Chapter 7
Temporary Policy*

Carlos A. Végh
University of Maryland and NBER

Current draft: January 2011

1 Introduction

Chapter 5 introduced money as a veil in the endowment model of Chapter 1. In the world of chapter 5, therefore, changes in monetary or exchange rate policy do not affect the real economy. Chapter 6 removed the veil by abstracting from interest-bearing bonds and showing how, in that context, exchange rate policy has real effects by affecting the desired holdings of real money balances and thereby inducing consumers to run trade imbalances to alter the level of real money balances. Monetary policy, however, still did not have real effects because desired changes in real money balances could be accommodated through changes in the price level. This chapter introduces a different friction into the model of Chapter 5, which will result in both exchange rate and monetary policy having real effects. Specifically, we will introduce a link between the nominal interest rate and consumption. With this channel present, we will be opening the door for *temporary* changes in monetary/exchange rate policy to affect consumption through changes in the nominal interest rate.

How can we model the link between nominal interest rates and consumption? The simplest way – but, as will become clear below, certainly not the only one – is to introduce money into the model via a cash-in-advance constraint. The cash-in-advance constraint, which goes back to Clower (1967), posits that goods must be bought with money (as opposed to, say, credit) and requires that consumers be in possession of the required real money balances *before* they enter the goods market. As already touched upon in Chapter 5, in discrete time there are two possible timings depending on whether asset markets open before or after goods markets. In the Lucas (1982) timing, asset markets open before goods markets (think of asset markets opening early in the morning and

*Comments very welcome. This chapter is part of a graduate textbook on "Open Economy Macroeconomics in Developing Countries" (forthcoming MIT Press), currently under preparation by the author and should be cited accordingly. I am very grateful to Daniel Hernaiz, Pablo Lopez Murphy, Rong Qiang, Agustin Roitman, Belen Sbrancia, and Igor Zuccardi for their help in the preparation of this chapter.

closing at noon, and goods markets opening at noon and closing at 5 p.m.). In this case, consumers do not need to carry overnight the money needed to buy goods tomorrow because they can procure the needed cash in the morning. As a result, the nominal interest rate does not affect consumption.[1]

Imagine, however, the opposite market timing: goods markets open and close in the morning followed by asset markets in the afternoon.[2] In such a case, consumers will need to acquire during the previous afternoon the cash balances needed to buy goods the following morning. By carrying these money balances overnight, they will be subject to the inflation tax. The inflation tax thus becomes part of the effective cost of consuming goods. This is the channel whereby the nominal interest rate will affect consumption. In particular, by reducing the inflation tax, a fall in the nominal interest rate will reduce the *effective* price of consumption. A temporary fall in the nominal interest rate will thus alter the intertemporal profile of the effective price of consumption and induce consumers to substitute future for present consumption à la Chapter 3. We will see that the continuous-time version of the cash-in-advance model – presented in Section 2 – is equivalent to assuming the Svensson timing. In this context, a temporary reduction in the rate of devaluation – which, by interest parity, is equivalent to a temporary reduction in the nominal interest rate – leads to a temporary increase in consumption of tradable goods and a temporary increase in the relative price of non-tradable goods. The temporary reduction in the nominal interest rate is welfare reducing because it induces a non-constant path of consumption while not changing the present discounted value of resources.

In Section 2, *permanent* changes in the rate of devaluation have no real effects because they do not affect the intertemporal profile of the effective price of consumption. In other words, starting with a stationary equilibrium, a permanent reduction in the rate of devaluation leads to a permanent reduction in the nominal interest rate but to no change in consumption because the path of the nominal interest rate remains flat over time. (Notice that there is no wealth effect either because proceeds from the inflation tax are rebated to the public in a lump-sum way.) This feature, however, critically depends on the absence of a labor/leisure choice. In the presence of a labor-leisure choice – as analyzed in Section 3 – a positive nominal interest rate constitutes an *intratemporal* distortion (à la Chapter 4) because it implies that the private relative price of consumption in terms of leisure is greater than one (as opposed to being one, which is the social cost of consumption in terms of leisure). As a result, by reducing the relative price of consumption in terms of leisure, even a permanent reduction in the nominal interest rate will have real effects and lead to a substitution away from leisure (which increases labor supply and hence the present discounted value of output) and towards consumption. In this con-

[1] Of course, households will still carry money balances overnight (as a result of selling their endownmentt for cash in the goods market) but – and this is the critical point – these money balances are not related to tomorrow's consumption.

[2] This is often referred to the Svensson timing (after Svensson (1985)) as opposed to the Lucas timing.

text, therefore, temporary changes in the nominal interest rate will have both intertemporal and intratemporal (i.e., static) effects. The welfare effects of a temporary fall in the nominal interest rate will thus depend on the relative strength of the intertemporal consumption substitution effect (which reduces welfare) and the wealth effect induced by higher labor supply (which increases welfare).

Aside from introducing a labor/leisure choice, there are other ways of incorporating wealth effects into this kind of model. Section 4 illustrates this idea by returning to the endowment economy of Section 2 and introducing money via a transactions costs technology. The idea is that real money balances reduce transactions costs (which use up resources) associated with the purchase of goods. A fall in the nominal interest rate will thus have a wealth effect because it induces consumers to hold more real money balances, which reduces transactions costs. Hence, a permanent reduction in the rate of devaluation – which, via interest parity, leads to a permanent fall in the nominal interest rate – leads to a permanently higher level of consumption due to a permanent reduction in transactions costs. A temporary reduction in the nominal interest rate, on the other hand, will lead to a temporary increase in consumption (by reducing the implicit effective price of consumption). It follows that while a permanent reduction in the rate of devaluation is clearly welfare improving, the welfare effects of a temporary reduction in the nominal interest rate will depend on the relative strength of the intertemporal consumption substitution and wealth effects.

Up to this point, the chapter will have dealt exclusively with an economy operating under predetermined exchange rates. The reason is that since controlling the rate of devaluation is tantamount to setting the nominal interest rate, this case is easier to solve than the flexible exchange rates case. Section 5 turns to the case of flexible exchange rates under a cash-in-advance constraint (i.e., the flexible exchange rate counterpart of the model of Section 2) and shows how the same basic results carry on in the sense that a temporary reduction in the rate of monetary growth also leads to a temporary consumption boom.

2 The basic monetary model with a cash-in-advance

This section considers the basic monetary model of Chapter 5 but introduces money through a cash-in-advance constraint instead of doing it via a MIUF (and we also add non-tradable goods). As in Chapter 5, consider a small open economy which is perfectly integrated in both goods and capital markets. The law of one price holds for the tradable good (i.e., $P^T = EP^{T*}$). Consumers derive utility from consuming both tradable (c^T) and non-tradable (c^N) goods. The economy is endowed with a constant stream of tradable (y^T) and non-tradable goods (y^N). The world real interest rate (r) is given and constant over time.

2.1 Consumer's problem

Preferences are given by

$$\int_0^\infty [u(c_t^T) + v(c_t^N)]e^{-\beta t}dt, \tag{1}$$

where $\beta > 0$ is the discount rate.

Let a denote real financial assets (in terms of tradable goods)

$$a_t \equiv m_t + b_t,$$

where $m_t (\equiv M_t/E_t P_t^*)$ denotes real money balances and b_t stands for net foreign bonds.

Proceeding as in Chapter 5, we know that the flow constraint will be given by

$$\dot{a}_t = ra_t + y^T + \frac{y^N}{e_t} + \tau_t - c_t^T - \frac{c_t^N}{e_t} - i_t m_t, \tag{2}$$

where $e_t (\equiv P_t^T/P_t^N)$ is the real exchange rate (defined as the relative price of tradable goods in terms of non-tradable goods), τ_t are lump-sum transfers from the government, and i_t is the nominal interest rate.

The cash-in-advance constraint requires that consumers hold a proportion α of real money balances to finance their consumption purchases in every period. Formally,[3]

$$m_t = \alpha \left(c_t^T + \frac{c_t^N}{e_t} \right). \tag{3}$$

It proves convenient to get rid of m as a choice variable for the consumer by substituting the cash-in-advance constraint into the flow constraint (2) to obtain:

$$\dot{a}_t = ra_t + y^T + \frac{y^N}{e_t} + \tau_t - \left(c_t^T + \frac{c_t^N}{e_t} \right)(1 + \alpha i_t). \tag{4}$$

Integrating this flow constraint forward and imposing the appropriate transversality condition, we obtain

$$a_0 + \int_0^\infty \left(y^T + \frac{y^N}{e_t} + \tau_t \right) e^{-rt}dt = \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t} \right)(1 + \alpha i_t)e^{-rt}dt. \tag{5}$$

Consumers then choose $\{c^T, c^N\}_{t=0}^\infty$ to maximize (1) subject to the intertemporal constraint (5). In terms of the Lagrangean:

$$\mathcal{L} = \int_0^\infty [u(c_t^T) + v(c_t^N)]e^{-\beta t}dt + \lambda \left\{ a_0 + \int_0^\infty \left[y^T + \frac{y^N}{e_t} + \tau_t - \left(c_t^T + \frac{c_t^N}{e_t} \right)(1 + \alpha i_t) \right] e^{-rt}dt. \right\}$$

[3] This cash-in-advance can be interpreted as a first-order approximation to the "true" cash-in-advance constraint (see Appendix 7.1).

In addition to (5), the first-order conditions are given by (assuming, as usual, that $\beta = r$):

$$u'(c_t^T) = \lambda(1 + \alpha i_t), \quad (6)$$

$$v'(c_t^N) = \lambda \frac{(1 + \alpha i_t)}{e_t}. \quad (7)$$

First-order condition (6) says that, at the margin, the consumer equates the marginal utility from consuming tradable goods to the marginal utility of wealth times the *effective price* of tradable goods (given by $1 + \alpha i_t$). Notice that it follows from the cash-in-advance constraint (3) that to purchase one unit of tradable goods, consumers need to hold α units of real money balances. Hence, the effective price of tradable goods comprises the real market price of the good (one) plus the opportunity cost of the α units of real money balances required to purchase one unit of the good (αi). In the same vein, the effective price of non-tradable goods is given by $1/e_t + (\alpha/e_t)i_t$. In other words, the effective price of non-tradable goods is equal to the market relative price ($1/e_t$) plus the opportunity cost of holding the α/e_t units of real money balances required to purchase one unit of non-tradables.

First-order condition (6) brings us back to the world of intertemporal distortions analyzed in Chapter 3. Clearly, if the nominal interest rate is constant along a perfect foresight equilibrium path, the path of consumption of tradable goods will also be constant over time. However, if the nominal interest rate is not constant over time, the path of consumption will not be constant over time either since the consumer will prefer to substitute consumption away from high interest rate periods (when the effective price consumption is relatively high) and towards low interest rate periods (when the effective price is relatively low). In other words, a non-constant path of the nominal interest rate will introduce an intertemporal distortion in much the same way as a non-constant tariff did in Chapter 3. For a given and constant path of the real exchange rate, the same is true of consumption of non-tradable goods.

Combining first-order conditions (6) and (7) yields the condition (with which we are familiar from Chapter 4):

$$\frac{u'(c_t^T)}{v'(c_t^N)} = e_t. \quad (8)$$

Since the path of the nominal interest rate affects both goods in the same way, it does not affect the marginal rate of substitution between the two goods.

2.2 Government

The government is the same as in Chapter 5. Its flow constraint is thus given by

$$\dot{h}_t = rh_t + \frac{\dot{M}_t}{P_t} - \tau_t. \quad (9)$$

The corresponding intertemporal constraint is given by

$$h_0 + \int_0^\infty \frac{\dot{M}_t}{P_t} e^{-rt} dt = \int_0^\infty \tau_t e^{-rt} dt. \tag{10}$$

2.3 Equilibrium conditions

Once again, perfect capital mobility implies that interest parity holds:

$$i_t = i_t^* + \varepsilon_t. \tag{11}$$

Equilibrium in the non-tradable goods market requires that consumption of non-tradables equal the constant endowment:

$$c_t^N = y^N. \tag{12}$$

Let $k(\equiv b + h)$ denote the economy's stock of net foreign assets. Combining the consumer's flow constraint (equation (2)) with the government's (equation (9)) yields the economy's flow constraint:

$$\dot{k}_t = rk_t + y^T - c_t^T. \tag{13}$$

Integrating forward the economy's flow constraint (equation (13)) and imposing the corresponding transversality condition yields the economy's resource constraint:

$$k_0 + \frac{y^T}{r} = \int_0^\infty c_t^T e^{-rt} dt. \tag{14}$$

Finally, notice for further reference that since, by definition, $e_t = P^T/P^N$, where $P^T = E_t P^{T*}$, it follows that

$$\frac{\dot{e}_t}{e_t} = \varepsilon_t + \pi_t^* - \pi_t, \tag{15}$$

where $\pi_t^*(\equiv \dot{P}^{T*}/P^{T*})$ and $\pi_t(\equiv \dot{P}_t^N/P_t^N)$ are, respectively, the foreign inflation rate and the rate of inflation of non-tradable goods. In equilibrium, therefore, the rate of change of the real exchange rate will be given by the difference between tradable goods inflation (given by $\varepsilon_t + \pi_t^*$) and non-tradable goods inflation (given by π_t).

2.4 Perfect foresight equilibrium

Suppose that the rate of foreign inflation is constant over time and equal to π^*. We proceed to characterize a perfect foresight equilibrium path for a constant rate of devaluation, $\bar{\varepsilon}$.

Given the constant rate of devaluation, interest parity (given by equation (11)) determines a constant nominal interest rate:

$$i = i^* + \bar{\varepsilon}.$$

Since the nominal interest rate is constant over time, first-order condition (6) tells us that c^T will also be constant over time. It then follows from the resource constraint (14) that

$$c^T = rk_0 + y^T. \tag{16}$$

Using (8), (12) and (16), we conclude that the real exchange rate will also be constant over time and given by

$$e = \frac{u'\left(rk_0 + y^T\right)}{v'(y^N)}. \tag{17}$$

Since $\dot{e}_t = 0$, it follows from (15) that

$$\pi = \pi^* + \bar{\varepsilon}. \tag{18}$$

2.5 Permanent changes in exchange rate policy

Having set-up the model, we now analyze two policy experiments: a permanent devaluation (i.e., an permanent increase in E) and a permanent increase in the rate of devaluation.

2.5.1 Permanent devaluation

Suppose that just before $t = 0$, the economy is in the stationary perfect foresight equilibrium just described. At $t = 0$, there is an unanticipated and permanent devaluation. Since there has been an unexpected change in the exchange rate, the consumer reoptimizes. In the new perfect foresight path, consumption of tradable goods will still be constant. Furthermore, since the devaluation does not affect the resources available to this economy, c^T will still be given by (16). The real exchange rate, therefore, will still be given by (17). In sum – and despite the potential effect of the nominal interest rate on consumption – a devaluation is neutral, as was the case in Chapter 5.

