Chapter 8
Sticky Prices*

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1 Introduction

After studying the frictionless world of Chapter 5 in which monetary and exchange rate policy had no effects on the real sector, Chapters 6 and 7 introduced frictions into the model – no interest-bearing bonds in the case of Chapter 6 and links between nominal interest rates and consumption in the case of Chapter 7 – that allowed us to focus on some of the key channels whereby monetary and exchange rate policy affect the real economy in a small open economy. This chapter introduces the last – and perhaps best-known – friction in the model of Chapter 5: sticky prices. This chapter's model can thus be viewed as an optimizing version of the Mundell-Fleming model.[1]

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[1] The Mundell-Fleming model is named after the contributions of Mundell (1963, 1964) and Fleming (1962). This was one of the two major contributions that earned Robert Mundell the 1999 Nobel Prize in Economics. The Mundell-Fleming model essentially refers to an open economy model with perfect capital mobility and sticky prices. Modern versions of the Mundell-Fleming model – couched in terms of Obstfeld and Rogoff's (1995) influential paper – have come to be known as "new open-economy macroeconomics". Since,

The main motivation for introducing sticky prices is that permanent changes in the level of the nominal money supply have no real effects in the context of the basic flexible-prices model introduced in Chapter 5 and modified in Chapters 6 and 7.[2] Once sticky prices come into the picture, however, monetary policy will have real effects and can thus potentially explain some key features of the real world. In particular, an expansion in the money supply will (i) lead to higher aggregate demand and hence output – since output is typically demand-determined in sticky-prices models – thus providing a possible explanation for business-cycle fluctuations; (ii) generate a positive co-movement between nominal and real exchange rates, thus explaining a well-documented stylized fact in open economies (see Mussa (1986)); and (iii) possibly result in exchange rate overshooting, in the sense that the short-run increase in the nominal exchange rate is larger than in the long-run, which could explain the high volatility of nominal exchange rates.

In contrast to flexible exchange rates, a permanent devaluation did have real effects in the world of Chapter 6. Indeed, by reducing real money balances on impact, a devaluation led to a fall in consumption in order to generate the trade surpluses required to replenish real money balances over time. In sharp contrast, we will see that, under sticky prices, a devaluation leads to higher output and consumption. These contrasting results illustrate a long-standing debate in development macroeconomics regarding the real effects of a devaluation, which is reviewed in Box 1.

The chapter proceeds as follows. Section 2 lays out the groundwork by introducing sticky prices into the model of Chapter 5. With the model in hand, Section 3 analyzes the effects of monetary policy (i.e., permanent changes in the money supply and the rate of growth of the money supply). In particular, a permanent increase in the level of the money supply leads to an expansion

conceptually, there is clearly nothing "new" in these models, this label presumably refers to the fact that explicit microfoundations are introduced both on the demand and the supply side. In fact, the title of Obtsfeld and Rogoff's paper ("Exchange rate dynamics redux") – "redux" meaning "redone" or "brought back" – pays hommage to Dornbusch's 1976 paper titled "Expectations and exchange rate dynamics", his most famous paper and arguably the most influential paper in the Mundell-Fleming tradition (see Rogoff (2002)). Rudi Dornbusch himself – a student of Robert Mundell at the University of Chicago – would have been a likely recipient of the Noble Prize in Economics had it not been for his untimely death of cancer in 2002 at the age of 60.

[2] There could be real effects, of course, if there is an endogenous labor/leisure choice or money were introduced through a transactions costs technology, but the adjustment would be instantaneous and there would be no dynamics.

in the non-tradables goods sector, higher inflation, a real depreciation of the domestic currency, and a fall in the domestic real interest rate. Intuitively, the presence of sticky prices prevents the price index (which comprises the prices of both tradables and non-tradable goods) to fully respond on impact to the increase in the money supply. As a result, there is an incipient excess supply in the money market, which requires an increase in consumption of non-tradable goods. This higher consumption is brought about by a fall in the relative price of non-tradable goods (i.e., a real depreciation).

In Section 4, we turn our attention to predetermined exchange rates. The main result is that, under sticky prices, a devaluation is expansionary. Intuitively, sticky prices imply that, on impact, a nominal devaluation leads to a real devaluation (i.e., a fall in the relative price of non-tradable goods). As a result, demand for non-tradable goods increases, which results in an output expansion.

Finally, in Section 5, we focus on one of the more influential results to come out of the Mundell-Fleming tradition: Dornbusch's (1976) exchange rate overshooting. In the model developed in Section 3, the nominal exchange rate increases on impact by the same proportion as the money supply and hence by the same amount that it will increase in the long-run. In other words, there is no overshooting in the sense of Dornbusch (1976). Section 5 studies a slightly more general version of the model of Section 3 which, by generating a money demand with a consumption-elasticity that is not necessarily equal to one, can yield both overshooting or undershooting of the nominal exchange rate. In fact, under the more relevant parameter configuration, the nominal exchange rate "overshoots" in the short-run its long-run level. This is a remarkable result because it can explain short-run volatility in nominal exchange rates that goes beyond that of the "fundamentals" (in this case the money supply).

2 A sticky prices model

We now incorporate sticky prices into the model of Chapter 5.[3] Sticky prices enter the picture through the supply side so there are no modifications of substance on the consumer side.

Consider a small open economy inhabited by a large number of identical, infinitely-lived consumers, who are endowed with perfect foresight. The economy is perfectly integrated with the rest of the world in both goods and capital markets. There exist two physical goods (one tradable and the other nontradable, both nonstorable). The law of one price holds for the tradable good and the foreign price of the tradable good is assumed to be one; hence, $P^T = E$. The economy can borrow/lend at a constant world real interest rate, r. As in Chapter 5, money is introduced in the utility function.

2.1 Consumers

The only two minor modifications to the consumer side of Chapter 5 are the adoption of logarithmic preferences and the introduction of a non-tradable good. Preferences are thus given by

$$\int_0^\infty \left[\log(c_t^T) + \log(c_t^N) + \log(z_t)\right] e^{-\beta t} dt, \tag{1}$$

where c_t^T and c_t^N denote consumption of tradable and non-tradable goods, respectively, and $z_t(\equiv M_t/P_t)$ denotes real money balances in terms of the price index P, given by[4]

$$P \equiv \sqrt{P^T P^N}. \tag{2}$$

Since money balances enter the utility function to capture the liquidity services provided by money, it seems natural to deflate nominal balances by a

[3] Here we follow Calvo and Vegh (1993). A similar model – but in discrete time and with a more refined supply-side – can be found in the appendix of Obtsfeld and Rogoff's (1995) paper. The short-run dynamics behind the two models – including the conditions for overshooting/undershooting examined in Section 5 – are the same because both models assume that output is demand-determined.

[4] Notice that, to be consistent with our notation in earlier chapters, we will use z to denote real money balances in terms of the price index and continue to use m to denote real money balances in terms of tradable goods. (We will continue to use tradable goods as the numeraire.)

price index because the consumer consumes both goods.[5]

Let $a_t(\equiv m_t + b_t)$ denote real financial assets in terms of tradable goods, where b_t denotes net foreign bonds held by consumers. The consumer's flow constraint is then given by

$$\dot{a}_t = ra_t + y_t^T + \frac{y_t^N}{e_t} + \tau_t - c_t^T - \frac{c_t^N}{e_t} - i_t \frac{z_t}{\sqrt{e_t}}, \tag{3}$$

where y_t^T and y_t^N denote output of tradable and non-tradable goods, respectively, e_t $(\equiv P^T/P^N)$ is the real exchange rate, and i_t is the nominal interest rate. To understand the last term on the RHS of constraint (3), notice that since we continue to use tradable goods as our numeraire, the opportunity cost of holding real money balances – given by $i(M/E)$ – can be expressed as $i(M/P)(P/E)$ which, taking into account (2) and the definition of the real exchange rate, equals $iz/\sqrt{e_t}$.

Integrating (3) and imposing the appropriate transversality condition, we obtain

$$a_0 + \int_0^\infty \left(y_t^T + \frac{y_t^N}{e_t} + \tau_t \right) e^{-rt} dt = \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t} + i_t \frac{z_t}{\sqrt{e_t}} \right) e^{-rt} dt. \tag{4}$$

The consumer chooses c^T, c^N, and z to maximize (1) subject to the intertemporal constraint (4). In terms of the Lagrangian:

$$\begin{aligned} \mathcal{L} &= \int_0^\infty [\log(c_t^T) + \log(c_t^N) + \log(z_t)] e^{-\beta t} dt \\ &+ \lambda \left[a_0 + \left(\int_0^\infty y_t^T + \frac{y_t^N}{e_t} + \tau_t \right) e^{-rt} dt - \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t} + i_t \frac{z_t}{\sqrt{e_t}} \right) e^{-rt} dt \right]. \end{aligned}$$

The first-order conditions with respect to c^T, c^N, and z are given, respectively, by (assuming, as usual, that $\beta = r$)

[5] This price index corresponds to the minimum nominal expenditure required to achieve a given level of utility; see Appendix 7.3 in Chapter 6 for the derivation. As also discussed in Chapter 6, in the case of logarithmic preferences it does not make a difference whether we introduce z or m in the utility function. We choose the specification with z because the model will be generalized to a CES specification in z below.

$$
\begin{aligned}
\frac{1}{c_t^T} &= \lambda, &(5)\\
\frac{1}{c_t^N} &= \frac{\lambda}{e_t}, &(6)\\
\frac{1}{z_t} &= \lambda\frac{i_t}{\sqrt{e_t}}. &(7)
\end{aligned}
$$

Combining (5) and (6), we obtain the condition :

$$
\frac{c_t^N}{c_t^T} = e_t. \tag{8}
$$

This condition should, of course, be familiar from Chapter 4 and says that, at an optimum, the marginal rate of substitution between tradables and non-tradables equals the relative price.

