

$$\begin{aligned} c_t - \bar{y} &= -(m_0 - m_{ss})\delta_1 e^{\delta_1 t}, & (85) \\ m_t - m_{ss} &= (m_0 - m_{ss})e^{\delta_1 t}. & (86) \end{aligned}$$

As a check, we verify that both variables converge asymptotically to their steady-state values:

$$\begin{aligned} \lim_{t\to\infty} c_t &= \bar{y}, \\ \lim_{t\to\infty} m_t &= m_{ss}. \end{aligned}$$

To fix ideas, suppose that $m_0 < m_{ss}$. Then, differentiating (85) and (86) with respect to time, we get

$$\begin{aligned} \dot{c}_t &= -(m_0 - m_{ss})\delta_1^2 e^{\delta_1 t} > 0, \\ \dot{m}_t &= (m_0 - m_{ss})\delta_1 e^{\delta_1 t} > 0, \end{aligned}$$

which tells us that both c and m increase over time towards their steady-state values. Alternatively, we can derive the analytical expression for the saddle path of the system by simply dividing (85) by (86) to obtain:

$$c_t - \bar{y} = -\delta_1(m_t - m_{ss}).$$

Since $\delta_1 < 0$, the saddle path is positively sloped (i.e., the slope is equal to $-\delta_1$) which is, of course, consistent with the phase diagram in Figure 1.

7.3 Derivation of price index

This appendix derives the price index used in the text, given by equation (22). Suppose preferences are given by

$$u(c^T, c^N) = \log(c^T) + \log(c^N),$$

which correspond to the consumption component of the preferences given by (21). Denote by X the nominal expenditure associated with consumption:

$$X = P^T c^T + P^N c^N.$$

Then P, the price index, is defined as the minimum expenditure needed to achieve a given level of utility. In other words, P solves the problem:

$$\begin{aligned} &\underset{\{c^T, c^N\}}{\text{Min}} \; X = P^T c^T + P^N c^N, \\ &\text{subject to} \\ &\log(c^T) + \log(c^N) = \bar{u}. \end{aligned} \tag{87}$$

The price index P will therefore be given by X evaluated at the optimal choices of c^T and c^N.

Setting up the Lagrangian,

$$\mathcal{L} = P^T c^T + P^N c^N + \lambda \left[\bar{u} - \log(c^T) + \log(c^N) \right].$$

The first-order conditions with respect to c^T and c^N are given by, respectively,

$$P^T = \frac{\lambda}{c^T}, \tag{88}$$

$$P^N = \frac{\lambda}{c^N}. \tag{89}$$

Adding up these two expressions, we obtain

$$P = 2\lambda. \tag{90}$$

To find λ, substitute (88) and (89) into (87) to obtain

$$\lambda = \sqrt{e^{\bar{u}}}\sqrt{P^T P^N}. \tag{91}$$

Substituting (91) into (90),

$$P = 2\sqrt{e^{\bar{u}}}\sqrt{P^T P^N}$$

which is expression (22) (ignoring, for notational simplicity, inconsequential constants).

7.4 Computations underlying Table 1

To construct Table 1, we need to solve explicitly for the jump in c^T at $t = 0$. The general solution for the dynamic system is given by

$$\begin{aligned} c_t^T - c_{ss}^T &= \omega_1 h_{11} e^{\delta_1 t}, \\ m_t - m_{ss} &= \omega_1 h_{12} e^{\delta_1 t}. \end{aligned} \tag{92}$$

where δ_1 is the negative root of the system. To compute the negative root, we solve for the root polynomial:

$$\begin{vmatrix} \beta + \varepsilon - \delta_1 & -(\beta+\varepsilon)^2 \\ \alpha Z^T (n^T)^{\alpha-1} \Psi' - 1 & -\delta_1 \end{vmatrix} = \delta_1^2 - (\beta+\varepsilon)\delta_1 + \left[\alpha Z^T \left(n^T\right)^{\alpha-1} \Psi' - 1\right](\beta+\varepsilon)^2 .$$

The negative root is thus given by:

$$\delta_1 = (\beta+\varepsilon)\frac{1-\sqrt{1-4\left[\alpha Z^T\left(n^T\right)^{\alpha-1}\Psi'-1\right]}}{2}.$$

We now solve for the elements of the eigenvector:

$$\begin{pmatrix} \beta+\varepsilon-\delta_1 & -(\beta+\varepsilon)^2 \\ \alpha Z^T\left(n^T\right)^{\alpha-1}\Psi'-1 & -\delta_1 \end{pmatrix}\begin{pmatrix} h_{11} \\ h_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Hence:

$$\frac{h_{11}}{h_{12}} = \frac{\delta_1}{\alpha Z^T\left(n^T\right)^{\alpha-1}\Psi'-1} < 0.$$

In response to a devaluation, at time 0 (setting $h_{12}=1$),

$$m_0 - m_{ss} = \omega_1.$$

Hence:

$$c_0^T - c_{ss}^T = \frac{(\beta+\varepsilon)}{2}(m_0-m_{ss})\frac{1-\sqrt{1-4\left[\alpha Z^T\left(n^T\right)^{\alpha-1}\Psi'-1\right]}}{\alpha Z^T\left(n^T\right)^{\alpha-1}\Psi'-1}$$

Dividing through by c_{ss}^T and taking into account that $m_{ss} = c_{ss}^T/(\beta+\varepsilon)$, we obtain

$$\frac{c_0^T - c_{ss}^T}{c_{ss}^T} = \frac{1}{2}\left(\frac{m_0-m_{ss}}{m_{ss}}\right)\frac{1-\sqrt{1-4\left[\alpha Z^T\left(n^T\right)^{\alpha-1}\Psi'-1\right]}}{\alpha Z^T\left(n^T\right)^{\alpha-1}\Psi'-1},$$

where

$$\alpha Z^T\left(n^T\right)^{\alpha-1}\Psi' = -\frac{1}{1+\frac{(1-\alpha)(\bar{n}-n^T)}{n^T}},$$

all evaluated at the steady-state.

Exercises[26]

1. The monetary approach model with a labor/leisure choice

   Consider the model of Section 2 with the following modifications. Preferences are now given by

$$\int_0^\infty [u(c_t) + z(\ell_t) + v(m_t)]e^{-\beta t}dt,$$

   where ℓ_t is leisure. Production takes the linear form:

$$y_t = \alpha(1 - \ell_t),$$

   where α is positive parameter. The rest of the model is unchanged.

   (a) In this context:

   (b) Solve the model by proceeding as in the text

   (c) Analyze the effects of an unanticipated and permanent devaluation. (In particular, show that the impact effect of the devaluation will be to increase output and reduce consumption.)

2. The monetary approach model with a cash-in-advance

   Consider the monetary model with no bonds developed in Section 2 of this chapter with the following modification. Instead of introducing money into the utility function, suppose that preferences are given by

$$\int_0^\infty u(c_t)e^{-\beta t}dt,$$

   and that consumption is subject to a cash-in-advance constraint of the form:

$$m_t = \alpha c_t.$$

   The rest of the model remains unchanged. In this context:

   (a) Analyze the effects of an unanticipated and permanent devaluation. Explain the intuition behind the results. Do the results differ from those in the text? Why or why not?

[26] An answer key is available from the author upon request.

(b) Analyze the effects of an unanticipated and permanent increase in the stock of domestic credit, D. Explain the intuition behind the results. Do the results differ from those in the text? Why or why not?

(c) Analyze the effects of an unanticipated and permanent increase in the rate of devaluation (ε). Explain the intuition behind the results. Do the results differ from those in the text? Why or why not?

3. ## The two-good model with linear production

Consider the model of Section 3 with a linear production function for tradable goods. In other words, production of tradable goods is given by:

$$y_t^T = Z^T n_t^T.$$

In the context of this model, analyze the economy's response to a permanent devaluation. Explain the intuition behind the results.