2.5.2 Permanent reduction in the rate of devaluation

Suppose now that, starting from the stationary equilibrium characterized above, there is an unanticipated and permanent reduction in the devaluation rate. Given the interest parity condition (11), the nominal interest rate will also fall permanently. After the consumer reoptimizes, the same first-order conditions will apply. Since it is still the case that the nominal interest rate is constant along the new perfect foresight equilibrium path (though it is now at a lower level than before), consumption of tradable goods will be constant over time and, in light of (14), equal to $rk_0 + y^T$. Hence, from (17), the real exchange rate will also be constant over time. Since the real exchange rate is constant over time, equation (18) indicates that the rate of inflation of non-tradable goods will fall instantaneously by the same amount as the rate of devaluation. In sum, a permanent change in the rate of devaluation reduces inflation with no real effects whatsoever.[4]

2.6 Temporary stabilization

Once again, suppose that just before $t = 0$, the economy is in the stationary perfect foresight equilibrium characterized above. At $t = 0$, there is an unanticipated and temporary fall in the rate of devaluation from $\bar{\varepsilon}$ to $\bar{\varepsilon}^L$, $\bar{\varepsilon}^L < \bar{\varepsilon}$ (Figure 1, Panel A). At time T, the rate of devaluation goes back to its initial level, $\bar{\varepsilon}$. Formally,

$$\varepsilon_t = \begin{cases} \bar{\varepsilon}^L, & 0 \leq t < T, \\ \bar{\varepsilon}, & t \geq T, \end{cases}$$

for some $T > 0$.

[Figure 1]

Naturally, the interest parity condition implies that the nominal interest rate behaves analogously (Figure 1, Panel B):

$$i_t = \begin{cases} i^* + \bar{\varepsilon}^L & 0 \leq t < T, \\ i^* + \bar{\varepsilon} & t \geq T. \end{cases}$$

What will happen to the path of consumption of tradable goods? From the first-order condition (6), we know that consumption will be constant within each subperiod (denote those levels by $(c^T)^1$ and $(c^T)^2$) respectively:

$$\begin{aligned} u'((c^T)^1) &= \lambda(1 + \alpha i^L), \quad 0 \leq t < T, \\ u'((c^T)^2) &= \lambda(1 + \alpha i), \quad\;\; t \geq T. \end{aligned}$$

Clearly, by the strict concavity of $u(.)$, $(c^T)^1 > (c^T)^2$. Furthermore, since the temporary stabilization does not affect the economy's resources, it follows from the resource constraint that $(c^T)^1$ will be higher than $rk_0 + y^T$ and $(c^T)^2$ will be lower (Figure 1, Panel C). Given this path of consumption, the trade balance will worsen at $t = 0$ and improve at time T (see Figure 1, Panel D, which assumes that $k_0 = 0$). The current account will therefore go into deficit at $t = 0$, worsen during the transition, and jump back to zero at time T.

What happens with the real exchange rate? From (8), it follows that

$$e_t = \frac{u'\left(c_t^T\right)}{v'(y^N)}. \tag{19}$$

[4] Notice, incidentally, that in this cash-in-advance set-up, real money balances do not change either in response to the permanent fall in the rate of devaluation. The reason is that money demand is not interest rate elastic. In contrast, if we introduced money in the utility function, the fall in the nominal interest rate that results from a fall in the rate of devaluation would lead to an increase in real money balances along the lines of Chapter 5.

It follows that e_t will fall on impact (real appreciation) and increase at T (real depreciation) (Figure 1, Panel E). Since c_t^T is below the pre-shock level after T, the real exchange rate will be above its pre-shock level.

Since $\dot{e}_t = 0$ for $0 \leq t < T$ and $t \geq T$, it follows from (15) that the path of inflation of non-tradables goods is given by

$$\begin{aligned} \pi_t &= \pi^* + \varepsilon^L \quad 0 \leq t < T, \\ \pi_t &= \pi^* + \bar{\varepsilon}, \quad t \geq T. \end{aligned}$$

Inflation of non-tradable goods thus falls on impact in line with the fall in the rate of devaluation and increases back to its pre-shock level at time T (Figure 1, Panel F).

In sum, a temporary exchange rate-based stabilization leads to an initial consumption boom, real appreciation, and trade deficits followed at time T by a consumption bust, real depreciation, and trade surpluses.[5] [6] What is the intuition behind the results? The fact that the nominal interest rate is lower during $[0, T)$ than afterwards implies that the effective price of consumption is lower during $[0, T)$ than afterwards. Hence – and for exactly the same reasons analyzed in Chapter 3 – consumers will engage in intertemporal consumption substitution by shifting consumption away from the relative expensive period (after T) towards the relatively cheap period ($[0, T)$). For the initial real exchange rate, non-tradable goods also become relatively cheaper and there is thus an excess demand for non-tradable goods. Since the endowment of non-tradables is fixed, however, the relative price (i.e., $1/e_t$) must increase to clear the market.

2.7 A reinterpretation of the results

As we did in Chapter 3 for the case of a trade liberalization, we can reinterpret the above results for the permanent and temporary reduction in the rate of devaluation as applying to a situation in which the same policy announcement may have different degrees of credibility. As discussed in Box 1, even though lack of credibility is hard to measure, it has undoubtedly been a perennial problem in developing countries.

Suppose that at time 0, policymakers announce a permanent reduction in the rate of devaluation. If the announcement is fully credible (i.e., if agents believe the government's announcement), then the economy will behave as in the permanent case analyzed above. Inflation will be stopped immediately with no output costs. As argued in Chapter 14, this type of experiment can be taken to apply to the end of hyperinflations in which inflation has been reduced immediately with little or no real effects.

[5] Not surprisingly, the real effects of a temporary exchange rate-based stabilization are the same that we encountered when we studied an anticipated fall in demand in Chapter 4. In fact, we can see this model as rationalizing changes in aggregate demand as a result of fluctuations in nominal interest rates.

[6] As Exercise 1 at the end of this chapter shows, the same results obtain in a money-in-the-utility-function model if the cross derivative between consumption and real money balances is positive.

Instead, suppose that the public does not believe that the government will stick to the stabilization announcement. This lack of credibility is very likely to arise in countries with a history of failed stabilization attempts. When the latest finance minister goes on television and announces a major exchange rate-based stabilization plan involving, say, a fixed exchange rate and swears that this parity will last for an eternity, the public – who has heard this before – will be rightly skeptical. To capture this lack of credibility in a simple way, suppose that the public expects that the government will abandon the stabilization plan at some time T in the future. Then all the real effects that we studied for the case of a temporary reduction in the devaluation rate would go through. We can thus reinterpret the exercise of a temporary reduction in the rate of devaluation as arising from a non-credible exchange rate-based stabilization.[7] This case is particularly relevant for exchange rate-based stabilization in chronic inflation countries, in which a history of failed stabilization attempts combined with the economy's ability to live with high inflation makes any attempt to stop inflation non-credible (see Chapter 14).

2.8 Welfare implications

What are the welfare implications of the above analysis? The analysis parallels that of a trade liberalization in Chapter 3. A permanent exchange rate-based stabilization has no welfare effects because the model does not incorporate any benefit of a lower *level* of inflation. Put differently, welfare in this model is the same for any constant level of the devaluation rate (and hence of the nominal interest rate). On the other hand, a temporary exchange rate-based stabilization is welfare-reducing because it introduces an intertemporal distortion. Hence, taken at face value, the model would have the uncomfortable implication that no stabilization attempt should ever be attempted!

Of course, the model ignores the fact that inflation stabilization plans are likely to bring about wealth effects through various channels (higher productivity, less time spent on financial engineering, and so forth). As shown in Section 4 below, a simple way of incorporating a wealth effect of lower inflation and studying the resulting trade-off is by introducing money through a transactions costs technology. In that case, a *permanent* exchange rate-based stabilization will always be welfare improving. Whether a *temporary* program is welfare-improving or not will depend on the relative strength of the intertemporal substitution effect and the wealth effect. The larger is T, the more likely that the wealth effect will dominate.

Another simple way of incorporating a wealth effect is to assume that the government uses the revenues from the inflation tax on unproductive government spending (think of the government simply throwing away into the sea the revenues from the inflation tax), a scenario analyzed in Exercise 2 at the end of the chapter. Government spending is assumed to be endogenous and hence

[7] Proceeding as in Chapter 3, it can be shown that the real effects at time T are independent of whether the government sticks to the plan (thus validating expectations) or not.

respond to available revenues. In this case, a permanent exchange rate-based stabilization will also be welfare improving because it leads to a permanently lower level of government spending. For the case of logarithmic preferences, a temporary exchange-rate based stabilization will also raise welfare as the wealth effect dominates the intertemporal substitution effect.

3 Labor supply

The model considered so far has assumed that there is a constant endowment path of both tradable and non-tradable goods. The model can therefore not explain a positive output response to an exchange rate-based stabilization. We now consider an extension of the model which incorporates a consumption/leisure choice.[8] (For simplicity, the model now abstracts from non-tradable goods.)

3.1 Consumer's problem

Consider a one-good economy in which the representative household maximizes

$$\int_0^\infty u(c_t, \ell_t) e^{-\beta t} dt, \tag{20}$$

where ℓ_t denotes leisure. The function $u(.)$ is assumed to be strictly increasing in both arguments and strictly concave:

$$u_c > 0,\ u_\ell > 0,\ u_{cc} < 0,\ u_{cc}u_{\ell\ell} - u_{c\ell}^2 > 0.$$

We also assume that both goods are normal, which requires (as shown in the appendix to Chapter 4):

$$u_\ell u_{c\ell} - u_c u_{\ell\ell} > 0, \tag{21}$$

$$u_c u_{\ell c} - u_\ell u_{cc} > 0. \tag{22}$$

Agents are endowed with one unit of time. Labor is thus $1 - \ell_t$. The production function is linear and given by

$$y_t = 1 - \ell_t. \tag{23}$$

The cash-in-advance constraint now takes the form

$$m_t = \alpha c_t. \tag{24}$$

The flow constraint is given by

$$\dot{a}_t = r a_t + y_t + \tau_t - c_t - i_t m_t. \tag{25}$$

[8] See Roldos (1995, 1997) and Lahiri (2000) for further exploration of this channel.

Integrating forward and imposing the corresponding transversality condition, we obtain:

$$a_0 + \int_0^\infty (y_t + \tau_t) e^{-rt} dt = \int_0^\infty (c_t + i_t m_t) e^{-rt} dt. \tag{26}$$

Substituting the production function (23) and the cash-in-advance constraint (24) into the intertemporal constraint, we obtain:

$$a_0 + \int_0^\infty (1 - \ell_t + \tau_t) e^{-rt} dt = \int_0^\infty (1 + \alpha i_t) c_t e^{-rt} dt. \tag{27}$$

The representative consumer chooses $\{c_t, \ell_t\}_{t=0}^\infty$ to maximize (20) subject to the intertemporal constraint (27). Setting up the Lagrangean:

$$\mathcal{L} = \int_0^\infty u(c_t, \ell_t) e^{-\beta t} dt + \lambda \left[a_0 + \int_0^\infty (1 - \ell_t + \tau_t) e^{-rt} dt - \int_0^\infty (1 + \alpha i_t) c_t e^{-rt} dt. \right].$$

First-order conditions imply that (assuming $\beta = r$):

$$u_c(c_t, \ell_t) = \lambda (1 + \alpha i_t), \tag{28}$$

$$u_\ell(c_t, \ell_t) = \lambda, \tag{29}$$

where λ is now the Lagrange multiplier associated with constraint (27).

Combining the two first-order conditions, we obtain:

$$\frac{u_c(c_t, \ell_t)}{u_\ell(c_t, \ell_t)} = 1 + \alpha i_t. \tag{30}$$

The nominal interest rate now introduces an *intratemporal distortion* between consumption and leisure.[9] It is a *distortion* because – as discussed in detail below – in an undistorted equilibrium, the relative price of consumption in terms of leisure should be equal to the marginal productivity of labor (which, in this model, is one). It is *intratemporal* because it affects the contemporaneous choice between consumption and leisure.

3.2 Government

The government's constraint remains given by equations (9) and (10).

3.3 Equilibrium conditions

The interest parity condition, given by (11), continues to hold. Combining the consumer's and the government's flow constraints (given by equations (25) and (9), respectively), we obtain the economy's flow constraint:

[9] You may remember that we introduced the concept of intratemporal distortion in Chapter 4, when we analyzed the effects of taxation on the real exchange rate.

$$\dot{k}_t = rk_t + 1 - \ell_t - c_t,$$

where $k_t (\equiv b_t + h_t)$ denotes the economy's total net foreign assets.

The corresponding resource constraint is given by

$$k_0 + \int_0^\infty (1 - \ell_t) e^{-rt} dt = \int_0^\infty c_t e^{-rt} dt. \tag{31}$$

3.4 Perfect foresight equilibrium

Suppose that the foreign rate of inflation is constant. Let us characterize a perfect foresight equilibrium path for a constant path of the rate of devaluation, $\bar{\varepsilon}$. From interest parity, the nominal interest rate will also be constant over time:

$$i_t = i^* + \bar{\varepsilon}.$$

Due to the non-separability of consumption and leisure, it is now a little bit more involved to show that consumption and leisure will also be constant along time. To this effect, totally differentiate first-order conditions (28) and (29) along a perfect foresight equilibrium path, taking into account that both i and λ are constant along such a path, to obtain:[10]

$$\begin{aligned} u_{cc} dc + u_{c\ell} d\ell &= 0, \\ u_{\ell c} dc + u_{\ell\ell} d\ell &= 0. \end{aligned}$$

Using the last equation to solve for $d\ell$ in terms of dc and substituting into the first equation, we obtain:

$$dc \left(\frac{u_{cc} u_{\ell\ell} - u_{\ell c}^2}{u_{\ell\ell}} \right) = 0.$$

Since the term in parenthesis is negative (by strict concavity of the utility function, the numerator is positive and the denominator is negative), it must be the case that $dc = 0$ along a perfect foresight equilibrium path. Hence, $d\ell = 0$.

The constant values of c and ℓ (denoted by $\bar{c}$ and $\bar{\ell}$) then satisfy condition (30):

$$\frac{u_c(\bar{c}, \bar{\ell})}{u_\ell(\bar{c}, \bar{\ell})} = 1 + \alpha \bar{\imath}. \tag{32}$$

From the resource constraint (31), it follows that:

$$\bar{c} = rk_0 + 1 - \bar{\ell}. \tag{33}$$

Equations (32) and (33) implicitly define $\bar{c}$ and $\bar{\ell}$ as a function of rk_0 and $\bar{\imath}$.

[10] From now on – and to avoid notational clutter – we omit the arguments of $u(.)$.