To obtain the money demand, combine (6) and (7) to obtain:

$$
z_t = \frac{c_t^N}{\sqrt{e_t} i_t}. \tag{9}
$$

The demand for real money balances in terms of the price index, z, depends positively on home goods consumption expressed in terms of the price index – given by $c^N P^N/P = c^N/\sqrt{e_t}$ – and negatively on the nominal interest rate.

For further reference, let us also derive the money demands in terms of tradable goods and non-tradable goods. By combining (5) and (7), we obtain the demand for real money balances in terms of tradable goods:

$$
m_t = \frac{c_t^T}{i_t}, \tag{10}
$$

which is, of course, familiar from previous chapters.

Finally, to derive the money demand in terms of non-tradable goods, notice that $z\sqrt{e_t} = M/P^N$ and rewrite (9) as:

$$
n_t = \frac{c_t^N}{i_t}, \tag{11}
$$

where $n_t \equiv M_t/P_t^N$. The usefulness of defining a money demand in terms of non-tradable goods will become apparent below.

2.2 Supply side

The supply of tradable goods is assumed to be constant over time and equal to y^T. The major departure from the frictionless world of Chapter 5 is that prices in the non-tradable goods sector (P^N) are assumed to be sticky (i.e., they cannot change at any given point in time but can, of course, change over time). Since prices are given at any point in time, they will not be able to adjust to clear the market in response to shocks that may lead to excess supply or demand of non-tradable goods. Instead, we will assume that quantities adjust to clear the market for non-tradable goods. More specifically, we will assume that output of non-tradable goods is *demand-determined* and hence supply always adjusts to demand.

Formally, sticky prices are introduced into the model via Calvo's (1983) staggered prices formulation, which is a continuous-time version of overlapping contracts models à la Fischer (1977)-Taylor (1979, 1980). In this formulation, the rate of change of the inflation rate is a *negative* function of excess aggregate demand:

$$\dot{\pi}_t = -\theta(y_t^N - y_f^N), \qquad \theta > 0, \tag{12}$$

where y_t^N is aggregate demand and y_f^N is the "full-employment" level of output. In this formulation, the price level is sticky (i.e., it is predetermined at each instant in time), but the inflation rate is fully flexible because it is a forward-looking variable.[6] Equation (12) can be derived by assuming that firms set prices in a non-synchronous manner taking into account the future path of aggregate demand and the average price level prevailing in the economy (see Appendix 8.1 for the derivation of equation (12)).

Intuitively, think of firms as being able to change their individual prices only if they receive some random signal. In this context, suppose that there is an increase in aggregate demand. Then some firms – those which do receive the random signal – will be able to change their individual price (and hence inflation will rise). Most firms, however, will not be able to change

[6] If this is the first time that you encounter Calvo's (1983) staggered-prices formulation, you may be somewhat surprised to see that the change in the rate of inflation is a *negative* function of excess aggregate demand. This, however, makes perfect sense in light of the intuition given below. The key is that in this set-up the inflation rate itself is fully flexible. If the inflation rate were sticky, then one would need to assume that the change in the inflation rate is a *positive* function of excess aggregate demand for the problem to be well-defined.

prices and hence the price level itself will not change. Next instant, there will be some firms that could not change their prices before that will be able to do so. But since the random signal follows an exponential distribution, the number of firms changing prices will be smaller than those that changed prices immediately after aggregate demand went up. Hence, the inflation rate will rise by less (relative to the pre-shock situation) than before, which explains why the rate of inflation falls over time.[7]

2.3 Government

The government is unchanged relative to Chapter 5. Its flow constraint is therefore given by:

$$\dot{h}_t = rh_t + \dot{m}_t + \varepsilon_t m_t - \tau_t. \tag{13}$$

The corresponding intertemporal constraint is

$$h_0 + \int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt = \int_0^\infty \tau_t e^{-rt} dt. \tag{14}$$

2.4 Equilibrium conditions

Since perfect capital mobility prevails, the interest parity condition holds:

$$i_t = r + \varepsilon_t. \tag{15}$$

Equilibrium in the non-tradable goods market requires that

$$c_t^N = y_t^N. \tag{16}$$

Recall that, in the current set-up, output of non-tradable goods is demand-determined, so "equilibrium" in the non-tradable goods market holds by construction.

By definition, $e = E/P^N$. Hence,

$$\frac{\dot{e}_t}{e_t} = \varepsilon_t - \pi_t. \tag{17}$$

[7] But how sticky are prices in practice? Box 2 reviews the available empirical evidence.

This dynamic equation simply states that if tradable goods inflation (given by ε_t) is, say, larger than non-tradables goods inflation (π_t), then the relative price of tradable goods will be increasing over time (i.e., $\dot{e}_t > 0$).

As in Chapter 4, let us now define the real interest rate in terms of non-tradable goods, r^d, as

$$r_t^d \equiv r + \frac{\dot{e}_t}{e_t}. \tag{18}$$

If, say, the relative price of tradable goods is rising over time (i.e., $\dot{e}_t > 0$), then $r^d > r$. Intuitively, if you have invested in a tradable bond, your return in terms of non-tradable goods will be higher if you are able to purchase more non-tradable goods later on.

To derive the Euler equation for non-tradable goods, totally differentiate (6) and use (18) to obtain

$$\frac{\dot{c}_t^N}{c_t^N} = r_t^d - r.$$

As discussed in Chapter 4, if, say, $r^d > r$, today's consumption of non-tradable goods is lower than "tomorrow's" because the return on postponing consumption (r^d) is higher than the utility cost of deferring consumption (β, which equals r).

Combining the consumers' flow constraint (given by (3)) with the government's (given by (13)) and imposing equilibrium in the non-tradable goods market, we obtain

$$\dot{k}_t = rk_t + y^T - c_t^T,$$

where $k(\equiv b + h)$ denotes the economy's total net foreign assets.

By the same token, combining the consumer's and the government's intertemporal constraints – given by (4) and (14), respectively – and imposing equilibrium in the home goods market, we obtain

$$k_0 + \frac{y^T}{r} = \int_0^\infty c_t^T e^{-rt} dt. \tag{19}$$

3 Flexible rates

3.1 Perfect foresight equilibrium

Suppose that the economy is operating under flexible exchange rates. Hence, we assume that $h_t = 0$ for all t. We will now solve for the perfect foresight equilibrium path for a constant rate of money growth, $\bar{\mu}$. To this effect, we proceed in three stages. In the first stage, we will show that consumption of tradables is constant (and, in fact, independent of monetary policy). In the second stage – and as we have done already in Chapters 5 and 7 – we will show that the path of m is governed by an unstable differential equation and argue that a convergent perfect foresight equilibrium path requires that m be constant over time. In the third and final stage, we will set a dynamic system in n (real money balances in terms of non-tradable goods) and π to solve for the rest of the model.

3.1.1 Consumption path of tradable goods

First-order condition (5) makes clear that c^T will be constant over time. Using the resource constraint (19), this constant value, denoted by $\overline{c^T}$, will be given by

$$\overline{c^T} = rk_0 + y^T. \tag{20}$$

For further reference, notice that the path of consumption of tradables will be given by (20) regardless of the path of the money supply and, furthermore, will not affected by any (anticipated or unanticipated) change in monetary policy. Hence, from (5), the same is true of the multiplier λ.

3.1.2 Real money balances

By definition, $m = M/E$. (Recall that, by assumption, the foreign price of the tradable good is unity.) Hence,

$$\frac{\dot{m}_t}{m_t} = \bar{\mu} - \varepsilon_t. \tag{21}$$

Solving for ε_t from the interest parity condition (15) and using (10) – taking also into account (20) – we obtain

$$\varepsilon_t = \frac{\overline{c^T}}{m_t} - r.$$

Substituting this last expression into (21), we obtain a linear differential equation for m:

$$\dot{m}_t = (r + \bar{\mu})m_t - \overline{c^T}, \tag{22}$$

where $\overline{c^T}$ is a constant given by (20). Given that

$$\frac{\partial \dot{m}_t}{\partial m_t} = r + \bar{\mu} > 0,$$

the differential equation (22) is unstable. It follows that for m_t to follow a convergent path, $\dot{m}_t = 0$ for all $t \geq 0$. Hence, along a perfect foresight path with a constant $\bar{\mu}$, m will be constant and equal to

$$\bar{m} = \frac{\overline{c^T}}{r + \bar{\mu}}. \tag{23}$$

An important implication is that, in response to unanticipated and permanent changes in $\bar{\mu}$, real money balances will need to adjust instantaneously to their new steady state. If they did not, they would diverge over time.