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Box 1. The monetary approach Southern-Cone style

Our model predicts that increases in Central Bank domestic credit should lead to losses in international reserves as the public gets rid of unwanted money balances. The final stages of the so-called Southern-Cone "tablitas" provide an ideal testing ground for this key prediction of the monetary approach. The Southern-Cone tablitas consisted of exchange rate-based stabilization plans implemented in Argentina, Chile, and Uruguay in the late 1970s that were based on the preannouncement of an exchange rate schedule (or table), which would specify the daily exchange rate several months in advance (and typically implied a declining rate of devaluation).[27] The tablitas, however, were short-lived and the exchange rate pegs were abandoned in the midst of costly balance of payment and financial crises.

In all three cases, a key factor contributing to the demise of the tablitas was a loose fiscal situation in the months prior to the abandonment of the peg. As Figure 12 shows, Central Bank domestic credit rose sharply in the final stages of the tablitas as Central Banks were forced to finance part of the fiscal deficits by printing money. In line with the predictions of the monetary approach, international reserves declined steadily reflecting the public's unwillingness to hold additional money balances. The steady loss of international reserves left Central Banks with no choice but to abandon the peg.

[Figure 12]

[27] See Chapter 14 for more details. The Argentinean tablita was implemented in December 1978 and abandoned in February 1981, the Chilean tablita was implemented in February 1978 and abandoned in June 1981; the Uruguayan tablita was implemented in October 1978 and abandoned in November 1982.

Box 2. IMF's financial programming

A key objective of the International Monetary Fund (IMF) is to provide financial assistance to member countries experiencing temporary balance of payments problems. Indeed, the IMF's Articles of Agreement state that one of the purposes of the IMF is to "[t]o give confidence to members by making the general resources of the institution available to them under adequate safeguards, thus providing them with opportunity to correct maladjustments in their balance of payments without resorting to measures destructive of national or international prosperity." The presumption is therefore that countries facing temporary financing problems would have no access to international credit (or that the costs of doing so would be prohibitively high) which – in the absence of IMF financing – would force them to costly adjustments (as analyzed in Chapter 2).

The IMF lends subject to some conditionality. In other words, the IMF and the country in question agree on certain corrective measures and performance criteria that must be met in order for the Fund to continue disbursements.[28] Identifying measures to correct balance of payments problems requires an analytical framework linking policy instruments with the balance of payments. The framework developed in the Fund – known as "financial programming" and first published in Polak (1957) – is intimately linked to the monetary accounting discussed in this chapter. IMF financial programming focuses on domestic credit creation on the part of the Central Bank, which is the key policy variable in a predetermined exchange rate regime. Targets for domestic credit creation are therefore central to IMF's conditionality.

The most basic version of the model underlying the IMF's financial programming – which was built having in mind a fixed exchange rate regime though it has been applied much more generally – has three building blocks. The first one is the Central Bank's balance sheet:

$$\Delta M^s = \Delta H + \Delta D, \tag{93}$$

where M is the stock of money, H is the domestic currency value of international reserves at the Central Bank, D is net domestic credit, and Δ denotes

[28] For a detailed analysis of IMF programs, see Mussa and Savastano (2000). (Michael Mussa, a prominent academic economist who was for many years at the University of Chicago, was the IMF's chief economist from 1991 to 2001.) See also Khan, Montiel, and Haque (1990) for an integration of the IMF and World Bank basic models and Easterly (2002) for a critique of IMF financial programming.

discrete changes.[29]

The second building block is a flow equilibrium condition in the money market:

$$\Delta M^d = \Delta M^s. \tag{94}$$

Combining (93) with (94), we obtain:

$$\Delta R = \Delta M^d - \Delta D. \tag{95}$$

This equation says that the change in net foreign assets will be positive to the extent that the change in money demand exceeds the change in domestic credit. Since the demand for money is not affected by the level of domestic credit, any increases in domestic credit above the desired increase in money will be offset by decreases in international reserves on a one-for-one basis.

The final building block is the demand for money, which can be specified in a variety of ways. For the sake of simplicity, we assume that the change in nominal money, ΔM^d, is a constant fraction of the change in nominal income, ΔY

$$\Delta M^d = k\Delta Y, \tag{96}$$

where k is the inverse of the income velocity of money.[30]

At its most basic core, the design of a financial program by the IMF in the context of this simple model requires three steps. First, it is necessary to set a target for the change in international reserves over some specified period, generally a year. Second, an estimate is made of the most likely path of the demand for money over the same period. Typically, velocity is assumed to remain constant over the relevant time period in which case – as equation (96) makes clear – we only need a projection of nominal income to estimate the change in money demand. Finally, given the target for international reserves and the estimate in the change of money demand, the change in domestic

[29]Notice that – reflecting common practices in central banks – capital gains/losses on the stock of international reserves are not incluced as they are not typically "monetized".

[30]Needless to say, for operational purposes this simple model needs to be cast in a more general framework (as the IMF does). Typically, this entails decomposing the balance of payments into its individual components (and explaining these items separately) and linking the growth of domestic credit to the fiscal accounts. See Easterly (2002).

credit needed to achieve the international reserves target can be computed as a residual using equation (95).[31]

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[31] In countries with underdeveloped financial systems and/or in which the monetary authority resorts to direct controls to influence total credit, targets for domestic credit could be set for the banking system as a whole (see Mussa and Savastano (2000) for a discussion).

Box 3. How do prices respond to a large devaluation?

In Subsection 3.6.4, we analyzed the impact of a devaluation on the real exchange rate, prices of non-tradable goods, and the price level. We showed how, depending on the supply-elasticity of tradable goods, prices of non-tradable goods may respond little to the devaluation in the short-run. In fact, for the extreme case of total inelastic supply, the price of non-tradable goods increases by only 2.7 percent in response to a 10 percent devaluation.

What do the data say? In a recent paper, Burstein, Eichenbaum, and Rebelo (2005) document the response of prices after five large devaluations in developing countries: Argentina (December 2001), Brazil (December 1998), Korea (September 1997), Mexico (December 1994), and Thailand (June 1997). Table 2 reports the increases in the nominal exchange rate, the real exchange rate, price of tradable goods, price of non-tradable goods, and a price index twelve months after the devaluation. The main findings are the following:

1. The increase in the price of tradable goods (proxied by import prices at the dock) generally matches the increase in the nominal exchange rate. This provides support for the assumption in our model – which is the standard assumption in small open economy models – that, to a first approximation, the law of one price holds for tradable goods.[32]

2. The price of non-tradable goods rises by considerably less than the nominal exchange rate. The smallest increase (relative to the magnitude of the devaluation) was in Argentina where prices of non-tradables rose by only 10.5 percent of the nominal devaluation; while the largest was in Mexico (40 percent).

3. A nominal devaluation leads to an increase in the real exchange rate (a real depreciation of the currency). The percentage of the nominal devaluation that gets reflected in the real exchange rate lies within a 50 to 80 percent range.

[32] As Burstein, Eichenbaum, and Rebelo (2005) persuasively argue, it would be misleading to look at retail prices of tradable goods (which rise by much less than the nominal exchange rate) and conclude that the law of one price does not hold. The problem is that retail prices of tradable goods include large distribution costs, which are non-tradable in nature (see Burstein, Neves, and Rebelo (2003)).

[Table 2]

In sum, the large real depreciation and the small response of non-tradable prices are consistent with our model and, as expected, would correspond to a case of low supply elasticity (i.e., a low α).[33]

[33] Needless to say, these findings might be consistent with other models as well, several of which we will examine as we progress in the book.