3.5 Permanent reduction in rate of devaluation

Suppose that just before $t = 0$, the economy is in the stationary equilibrium just described. At $t = 0$, there is an unanticipated and permanent reduction in the devaluation rate from $\bar{\varepsilon}$ to $\bar{\varepsilon}^L$ (Figure 2, Panel A). In response, the nominal interest rate falls permanently as well (Figure 2, Panel B). Clearly, both consumption and leisure will be constant along the new perfect foresight equilibrium path as shown above for the initial perfect foresight path. To find out how the fall in the nominal interest rate affects consumption and leisure, totally differentiate equations (32) and (33) (rewritten as $1-\bar{\ell}-\bar{c} = -rk_0$) with respect to $\bar{c}$, $\bar{\ell}$, and $\bar{\imath}$ to obtain, in matrix form,

$$\begin{bmatrix} \frac{-(u_c u_{\ell c} - u_\ell u_{cc})}{u_\ell^2} & \frac{u_\ell u_{c\ell} - u_c u_{\ell\ell}}{u_\ell^2} \\ -1 & -1 \end{bmatrix} \begin{bmatrix} d\bar{c} \\ d\bar{\ell} \end{bmatrix} = \begin{bmatrix} \alpha \\ 0 \end{bmatrix} di.$$

[Figure 2]

Applying Cramer's rule, we obtain:

$$\frac{d\bar{c}}{di} = -\frac{\alpha}{\Delta} < 0, \qquad (34)$$

$$\frac{d\bar{\ell}}{di} = \frac{\alpha}{\Delta} > 0, \qquad (35)$$

where

$$\Delta \equiv \frac{1}{u_\ell^2}\left(\underbrace{u_c u_{\ell c} - u_\ell u_{cc}}_{+} + \underbrace{u_\ell u_{c\ell} - u_c u_{\ell\ell}}_{+}\right) > 0.$$

The determinant Δ is positive – as indicated – due to the assumption of normality of c and ℓ (recall conditions (21) and (22)). Hence, in response to a permanent reduction in the nominal interest rate, equations (34) and (35) tell us that consumption increases and leisure falls (i.e., output increases), as illustrated in Figure 2, Panels C and D, respectively. The trade balance and current account, of course, do not change (Figure 2, Panel E and F).

In sum, a permanent reduction in the devaluation rate leads to a permanent increase in consumption and output. Intuitively, the corresponding fall in the nominal interest rate makes consumption more attractive in relation to leisure. Consumers thus consume more and work more.

3.6 Temporary reduction in rate of devaluation

Suppose that, just before $t = 0$, the economy is in the stationary equilibrium just described. Further, suppose $u_{\ell c} < 0$.[11] At $t = 0$, there is an unanticipated

[11] As will become clear below, the assumption that consumption and leisure are Edgeworth substitutes is critical to obtain a fall in ouput at time T. Exercise 3 at the end of this chapter examines the cases in which $u_{\ell c} \geq 0$.

and temporary reduction in the devaluation rate from $\bar{\varepsilon}$ to $\bar{\varepsilon}^L$ (Figure 3, Panel A). In light of the interest parity condition, the nominal interest rate follows the path depicted in Figure 3, Panel B.

[Figure 3]

We begin by characterizing the change in consumption and leisure at time T. To this effect, totally differentiate equations (28) and (29) at time T obtain, in matrix form,

$$\begin{bmatrix} u_{cc} & u_{c\ell} \\ u_{\ell c} & u_{\ell\ell} \end{bmatrix} \begin{bmatrix} dc \\ d\ell \end{bmatrix} = \begin{bmatrix} \lambda\alpha \\ 0 \end{bmatrix} di.$$

Applying Cramer's rule, we obtain:

$$\left.\frac{dc}{di}\right|_{t=T} = \frac{\lambda\alpha u_{\ell\ell}}{u_{cc}u_{\ell\ell} - u_{c\ell}^2} < 0, \tag{36}$$

$$\left.\frac{d\ell}{di}\right|_{t=T} = -\frac{\lambda\alpha u_{\ell c}}{u_{cc}u_{\ell\ell} - u_{c\ell}^2} > 0. \tag{37}$$

While consumption unambiguously falls at time T in response to the increase in the nominal interest rate, the change in leisure at time T depends on the sign of the cross-derivative between consumption and leisure. Under the maintained assumption that $u_{\ell c} < 0$, the increase in the nominal interest rate at time T will lead to an increase in leisure (i.e., a reduction in labor). Hence, at time T, both consumption and output fall.

The changes in consumption and leisure at time 0 can be found by contradiction (see Appendix 7.2). Consumption increases at time 0 while leisure falls (and hence output increases). Panels C and D in Figure 3 illustrate the paths of consumption and leisure, respectively.

What will happen to the trade balance? The behavior of the trade balance is in principle ambiguous. However, it is easy to establish that for the separable case (i.e., $u_{\ell c} = 0$), consumption increases by more than output on impact, which leads to a trade deficit.[12] Using a continuity argument, this will also be true for functions with $u_{\ell c} < 0$ which are "close" to the separable case. For such cases, the trade balance will go into deficit at time 0 (assuming initial net assets are zero), as illustrated in Figure 3, Panel E. The corresponding behavior of the current account is illustrated in Figure 3, Panel F.

In sum, under the assumption that $u_{\ell c} < 0$, a temporary reduction in the devaluation rate will lead to a boom-bust cycle in both consumption and output, which is consistent with the stylized facts associated with exchange rate-based stabilization (see Chapter 14 on inflation stabilization).

[12] Notice that in the separable case, labor goes up permanently at $t = 0$. To satisfy the intertemporal constraint, consumption must go up by more than output at $t = 0$ (for details, see Exercise 3 at the end of this chapter).

3.7 Welfare effects

The key idea is that, unlike the model with no labor supply, a constant but positive nominal interest rate introduces a *static (i.e., intratemporal) distortion* in this model. To see this, notice that a planner would set the relative price between consumption and leisure equal to the marginal productivity of labor (which, given the linear production function (23), is one). In addition, a non-constant nominal interest rate would introduce the *intertemporal distortion* analyzed above. The first-best in this economy is therefore to have a constant nominal interest rate equal to zero.[13] This eliminates both the intratemporal and intertemporal distortions.

In light of this discussion, it should be clear that a permanent reduction in the rate of devaluation will always be welfare improving because it reduces the intratemporal distortion between consumption and leisure. A temporary stabilization, on the other hand, will have ambiguous welfare implications because the reduction in the intratemporal distortion during the period $[0, T)$ – which is welfare improving – must be traded-off against the intertemporal distortion introduced by a non-constant path of the nominal interest rate (which is welfare reducing).

To illustrate these ambiguous welfare implications, consider the case in which preferences are given by the following CES specification:

$$u(c_t, \ell_t) = \frac{c^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}, \tag{38}$$

where c is a composite defined as

$$c = z^{\frac{\rho}{\rho-1}}, \tag{39}$$

where

$$z = q \log(c_t) + (1 - q) \log(\ell_t). \tag{40}$$

Figure 4 plots the change in welfare (relative to an initial stationary equilibrium) resulting from a temporary stabilization as a function of T and for three different values of the intertemporal elasticity of substitution ($\sigma = 1$, 2, and 5)[14] The first observation is that, as we expected, the temporary stabilization is always welfare improving for large values of T. We knew this would be the case because, as analyzed above, a permanent stabilization is welfare improving due to the reduction in the intratemporal distortion. For small values of T, the welfare change depends on the value of σ. We can see that, for the separable case (i.e., $\sigma = 1$), the temporary stabilization is always welfare improving as the wealth effect dominates the intertemporal substitution effects. For $\sigma = 2$, however, the temporary stabilization is welfare reducing for low values of T, reflecting the larger intertemporal distortion. The negative welfare effects for

[13] This is, of course, the celebrated Friedman rule which, in this case, would call for setting the rate of devaluation equal to $-i^*$ so that, through the interest parity condition, $i = 0$.

[14] The parameters underlying Figure 4 are the following: $r = 0.01$, $\varepsilon^L = 0.01$, $\varepsilon^H = 0.9$, $\alpha = 0.5$, and $q = 0.5$.

small values of T are even larger for $\sigma = 5$, as the higher elasticity of substitution implies a larger intertemporal distortion.

[Figure 4]

In sum, when a labor/leisure choice is introduced into our model, the welfare implications of a temporary stabilization will depend on the relative strength of the intertemporal distortion effect and the wealth effect. For large values of T, however, the latter will always dominate.

4 A transactions costs model

An important variation of the cash-in advance model – which also introduces a link between changes in the nominal interest rate and consumption – is the transactions costs model. Unlike the cash-in-advance model, however, the transactions costs model exhibits wealth effects resulting from changes in the nominal interest rate that have important implications from a welfare point of view.

4.1 Consumer's problem

Let preferences be given by

$$\int_0^\infty \frac{c_t^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} e^{-\beta t} dt, \tag{41}$$

where c is consumption of the only (tradable) good and $\sigma(> 0)$ denotes the intertemporal elasticity of substitution. Adopting these CES preferences will allow us to derive closed-form solutions that will nicely illustrate the strength of the intertemporal consumption substitution effects relative to the wealth effect.

Transacting is assumed to be a costly activity. Real money balances are assumed to reduce transactions costs, denoted by s, according to the following transactions technology:

$$s_t = c_t v\left(\frac{m_t}{c_t}\right), \qquad v'(.) \leq 0,\ v''(.) > 0. \tag{42}$$

The consumer's flow constraint is therefore given by:

$$\dot{a}_t = r a_t + y + \tau_t - c_t - s_t - i_t m_t, \tag{43}$$

where y is the constant endowment of the good. After substituting (42) into (43), we can express the consumer's intertemporal constraint (imposing, of course, the standard transversality condition) as

$$a_0 + \frac{y}{r} + \int_0^\infty \tau_t e^{-rt} dt = \int_0^\infty \left\{ c_t \left[1 + v\left(\frac{m_t}{c_t}\right)\right] + i_t m_t \right\} e^{-rt} dt. \tag{44}$$

Consumers choose $\{c_t, m_t\}_{t=0}^{\infty}$ to maximize lifetime utility (41) subject to (44). The first-order condition (assuming $\beta = r$) are given by

$$
\begin{aligned}
c_t^{-\frac{1}{\sigma}} &= \lambda \left[1 + v\left(\frac{m_t}{c_t}\right) - \frac{m_t}{c_t} v'\left(\frac{m_t}{c_t}\right)\right], \qquad (45) \\
-v'\left(\frac{m_t}{c_t}\right) &= i_t.
\end{aligned}
$$

The last equation implicitly defines

$$
\frac{m_t}{c_t} = L(i), \qquad L'(i) < 0. \tag{46}
$$

Substituting (46) into the RHS of (45), we can define the effective price of consumption as:

$$
p(i) \equiv 1 + v\left(L(i)\right) - L(i)v'\left(L(i)\right), \tag{47}
$$

which is an increasing function of the nominal interest rate:

$$
p'(i) = -L(i)v''\left(L(i)\right)L'(i) > 0.
$$

Using (47), we can write first-order condition (45) as:

$$
c_t^{-\frac{1}{\sigma}} = \lambda p(i), \tag{48}
$$

which is, of course, reminiscent of first-order condition (6) in the cash-in-advance model.

4.2 Government

The government budget constraints continue to be given by equations (9) and (10).

4.3 Equilibrium conditions

Interest parity holds:

$$
i_t = r + \pi^* + \varepsilon.
$$

Combining the households' and the government's flow constraints – given by (43) and (9) – and using (42) and (46) – we obtain

$$
\dot{k}_t = rk_t + y - c_t\left[1 + v(L(i_t))\right]. \tag{49}
$$

As expected – since transactions costs capture a social loss – they appear in the economy's flow constraint (i.e., the current account).

Combining the consumer's and the government's intertemporal constraints – given by equations (44) and (10), respectively – and using (42) and (46) – we obtain

$$k_0 + \frac{y}{r} = \int_0^{\infty} \{c_t [1 + v(L(i_t))]\} e^{-rt} dt. \tag{50}$$

4.4 Perfect foresight equilibrium

Let us characterize the perfect foresight equilibrium path corresponding to a constant rate of devaluation, $\bar{\varepsilon}$. By interest parity, the nominal interest rate will also be constant and equal to

$$\bar{\imath} = r + \bar{\varepsilon}.$$

The constancy of the nominal interest rate implies, by (47), that the effective price of consumption is also constant at a value given by $p(\bar{\imath})$. Then, by (48), consumption is also constant along a perfect foresight equilibrium path. Hence, taking into account the resource constraint (50) and (42),

$$\bar{c} = \frac{rk_0 + y}{1 + v\left(L(\bar{\imath})\right)}. \tag{51}$$

Finally, real money balances will be given by the money demand (46):

$$\bar{m} = \bar{c}L(\bar{\imath}). \tag{52}$$

4.5 Permanent reduction in the devaluation rate

Suppose that for $t < 0$ the economy is in the stationary equilibrium characterized above. At $t = 0$, there is an unanticipated and permanent reduction in the devaluation rate (Figure 4, Panel A). How will the economy respond?

[Figure 5]

By interest parity, the nominal interest rate falls permanently as well (Panel B). As the consumer reoptimizes at $t = 0$, the expression for consumption given in (51) continues to be valid. Hence, we infer that consumption will be higher in the new equilibrium path (Panel C). By the same token, real money demand will be higher because of both higher consumption and a lower nominal interest rate (Panel D).

Intuitively, the fall in the nominal interest rate reduces the opportunity cost of holding real money balances. Increased real money balances reduce transactions costs for any given level of consumption. The resulting wealth effect induces households to increase consumption.

The higher path of consumption implies that lifetime utility is now higher. The economy's resources have increased because of the reduction in transactions costs. This benefit of lower inflation was not present in the cash-in-advance model.

4.6 Temporary reduction in the devaluation rate

Suppose now that at $t = 0$ there is an unanticipated and temporary reduction in the devaluation rate (Figure 5, Panel A). By interest parity – and denoting by i^1 and i^2 the values of i between $[0, T)$ and $[T, \infty)$, respectively – the path of the nominal interest rate mimics the path of the devaluation rate (Figure 5, Panel B).

[Figure 6]

From the first-order condition (48), it follows that consumption between 0 and T (denoted by c^1) will be higher than after T (c^2). Notice, however, that the economy's resources are now higher since the temporary fall in the nominal interest rate will reduce transactions costs for a given level of consumption. Intuitively, we expect c^1 to be higher than consumption before the shock (c_{0^-}). We can, in fact, compute a reduced-form for c^1 to verify this conjecture. To this end, use (48) to solve for c and substitute in the resource constraint to solve for λ to obtain:

$$\lambda^\sigma = \frac{\int_0^\infty \frac{1}{[p(i_t)]^\sigma}\{1 + v(L(i_t)\} e^{-rt} dt}{k_0 + \frac{y}{r}}.$$

Now substitute back this expression into the first-order condition (48) to obtain:

$$c_t = \frac{1}{[p(i_t)]^\sigma} \frac{k_0 + \frac{y}{r}}{\int_0^\infty \frac{1}{[p(i_t)]^\sigma}\{1 + v(L(i_t))\} e^{-rt} dt}. \tag{53}$$

We can now use the path of the nominal interest rate depicted in Figure 5, Panel B, to compute c^1:

$$c^1 = c_{0^-} \frac{1}{\left[\frac{1+v(L(i^1))}{1+v(L(i^2))}\right](1 - e^{-rT}) + \left[\frac{p(i^1)}{p(i^2)}\right]^\sigma e^{-rT}} > c_{0^-},$$

where

$$c_{0^-} = \frac{rk_0 + y}{1 + v(L(i^2))}$$

is the pre-shock value of the nominal interest rate. As expected, $c^1 > c_{0^-}$ because both the intertemporal consumption substitution effect and the positive wealth effect call for higher consumption between 0 and T.