Finally, notice that since $\dot{m}_t = 0$ along a perfect foresight path with constant $\bar{\mu}$, it follows from (21) that ε_t will also be constant:

$$\bar{\varepsilon} = \bar{\mu}.$$

Hence, the nominal interest rate is also constant and given by

$$\bar{\imath} = r + \bar{\mu}.$$

3.1.3 Dynamic system

To solve for the rest of the system, we will set up a dynamic system in n (real money balances in terms of non-tradable goods, as defined in (11)) and π_t.[8] The variable n is a predetermined variable since M is exogenous (i.e.,

[8] The reader may wonder why we need to introduce a different measure of real money balances for the purposes of setting up the dynamic model. The answer is that when it comes to setting up a dynamic system, it is convenient to have a predetermined variable. In this case, n is such a variable.

controlled by the monetary authority) and prices of non-tradable goods are sticky (i.e., cannot jump at any point time). Since, by definition, $n \equiv M/P^N$, it follows that

$$\dot{n}_t = n_t(\bar{\mu} - \pi_t), \tag{24}$$

where, by definition, π_t ($\equiv \dot{P}^N/P^N$) is the rate of inflation of non-tradable goods.

To derive our second dynamic equation, first use (16) to rewrite (12) as

$$\dot{\pi}_t = \theta \left(y_f^N - c^N \right). \tag{25}$$

Hence, inflation will be rising if consumption of non-tradable goods is below its full-employment level and viceversa.

Taking into account (8) and noting that $e_t = n_t/m_t$, equation (25) can be rewritten as as:

$$\dot{\pi}_t = \theta \left(y_f^N - \frac{\overline{c^T}}{\bar{m}} n_t \right). \tag{26}$$

Equations (24) and (26) constitute a dynamic system in n and π, for given values of $\bar{\mu}$ and $\bar{m}$.[9]

To characterize the steady-state of the dynamic system, set $\dot{n}_t = \dot{\pi}_t = 0$ in (24) and (26), respectively, to obtain:

$$\pi_{ss} = \bar{\mu}, \tag{27}$$

$$n_{ss} = \frac{y_f^N \bar{m}}{\overline{c^T}}. \tag{28}$$

Linearizing the system around the steady-state, we obtain:

$$\begin{bmatrix} \dot{n}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} 0 & -n_{ss} \\ -\theta \frac{\overline{c^T}}{\bar{m}} & 0 \end{bmatrix} \begin{bmatrix} n_t - n_{ss} \\ \pi_t - \bar{\mu} \end{bmatrix}.$$

The determinant of the matrix associated with the linear approximation is negative:

[9] Notice that, as far as the dynamic system is concerned, m is like a parameter since it will adjust instantaneously in response to permanent changes in $\bar{\mu}$.

$$\Delta = -\theta \frac{n_{ss}\overline{c^T}}{\bar{m}} < 0.$$

The system has therefore one positive and one negative root and thus exhibits saddle-path stability.

As in Chapter 6, we now proceed to characterize the qualitative behavior of this dynamic system by resorting to a phase-diagram. To construct the phase diagram, we first draw the $\dot{n}_t = 0$ and $\dot{\pi}_t = 0$ loci. To obtain these curves, set $\dot{n}_t = 0$ in equation (24) and $\dot{\pi}_t = 0$ in (26) to obtain, respectively,

$$\begin{aligned} \pi_t &= \bar{\mu}, \\ n_t &= \frac{y_f^N \bar{m}}{\overline{c^T}}. \end{aligned}$$

Hence, the $\dot{n}_t = 0$ locus shows up in the phase diagram (Figure 1) as a horizontal line, while the $\dot{\pi}_t = 0$ locus shows as a vertical line. The intersection of both loci at point A determines the steady-state of the system. As we saw in Chapter 6, the $\dot{n}_t = 0$ and $\dot{\pi}_t = 0$ curves define four regions. Proceeding in analogous fashion, we can draw the arrowheads shown in Figure 1 and conclude that the saddle path will be positively-sloped.

[Figure 1]

The variable n is predetermined in the sense that it cannot jump in an endogenous way. Hence, if n at time 0 were above n_{ss} (denoted by n_0 in Figure 1), then inflation at 0 would also be above $\bar{\mu}$ (at a value given by π_0 in Figure 1) so as to position the system along the saddle path at a point like B. Hence, both π and n would fall during the adjustment process towards point A. Since $\dot{\pi} < 0$ during the adjustment, c^N would be above its full-employment level, as equation (25) makes clear. Conversely, if n at time 0, were below n_{ss}, (at a value given by, say, $n_0^{'}$ in Figure 1), then the rate of inflation would adjust endogenously to a value below $\bar{\mu}$ ($\pi_0^{'}$ in Figure 1) so as to position the system on the saddle path at a point like C. From then on, the system would travel along the saddle path towards point A. Both π and n would thus increase during this adjustment process. Since $\dot{\pi} > 0$ during the adjustment to the steady-state, c^N would be below its full-employment level, as equation (25) makes clear.

We thus conclude that the model endogenously generates a Phillips-curve type relationship in the sense that when inflation is above $\bar{\mu}$, the economy is operating above its full-employment level (i.e., $c_t^N > y_f^N$) and when inflation is below $\bar{\mu}$, the economy is operating below its full-employment level (i.e., $c_t^N < y_f^N$). While traditional sticky-prices model postulate such a relationship, here it arises as an equilibrium phenomenon.[10]

3.2 Permanent increase in the money supply

Suppose that, just before $t = 0$, the economy is in the stationary equilibrium corresponding to $\bar{\mu} = 0$ and point A in Figure 1. At time 0, there is an unanticipated and permanent increase in the stock of money supply, M (Figure 2, Panel A). How does the economy react?

[Figure 2]

As we have already established, c^T will not change. It is also the case that an increase in M will not affect the steady-state value of m, as (23) makes clear. Hence, the nominal exchange rate will increase by the same proportion as the nominal money supply so as to keep m constant.

In terms of the dynamic system, the increase in M will not affect the steady-state values of n and π, as follows from equations (27) and (28). Hence, the steady-state of the system will remain at point A in Figure 1. On impact, however, n will increase to a value like n_0 in Figure 1 because the nominal money supply goes up and the price of non-tradables goods is sticky. Given n_0, the inflation rate will have to jump to π_0 so as to position the system on the saddle path (point B in Figure 1). After this initial jump from point A to point B, the system travels back to point A along the saddle path. The corresponding paths of n and π as a function of time are depicted in Figure 2, Panels B and C.

To find out the path of the real exchange rate, recall that $e = n/m$. Since m does not change, e will behave in the same way as n, jumping up on impact (real depreciation) and then falling back to its initial steady-state (Figure 2, Panel D).

[10] This is true, of course, as long as the system is on the saddle-path. Interestingly, if the economy is not on the saddle-path (which can happen in response to a temporary shock), then this Phillips-curve relationship will not necessarily hold, which could explain, for example, periods of "stagflation" (i.e., high inflation and underutilization of resources) as illustrated by Exercise 1 at the end of this chapter.

Given condition (8) and the fact that c^T does not change, the behavior of c^N will mimic that of e, as illustrated in Figure 2, Panel E.

Finally, what will happen to the domestic real interest rate, $r_t^d \equiv i_t - \pi_t$? Since the nominal interest rate remains constant, the behavior of r_t^d will be dictated by the behavior of inflation. Hence, the domestic real interest rate falls on impact and then gradually reverts back to its initial steady-state, r (Figure 2, Panel F).

In sum, a permanent increase in the stock of the money supply leads to higher inflation, a real depreciation, an expansion in the non-tradable goods sector, and a fall in the domestic real interest rate. This is, of course, in sharp contrast to the world of Chapter 5 where a permanent increase in the money supply had no real effects.

What is the economic intuition behind these effects? For these purposes – and using (11) –let us focus on the money market equilibrium for n at time 0:

$$\underbrace{\frac{M_0}{P_0^N}}_{\text{real money supply}} = \underbrace{\frac{c_0^N}{i_0}}_{\text{real money demand}}. \tag{29}$$

As indicated below the equation, think of the LHS as real money supply and the RHS as real money demand. The critical implication of price stickiness is that an increase in the *nominal* money supply translates into an increase in the *real* money supply. Hence, for unchanged real money demand, there would be an *incipient* excess supply in the money market. In their attempt to get rid of unwanted money balances by purchasing foreign bonds, households would bid up the domestic price of foreign bonds (E). Since P^N is sticky, this nominal depreciation translates into a real depreciation (i.e., a fall in the relative price of non-tradable goods). As a result, demand for non-tradables increases which – given that output is demand-determined – leads to an output expansion in the non-tradable sector.