Figure 1. Phase diagram

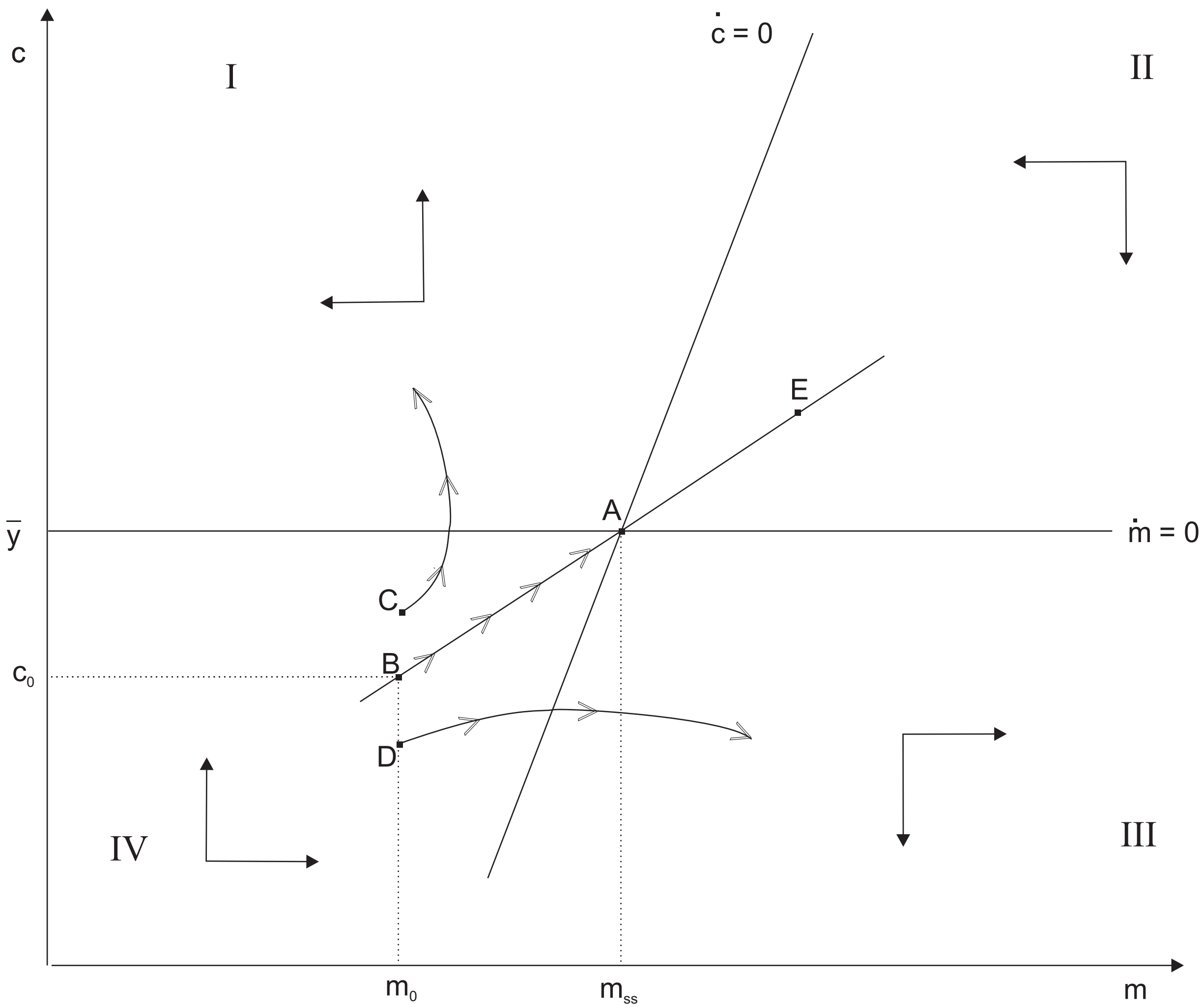


Figure 2. Permanent devaluation

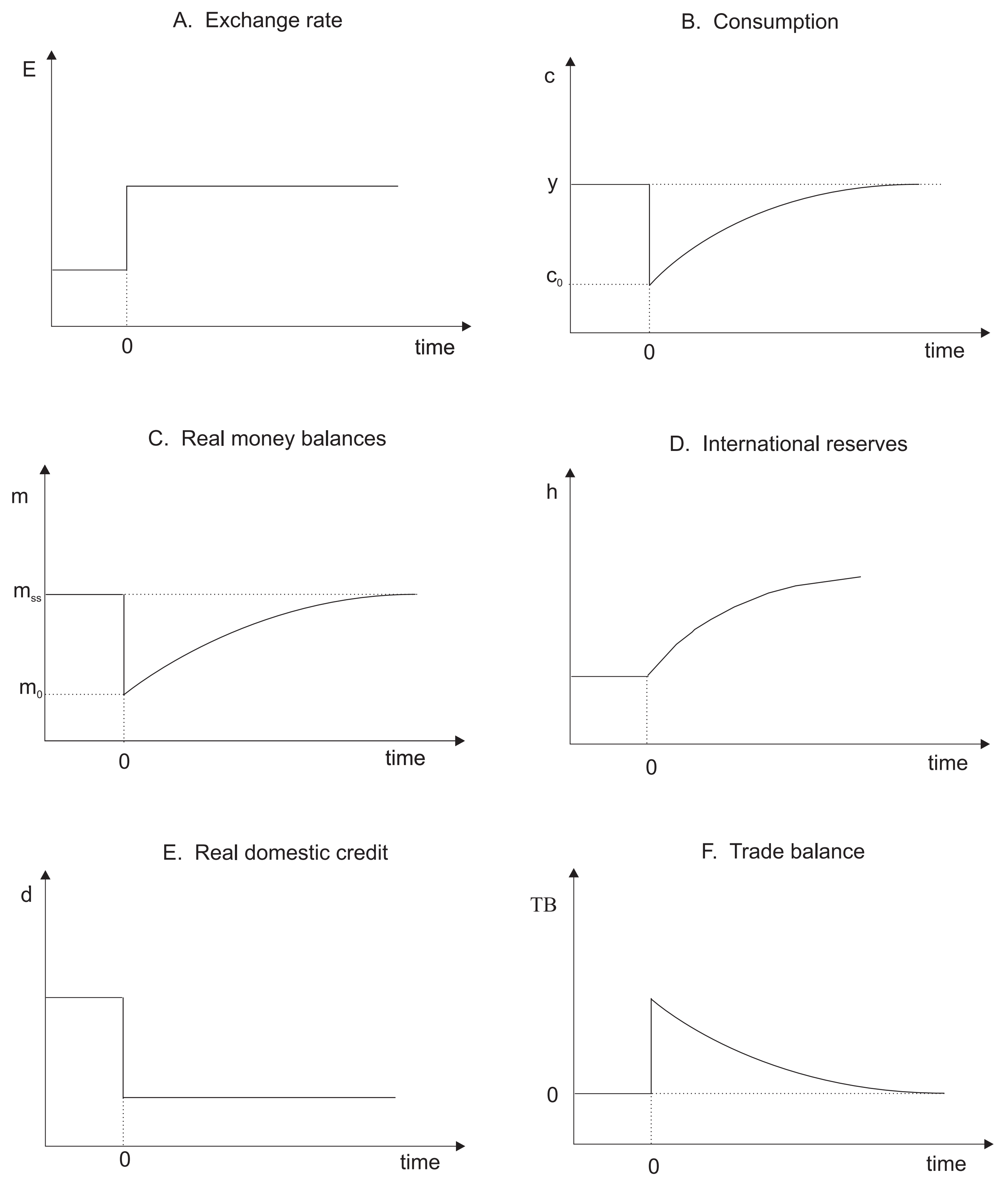


Figure 3. Permanent increase in domestic credit