What about c^2? Using (53), we obtain

$$c^2 = c_{0^-} \frac{1}{\left[\frac{1+v(L(i^1))}{1+v(L(i^2))}\right]\left[\frac{p(i^2)}{p(i^1)}\right]^\sigma (1 - e^{-rT}) + e^{-rT}} \gtreqless c_{0^-}. \tag{54}$$

As expected, the value of c^2 may be equal, higher, or lower than the pre-shock level of consumption, c_{0^-}. Intuitively, the two effects – intertemporal consumption substitution and wealth effect – affect c^2 in opposite directions. The intertemporal consumption substitution effect calls for a lower c^2 since c^2 has become more expensive relative to c^1. On the other hand, the positive wealth effect, stemming from reduced transaction costs, calls for a higher c^2. The relative strength of these two effects will determine the level of c^2 relative to c_{0^-}.

It is easy to see from (54) that a sufficient condition for $c^2 \geq c_{0^-}$ is that

$$\underbrace{\left[\frac{p(i^2)}{p(i^1)}\right]^{\sigma}}_{\text{intertemporal subst. effect}} \leq \underbrace{\frac{1+v(L(i^2))}{1+v(L(i^1))}}_{\text{wealth effect}},$$

where, as indicated, the LHS captures the intertemporal substitution effect while the RHS captures the wealth effect. (Notice that both sides of this inequality are greater or equal than one.) As a particular case, if transactions costs were not a social cost (because, say, these transactions costs were provided at no cost by some government agency), then the RHS would be equal to 1 and the inequality would never hold. In other words, there would be no wealth effect and we would be back to the cash-in-advance world in which a temporary fall in the nominal interest rate induces a pure intertemporal substitution effect. At the other extreme, if σ were close to zero (almost no intertemporal substitution), then the LHS would be close to one, which implies that the wealth effect would dominate and c^2 would be larger than c_{0^-}.

Figure 5, Panel C, illustrates the path of consumption (assuming that the intertemporal substitution effect dominates). This implies that the trade balance follows the path illustrated in Panel D (assuming initial net foreign assets equal to zero). To see this, refer to equation (50) and notice that, if $c^2 < c_{0^-}$, then $c^2\left[1+v(L(i^2)\right] < c_{0^-}\left[1+v(L(i^2)\right]$ and hence the trade balance is in surplus for $t \geq T$. Since the present value of output has not changed, the trade balance must be in deficit between time 0 and time T.

4.7 Welfare

Clearly, a permanent reduction in the devaluation rate increases welfare due to the wealth effect. What about a temporary fall in the devaluation rate? The welfare implications depend on the relative strength of the intertemporal consumption substitution effect and the wealth effect. To illustrate this, suppose that the transactions technology is given by

$$s_t = c_t\left[\left(\frac{m_t}{c_t}\right)^2 - \frac{m_t}{c_t} + \frac{1}{4}\right], \qquad \frac{m}{c} \leq \frac{1}{2}. \tag{55}$$

Figure 7 depicts the change in welfare (relative to the initial stationary equilibrium) resulting from a temporary reduction in the devaluation rate as a function of T and for three different values of the intertemporal elasticity of substitution ($\sigma = 1$, 3, and 5).[15] As expected, for large values of T, the wealth effect dominates (in the limit, the reduction in the devaluation rate is permanent) and

[15] Parameter values are as follows: $k_0 = 0$, $r = 0.01$, $i^1 = 0.001$, $i^2 = 0.05$, and $y = 10$.

welfare increases regardless of the value of σ. For smaller values of T, however, whether the temporary stabilization is welfare improving or not depends on the value of σ. We can see in the figure that for $\sigma = 1$ (the logarithmic case), the temporary stabilization is always welfare improving. For $\sigma = 3$, however, the intertemporal distortion becomes larger and stabilization is welfare reducing for small values of T. For $\sigma = 5$, the fall in welfare for small values of T is, as expected, larger.

[Figure 7]

5 Monetary policy under flexible exchange rates

So far we have dealt with the case of predetermined exchange rates. We now study the same cash-in-advance model of Section 2 for an economy operating under flexible exchange rates. We will see that a permanent change in the rate of monetary growth has no real effects but a temporary reduction will lead, as in the predetermined exchange rates case, to an initial consumption boom and real appreciation but with a different dynamic pattern.

To simplify the analysis, we consider a logarithmic version of the separable preferences given in (1):

$$\int_0^\infty [\log(c_t^T) + \log(c_t^N)]e^{-\beta t}dt.$$

The rest of the model remains unchanged.

Under these logarithmic preferences, first-order conditions (6) and (7) reduce to:

$$\frac{1}{c_t^T} = \lambda(1 + \alpha i_t), \tag{56}$$

$$\frac{1}{c_t^N} = \lambda\frac{(1 + \alpha i_t)}{e_t}. \tag{57}$$

Combining these two first-order conditions yields:

$$\frac{c_t^N}{c_t^T} = e_t. \tag{58}$$

Substituting this expression into the cash-in-advance constraint (3), we obtain

$$m_t = 2\alpha c_t^T. \tag{59}$$

With these preliminary steps in place, we are ready to characterize a stationary perfect foresight equilibrium path.

5.1 Perfect foresight equilibrium path

Let us characterize the perfect foresight equilibrium path for this economy for a constant rate of money growth, $\bar{\mu}$. We will first show that the nominal interest rate will be constant. To this effect, we will derive an unstable differential equation in the nominal interest rate.[16] Differentiating first-order condition (56) along a perfect foresight path (recall that λ is constant along such a path), we obtain

$$\frac{\dot{c}_t^T}{c_t^T} = -\frac{\alpha}{1+\alpha i_t}\dot{i}_t.$$

Since it follows from (59) that $\dot{m}_t/m_t = \dot{c}_t^T/c_t^T$, we can rewrite the last expression as

$$\frac{\dot{m}_t}{m_t} = -\frac{\alpha}{1+\alpha i_t}\dot{i}_t. \tag{60}$$

Recall that, by definition, $m_t = M_t/E_t P_t^{T*}$. Hence,

$$\frac{\dot{m}_t}{m_t} = \bar{\mu} - \varepsilon_t - \pi^*. \tag{61}$$

Substituting (61) into (60),

$$\dot{i}_t = \frac{1+\alpha i_t}{\alpha}(\varepsilon_t + \pi^* - \bar{\mu}).$$

But, from the interest parity condition (equation (11)) and the Fisher equation in the rest of the world ($i^* = r + \pi^*$), $\varepsilon_t + \pi^* = i_t - r$. Hence,

$$\dot{i}_t = \frac{1+\alpha i_t}{\alpha}(i_t - r - \bar{\mu}). \tag{62}$$

This is an unstable differential equation. Hence, unless the nominal interest rate starts at the value $i_t = r + \bar{\mu}$, it will diverge over time. Since we wish to rule out non-convergent paths, we conclude that the only convergent perfect foresight equilibrium path is the one in which $i_t = r + \bar{\mu}$ for all t.

Since the nominal interest rate is constant along a perfect foresight equilibrium path – and so is, of course, λ – it follows from first-order condition (56) that consumption of tradables will also be constant along such a path. Hence, from the resource constraint (14),

$$c^T = rk_0 + y^T. \tag{63}$$

[16] We could instead proceed as in Chapter 5 and derive a differential equation in m which, would, of course, lead to the same results. In this case, however, deriving a differential equation in i is more convenient because it completely pins down the stationary value of the nominal interest rate. The differential equation in m (which the reader may want to derive as an exercise) would not, in and of itself, pin down the stationary value of m because the Lagrange multiplier would show up.

Since the endowment of non-tradables is fixed, $c_t^N = y^N$ for all $t \geq 0$. From the static condition (58) and using (63), the real exchange rate will also be constant over time and given by

$$e = \frac{y^N}{rk_0 + y^T}.$$

5.2 Permanent reduction in the rate of growth of money supply

Suppose that at time 0 there is an unanticipated and temporary reduction in the rate of money growth, $\bar{\mu}$. Clearly, differential equation (62) remains valid. The stationary value of the nominal interest rate falls. The nominal interest rate must adjust immediately to its lower stationary value for, if it did not, it would diverge over time. Since i is still constant over time (though at a lower level), first-order condition (56) implies that c_t^T will be constant as well and hence continues to be given by (63). Clearly, consumption of non-tradables and the real exchange rate are not affected either. In sum, a permanent reduction in the rate of monetary growth leads to an immediate fall in the nominal interest rate without any real effects.

5.3 Temporary reduction in rate of growth of money supply

Suppose now that, starting from the stationary perfect foresight equilibrium characterized above, there is an unanticipated and temporary reduction in the rate of money growth (Figure 8, Panel A).

[Figure 8]

To figure out the path of the nominal interest rate, consider the differential equation for i_t. As always when dealing with a temporary change, we need to first check whether i_t will jump or not at time T. The first importance piece of information is that, from equation (59), we can infer that c^T will not jump at T (since M_T does not jump by construction and E_T cannot jump because, if it did, it would give rise to infinite profit opportunities). Since c^T does not jump at T, it follows from first-order condition (56) that i will not jump either at T. The second piece of information is that during $[0, T)$ the differential equation in i_t will be governed by the stationary state corresponding to μ^L. Given these two pieces of information, it follows that the only converging path is the one illustrated in Figure 8, Panel B.

From the path for the nominal interest rate and the first-order condition (56), we infer that consumption must be falling over time. Since the temporary reduction in μ does not affect the economy's resources, consumption of tradable goods must follow the path illustrated in Figure 8, Panel C. The corresponding path of the trade balance is shown in Figure 8, Panel D. The path of the real exchange rate – illustrated in Figure 8, Panel E – then follows from equation (58) and the fact that consumption of non-tradable goods is, of course, constant over time.

Finally, notice that the path of inflation of home goods – illustrated in Figure 8, Panel F – follows the path of μ. To see this, rewrite (59) as (defining $n \equiv M/P^N$)

$$n_t = 2\alpha c_t^N,$$

which implies that since $c_t^N = y^N$ for all t, then

$$\pi_t = \mu_t.$$

Notice that the recession starts earlier than T since consumption falls below its preshock value before T. The same is true of the nominal interest rate which rises continuously before the rate of money growth increases. All this occurs *in anticipation* of the end of the stabilization.

6 Final remarks

This chapter has analyzed models in which monetary and exchange rate policy have real effects because of the impact of the path of the nominal interest rate on equilibrium allocations. A simple way of modelling this interaction is to introduce money via a cash-in-advance constraint. In such a world, the effective price of consumption includes the nominal interest rate because it represents the opportunity cost of holding the real money balances needed to purchase goods. A temporary reduction in the nominal interest rate thus makes today's consumption cheaper relative to tomorrow's and induces consumers to engage in intertemporal consumption substitution as in Chapter 3. This is true regardless of the exchange rate regime under which the economy is operating.

In the absence of wealth effects, temporary changes in monetary or exchange rate policy will be welfare reducing because there is no intrinsic benefit of temporarily lower inflation. When labor supply is introduced into the picture, however, permanent reductions in either the rate of devaluation or the rate of money growth are welfare improving because they reduce the intratemporal distortion between consumption and leisure and lead to higher labor supply and thus output. Temporary reductions in the rate of devaluation or money supply growth can thus be welfare improving if the labor supply effect dominates the intertemporal distortion effect. In a similar vein, if money plays a productive role such as reducing transaction costs, then a temporary reduction in, say, the rate of devaluation may be welfare improving if the resulting wealth effect dominates the intertemporal distortion effect.

In the next chapter, we will introduce yet another friction – sticky prices – that will cause monetary and exchange rate policy to have real effects.

7 Appendices

7.1 The cash-in-advance constraint in continuous time

This appendix – based on Feenstra (1985) – shows how the cash-in-advance given by (3) can be interpreted as a first-order approximation to the "true" cash-in-advance in continuous time.

For notational convenience, denote total consumption by c:

$$c \equiv c^T + \frac{c^N}{e_t}.$$

The natural counterpart in continuous time of a discrete-time cash-in-advance would be given by

$$m_t = \int_t^{t+\alpha} c_s ds, \tag{64}$$

which says that at the beginning of the period $[t, t+\alpha]$, consumers are required to hold the real money balances necessary to purchase consumption during this period of length α.

Notice that the RHS of equation (64) is a function of the parameter α. Hence, we can write

$$F(\alpha) \equiv \int_t^{t+\alpha} c_s ds. \tag{65}$$

Taking a first-order approximation of $F(\alpha)$ around $\alpha = 0$, we obtain

$$F(\alpha)|_{\alpha=0} \approx F(0) + F'(0)\alpha. \tag{66}$$

Clearly, $F(0) = 0$.

To find out $F'(0)$, we need to first differentiate the function $F(\alpha)$ with respect to α by applying Leibniz's formula.[17] Leibniz's formula tells us that if we have an integral that depends on a parameter of the form

$$\varphi(\gamma) = \int_{\beta(\gamma)}^{\alpha(\gamma)} f(x, \gamma) dx.$$

Then, under appropriate regularity conditions,

$$\varphi'(\gamma) = f(\alpha(\gamma), \gamma)\alpha'(\gamma) - f(\beta(\gamma), \gamma)\beta'(\gamma) + \int_{\beta(\gamma)}^{\alpha(\gamma)} f_\gamma(x, \gamma) dx. \tag{67}$$

Applying Leibniz formula to (65), we obtain (notice that the second and third terms of the Leibniz rule are equal to zero in this particular case):

$$F'(\alpha) = c_{t+\alpha}.$$

Evaluating this expression at $\alpha = 0$, we obtain:

[17] See, for instance, Bartle (1976), p. 244-247.

$$F'(0) = c_t.$$

Substituting this last expression into (66) and recalling that $F(0) = 0$, we get

$$F(\alpha)|_{\alpha=0} = \alpha c_t. \tag{68}$$

Hence, given (65) and (68), we can rewrite (64) as

$$m_t = \alpha c_t.$$

We conclude that the cash-in-advance constraint in the text (equation (3)) can thus be viewed as a first-order approximation to the "true" cash-in-advance in continuous time given by (64).

7.2 Changes in consumption and leisure at $t = 0$

This appendix establishes the changes in consumption and leisure at time $t = 0$ that underlie Figure 3, Panels C and D. Recall that we have already established in the text that at time T leisure increases and consumption falls. Our intuition would tell us that, since the fall in i at time 0 makes consumption cheaper relative to leisure, we expect consumption to increase and leisure to fall at $t = 0$.

Claim Leisure falls at $t = 0$.

Proof To prove it, we proceed by contradiction. In other words, we will assume that leisure either remains constant or increases and establish a contradiction.

Suppose that leisure does not change at $t = 0$. Since it goes up at time T, it follows that the present discounted value of leisure increases or, which is the same, the PDV of output falls relative to its pre-shock value.

What about consumption? Consumption at T will be higher than before the shock. To show this, differentiate (30) with respect to c and ℓ, holding i constant to obtain:

$$\left.\frac{dc}{d\ell}\right|_{\text{constant } i} = \frac{u_{c\ell}u_\ell - u_c u_{\ell\ell}}{u_c u_{\ell c} - u_{cc}u_\ell} > 0, \tag{69}$$

where the sign follows from the normality conditions, given by (21) and (22). Since ℓ_t for $t \geq T$ is higher than before the shock, so will c. This implies that consumption will go up at $t = 0$ and fall at time T to a value higher than before the shock. Hence, the PDV value of consumption will be higher than before the shock, which is a contradiction since we have established that the PDV of output will be lower than before the shock.