In the long-run, however, real money demand will not change relative to its pre-shock value. Hence, real money supply must fall over time to its pre-shock value. For this to happen, inflation of non-tradable goods, π, must increase on impact above the (unchanged) rate of money growth, $\bar{\mu}$. This high inflation – coupled with no changes in the nominal exchange rate after the initial jump – explains the real appreciation that takes place over time.

Finally, notice that in this model the nominal exchange rate increases by the same proportion as the nominal money supply. There is thus no overshooting in the sense of Dornbusch (1976). Overshooting would occur if, on impact, the nominal exchange rate increased by more than the money supply does. Why does this model not generate overshooting? The answer to this question will become clear once we show in Section 5 how a version of this model with more general preferences can indeed generate both overshooting and undershooting.

3.3 Permanent reduction in the rate of money growth

Suppose, once again, that an instant before time 0, the system is in the steady state characterized above (with the rate of money growth given by $\bar{\mu}^H$). At $t = 0$, there is an unanticipated and permanent reduction in the rate of money growth from $\bar{\mu}^H$ to $\bar{\mu}^L$ ($\bar{\mu}^L < \bar{\mu}^H$) (Figure 3, Panel A). Consumption of tradable goods – given by (20) – does not change because, as discussed above, its level is independent of monetary policy. From (23), we can see that real money balance in terms of tradable goods (m) will be higher in the new stationary state because the opportunity cost of holding money has fallen. Given the instability of the differential equation governing the behavior of real money balances, m must adjust instantaneously to its higher value. If it did not, it would diverge over time. The rate of depreciation will therefore also fall down immediately to its new steady-state value and so will the nominal interest rate.

[Figure 3]

In terms of the dynamic system, suppose that the system is initially at point A in Figure 4. At point A, the steady-state rate of inflation of non-tradable goods is equal to $\bar{\mu}^H$ and the corresponding real money balances are $n_{ss}(\bar{\mu}^H)$. In the new steady-state – and as (27) and (28) make clear – n will be higher and π will be lower (point B in Figure 4). Hence, on impact the system must jump from point A to point C and then travel along the saddle-path towards point B. The corresponding paths of n and π are illustrated in Figure 3, Panels B and C, respectively. On impact, the inflation rate falls by more than it will in the long-run.

[Figure 4]

To derive the path of the real exchange rate, recall that $e = n/m$. Since m increases at time 0, e will jump down at time 0 (real appreciation), as illustrated in Figure 3, Panel E. Given that the real exchange rate does not change across steady-states, it will need to gradually rise back to its initial steady-state. Given equation (8), consumption of non-tradable goods follows the same path as the real exchange rate (Figure 3, Panel D). Finally, the domestic real interest rate will increase on impact because the fall in inflation is larger than the fall in the nominal interest rate (Figure 3, Panel F). It then falls gradually to its unchanged steady-state.

Summing up, an unanticipated reduction in μ leads to a recession, real exchange rate appreciation, and higher real interest rates. Interestingly – and as documented in Chapter 14 – these are the main stylized facts associated with money based stabilizations.

The economic intuition behind the results just discussed is as follows. Think again in terms of the money market equilibrium described by equation (29). The reduction in the money growth rate lowers the nominal interest rate and hence increases real money demand. Real money supply, however, does not change on impact. Hence, for the initial level of c^N, there is an excess demand for money. As a result, the public will try to get rid of foreign bonds in order to acquire money, which pushes down the domestic price of foreign bonds, E. Since the price of non-tradable goods is sticky, the fall in E translates into a reduction in e. This increase in the relative price of non-tradable goods reduces their demand, which leads to an output fall.

4 Predetermined exchange rates

We now solve the model for the case of predetermined exchange rates and use it to ask the question: How does the economy respond to a permanent devaluation? Contrary to our results in Chapter 6 – where a devaluation led to a reduction in consumption – in this sticky-prices model a devaluation will lead to an expansion in aggregate demand for non-tradable goods and therefore output.

We begin by solving for the perfect foresight equilibrium corresponding to a constant rate of devaluation. We then turn our attention to a permanent devaluation and a permanent reduction in the rate of devaluation.

4.1 Perfect foresight equilibrium

Let us now characterize the perfect foresight equilibrium for a constant value of the rate of devaluation, $\bar{\varepsilon}$.

First, notice that, as in the flexible exchange rates case, consumption of tradables will be constant and given by (20). Furthermore, consumption of tradables will not be affected by changes in the rate of devaluation.

From the interest parity condition (15), the nominal interest rate will be constant as well and given by

$$\bar{\imath} = r + \bar{\varepsilon}.$$

Given that both c^T and $\bar{\imath}$ are constant, the money demand equation (10) tells us that m will be constant as well.

To solve for the rest of the variables, we need to set up a different dynamic system from the one we used above for flexible exchange rates for the following reasons. For starters, that system has the variable μ which is now an endogenous variable. Furthermore, real money balances in terms of non-tradables goods (n) are no longer a predetermined variable under predetermined exchange rates because the nominal money supply is an endogenous variable. As a methodological matter, it would not be wise to set a dynamic system with two jumping variables because it would be harder to solve. We thus need to find a variable that will be predetermined under predetermined exchange rates and sticky prices. A moment's reflection should reveal that the obvious candidate is the real exchange rate, e. Since, by definition, $e = E/P^N$, the real exchange rate will be a predetermined variable in standard models of predetermined exchange rate under sticky prices.

We will therefore set up a dynamic system in π and e. To obtain the first dynamic equation, substitute (8) into (25) to obtain:

$$\dot{\pi}_t = \theta \left[y_f^N - e_t c^T \right]. \tag{30}$$

The second dynamic equation is given by (17).

The system's steady state is given by:

$$\pi_{ss} = \bar{\varepsilon}, \tag{31}$$

$$e_{ss} = \frac{y_f^N}{\overline{c^T}}. \tag{32}$$

Linearizing the system around the steady-state, we obtain:

$$\begin{bmatrix} \dot{e}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} 0 & -e_{ss} \\ -\theta\overline{c^T} & 0 \end{bmatrix} \begin{bmatrix} e_t - e_{ss} \\ \pi_t - \bar{\varepsilon} \end{bmatrix}.$$

The determinant of the matrix associated with the linear approximation of the system is given by

$$\Delta = -\theta\overline{c^T}e_{ss} < 0,$$

which implies that the system is saddle-path stable.

Proceeding as before, it is easy to construct the phase diagram illustrated in Figure 5. As depicted, the saddle-path is positively sloped.

[Figure 5]

The path of the remaining variables along a perfect foresight equilibrium will depend on the initial value of the real exchange rate. Suppose that the initial value of the real exchange rate were given by e_0 in Figure 5. Then the inflation rate would need to be π_0 so as to position the system at point B in Figure 5. Both the real exchange rate and inflation would fall over time towards the steady-state, given by point A. Given condition (8), consumption of non-traded goods would behave in the same way as the real exchange rate and hence fall over time.

4.2 Permanent devaluation

Suppose that, just before $t = 0$, the economy is in the steady-state characterized above. At $t = 0$, there is an unanticipated and permanent devaluation (Figure 6, Panel A). How does the economy react?

[Figure 6]

In terms of the dynamic system, it is clear from equations (31) and (32) that the devaluation does not affect the steady-state values of the inflation rate of non-tradable goods or the real exchange rate. On impact, however, the real exchange rate will increase. In other words, in this particular model, a nominal depreciation leads on impact to a real depreciation.[11] The real

[11] The model is thus able to explain the high correlation between nominal and real exchange rates, as documented by Mussa (1986). See Chari, Kehoe, and McGrattan (2002)

exchange rate thus jumps on impact to a value such as e_0 in Figure 5. The inflation rate must then adjust so that the system positions itself along the saddle path (at point B in Figure 5). The system then travels along the saddle path back to its initial steady-state, point A. The corresponding path of π and e are illustrated in Figure 6, Panels C and D.

The path of consumption of non-tradable goods follows from condition (8). Consumption of non-tradable goods increases on impact and then falls back towards its full-employment level (Figure 6, Panel E). The domestic real interest rate falls on impact as a result of the increase in inflation and then gradually reverts back to its initial steady-state (Figure 6, Panel F). The path of n – illustrated in Figure 6, Panel B – follows from the fact that $n = em$.

We thus conclude that a devaluation is expansionary. Intuitively, the key is that, due to sticky prices, a nominal devaluation translates into a real devaluation. The increase in the relative price of tradable goods induces consumers to switch expenditures towards non-tradable goods. Since output is demand-determined, output of non-tradable goods responds immediately.

The expansionary effects of a devaluation under sticky prices stand in sharp contrast to the results that we obtained in Chapter 6 where a devaluation – by reducing real money balances and forcing consumers to reduce consumption to replenish real money balances – was actually contractionary. We have thus illustrated two possible channels through which a devaluation may impact the real economy. In fact – and as argued in Box 1 – there are other plausible channels, which makes the overall impact of a devaluation an empirical matter.