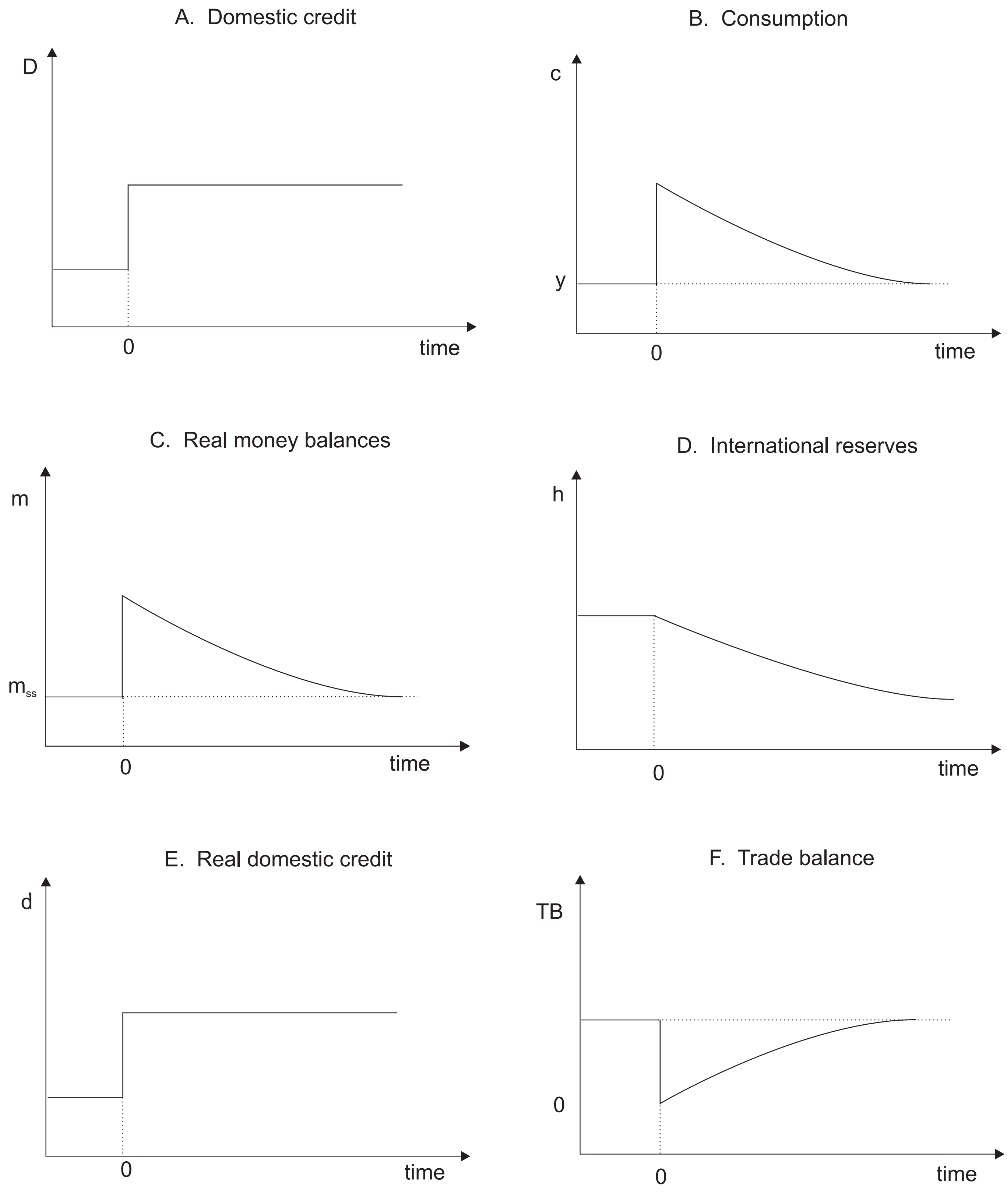


Figure 4. Permanent increase in devaluation rate: Phase diagram

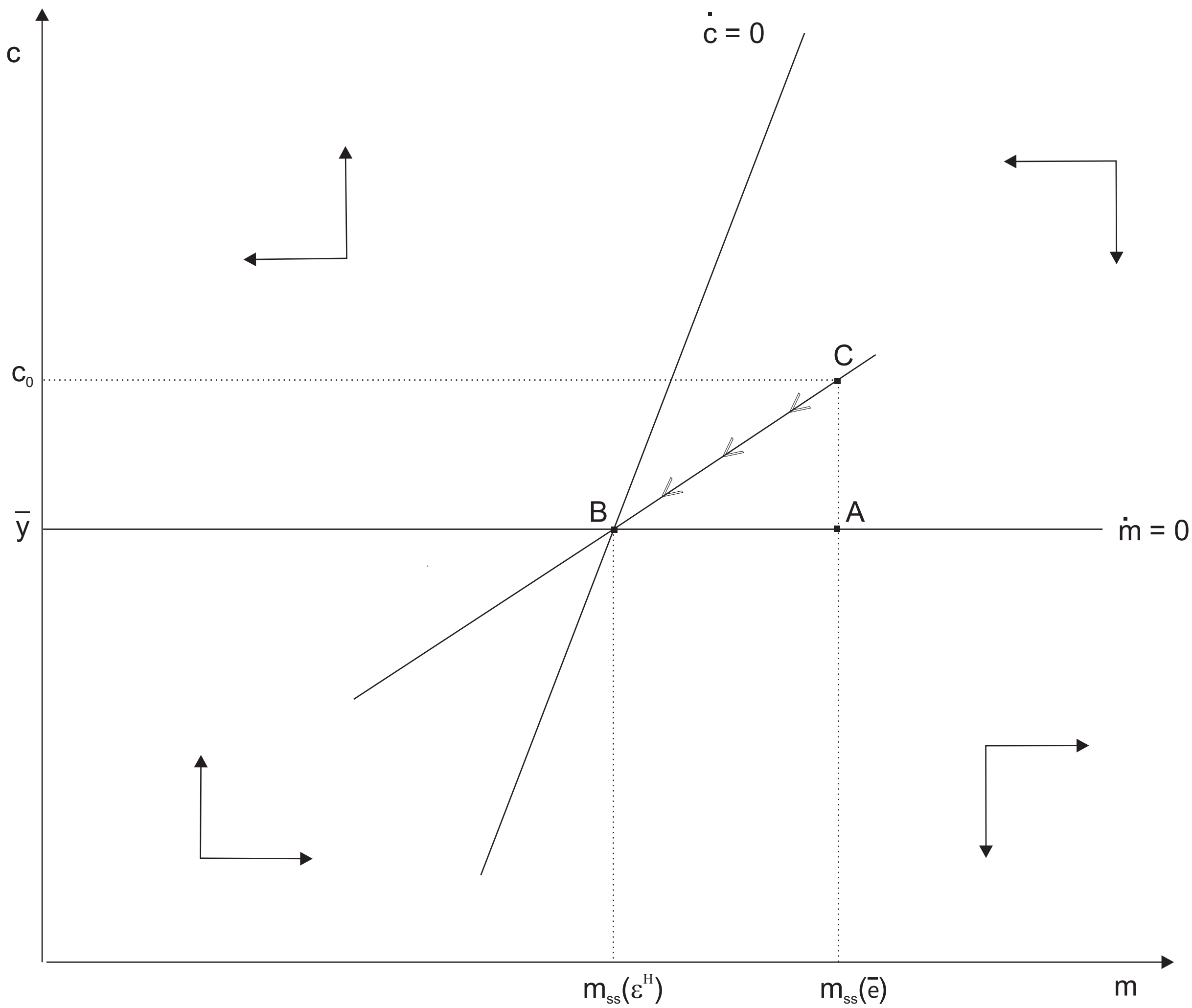


Figure 5. Permanent increase in devaluation rate