Suppose now that leisure goes up at $t = 0$. Since it will also go up at time T, the PDV of leisure is higher, and then the PDV of output is lower, than before the shock.

Since leisure from T onwards is higher than before the shock, so will consumption, as follows from (69). This implies that consumption will go up at $t = 0$ and fall at time T to a value higher than before the shock. Hence, the PDV value of consumption will be higher than before the shock, which is a contradiction since we have established that the PDV of output will be lower than before the shock. QED.

We have thus proved that consumption increases at $t = 0$ while leisure falls (i.e., output increases). As drawn in Figure 3, Panels C and D, from T onwards, consumption is below its pre-shock value and output is above. This is not the only possible case but consumption and leisure for T onwards must satisfy condition (69) (which tells us that if, say, consumption is above its pre-shock value, leisure must also be above). Notice that, by a continuity argument, the case depicted in Figure 3 will be the one relevant for preferences with $u_{c\ell} < 0$ which are "close" to the separable case. We know this because we can easily show that for the separable case, labor goes up permanently at $t = 0$ and consumption increases at $t = 0$ and falls at time T *below* its pre-shock value.

7.3 The cash-in-advance model in discrete time

We have studied in this chapter a continuous-time cash-in-advance model in which the cash-in-advance constraint imposes an additional cost of consuming, captured by the opportunity cost of holding the money balances that are needed to purchase the goods. In such a model, a non-constant path of the nominal interest rate imposes an intertemporal distortion by affecting the path of the effective price of consumption.

What would be the discrete-time version of a cash-in-advance model that would capture this effect? From the cash-in-advance model in discrete time studied in Chapter 5, we may already guess that whether this distortion is present or not will depend on the timing assumptions underlying the model. In chapter 5, we adopted the more common specification of CIA models, in which asset markets open *before* goods markets. We saw that, in that context, the nominal interest rate does not affect consumption and therefore there will not be an intertemporal distortion even if the nominal interest rate fluctuates over time.

We will now see that if instead we assume that goods markets open before asset markets, there will be an intertemporal distortion. In other words, this timing would be the one implicitly corresponding to the continuous-time CIA version that we studied earlier in this chapter.

Preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \log(c_t). \tag{70}$$

The consumer's flow budget constraint remains the same as in Chapter 5:

$$b_t + m_t = (1 + r)b_{t-1} + \frac{m_{t-1}}{1 + \pi_t} + \tau_t + y_t - c_t. \tag{71}$$

The cash-in-advance constraint is given by

$$M_{t-1} + P_t\tau \geqslant P_t c_t. \tag{72}$$

Notice that we are assuming that the consumer' receives the government's transfers at the beginning of the period. Defining $m_{t-1} \equiv M_{t-1}/P_{t-1}$ and $P_t/P_{t-1} \equiv 1+\pi_t$, we can rewrite the cash-in-advance constraint as

$$\frac{m_{t-1}}{1+\pi_t} + \tau \geqslant c_t \tag{73}$$

The consumer chooses $\{c_t, m_t, b_t\}_{t=0}^{\infty}$ to maximize lifetime utility (70) subject to a sequence of flow constraints given by (71) and a sequence of cash-in-advance constraints given by (73). In terms of the Lagrangian,

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty}\beta^t \log(c_t) + \sum_{t=0}^{\infty}\beta^t\lambda_t\left\{(1+r)b_{t-1} + \frac{m_{t-1}}{1+\pi_t} + \tau_t + y_t - c_t - b_t - m_t\right\} \\ &\quad + \sum_{t=0}^{\infty}\beta^t\Psi_t\left(\frac{m_{t-1}}{1+\pi_t} - c_t\right)\end{aligned}$$

The first-order conditions with respect to c_t, m_t, and b_t are given, respectively, by (under our usual assumption that $\beta(1+r)=1$):

$$\frac{1}{c_t} = \lambda_t + \Psi_t, \tag{74}$$

$$\frac{\beta}{1+\pi_{t+1}}\left(\lambda_{t+1} + \Psi_{t+1}\right) = \lambda_t, \tag{75}$$

$$\lambda_t = \lambda_{t+1}. \tag{76}$$

From the law of one price, $P_t = E_t P_t^*$, it follows that $1+\pi_{t+1} = (1+\varepsilon_{t+1})(1+\pi_{t+1}^*)$ where $1+\pi_{t+1} \equiv P_{t+1}/P_t$, $1+\varepsilon_{t+1} \equiv E_{t+1}/E_t$, and $1+\pi_{t+1}^* \equiv P_{t+1}^*/P_t^*$. Using this piece of information, together with the interest parity condition

$$1+i_t = (1+r)(1+\pi_{t+1}^*)(1+\varepsilon_{t+1}),$$

we can rewrite optimality condition (75) as

$$\lambda_{t+1} + \Psi_{t+1} = \lambda_t\left(1+i_t\right). \tag{77}$$

Denote the constant Lagrange multiplier by λ. Using (76), we can express the nominal interest rate as

$$i_t = \frac{\Psi_{t+1}}{\lambda}, \tag{78}$$

which indicates that the cash-in-advance will be binding (i.e., $\Psi_{t+1} > 0$) as long as the nominal interest rate is positive. Substituting (78) into first-order condition (98),

$$\frac{1}{c_t} = \lambda(1 + i_{t-1}). \tag{79}$$

As in the continuous-time case, the effective price of consumption, $1 + i_{t-1}$, includes the opportunity cost of holding the real money balances needed to purchase the consumption good. Hence, a non-constant path of the nominal interest rate imposes intertemporal distortions that will lead to a non-constant path of consumption.

To solve the model, use (79) to obtain

$$\frac{c_{t+1}}{c_t} = \frac{1 + i_{t-1}}{1 + i_t} \tag{80}$$

The government budget constraint implies that $P_t\tau_t = M_t - M_{t-1}$. This, together with the cash-in-advance constraint (72), implies

$$M_t = P_t c_t. \tag{81}$$

Denote the money growth rate by μ_t; that is, $M_{t+1}/M_t = 1 + \mu_{t+1}$. Then, using (81),

$$\mu_{t+1} = \frac{P_{t+1}}{P_t}\frac{c_{t+1}}{c_t}.$$

Using (80) and recalling that $1 + \pi_{t+1} = (1 + i_t)/(1 + r)$, we can rewrite the last equation as

$$1 + i_{t-1} = (1 + \mu_{t+1})(1 + r).$$

The path of the nominal interest rate is fully determined by the path of the money growth rate. In particular, if the rate of money growth is constant over time, then the nominal interest rate will also be constant over time. If the rate of money growth were to increase along a perfect foresight path, so would the nominal interest rate. Hence, unlike what would occur in the continuous-time case, the nominal interest rate would not begin to increase in anticipation of the increase in μ. This difference is due to the fact that, in discrete time, the relationship between consumption and the nominal interest rate involves a lag (i.e., the effective price of consumption for c_t is given by $1+i_{t-1}$ as evidenced by equation (79)) whereas in continuous-time that relationship is contemporaneous (i.e., the effective price of c_t is $1 + \alpha i_t$).

7.4 MATLAB code for predetermined exchange rates

For the purposes of coding this model in MATLAB – and following the procedure outlined in Appendix 7.6 of Chapter 1 – we will have four control variables (c^T, c^N, i, and e); one co-state variable (λ); one state variable (k) and one flow variable (TB).

7.4.1 System of equations

The system of equations to be linearized is the following:

$$\frac{1}{c_t^T} = \lambda_t(1+i_{t-1}), \tag{C1}$$

$$c_t^T = \frac{c_t^N}{e_t}, \tag{C2}$$

$$1+i_{t-1} = (1+r)(1+\varepsilon_{t-1}), \tag{C3}$$

$$c_t^N = y^N, \tag{C4}$$

$$\lambda_t = \lambda_{t+1}, \tag{S1}$$

$$k_t = (1+r)k_{t-1} + y_t^T - c_t^T, \tag{S2}$$

$$TB_t = y_t^T - c_t^T. \tag{FV1}$$

7.4.2 Steady state

The steady-state, which follows from equations C1-C4, S1 and S2, and FV1 is given by

$$c_{ss}^T = y^T + rk_0, \tag{82}$$

$$c_{ss}^T = \frac{1}{\lambda_{ss}(1+i_{ss})},$$

$$c_{ss}^N = y^N, \tag{83}$$

$$e_{ss} = \frac{c_{ss}^N}{c_{ss}^T}, \tag{84}$$

$$i_{ss} = (1+r)(1+\varepsilon) - 1, \tag{85}$$

$$TB_{ss} = y^T - c_{ss}^T.$$

7.4.3 Linearization

The linearized system is given by:

$$
\begin{aligned}
-\widehat{c_t^T} - \frac{i_{ss}}{1+i_{ss}}\widehat{i_{t-1}} &= \widehat{\lambda}_t, \\
\widehat{c_t^N} - \widehat{c_t^T} - \widehat{e}_t &= 0, \\
\widehat{i_{t-1}} &= \frac{\varepsilon_{ss}}{i_{ss}}(1+r)\widehat{\varepsilon_{t-1}}, \\
\widehat{c_t^N} &= 0, \\
\widehat{\lambda}_t - \widehat{\lambda_{t+1}} &= 0, \\
\widehat{k}_t - (1+r)\widehat{k_{t-1}} &= -\frac{c_{ss}^T}{k_{ss}}\widehat{c_t^T}, \\
\widehat{TB_t} &= -\frac{c_{ss}^T}{TB_{ss}}\widehat{c_t^T}.
\end{aligned}
$$

Based on these linearized equations – and following Chapter 1, Appendix 7.6 – the coefficients to be inputed in the program *dynerbsflexprices.m* can be easily derived (the reader is referred to this program, which is posted on the book website).

MATLAB program The above system has been inputed into MATLAB and the corresponding program files can be found on the book website at http://www.econ.umd.edu/~vegh/book/book.htm. There are three files: (1) *dynerbsflexprices.m*; (2) *patherbsflexprices.m*; and (3) *plotimperbsflexprices.m*. To run this routine, you need to first run *dynerbsflexprices.m*, then *patherbsflexprices.m*, and finally *plotimperbsflexprices.m*.[18] Figure 9 shows the output of this routine for a temporary reduction of 10 percent in the devaluation rate.[19]

[Figure 9]

7.5 MATLAB code for MIUF, flexible exchange rates model

This appendix develops a discrete-time version of a MIUF model that will replicate the results obtained in the text for the continous-time cash-in-advance model (unless otherwise noticed, the same notation as in the text is used).

7.5.1 Model

Preferences are given by

$$
\sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left[(c_t)^{\eta} (z_t)^{1-\eta} \right]^{1-1/\sigma} - 1}{1-1/\sigma} \right\}, \qquad 0 < \eta < 1, \quad \sigma > 0 \tag{86}
$$

[18] See Chapter 1, Appendix 7.6, for details on each particular program.

[19] Parameters values are as follows: $r = 0.015$; $y^T = y^N = 1$; $k_0 = 5$, and $\varepsilon = 0.5$.

where

$$c_t \equiv \left(c_t^T\right)^{\gamma} \left(c_t^N\right)^{1-\gamma} \tag{87}$$

where c is a consumption aggregator and z denotes real money balances in terms of a price index, given by $(P^T)^{\gamma}\left(P^N\right)^{1-\gamma}$. The parameter σ is the intertemporal elasticity of substitution.

The flow constraint takes the form

$$a_t = (1+r)a_{t-1} + y_t^T + \frac{y_t^N}{e_t} + \tau_t - \left(c_t^T + \frac{c_t^N}{e_t}\right) - \frac{i_{t-1}}{1+\varepsilon_t} m_{t-1}. \tag{88}$$

Consumers choose $\{c_t^T, c_t^N, z_t, a_t\}$ to maximize (86) subject to a sequence of flow constraints given by (88). In terms of the Lagrangean:

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left[(c_t)^{\eta} (z_t)^{1-\eta}\right]^{1-1/\sigma} - 1}{1-1/\sigma} \right\} \\ &\quad + \sum_{t=0}^{\infty} \beta^t \lambda_t \left[(1+r)a_{t-1} + y_t^T + \frac{y_t^N}{e_t} + \tau_t - \left(c_t^T + \frac{c_t^N}{e_t}\right) - \frac{i_{t-1}}{1+\varepsilon_t} \frac{z_{t-1}}{e_{t-1}^{1-\gamma}} - a_t \right].\end{aligned}$$

Deriving and rearranging first-order conditions, we obtain (assuming, as usual, that $\beta(1+r)=1$):

$$\left[(c_t)^{\eta} (z_t)^{1-\eta}\right]^{-1/\sigma} \eta (z_t)^{1-\eta} (c_t)^{\eta-1} \left(\frac{c_t^N}{c_t^T}\right)^{1-\gamma} \gamma = \lambda_t \tag{89}$$

$$\left(\frac{\gamma}{1-\gamma}\right) \frac{c_t^N}{c_t^T} = e_t, \tag{90}$$

$$\left[(c_t)^{\eta} (z_t)^{1-\eta}\right]^{-1/\sigma} (c_t)^{\eta} (1-\eta) (z_t)^{-\eta} = \beta \lambda_{t+1} \frac{i_t}{1+\varepsilon_{t+1}} \frac{1}{e_t^{1-\gamma}} \tag{91}$$

$$\lambda_t = \lambda_{t+1} \tag{92}$$

where λ_t is the multiplier associated with constraint (88).