4.3 Permanent reduction in devaluation rate

Suppose that the economy is initially in a situation of high inflation at a point like C in Figure 5. At $t = 0$, there is an unanticipated and permanent reduction in the rate of devaluation. In the new steady-state (point A in Figure 5), the rate of inflation is lower and the real exchange rate remains unchanged. How will the economy adjust from point C to point A? A moment's reflection reveals that the economy must adjust instantaneously to its new steady state. Since e is a predetermined variable, the system must

for a quantitative analysis. Remember, however, from Chapter 6 that other frictions could generate the same outcome.

remain along the vertical line corresponding to e_{ss} at time 0. But if the system placed itself at any point other than A along that vertical line, it would diverge over time. Hence, the only possible equilibrium is for the system to jump from point C to point A. The reduction in the devaluation rate is thus super-neutral.

This is a remarkable result because it says that, even if prices are sticky, a permanent reduction in the devaluation rate might reduce inflation at no real costs. This model may thus be used to think about the end of hyperinflations which, by and large, have involved large and sudden reductions in inflation at little real costs (see Chapter 14).[12]

5 Overshooting

We mentioned earlier that our basic sticky–prices model does not generate over/undershooting. This section analyzes a more general version of the model in which both overshooting and undershooting are possible.

5.1 Consumers

The only change in the model is that the sub-utility for real money balances now takes a CES form:

$$\int_0^\infty \left[\log(c_t^T) + \log(c_t^N) + \frac{z_t^{1-1/\sigma} - 1}{1 - 1/\sigma}\right] e^{-\beta t} dt, \tag{33}$$

where σ is a positive parameter that, as will become clear below, will capture the consumption and interest-rate elasticity of real money demand.

The intertemporal constraint remains given by (4). The first-order conditions for c^T and c^N continue to be given by (5) and (6). The first-order condition for z now reads:

$$z_t^{-1/\sigma} = \lambda \frac{i_t}{\sqrt{e_t}}. \tag{34}$$

[12] We should notice the economy's very different response to a permanent reduction in the rate of devaluation compared to the response to a permanent reduction in the rate of money growth. Interestingly enough – and as shown in Exercise 2 at the end of this chapter – under logarithmic preferences the response to a change in fiscal policy would be the same under either regime.

Using this first-order condition to solve for z – taking into account (6) – we obtain the real money demand (in terms of the price index):

$$z_t = \left(\frac{c_t^N}{\sqrt{e_t} i_t} \right)^{\sigma}. \tag{35}$$

The parameter σ thus denotes the consumption and interest-rate elasticity of money demand. When $\sigma = 1$, the model reduces to the case analyzed in Section 3 (recall equation (9)).

For further reference, it is also convenient to obtain the real money demand in terms of tradable goods, m. Recalling that $m \equiv M/E$ and taking into account (2), equation (35) can be rewritten as:

$$m = \left[\frac{e_t^{(1/2)(1-1/\sigma)} c_t^T}{i_t} \right]^{\sigma}. \tag{36}$$

Once again, notice that when $\sigma = 1$, equation (36) reduces to (10).

5.2 Dynamic system

We now proceed to solve this model for the case of flexible exchange rates in the same way as we did before. A key difference, however, will be that, due to the CES preferences for money, the resulting dynamic system in m, n, and π will *not* be block recursive. In other words, with logarithmic preferences for z, the system breaks down into a single differential equation for m and a system of two differential equations for n and π. This will not be the case for CES preferences. As a result, we will have no choice but to set up a three-equation differential-equation system in m, n, and π.

For starters, solve for i_t from (34) – taking into account that $E_t/P_t = \sqrt{e}$to obtain:

$$i_t = \frac{e_t^{(1/2)(1-1/\sigma)}}{\lambda m_t^{1/\sigma}}.$$

Substituting this last equation into (21) – taking into account the interest parity condition– we obtain

$$\dot{m}_t = m_t \left[\bar{\mu} + r - \overline{c^T} \frac{n_t^{(1/2)(1-1/\sigma)}}{m_t^{1/\sigma+(1/2)(1-1/\sigma)}} \right], \tag{37}$$

where $\overline{c^T}$ is the constant value of consumption of tradable goods given by (20). This differential equation, together with (24) and (26), constitute a dynamic system of three differential equations in n, π, and m.

To characterize the system's steady-state, set $\dot{m}_t = \dot{n}_t = \dot{\pi}_t = 0$ in (24), (26), and (37) to obtain:

$$m_{ss} = \left[\frac{\left(\overline{c^T}\right)^{1-\Phi} \left(y_f^N\right)^{\Phi}}{\mu + r} \right]^{\sigma}, \tag{38}$$

$$n_{ss} = \frac{y_f^N m_{ss}}{\overline{c^T}}, \tag{39}$$

$$\pi_{ss} = \bar{\mu}, \tag{40}$$

where

$$\Phi \equiv \frac{1}{2}\left(1 - \frac{1}{\sigma}\right) \tag{41}$$

is a parameter that will be critical for the dynamics of m_t.

Linearizing the system around the steady-state, we obtain:

$$\begin{pmatrix} \dot{m}_t \\ \dot{n}_t \\ \dot{\pi}_t \end{pmatrix} = \begin{pmatrix} (1/\sigma + \Phi)\overline{c^T}\frac{n_{ss}^{\Phi}}{m_{ss}^{1/\sigma+\Phi}} & -\overline{c^T}\Phi\frac{n_{ss}^{\Phi-1}}{m_{ss}^{1/\sigma+\Phi-1}} & 0 \\ 0 & 0 & -n_{ss} \\ \frac{\theta\overline{c^T} n_{ss}}{m_{ss}^2} & -\theta\frac{\overline{c^T}}{m_{ss}} & 0 \end{pmatrix} \begin{pmatrix} m_t - m_{ss} \\ n_t - n_{ss} \\ \pi_t - \bar{\mu} \end{pmatrix}.$$

Taking into account (38) and (39), we can rewrite this dynamic system as

$$\begin{pmatrix} \dot{m}_t \\ \dot{n}_t \\ \dot{\pi}_t \end{pmatrix} = \begin{pmatrix} (1/\sigma + \Phi)(\mu + r) & -(\mu + r)\Phi\frac{m_{ss}}{n_{ss}} & 0 \\ 0 & 0 & -n_{ss} \\ \frac{\theta y_f^N}{m_{ss}} & -\frac{\theta y_f^N}{n_{ss}} & 0 \end{pmatrix} \begin{pmatrix} m_t - m_{ss} \\ n_t - n_{ss} \\ \pi_t - \bar{\mu} \end{pmatrix}.$$

The trace and the determinant of the matrix associated with the linear approximation are given by, respectively,

$$Tr = (1/\sigma + \Phi)(\mu + r) > 0,$$

$$\Delta = -(\mu + r)\frac{\theta}{\sigma} y_f^N < 0,$$

where we have used (39) to simplify the expression for the determinant. Since the determinant is negative (recall that the determinant is equal to the product of the roots), the system could have either three negative roots or one negative and two positive roots. The fact that the trace (which equals the sum of the roots) is positive, however, rules out the case of three negative roots. We thus conclude that the system has one negative and two positive roots.

Let δ denote the negative root associated with this system. Denoting by (h_1, h_2, h_3) the characteristic vector associated with the root δ, we can write:

$$\begin{pmatrix} (1/\sigma+\Phi)(\mu+r)-\delta & -(\mu+r)\Phi\frac{m_{ss}}{n_{ss}} & 0 \\ 0 & -\delta & -n_{ss} \\ \frac{\theta y_f^N}{m_{ss}} & -\frac{\theta y_f^N}{n_{ss}} & -\delta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

From the first and second rows, respectively, it follows that

$$\begin{aligned} \frac{h_1}{h_2} &= \frac{(\mu+r)\Phi\frac{m_{ss}}{n_{ss}}}{(1/\sigma+\Phi)(\mu+r)-\delta}, \\ \frac{h_2}{h_3} &= -\frac{n_{ss}}{\delta} > 0. \end{aligned}$$

Notice that, as discussed in detail below, the sign of h_1/h_2 depends on the sign of Φ and hence on σ.

Setting to zero the constants corresponding to the positive roots, the solution to the dynamic system is given by:

$$m_t - m_{ss} = \omega h_1 e^{\delta t}, \tag{42}$$

$$n_t - n_{ss} = \omega h_2 e^{\delta t}, \tag{43}$$

$$\pi_t - \bar{\mu} = \omega h_3 e^{\delta t}. \tag{44}$$

Combining (43) and (44), we get

$$\frac{n_t - n_{ss}}{\pi_t - \bar{\mu}} = \frac{h_2}{h_3} > 0, \tag{45}$$

which tells us that, along a perfect foresight path, n_t and π will move in the same direction. This should not come as a surprise since, in the logarithmic version of this model studied above, this was also the case.

Combining (42) and (44), we obtain

$$\frac{m_t - m_{ss}}{n_t - n_{ss}} = \frac{h_1}{h_2}.$$

Based on this expression, we can distinguish three possible cases:

1. $\sigma = 1$. In this case, $\Phi = 0$, as follows from (41). Hence $h_1/h_2 = 0$, which implies that m is always equal to its steady-state value. This is, of course, the case analyzed earlier in this chapter.