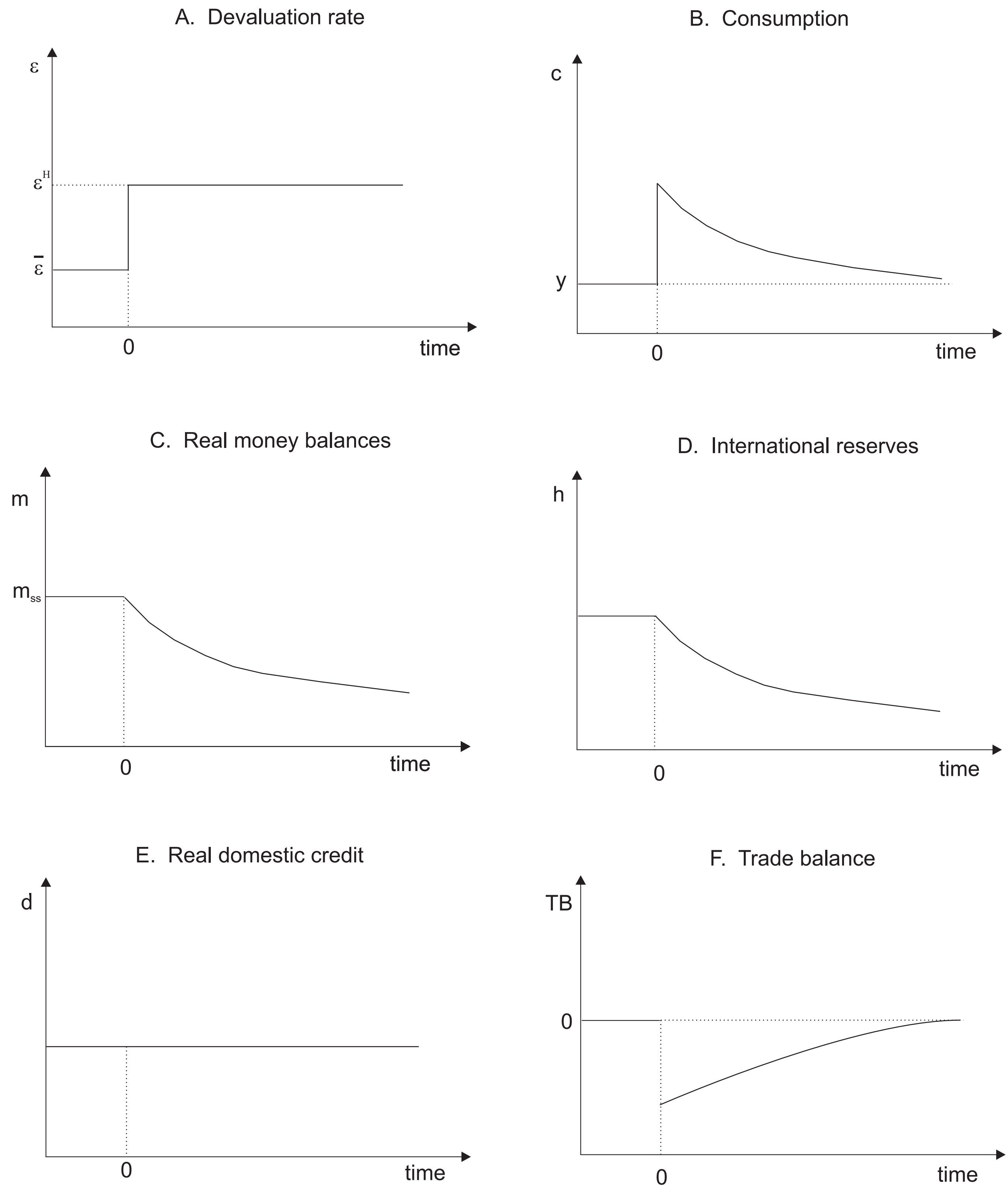


Figure 6. Expenditure reducing and expenditure switching

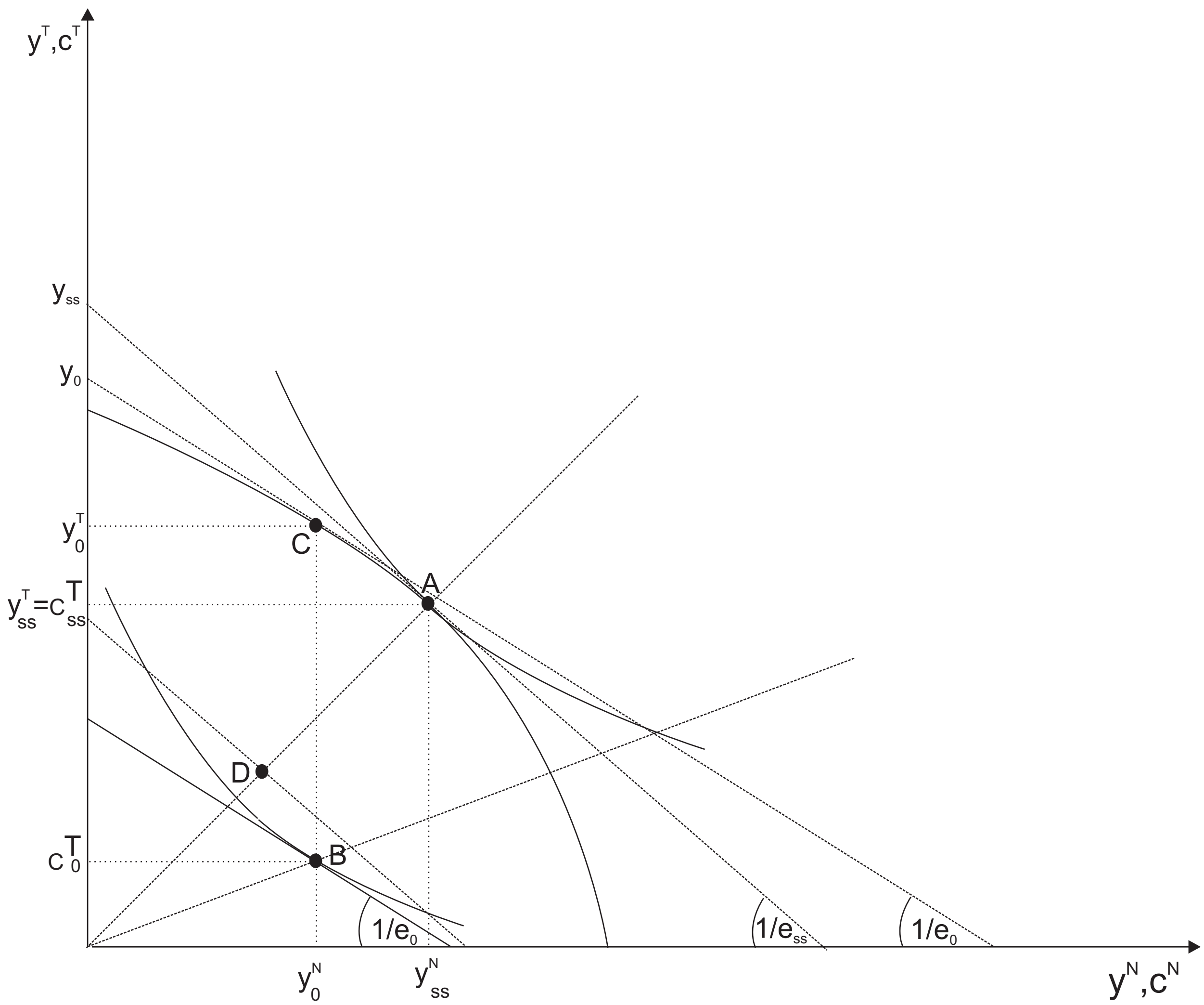


Figure 7. Permanent devaluation in two-good model