7.5.2 System of equations

The system of equations to be linearized comprises the following 10 equations:

$$\left[(c_t)^{\eta}(z_t)^{1-\eta}\right]^{-1/\sigma}\eta\,(z_t)^{1-\eta}(c_t)^{\eta-1}\left(\frac{c_t^N}{c_t^T}\right)^{1-\gamma}\gamma=\lambda_t, \tag{C1F}$$

$$\left(\frac{\gamma}{1-\gamma}\right)\frac{c_t^N}{c_t^T}=e_t, \tag{C2F}$$

$$m_t=\frac{c_t^T}{\gamma}\left(\frac{1-\eta}{\eta}\right)\frac{1}{\frac{i_t}{1+i_t}}, \tag{C3F}$$

$$1+i_t=(1+r)(1+\varepsilon_{t+1}), \tag{C4F}$$

$$c_t^N=y_t^N, \tag{C5F}$$

$$c_t=\left(c_t^T\right)^{\gamma}\left(c_t^N\right)^{1-\gamma}, \tag{C6F}$$

$$\lambda_t=\lambda_{t+1}, \tag{S1F}$$

$$\frac{m_{t+1}}{m_t}=\frac{1+\mu_{t+1}}{1+\varepsilon_{t+1}}, \tag{S2F}$$

$$a_t=(1+r)a_{t-1}+y_t^T-c_t^T, \tag{S3F}$$

$$TB_t-y_t^T+c_t^T=0. \tag{FV1F}$$

7.5.3 Steady-state

The steady-state is given by:

$$\begin{aligned}
c_{ss}^N &= y^N,\\
\varepsilon_{ss} &= \mu,\\
c_{ss}^T &= ra+y^T,\\
e_{ss} &= \frac{\gamma}{1-\gamma}\frac{y^N}{c_{ss}^T},\\
i_{ss} &= (1+r)(1+\mu)-1,\\
\lambda_{ss} &= \left[(c_{ss})^{\eta}\left(m_{ss}e_{ss}^{1-\gamma}\right)^{1-\eta}\right]^{-1/\sigma}\eta\left(m_{ss}e_{ss}^{1-\gamma}\right)^{1-\eta}(c_{ss})^{\eta-1}\left(\frac{c_{ss}^N}{c_{ss}^T}\right)^{1-\gamma}\gamma,\\
m_{ss} &= \frac{c_{ss}^T}{\gamma}\left(\frac{1-\eta}{\eta}\right)\frac{1}{\frac{i_{ss}}{1+i_{ss}}},\\
TB_{ss} &= y^T-c_{ss}^T,\\
c_{ss} &= \left(c_{ss}^T\right)^{\gamma}\left(c_{ss}^N\right)^{1-\gamma}.
\end{aligned}$$

7.5.4 Linearization

The linearized system takes the form

$$
\begin{aligned}
-\left[1+\eta\left(\frac{1-\sigma}{\sigma}\right)\right]\widehat{c_t}+(1-\gamma)\left[(1-\eta)\left(\frac{1-\sigma}{\sigma}\right)\right]\widehat{e_t}+(1-\gamma)\widehat{c_t^N}-(1-\gamma)\widehat{c_t^T} &= \widehat{\lambda}_t+\left[(1-\eta)\left(\frac{1-\sigma}{\sigma}\right)\right] \\
\widehat{c_t^N}-\widehat{c_t^T}-\widehat{e_t} &= 0, \\
\frac{1}{1+i}\widehat{i_t}-\widehat{c_t^T} &= -\widehat{m_t}, \\
\frac{i_t}{1+i_t}\widehat{i_t}-\frac{\varepsilon_{t+1}}{1+\varepsilon_{t+1}}\widehat{\varepsilon_{t+1}} &= 0, \\
\widehat{c_t^N} &= \widehat{y_t^N}, \\
\widehat{c_t}-\gamma\widehat{c_t^T}-(1-\gamma)\widehat{c_t^N} &= 0, \\
\widehat{\lambda}_t-\widehat{\lambda_{t+1}} &= 0, \\
\widehat{m_{t+1}}-\widehat{m_t} &= \frac{\mu_{t+1}}{1+\mu_{t+1}}\widehat{\mu_{t+1}}-\frac{\varepsilon_{t+1}}{1+\varepsilon_{t+1}} \\
\widehat{a_t}-(1+r)\widehat{a_{t-1}} &= \frac{y^T}{a_t}\widehat{y_t^T}-\frac{c_t^T}{a_t}\widehat{c_t^T}, \\
\widehat{TB_t} &= -\frac{c^T}{TB}\widehat{c_t^T}+\frac{y^T}{TB}\widehat{y_t^T}.
\end{aligned}
$$

MATLAB program The above system has been inputed into MATLAB and the corresponding program files can be found on the book website at http://www.econ.umd.edu/~vegh/book/book.htm. There are three files: (1) *dynflexiblerates.m*; (2) *pathflexiblerates.m*; amd (3) *plotimpflexiblerates.m*. To run this routine, you need to first run *dynflexiblerates.m*, then *pathflexiblerates.m*, and finally *plotimpflexiblerates.m*.[20] Figure 10 shows the output of this routine for a temporary reduction of 50 percent in the money growth rate.[21] Exercise 4 at the end of the chapter asks you to perform several additional experiments.

[Figure 10]

[20] See Chapter 1, Appendix 7.6, for details on each particular program.

[21] Parameters values are as follows: $\sigma = 1.5$, $\eta = \gamma = 0,5$, $r = 0.015$; $y^T = y^N = 1$; $k_0 = 5$, and $\mu = 0.5$.

Exercises

1. Exchange rate-based inflation stabilization with MIUF

This exercise shows that if money is introduced in the utility function and the cross derivative between money and consumption is positive, the same results of a temporary exchange rate-based stabilization with a cash-in-advanced derived in the text go through.

Consider the same economy analyzed in Section 2 with the only difference that preferences are given by

$$\int_0^\infty u(c_t, m_t)e^{-\beta t}dt, \tag{93}$$

where c is consumption of a tradable good, m denotes real money balances, and $\beta(>0)$ is the rate of time preference. The utility function $u(c_t, m_t)$ is increasing in both arguments and strictly concave. Specifically, it satisfies:

$$u_c > 0,\ u_m > 0,\ u_{cc} < 0,\ u_{mm} < 0,\ u_{cc}u_{mm} - u_{cm}^2 > 0. \tag{94}$$

(Notice that we have not assume any particular sign for u_{cm}, so $u_{cm} \gtreqless 0$.) Naturally, the cash-in-advance constraint is dropped since we have introduced money in the utility function.

In addition, assume that both good are normal, which implies that

$$\begin{aligned} u_m u_{cc} - u_c u_{mc} &< 0, \\ u_c u_{mm} - u_m u_{cm} &< 0. \end{aligned}$$

The rest of the model remains unchanged.

In the context of this model:

(a) Derive the first-order conditions. Show that they yield a standard money demand function.

(b) Characterize a perfect foresight equilibrium path for a constant rate of devaluation.

(c) Analyze the effects of an unanticipated and temporary exchange rate-based stabilization. In particular, show how the results critically depend on the sign of u_{cm}. Discuss the intuition behind the results.

(d) Suppose that preferences are given by

$$u(c_t, m_t) = (c^\alpha + m^\beta)^{\frac{1}{\sigma}}.$$

Show that u_{cm} is zero if $\sigma = 1$, positive if $\sigma < 1$ and negative if $\sigma > 1$.

2. Exchange rate-based inflation stabilization with fiscal implications

This exercise analyzes the case in which, instead of rebating the proceeds from the inflation tax and interest on reserves to the consumer, the government spends those proceeds (wasteful spending). We will see how in this case there is a wealth effect associated with either a permanent or temporary reduction in the devaluation rate.

The model is a one-good version of the cash-in-advance model of Section 2. Unless otherwise noticed, the notation remains the same. Preferences are given by

$$\int_0^\infty \log(c_t)e^{-\beta t}dt,$$

where c is consumption of the tradable good. Consumers' flow constraint takes the form:

$$\dot{a}_t = ra_t + y - c_t - i_t m_t, \tag{95}$$

while the intertemporal constraint reads as

$$a_0 + \frac{y}{r} = \int_0^\infty (c_t + i_t m_t)e^{-rt}dt.$$

The cash-in-advance constraint is given by

$$m_t = \alpha c_t.$$

The government's flow constraint is given by

$$\dot{h}_t = rh_t + \dot{m}_t + \varepsilon_t m_t - g_t,$$

where g denotes government spending.

The economy's flow constraint is given by

$$\dot{k}_t = rk_t + y - c_t - g_t,$$

while the economy's resource constraint is given by

$$k_0 + \frac{y}{r} = \int_0^\infty (c_t + g_t)e^{-rt}dt.$$

In the context of this model:

(a) Characterize a perfect foresight equilibrium path for a constant path of the rate of devaluation.

(b) Suppose that, starting from the perfect foresight equilibrium path characterized above, there is an unanticipated and permanent reduction in the rate of devaluation. Derive (and plot) the paths of all endogenous variables. What are the welfare implications? Explain the intuition behind your results.

(c) Suppose that, starting from the perfect foresight equilibrium path characterized above, there is an unanticipated and temporary reduction in the rate of devaluation. Derive (and plot) the paths of all endogenous variables. What are the welfare implications? Explain the intuition behind your results.

3. ## Temporary reduction in devaluation rate under alternative assumptions about the consumption-leisure cross-derivative

Consider the model with labor supply analyzed in the text. When it comes to a temporary reduction in the devaluation rate, we solved for the case in which $u_{c\ell} < 0$. You are asked to:

(a) Solve for the separable case (i.e., $u_{c\ell} = 0$). In particular, show that labor rises permanently at $t = 0$ and that consumption rises at $t = 0$ and falls at time T below its pre-shock value. Discuss the intuition behind he results.

(b) Solve for the case in which $u_{c\ell} > 0$. In particular, show that consumption always rises at $t = 0$ whereas labor could either increase, remain the same, or fall.

4. ## Numerical solution of MIUF flexible exchange rates model

Consider the discrete-time MIUF model developed in Appendix 7.5. Perform the following numerical exercises using the corresponding MATLAB programs:

(a) A temporary fall in the money growth rate for a value of $\sigma = 1$.(This is the separable case analyzed in Chapter 5.)

(b) A temporary fall in the money growth rate for a value of $\sigma = 0.5$. (Notice how the response in consumption of tradables is the opposite to the case of $\sigma = 1.5$ depicted in Figure 10).

(c) Compute analytically U_{cz} and show how it depends on the value of σ. Explain intuitively why the sign of U_{cz} critically affects the response of c^T. What is the case that replicates the cash-in-advance results obtained in the text?

(d) A temporary fall in y^T (for $\sigma = 1$). Explain the intuition behind the results.

(e) A temporary fall in y^N (for $\sigma = 1$). Explain the intuition behind the results.

References

[1] Bartle, Robert G. 1976. *The elements of real analysis*. Second ed. New York: John Wiley & Sons.

[2] Calvo, Guillermo A. 1986. Temporary stabilization: Predetermined exchange rates. *Journal of Political Economy* 94 (6): 1319-1329.

[3] Calvo, Guillermo A., and Carlos A. Végh. 1999. Inflation stabilization and BOP crises in developing countries. In *Handbook of Macroeconomics*, ed. John Taylor and Michael Woodford, Volume C, 1531-1614. Amsterdam: North-Holland.

[4] Clower, Robert W. 1967. A reconsideration of the microfoundations of monetary theory. *Western Economic Journal* 6 (1): 1-8.

[5] Drazen, Allan and Elhanan Helpman. 1987. Stabilization with exchange rate management. *Quarterly Journal of Economics* 102 (4): 835-855.

[6] Feenstra, Robert C. 1985. Anticipated devaluations, currency flight, and direct trade controls in a monetary economy. *American Economic Review* 75 (3): 386-401.

[7] Lahiri, Amartya. 2000. Disinflation programs under policy uncertainty. *Journal of International Economics* 50 (2): 351–373.

[8] Lucas, Robert E., Jr. 1982. Interest rates and currency prices in a two-country world. *Journal of Monetary Economics* 10 (3): 335-359.

[9] Reinhart, Carmen M. and Carlos A.Végh. 1995. Nominal interest rates, consumption booms, and lack of credibility: A quantitative examination. *Journal of Development Economics* 46 (2): 357-378.

[10] Roldos, Jorge. 1995. Supply-side effects of disinflation programs. *IMF Staff Papers* 42: 158-183.

[11] Roldos, Jorge. 1997. On gradual disinflation, the real exchange rate, and the current account. *Journal of International Money and Finance* 16: 37-54.

[12] Sargent, Thomas J. 1982. The ends of four big inflations. In *Inflation: Causes and consequences*, ed. Robert E. Hall. Chicago: University of Chicago Press.

[13] Svensson, Lars. 1985. Money and asset prices in a cash-in-advance economy. *Journal of Political Economy* 93 (5): 919-944.