2. $\sigma < 1$. In this case, $\Phi < 0$, as follows from (41). Hence, $h_1/h_2 < 0$. This implies that, along a perfect foresight path, m_t and n_t will move in opposite directions. Together with (45), this implies that m_t and π_t also move in opposite directions.

3. $\sigma > 1$. In this case, $\Phi > 0$, as follows from (41). Hence, $h_1/h_2 > 0$. This implies that, along a perfect foresight path, m_t and n_t will move in the same direction. Together with (45), this implies that m_t and π_t also move in the same direction.

5.3 Permanent increase in the money supply

Suppose that the economy is initially at the steady-state given by (38), (39), and (40), with $\mu = 0$. At $t = 0$, there is an unanticipated and permanent increase in the nominal money supply. To fix ideas, suppose that the money supply doubles from $\bar{M}$ to $2\bar{M}$. How will this economy react?

The first observation is that such a change does not alter the system's steady-state.

On impact, n will increase because P^N is a sticky variable. Further, since the system has only one negative root, it will adjust monotonically to its unchanged steady-state. To show this formally, normalize h_2 to one, and evaluate (43) at $t = 0$ to obtain:

$$n_0 - n_{ss} = \omega > 0.$$

Substituting this last piece of information into (43) and differentiating with respect to time, we obtain:

$$\dot{n}_t = (n_0 - n_{ss})\delta e^{\delta t} < 0.$$

We have already established that π will move in the same direction as n. Hence, π will increase on impact and fall gradually over time.

How will m behave? We need to consider three cases:

1. $\sigma = 1$. In this case, $m_t = m_{ss}$ for all t. This case corresponds to the one analyzed in Subsection 3.2. Since m does not change on impact, the nominal exchange rate increases by the same proportion as the nominal money supply. In terms of Figure 7, the nominal exchange rate will double on impact from $\bar{E}$ to $2\bar{E}$ (point A) and stay there. Since the rate of devaluation is zero, there is no change in the nominal interest rate.

2. $\sigma < 1$. In this case – and as established above – m_t and π_t move in opposite directions. It follows that m_t will fall on impact and then rise over time. The fall on impact in m_t implies that the nominal exchange rate rises by more than the nominal money supply. In the long run, however, the nominal exchange rate increases by the same proportion. Hence, on impact, the nominal exchange rate *overshoots* its long-run level. In terms of Figure 7, the nominal exchange rate jumps on impact to a point such as B and then falls over time to its long-run level. Since the nominal exchange rate falls over time, $\varepsilon_t < 0$, which implies that the nominal interest rate falls on impact.

3. $\sigma > 1$. In this case – and as established above – m_t and π_t move in the same direction. It follows that m_t will increase on impact and then fall over time. Hence, on impact, the nominal exchange rate increases by less than the nominal money supply. The exchange rate thus *undershoots* its long-run level. In terms of Figure 7, the nominal exchange rate jumps to a point like C and then increases over time. Since the nominal exchange rate increases over time, $\varepsilon_t > 0$, which implies that the nominal interest rate increases on impact.

[Figure 7]

What is the intuition behind the over/undershooting results? Recall the real money demand equation given by (35) – rewritten below taking into account that $c^N/\sqrt{e} = P^N c^N/P$ – and, once again, interpret it as the equilibrium condition in the money market with the LHS capturing real money supply and the RHS denoting real money demand:

$$\underbrace{\frac{M_t}{P_t}}_{\text{real money supply}} = \underbrace{\left(\frac{P_t^N c_t^N / P_t}{i_t}\right)^{\sigma}}_{\text{real money demand}}. \tag{46}$$

At $t = 0$, the nominal money supply doubles. Suppose that, in response to this increase in the money supply, the nominal exchange rate also doubled (i.e., $\hat{E} = \hat{M}$), thereby adjusting instantaneously to its long-run equilibrium level.[13] This implies, of course, that the rate of depreciation would be zero and that the nominal interest rate would not change. Would the money market be in equilibrium? To answer this question, first notice that since $P = \sqrt{P^T P^N}$ and P^N cannot jump, the real money supply would increase by $\hat{M} - \hat{E}/2 = \hat{M}/2$. In other words, the real money supply would increase by 50 percent.

To find out the change in real money demand, we need to establish the change in the demand for non-tradable goods. Since P^N does not change, the real exchange rate, $e_t(\equiv P^T/P^N)$, increases by the same proportion as the nominal money supply (i.e., it doubles). Given equation (8), the demand for non-tradable goods also doubles (recall that c^T is invariant). In terms of the price index, however, real consumption of non-traded goods increases by $\hat{M} - \hat{E}/2 = \hat{M}/2$. In other words, real consumption of non-traded goods less than doubles. Since the consumption-elasticity is σ, the resulting increase in real money demand is therefore $\sigma\hat{M}/2$.

We thus conclude that, if the nominal exchange rate increased by the same proportion as the money supply, real money supply would increase by $\hat{M}/2$ and real money demand by $\sigma\hat{M}/2$. Hence, the excess supply in the money market would be given by $\hat{M}/2 - \sigma\hat{M}/2$, which is simply $(1-\sigma)\hat{M}/2$. Based on this simple expression, it follows immediately that:

- If $\sigma = 1$, both real money supply and real money demand increase by the same amount and therefore the equiproportional increase in the nominal exchange rate (i.e., no overshooting or undershooting) is consistent with money market equilibrium.

- If $\sigma < 1$, real money supply would increase by more than real money demand and there would be excess supply in the real money market.

[13] A "hat" over a variables denotes proportional change.

This, of course, is not an equilibrium. The excess supply of money requires a fall in the nominal interest rate. For this to happen, the rate of depreciation must become negative (i.e., agents must expect a nominal *appreciation* of the currency). For the rate of depreciation to become negative, the nominal exchange rate must overshoot its long-run level and fall over time.

- If $\sigma > 1$, real money supply would increase by less than real money demand and there would be excess demand for money. To equilibrate the money market, the nominal interest rate needs to increase. From the interest parity condition, this requires a depreciation of the currency. For this to happen, the nominal exchange rate must jump by less than its long-run level.

  Finally, two observations are worth making. First, in general equilibrium, a sticky-prices model does not *necessarily* lead to a liquidity effect (i.e., an increase in M leading to a fall on impact of the nominal interest rate). In fact –and as we just saw– an increase in M is consistent with i falling, increasing, or remaining unchanged. Second, in the model the co-movement on impact between the nominal interest rate and the level of the exchange rate is also ambiguous. The model does not support the notion – typically found in the financial press and undergraduate textbooks – that a depreciation of the currency will be necessarily associated with a fall in nominal interest rates.

6 A model of sticky wages

While the model with sticky-prices developed above is extremely useful to ask a myriad of *positive* (as opposed to *normative*) questions – such as what happens when there is an increase in the money supply or a devaluation – it is not well-suited to ask *normative* questions. The reason is that, outside the steady-state, output is demand-determined and hence the present discounted value of output (which would determine the average level of consumption) does not obey any physical constraints. To address this shortcoming, this section develops a model with a fully-specified supply side in which nominal wages – rather than prices – are sticky.[14] This will provide us with a model

[14] The model is a simpler version of Lahiri and Vegh (2002). See Barro and Grossman (1971) for an early and highly influential contribution on disequilibrium models.

that, in addition to capturing the key dynamics of an economy with nominal rigidities, is perfectly suited to respond to normative questions.

What are the kind of normative questions that we would like to ask? Perhaps one of the most important – and most hotly debated by policy-oriented academic economists – is under what circumstances a country would find it optimal to devalue its currency. In times of low growth and trade deficits, economists often call for a devaluation to address such imbalances. An excellent case in point is Rudiger Dornbusch's forceful advocacy of a devaluation in Mexico during 1994 (see Box 3). In fact, the model developed in this section may be viewed as the best-case scenario for a Dornbusch-type argument since, for conceptual clarity, the model focuses exclusively on nominal rigidities and ignores other potential problems that may be associated with a devaluation (like credibility problems). In fact, we will see that in this model it is optimal to devalue in response to a negative real shock.

To simplify the presentation, we will consider a one-good model and thus abstract from non-tradable goods. We will, however, introduce a labor/leisure choice and, hence, endogenous production. The law of one price holds for the only good (i.e., $P_t = E_t P_t^*$). There is no foreign inflation and, to simplify notation, the foreign nominal price is taken to be unity (i.e., $P^* = 1$). Hence, $P_t = E_t$. (Unless otherwise noticed, the notation is the same as above.) The economy operates under predetermined exchange rates and, for simplicity, the rate of devaluation is taken to be zero (i.e., the exchange rate is fixed).