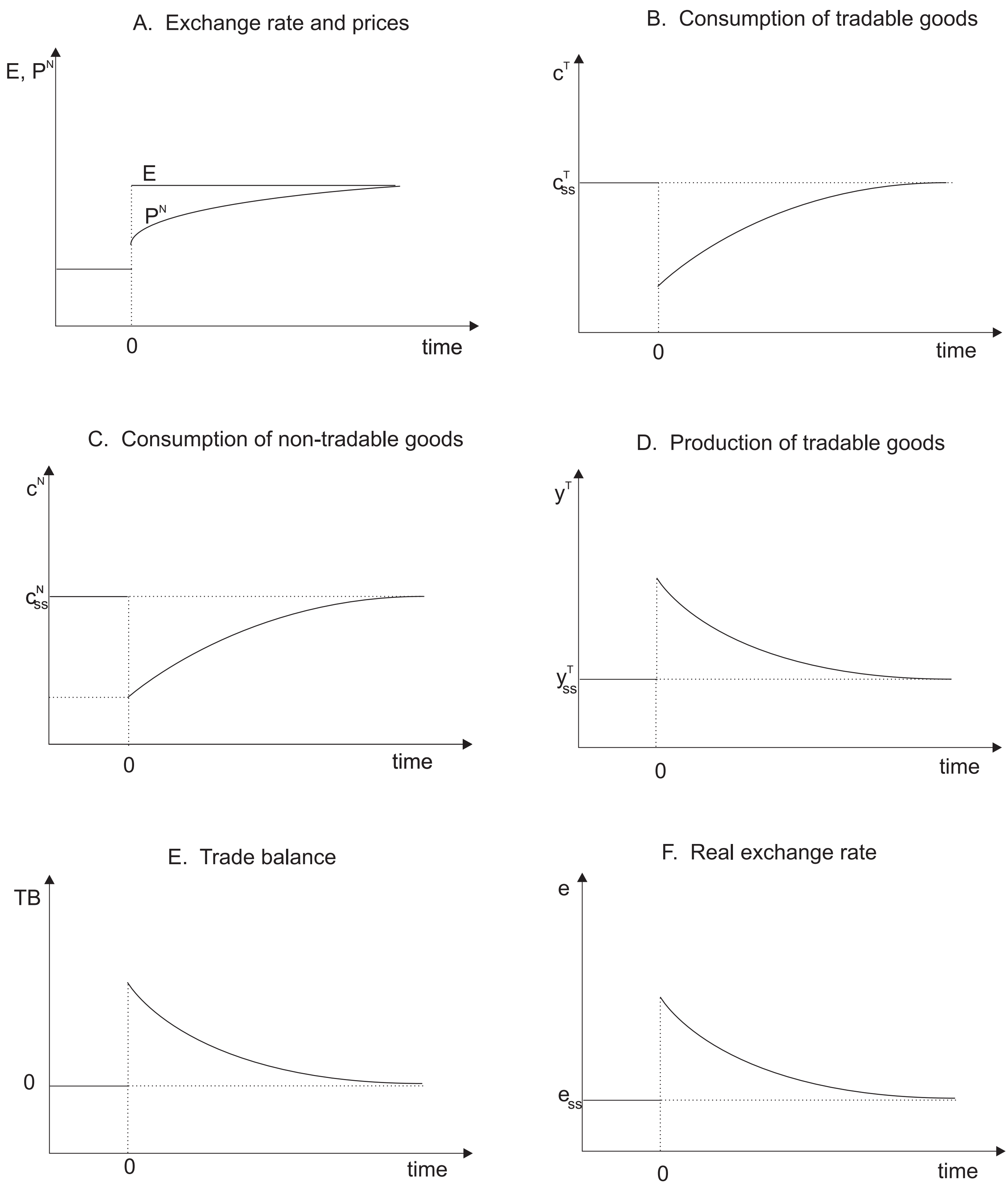


Figure 8. Output effect of a devaluation

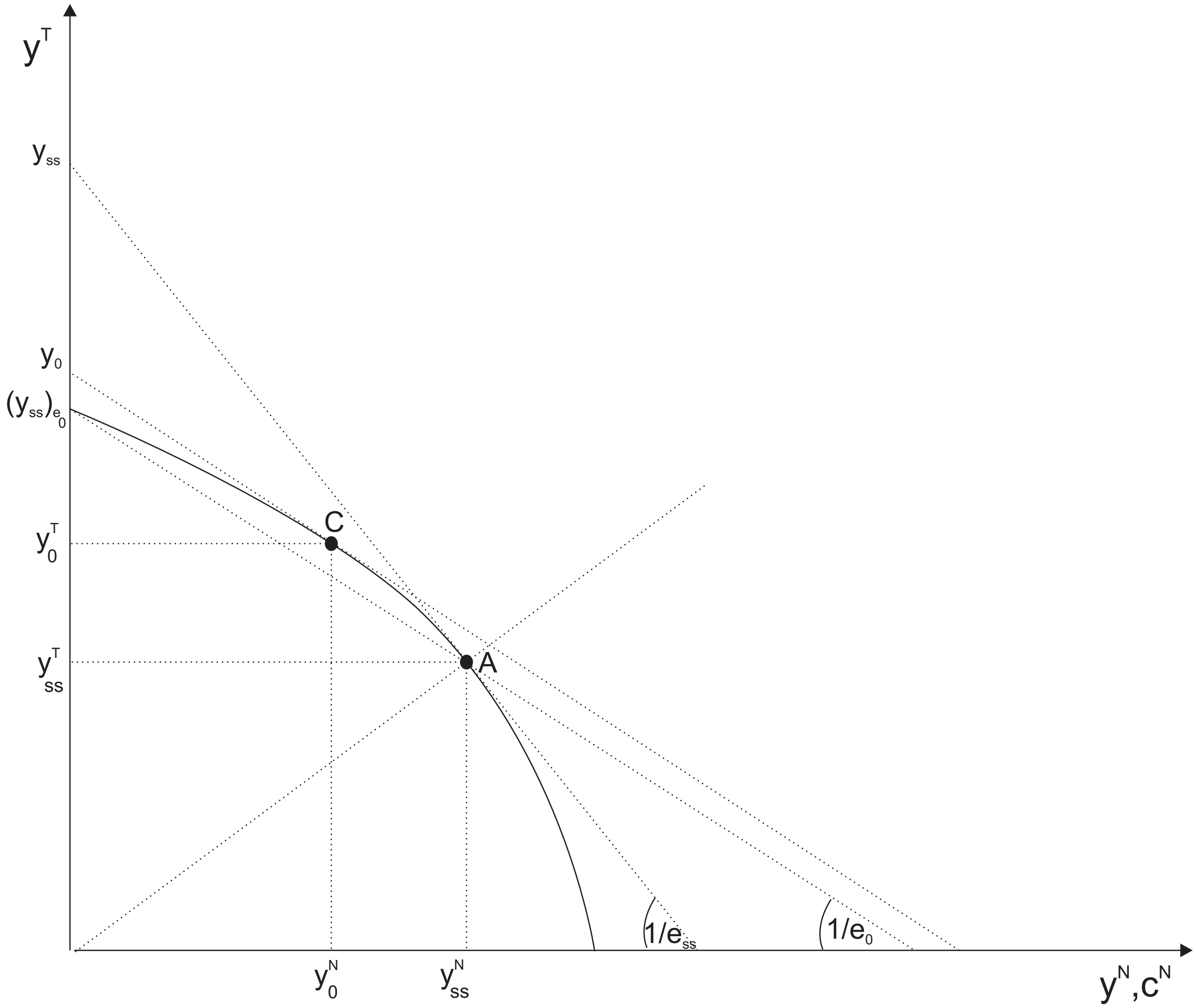
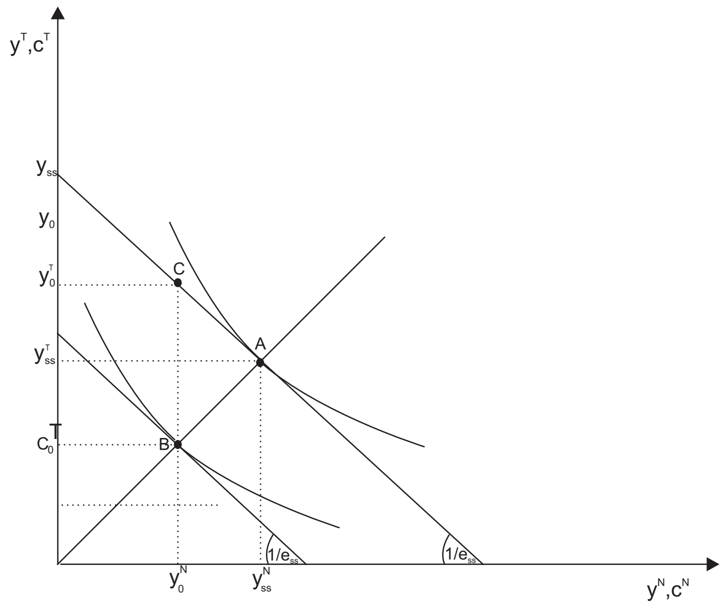