Figure 1. Temporary reduction in devaluation rate

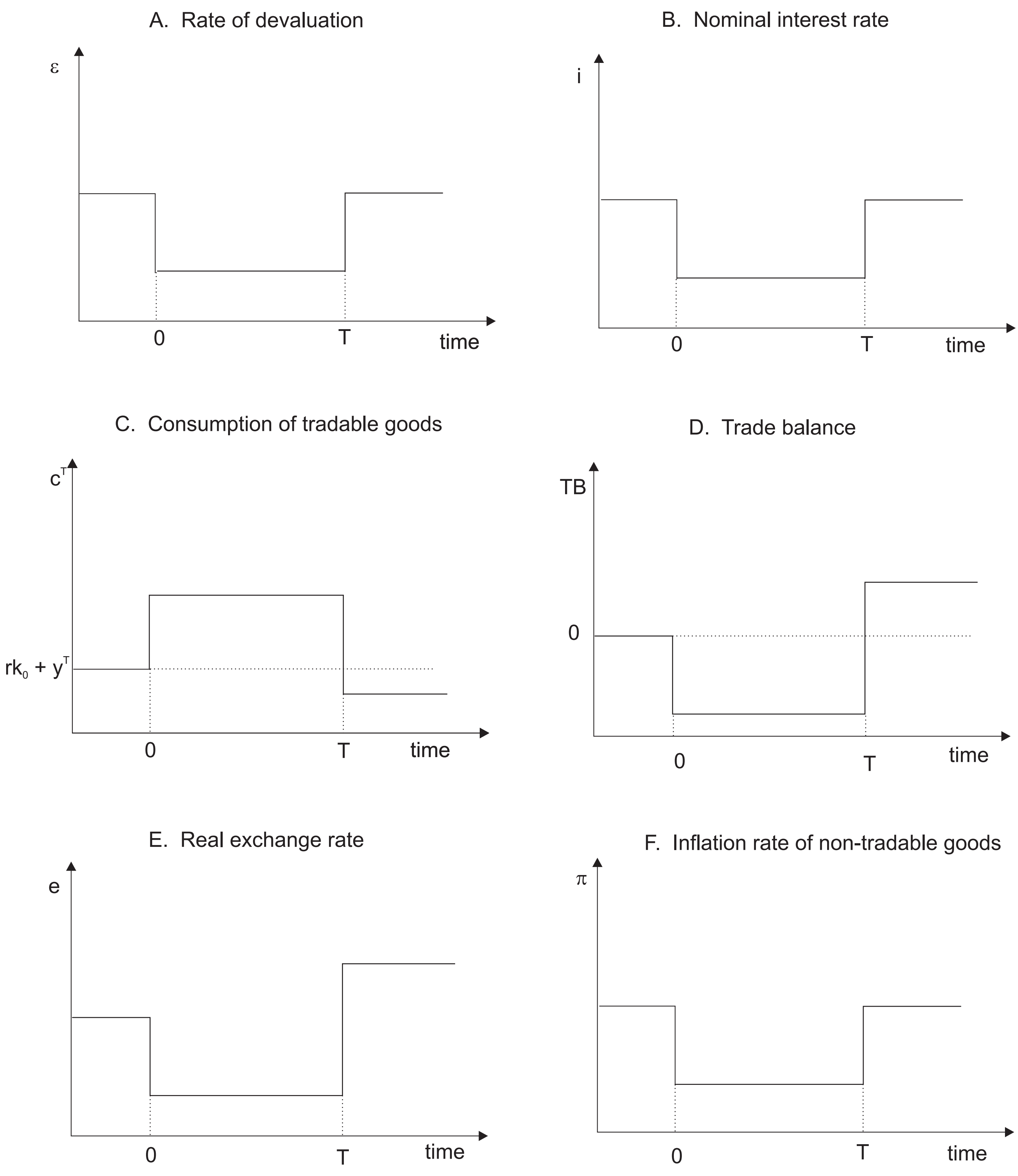

Figure 2. Permanent reduction in devaluation rate with labor supply

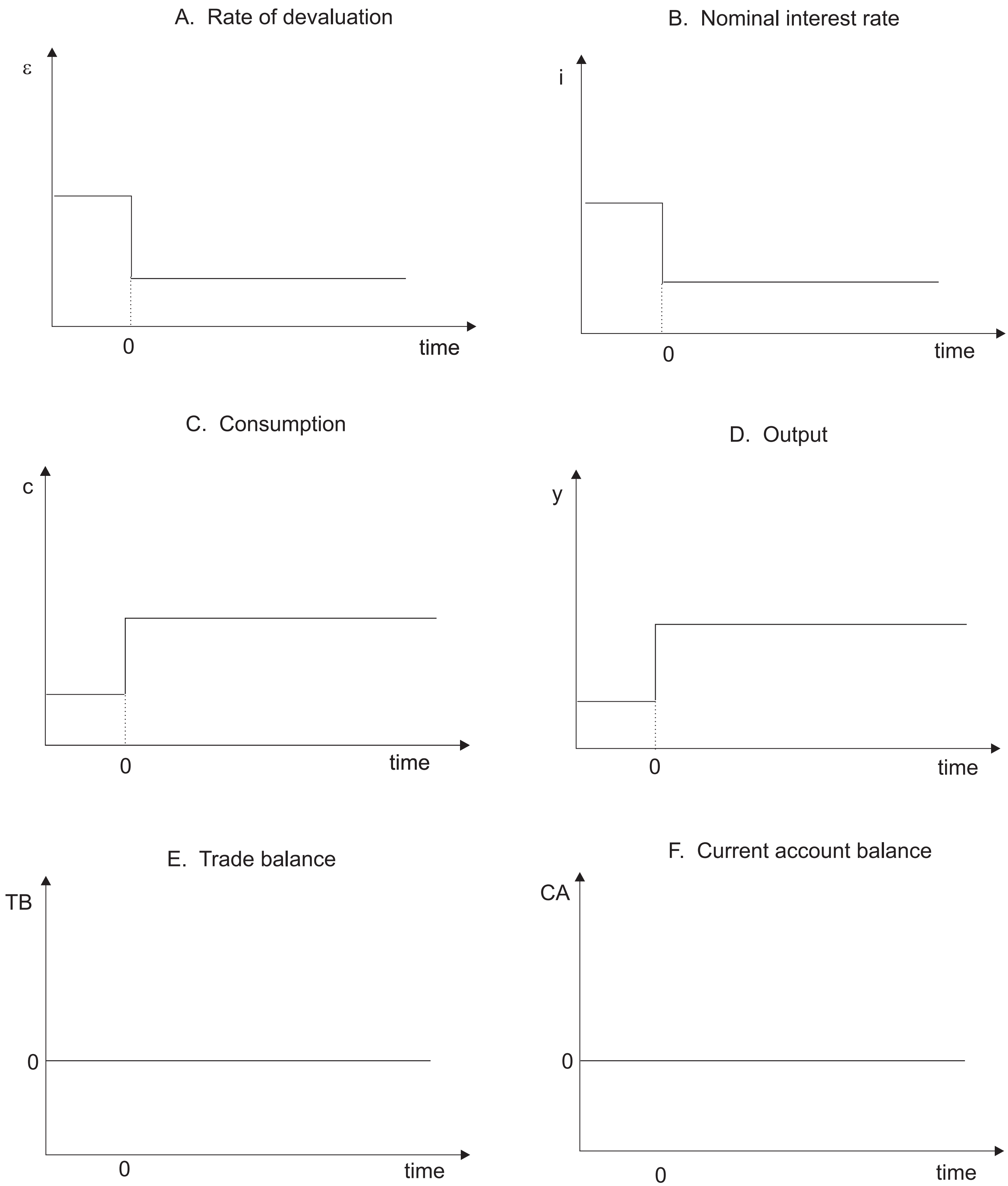

Figure 3. Temporary reduction in devaluation rate with labor supply

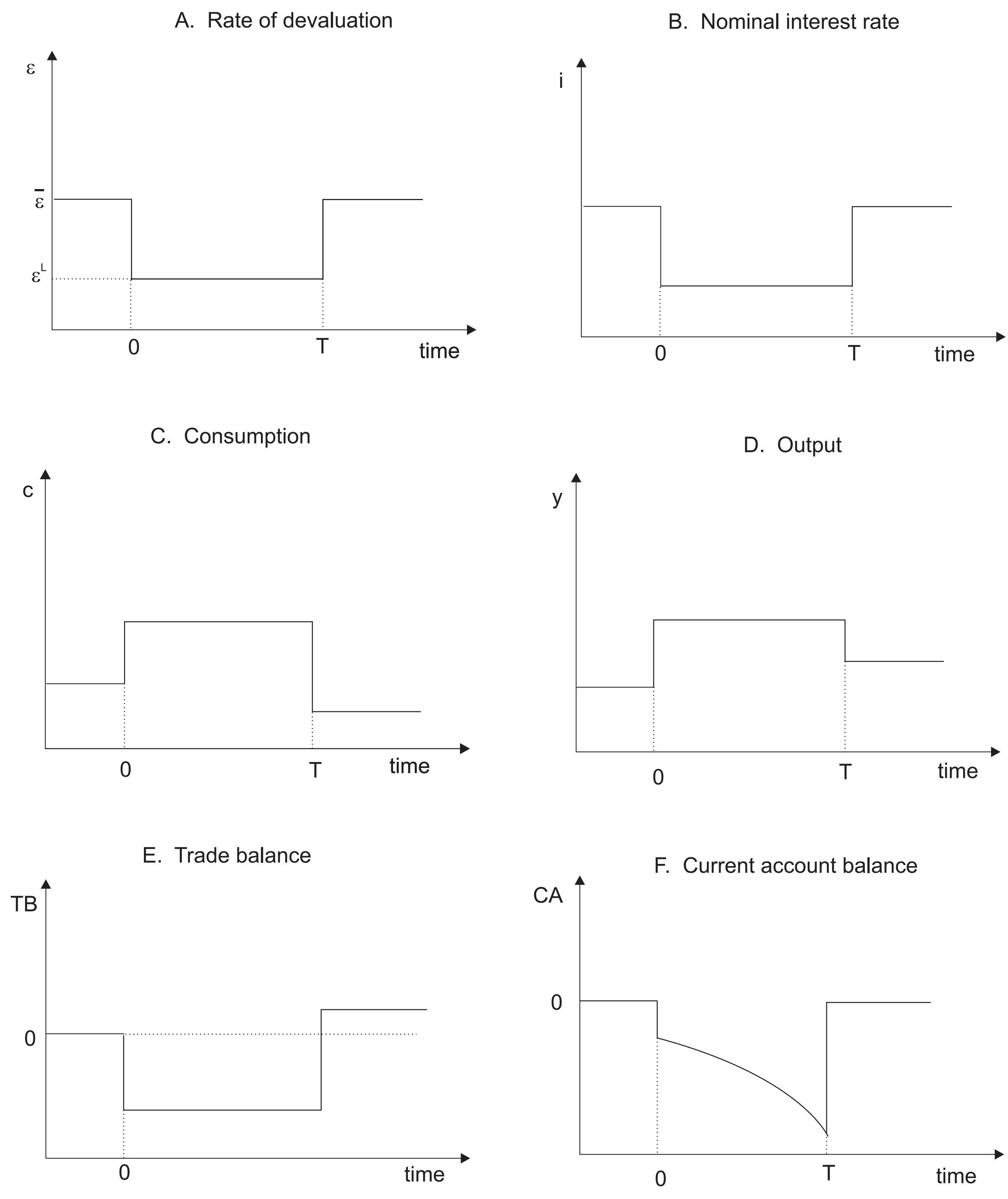

Figure 4. Welfare change as function of T

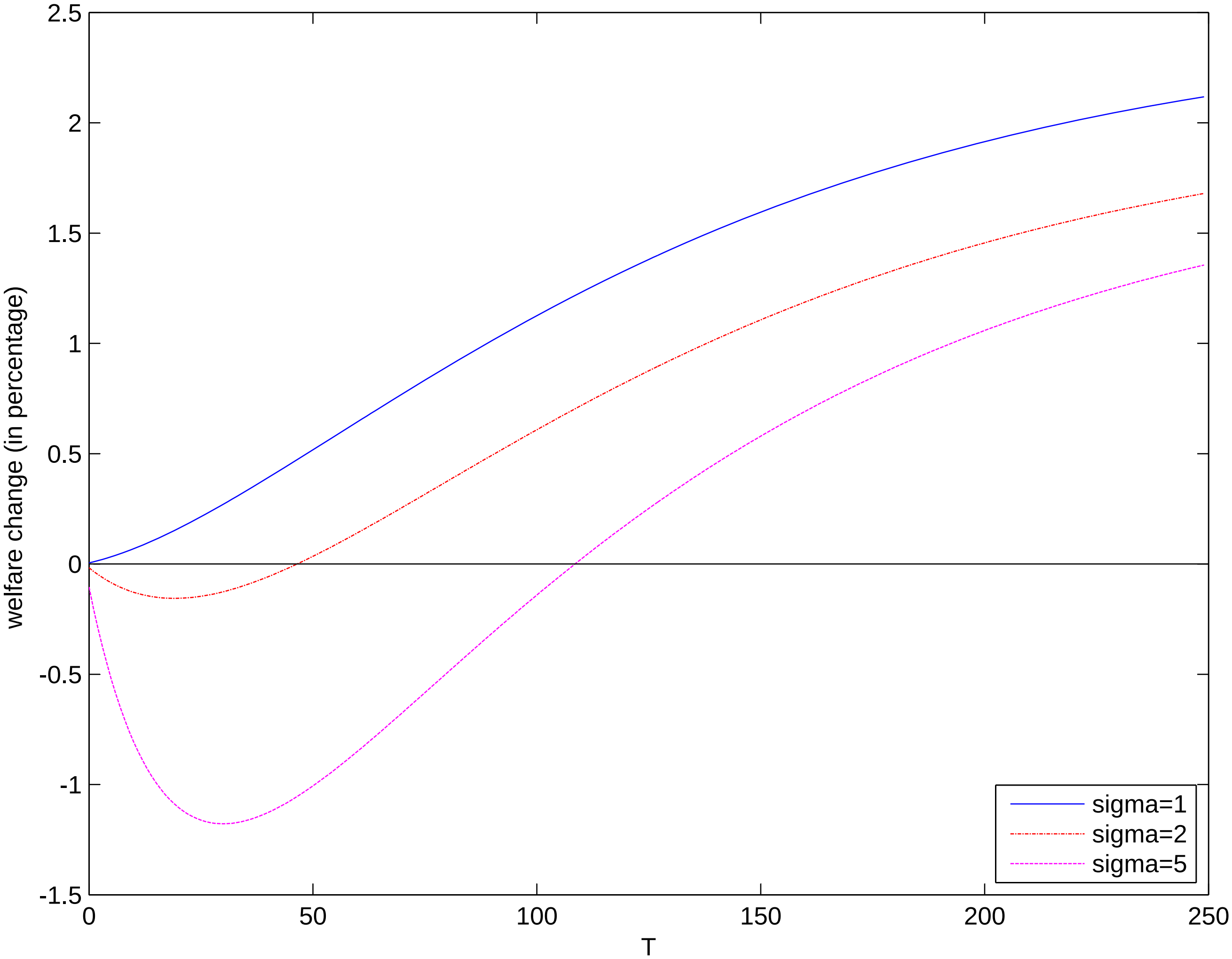

Figure 5. Transactions costs model: Permanent fall in devaluation rate