6.1 Households

Preferences are now given by

$$\int_0^\infty \{\log[c_t - \phi(\ell_t^s)^\upsilon] + \log(m_t)\} e^{-\beta t} dt, \qquad \phi > 0,\ \upsilon > 1. \tag{47}$$

where c is consumption of tradable goods (the only good in this world), ℓ^s denotes labor supply, and m denotes real money balances ($m \equiv M/E$).[15] These are the so-called GHH preferences – after the paper by Greenwood, Hercowitz and Huffman (1988) – that generate a labor supply function that

[15] In this model, it is critical to distinguish between labor supply and labor demand because, as discussed in detail below, the labor market may be in disequilibrium (i.e., labor supply may not be equal to labor demand at all points in time).

depends only on the real wage. In other words, there is no wealth effect on leisure, which greatly simplifies the solution of the model.[16]

The household's flow budget constraint is given by

$$\dot{a}_t = ra_t + w_t\ell_t^s + \tau_t + \Omega_t - c_t - i_t m_t, \tag{48}$$

where $a(\equiv m+b)$ denotes real financial wealth, $w(\equiv W/E)$ denotes the real wage, and Ω_t are dividends from firms (which are owned by households). The corresponding lifetime constraint reads as

$$a_0 + \int_0^\infty (w_t\ell_t^s + \Omega_t + \tau_t)\, e^{-rt}dt = \int_0^\infty (c_t + i_t m_t)\, e^{-rt}dt. \tag{49}$$

The household chooses $\{c_t, \ell_t^s, m_t\}_{t=0}^\infty$ to maximize (47) subject to lifetime constraint (49). In terms of the Lagrangian,

$$\begin{aligned}\mathcal{L} \;=\; & \int_0^\infty [\log[c_t - \phi(\ell_t^s)^\upsilon] + \log(m_t)]\, e^{-\beta t}dt \\ & +\lambda\left[a_0 + \int_0^\infty (w_t\ell_t^s + \Omega_t + \tau_t)\, e^{-rt}dt - \int_0^\infty (c_t + i_t m_t)\, e^{-rt}dt\right].\end{aligned}$$

The first-order conditions with respect to c_t, ℓ_t^s, and m_t are given by, respectively,

$$\frac{1}{c_t - \phi(\ell_t^s)^\upsilon} = \lambda, \tag{50}$$

$$\frac{\phi\nu(\ell_t^s)^{\nu-1}}{c_t - \phi(\ell_t^s)^\upsilon} = \lambda w_t, \tag{51}$$

$$\frac{1}{m_t} = \lambda i_t. \tag{52}$$

Condition (50) is the familiar condition in models with no intertemporal distortions that says that, along a perfect foresight path, the marginal utility of consumption will be constant. In this formulation, however, the marginal utility of consumption depends on labor supply. Hence, consumption smoothing will not necessarily obtain in this model.[17]

[16] You may recall that we have already encountered these preferences in Exercise 6 at the end of Chapter 1.

[17] As you may recall, this point was emphasized in Exercise 6 in Chapter 1.

Substituting equation (50) into equation (51), we obtain:[18]

$$\phi\nu(\ell_t^s)^{\nu-1} = w_t. \tag{53}$$

To obtain the labor supply schedule, solve for ℓ^s from the last equation to obtain:

$$\ell_t^s = \left(\frac{w_t}{\phi\nu}\right)^{\frac{1}{\nu-1}}. \tag{54}$$

As anticipated, labor supply depends solely on the real wage, w. Labor supply is an increasing function of the real wage, with the elasticity given by $1/(\nu-1)$.

The money demand follows from combining (50) and (52):

$$m_t = \frac{c_t - \phi(\ell_t^s)^{\upsilon}}{i_t}. \tag{55}$$

GHH preferences thus generate a non-standard money demand, in that the scale variable is consumption *net* of the disutility of labor.

6.2 Supply side

Firms produce tradable goods according to the following technology:

$$y_t = \psi\left(\ell_t^d\right)^{\alpha}, \qquad \alpha < 1, \tag{56}$$

where ℓ^d denotes labor demand.

The representative firm's profits are given by:

$$\Omega_t = y_t - w_t\ell_t^d. \tag{57}$$

Substituting (56) into (57) yields:

$$\Omega_t = \psi\left(\ell_t^d\right)^{\alpha} - w_t\ell_t^d. \tag{58}$$

Firms choose ℓ_t^d to maximize (58). The first-order condition is given by

$$\psi\alpha\left(\ell_t^d\right)^{\alpha-1} = w_t. \tag{59}$$

[18] As discussed in detail below, in this disequilibrium model, actual employment may not coincide with labor supply, in which case condition (51) will *not* hold. In that case, the marginal disutility of labor will fall short of the real wage.

Production efficiency requires that the marginal productivity of labor be equated to the real wage. Solving for ℓ_t^d from equation (59) yields the labor demand equation:

$$\ell_t^d = \left(\frac{\alpha\psi}{w_t}\right)^{\frac{1}{1-\alpha}}. \tag{60}$$

Labor demand is a decreasing function of the real wage because a higher real wage induces firms to shed labor to increase its marginal productivity.

6.3 Labor market

The key action in this model takes place in the labor market. To focus the discussion, Figure 8 illustrates the labor market. Labor demand – given by equation (60) – is shown as a decreasing function of the real wage whereas labor supply – given by equation (54) – is an increasing function of the real wage. As a benchmark, we will first discuss the flexible-wages case and then turn to the sticky-wages case.

[Figure 8]

6.3.1 Flexible wages

In the flexible-wages version of this model, labor market equilibrium requires that

$$\ell^s = \ell^d.$$

Graphically, the economy is always at point A in Figure 8. In other words, the economy always enjoys full-employment in the sense that all workers that are willing to work at the prevailing real wage are indeed employed. As indicated in Figure 8, we will denote the flexible-wages equilibrium values of labor and the real wage by, respectively, ℓ^f and w^f. Formally, using (54) and (60), it follows that

$$w^f = \left[(\alpha\psi)^{\frac{1}{1-\alpha}} (\phi\nu)^{\frac{1}{\nu-1}}\right]^{\frac{(\nu-1)(1-\alpha)}{\nu-\alpha}}. \tag{61}$$

Using (54), equilibrium labor can be expressed as:

$$\ell^f = \left(\frac{w^f}{\phi\nu}\right)^{\frac{1}{\nu-1}}. \tag{62}$$

Hence, in equilibrium, both the real wage and labor are an increasing function of the productivity parameter, ψ.

6.3.2 Sticky wages

If the nominal wage is sticky (i.e., it is a predetermined variable), the labor market will not necessarily be in equilibrium.[19] The reason is that, since the economy is operating under a fixed exchange rate, a sticky *nominal* wage implies a sticky *real* wage. To illustrate the concept of labor market disequilibrium, suppose that the prevailing real wage is above the equilibrium real wage and given by w^H in Figure 8. At the real wage w^H, labor supply (point C) exceeds labor demand (point B). Graphically, the excess supply of labor is given by the segment BC in Figure 8. Conversely, suppose that the prevailing real wage is below the equilibrium real wage and given by w^L in Figure 8. At this real wage, there is excess demand for labor – given by the segment DE.

If there is excess labor supply or demand, what will *actual* employment be? The more natural assumption – and the one commonly adopted in the literature – is that the *short-end* of the market prevails. Specifically, if labor demand falls short of labor supply, actual employment is given by labor demand (point B in Figure 8). If, on the other hand, labor supply falls short of labor demand, actual employment is given by labor supply (point D in Figure 8). Formally, denoting actual labor by ℓ^a, we have that:

$$\ell_t^a = \begin{cases} \ell_t^f, & \text{if } \ell_t^d = \ell_t^s, \\ \ell_t^d, & \text{if } \ell_t^d < \ell_t^s, \\ \ell_t^s, & \text{if } \ell_t^d > \ell_t^s. \end{cases}$$

As a result, if the real wage is w^H, there is involuntary unemployment in the sense that not all workers willing to work at the prevailing wage are

[19] To fix ideas, the discussion will assume that the nominal wage is sticky both upwards and downwards. An alternative assumption would be that the nominal wage is sticky downwards but not upward. In that case, sticky wages would not be a binding constraint for the economy's response to any shock that requires an *increase* in the equilibrium real wage.

employed (i.e., $\ell_t^a < \ell_t^s$). If the wage is w^L, firms would not be able to hire all the workers that they would like at the prevailing real wage (i.e., $\ell_t^a < \ell_t^d$). In either case, the labor market is in *disequilibrium* because demand and supply are not the same. This is, of course, equivalent to saying that one of the two marginal conditions related to the labor market is not holding. If $\ell_t^a = \ell_t^d < \ell_t^s$, firms are operating on their demand curve (i.e., marginal condition (59) holds) but households are not on their labor supply curve (i.e., condition (53) does not hold because the marginal disutility of labor, evaluated at ℓ_t^a, falls short of the real wage). Conversely, if $\ell_t^a = \ell_t^s < \ell_t^d$, then households are on their labor supply curve, but firms are not on their demand curve (in fact, the marginal productivity of labor evaluated at ℓ_t^a exceeds the real wage).