Figure 9. Impact effect of devaluation

A. Linear case



B. Endowment case

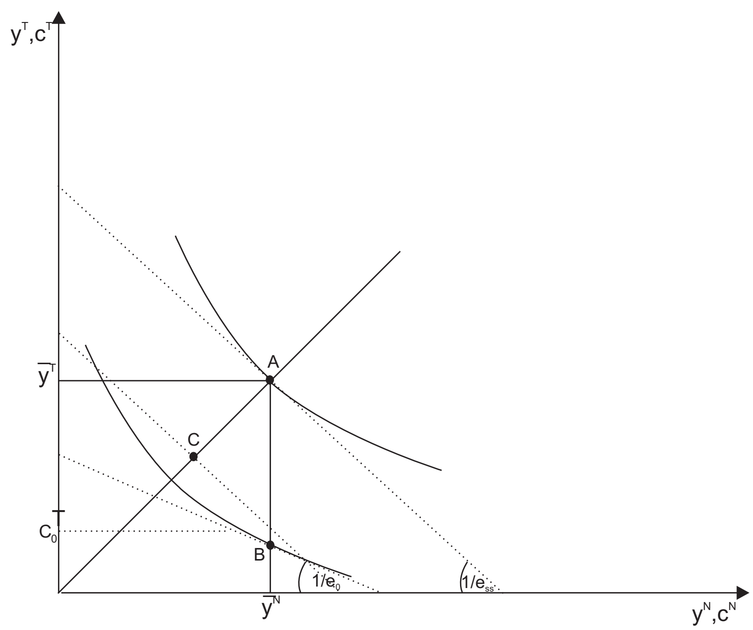


Figure 10. Currency substitution model: Phase diagram

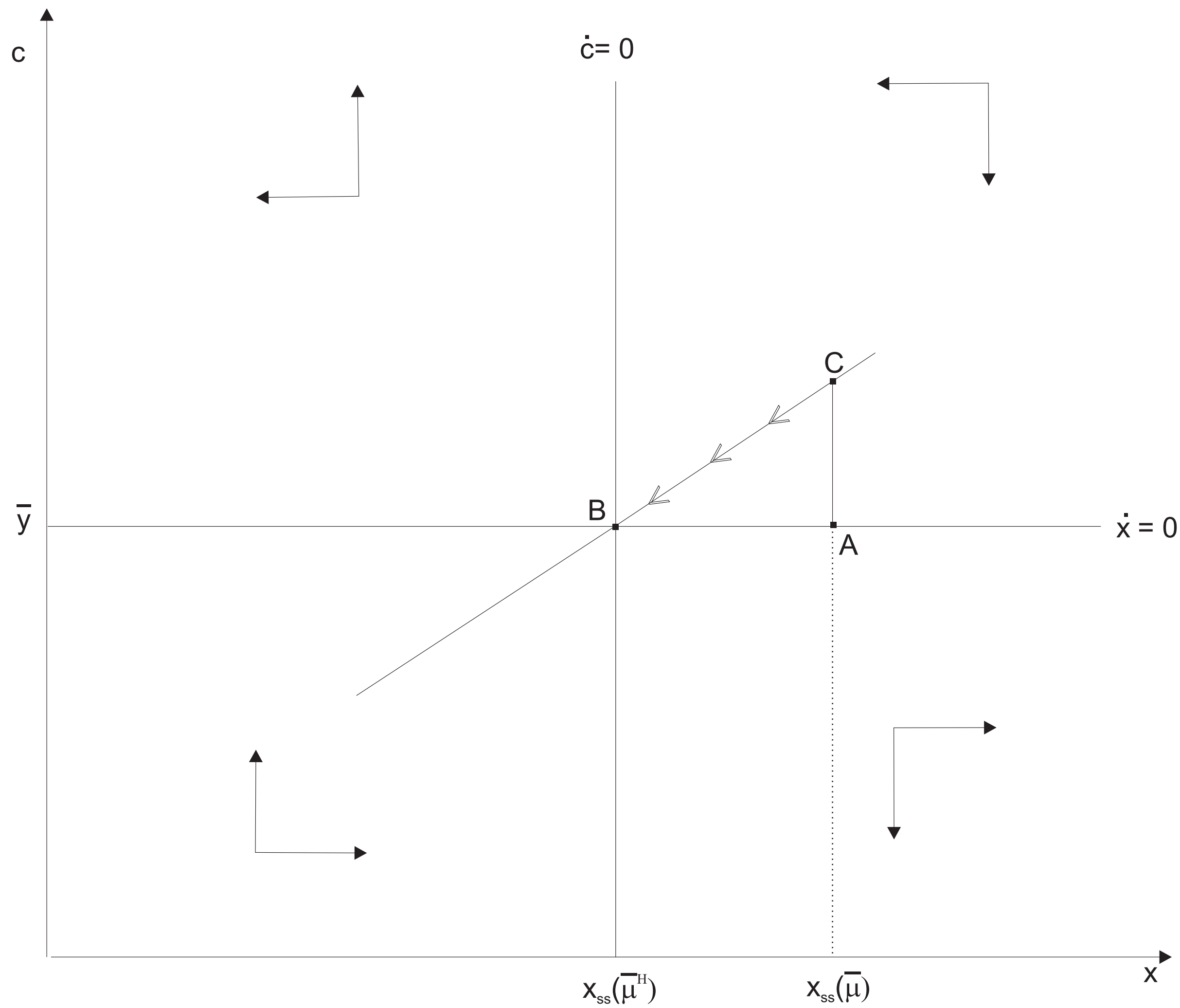


Figure 11. Permanent increase in rate of money growth

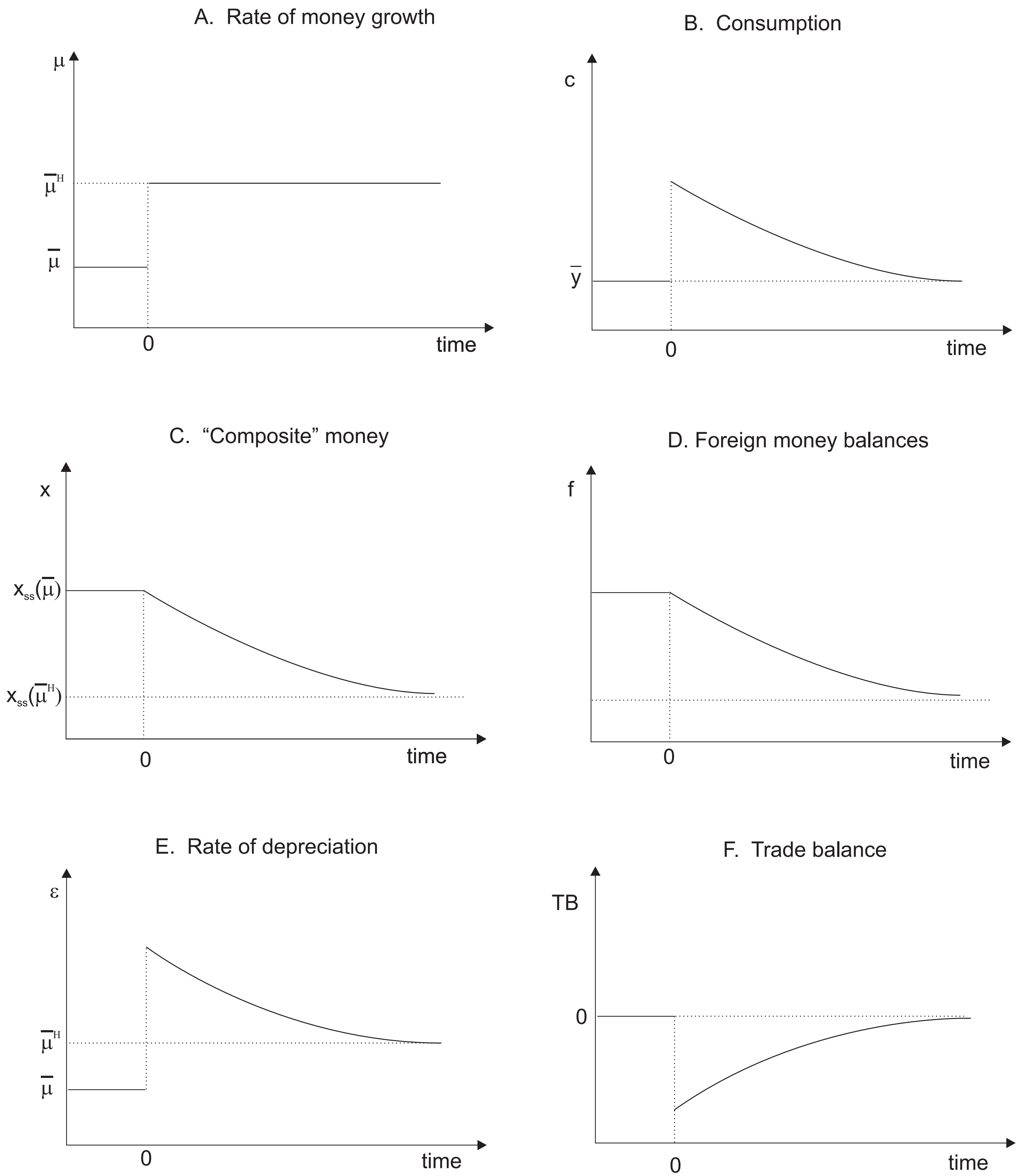
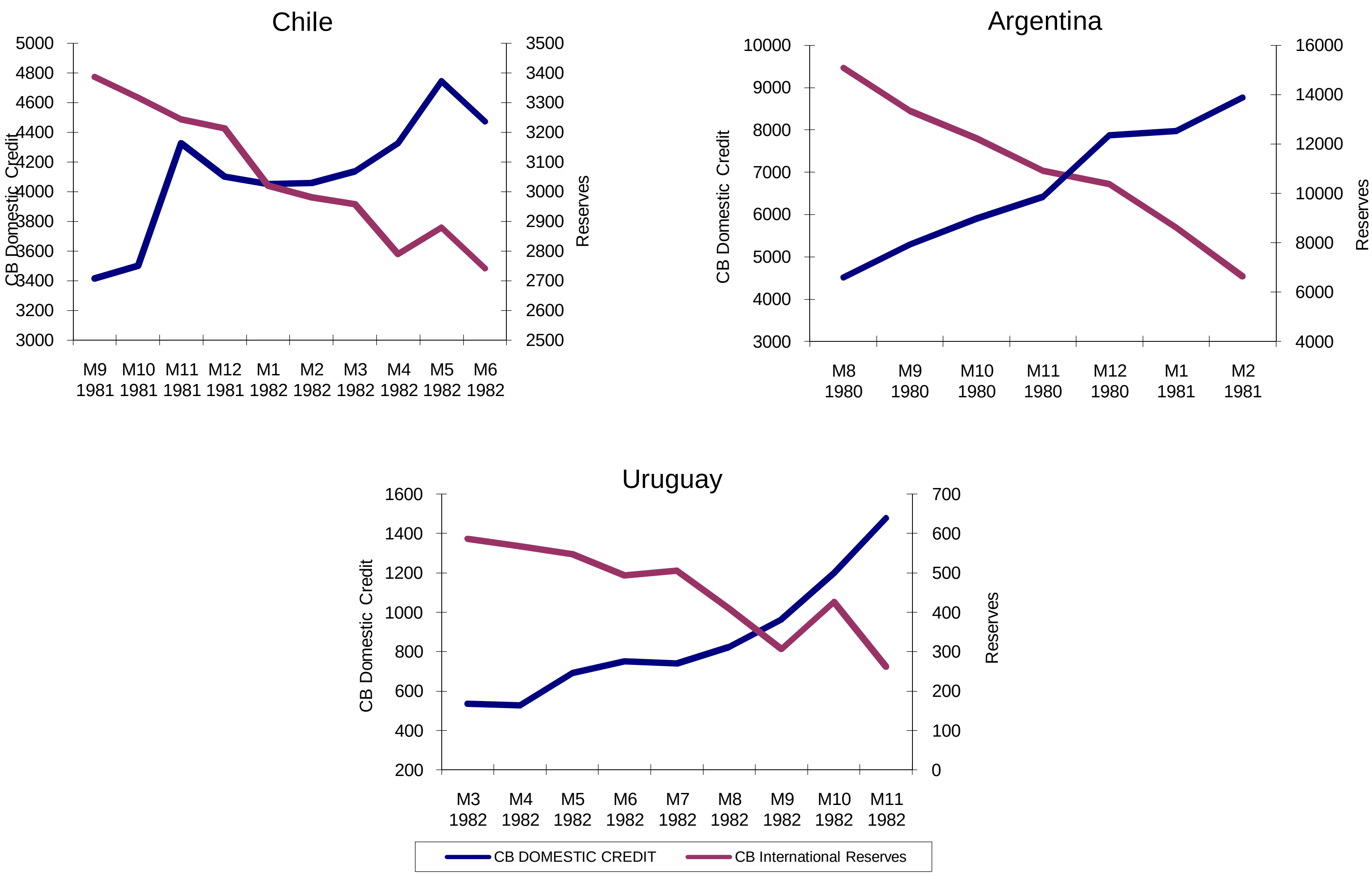


Figure 12. The monetary approach Southern-Cone style



Source: CB Domestic Credit: IFS. Reserves: IFS for Argentina and Chile (Total Reserves minus gold). Central Bank of Uruguay (Net International Reserves)

Table 1: Impact response to a 10 percent devaluation

α	e	P^N	P	y
1	0.0	10.0	10	0.0
0.8	1.1	8.8	9.4	-0.6
0.6	2.4	7.4	8.7	-1.2
0.4	3.8	6.0	8.0	-1.8
0.2	5.4	4.4	7.2	-2.5
Endowment	7.1	2.7	6.3	-3.3

Table 2: Price responses following large devaluations
(Logarithmic change after 12 months of devaluation)

Episode	E	e	P^T	P^N	P
Argentina (Dec. 2001)	123.5	82.2	111.3	13.0	34.3
Brazil (Dec. 1998)	42.4	32.7	43.1	5.1	8.6
Korea (Sept. 1997)	41.2	30.4	21.5	5.1	6.6
Mexico (Dec.1994)	80.0	42.7	84.0	31.6	39.5
Thailand (June 1997)	49.7	26.2	40.4	n/a	10.1

Source: Burstein, Einchenbaum, and Rebelo (2005)
Notes: Month and year of devaluation in parentheses.
P^T refers to import prices at the dock; e is a CPI-based measure of the real exchange rate, P is the CPI.