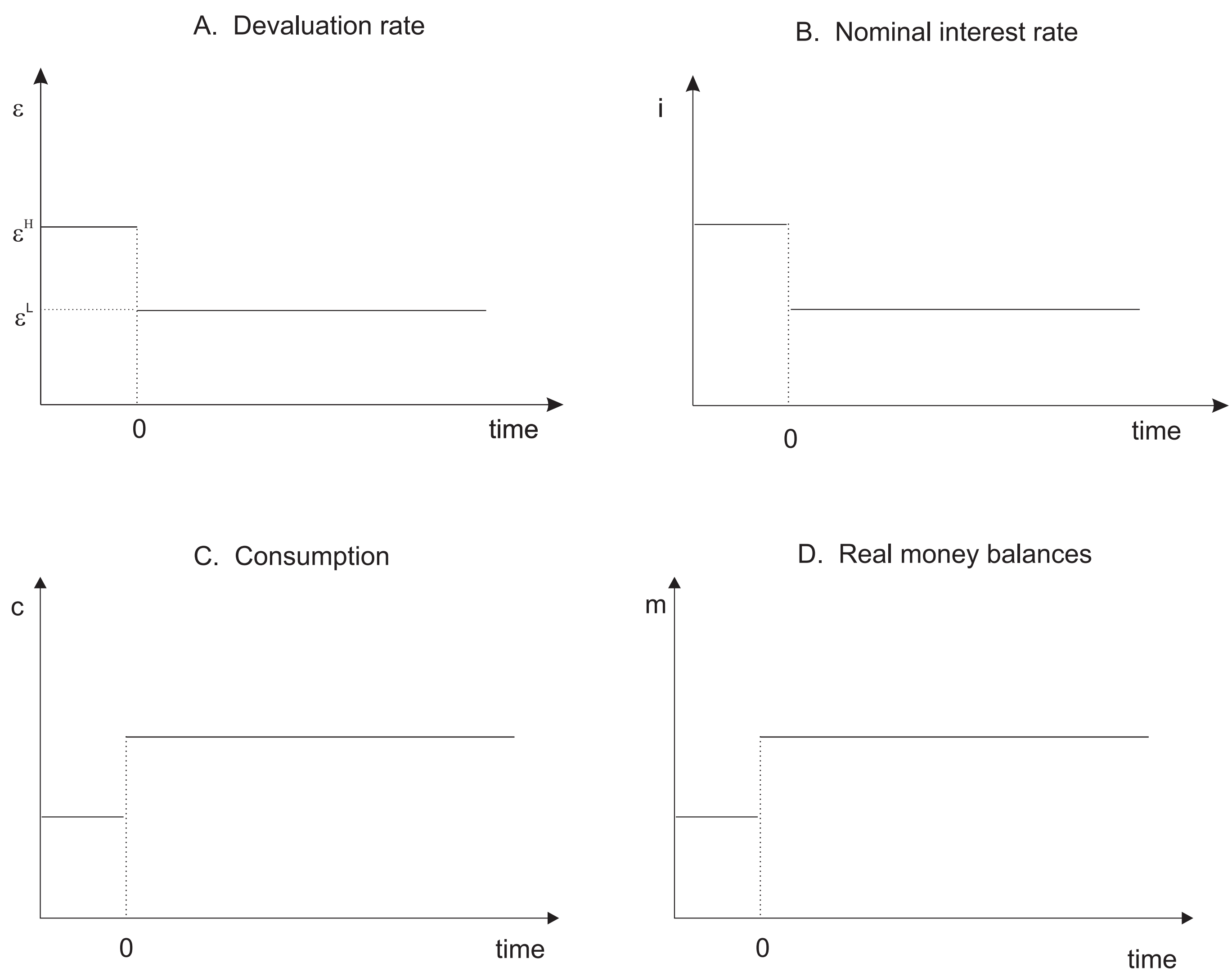

Figure 6. Transactions costs model: Temporary reduction in devaluation rate

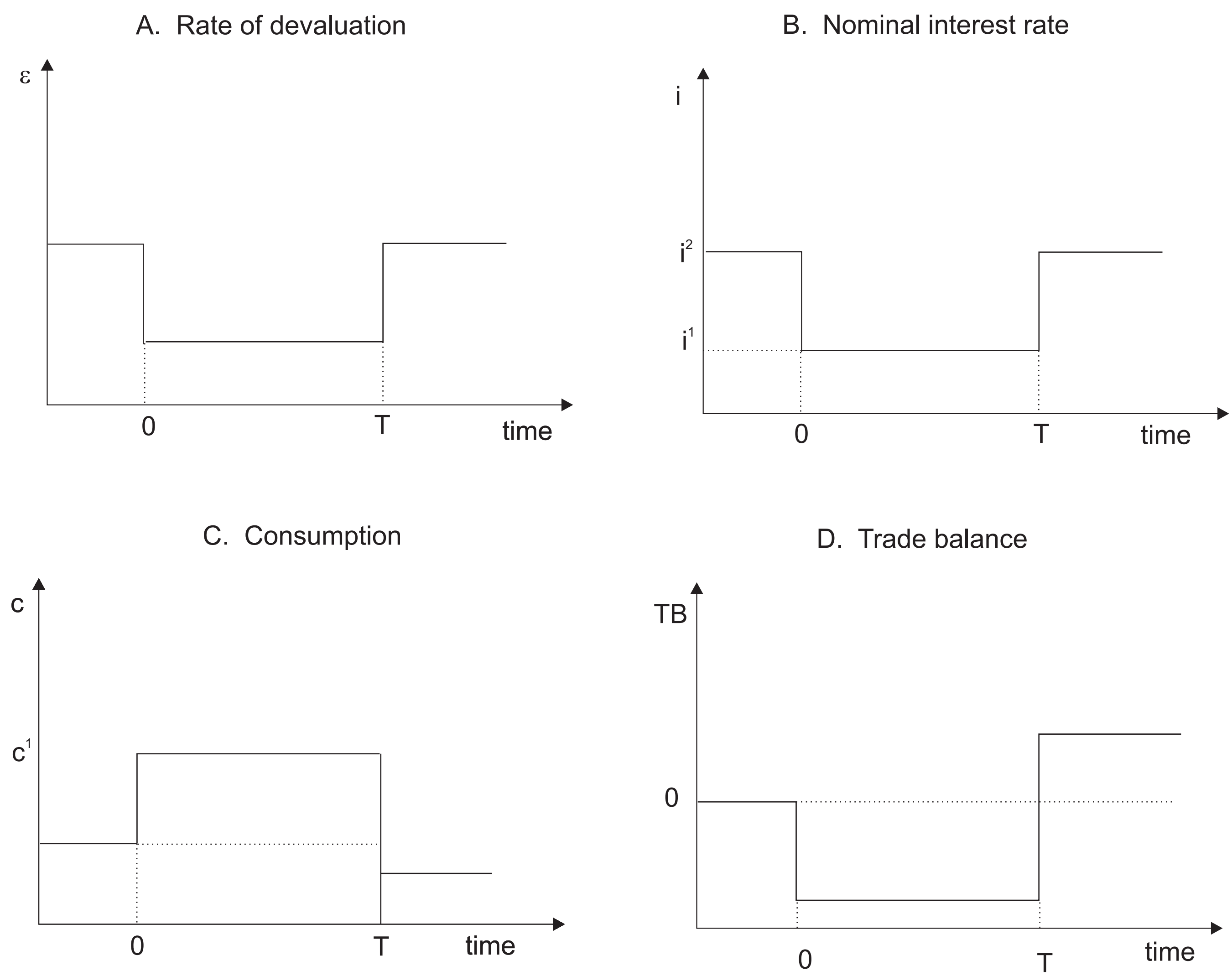

Figure 7. Welfare change as function of T

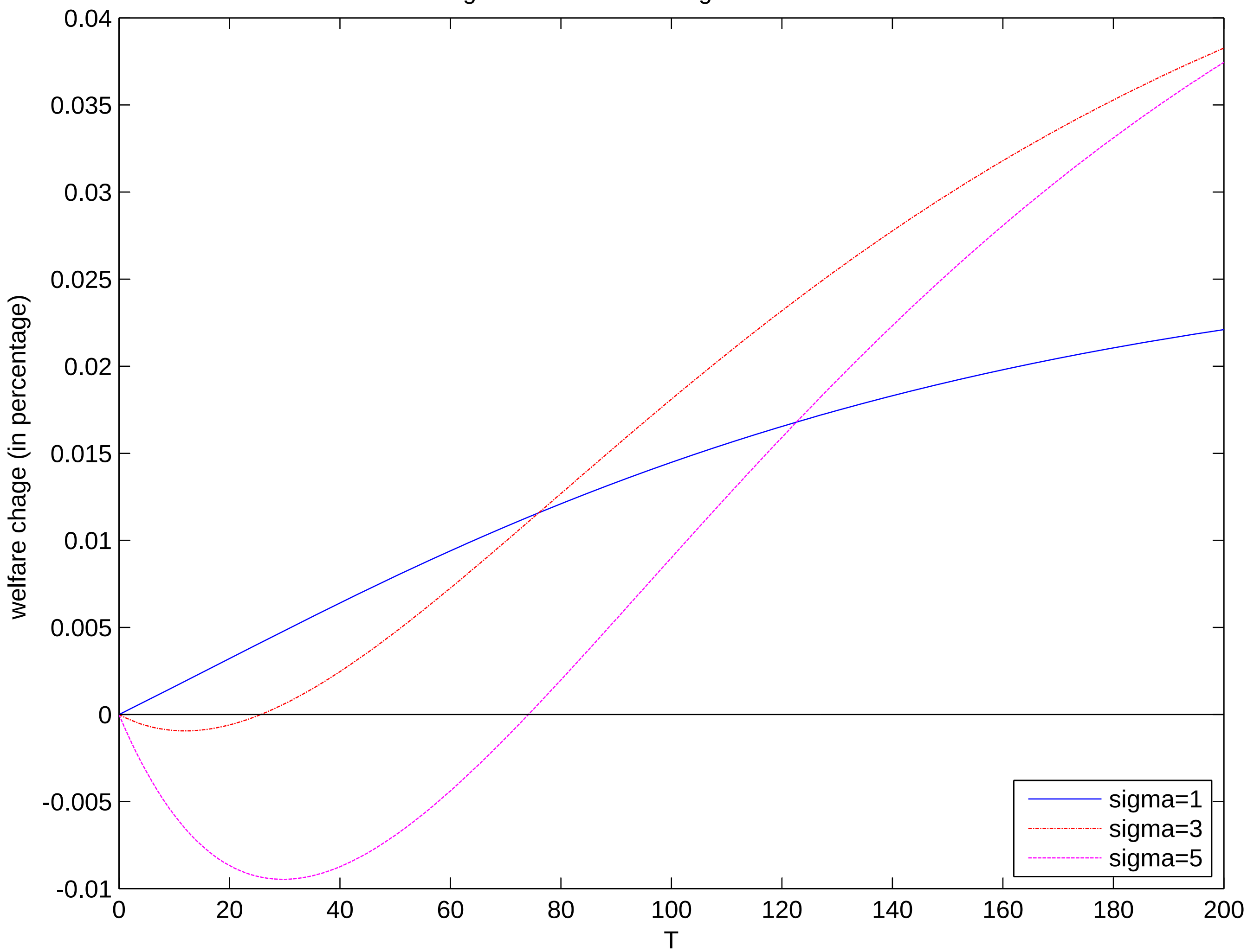

Figure 8. Temporary reduction in money growth rate

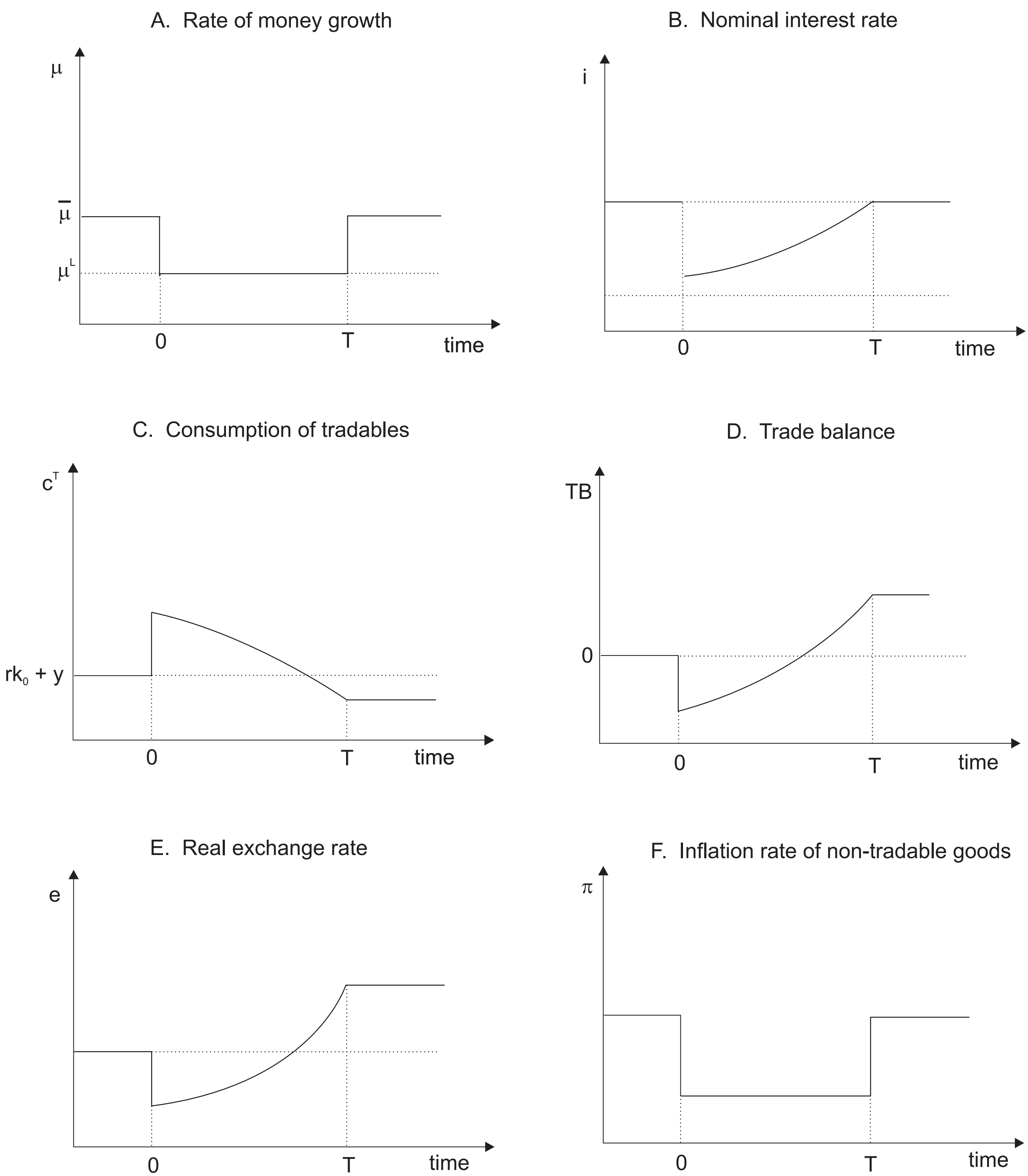

Figure 9. Temporary reduction in money growth rate

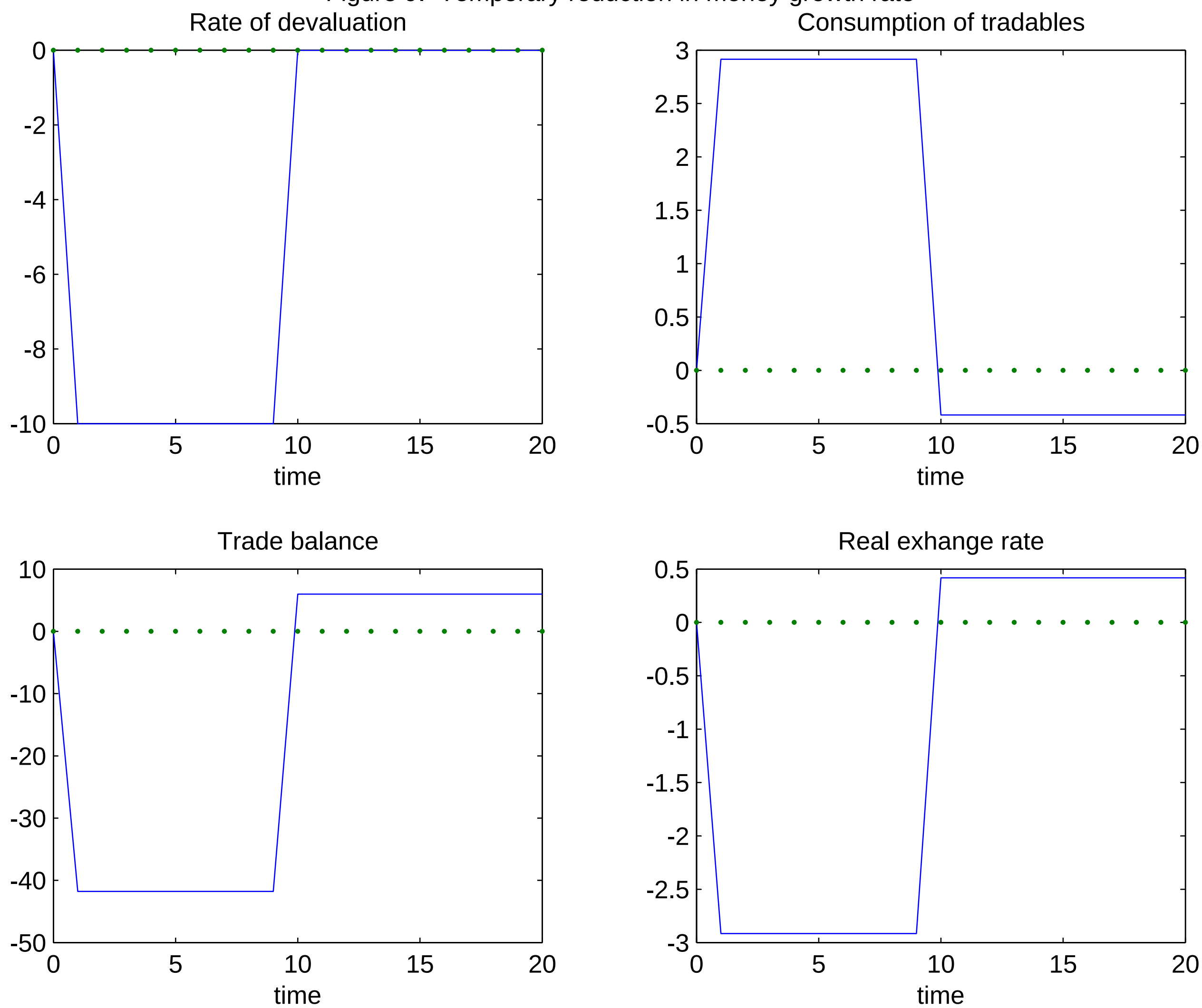

Figure 10. Temporary reduction in money growth rate