Finally, we need to ask: if the labor market is in disequilibrium, how will it adjust over time to reach equilibrium? A natural assumption regarding the adjustment in the labor market is to posit that the nominal wage evolves according to the deviation of the actual real wage from the full-employment real wage:

$$\dot{W}_t = \phi\left(w^f - \frac{W_t}{E_t}\right), \qquad \phi > 0,\ W_0 \text{ given.} \tag{63}$$

Hence, if the prevailing real wage is above the full-employment level (i.e., $\frac{W_t}{E_t} > w^f$), the nominal wage falls over time. The idea is that the excess labor supply leads to a gradual fall in nominal wages, as unemployed workers become more willing over time to take jobs at lower nominal wages. On the other hand, if the prevailing real wage is below the full-employment level (i.e., $\frac{W_t}{E_t} < w^f$) and there is thus excess demand for labor, the nominal wage increases over time reflecting the willingness of firms to pay higher nominal wages due to the tight labor market conditions.

6.4 Government

The government's budget constraints are unchanged relative to our previous model and continue to be given by equations (13) and (14).

6.5 Equilibrium conditions

Given perfect capital mobility and a fixed exchange rate, it follows that

$$i_t = r. \tag{64}$$

Before aggregating the households' and firms' constraints, we need to take into account that when the labor market is in disequilibrium, the relevant labor variable for either households' wage income or firm's productive purposes is ℓ^a. Hence, we can replace ℓ^s with ℓ^a in the households' budget constraint (48) and ℓ^d with ℓ^a in the firms' profits (58). We can then combine both to obtain:

$$\dot{a}_t = ra_t + \psi\left(\ell_t^a\right)^{\alpha} + \tau_t - c_t - i_t m_t.$$

Combining this constraint with the government's flow constraint – given by (13) – yields:

$$\dot{k}_t = rk_t + \psi\left(\ell_t^a\right)^{\alpha} - c_t.$$

Proceeding in the same way, we can combine the household's intertemporal constraint (49) with the firm's constraint and then with the government's lifetime constraint (14) to obtain:

$$k_0 + \int_0^{\infty} \psi\left(\ell_t^a\right)^{\alpha} e^{-rt} dt = \int_0^{\infty} c_t e^{-rt} dt. \tag{65}$$

6.6 Initial stationary equilibrium

Consider an initial perfect foresight equilibrium in which the economy is in a full-employment equilibrium. The equilibrium real wage and labor are thus given by expressions (61) and (62), respectively. From (56), it follows that full-employment output is given by

$$y^f = \psi\left(\ell^f\right)^{\alpha}. \tag{66}$$

From the resource constraint (65), it follows that full-employment consumption is given by

$$c^f = rk_0 + \psi\left(\ell^f\right)^{\alpha}. \tag{67}$$

Finally, we can derive the real money demand from (55) and (64):

$$m^f = \frac{c^f - \phi(\ell^f)^\upsilon}{r}. \tag{68}$$

6.7 Permanent fall in productivity

Suppose that up to time $t = 0^-$ the economy is in the full-employment stationary state just described. At time 0, there is an unanticipated and permanent reduction in ψ. As a benchmark, we will first analyze the adjustment that would take place under flexible wages and then focus on the sticky-wages case.

6.7.1 Flexible wages

As (60) makes clear, the fall in ψ reduces labor demand for a given level of the real wage. In terms of Figure 9, the initial equilibrium is given by point A. The fall in productivity shifts the labor demand to the left (from ℓ^d to $\left(\ell^d\right)'$). The labor market adjusts instantaneously from its initial equilibrium, point A, to the new equilibrium, point B, at which both the real wage $(w^f)'$ and employment are lower $\left(\ell^f\right)'$. Since the exchange rate is fixed, the fall in the real wage is effected through a fall in the *nominal* wage, W. Both output and consumption fall, as follows from (66) and (67). (This adjustment is captured by the full lines in Figure 10.)

[Figure 9]

[Figure 10]

Although it seems intuitive that, in response to the fall in consumption, real money demand should fall, this is not formally obvious from just glancing at equation (68). To formally show this, we compute the change in $c^f - \phi(\ell^f)^\upsilon$ in response to a small change in ψ which, using (67), leads to:

$$\frac{d\left[c^f - \phi(\ell^f)^\upsilon\right]}{d\psi} = \left(\ell^f\right)^\alpha + \frac{d\ell^f}{d\psi}\left[\psi\alpha\left(\ell^f\right)^{\alpha-1} - \phi\upsilon(\ell^f)^{\upsilon-1}\right].$$

The term in square brackets on the RHS is simply the difference between the marginal productivity of labor and the marginal disutility of labor, which is always zero under flexible wages. Hence:

$$\frac{d\left[c^f - \phi(\ell^f)^v\right]}{d\psi} = \left(\ell^f\right)^{\alpha} > 0. \tag{69}$$

Intuitively, this is an envelope condition that says that, at an optimum, consumption net of the disutility of labor falls by the direct effect of the fall in productivity on output since, at the margin, production efficiency always implies that the marginal productivity of labor is equated to the marginal disutility of labor. In light of (69), it follows that real money demand falls.

Finally, we verify that, as one should expect, welfare falls. Given that the economy jumps from one stationary state to the next, we infer from (47) that the change in welfare depends on the change in $c^f - \phi(\ell^f)^v$ and in m. Since we have shown that both fall, welfare also falls.

6.7.2 Sticky wages

Suppose now that the nominal wage is sticky. Since the economy is operating under a fixed exchange rate, the real wage cannot change on impact. Hence, on impact, the economy finds itself at point C in Figure 9. At the prevailing wage, w^f, there is excess supply of labor and actual employment, ℓ^a, is given by labor demand at the level ℓ_0. We thus see how, in the presence of sticky nominal wages, this negative supply shock leads to involuntary unemployment, in the amount CA in Figure 9.

How will the nominal wage evolve over time? At $t = 0$, we know, using (63),

$$\dot{W}_0 = \phi\left[\left(w^f\right)' - \frac{W_0}{\bar{E}}\right] < 0.$$

The nominal wage begins to fall at time 0 and, in fact, will continue falling over time. Since the exchange rate is fixed, the real wage, which does not change on impact, also falls over time towards its full-employment level (Figure 10, Panel B). As the real wage falls over time, actual employment will continue to be given by the short-end of the market. In terms of Figure 9, this means that actual employment will increase over time along the arrowed path from point C to point B as the nominal – and hence real – wage fall over time. Formally, it follows from (60) that

$$\dot{\ell}_t^a = \dot{\ell}_t^d = -\frac{\ell_t^d}{(1-\alpha)w_t}\dot{w}_t > 0.$$

The path of labor is thus given by Figure 10, Panel C. On impact, therefore, the fall in employment is larger than under flexible wages. Given the production function (56), the path of output follows that of labor (Figure 10, Panel D). The initial fall in output is thus also larger under sticky wages than under flexible wages.

Let us turn to the behavior of consumption.[20] From first-order condition (50) and the fact that labor increases over time, it follows that consumption also increases over time:

$$\dot{c}_t = \phi\upsilon \left(\ell_t^a\right)^{\upsilon-1} \dot{\ell}_t^a > 0 \tag{70}$$

To find out the change on impact, we need to look at the resource constraint. The output path illustrated in Figure 10, Panel D, indicates that the present discounted value of output falls. Hence, consumption can neither increase nor stay the same for, if it did, the present discounted value of consumption would increase and thus violate the resource constraint. We thus conclude that consumption falls on impact. Figure 10, Panel E illustrates the path of consumption.[21]

What happens to the trade balance? By definition, $TB_t = y_t - c_t$. Hence, taking into account (56) and (70),

$$\dot{TB}_t = \left[\psi\alpha \left(\ell_t^a\right)^{\alpha-1} - \phi\upsilon \left(\ell^a\right)^{\upsilon-1}\right] \dot{\ell}_t^a > 0,$$

where the sign follows from the fact that at point C in Figure 9 (and any other point along the arrowed path CB), the marginal productivity of labor is greater than the marginal disutility of labor. The trade balance thus improves over time. It immediately follows that, on impact, the trade balance must fall. If it did not change or increase on impact – and since it then increases

[20] Under sticky wages, one way to think about the household's optimization problem is that the household is now taking as given the path of labor illustrated in Figure 10, Panel C, and optimally choosing consumption and real money balances (see Barro and Grossman (1971) for a detailed discussion). In other words, while first-order conditions (50) and (52) continue to hold, first-order condition (51) does no longer hold. In fact, when actual labor is dictated by labor demand (i.e., there is an excess supply of labor), the marginal disutility of labor will be lower than the prevailing real wage (indicating an unsatisfied desire to work).

[21] To show that consumption ends up below its full-employment level, notice that, as shown below, net consumption (i.e., $c - \phi(\ell_t^a)^\upsilon$) is lower under sticky wages than under flexible wages. Since labor eventually converges to the same value in both cases, consumption must be lower in the long-run under sticky wages than under flexible wages.

over time – it would violate the resource constraint. The path of the trade balance is illustrated in Figure 10, Panel F (assuming $k_0 = 0$). It follows that there is some consumption smoothing taking place as the trade deficit is largest early on when the production is at its lowest point.

We now turn to the path of real money balances which, from (55) and (64), is given by

$$m_t = \frac{c_t - \phi(\ell_t^a)^\upsilon}{r}. \tag{71}$$

We know from first-order condition (50) that $c_t - \phi(\ell_t^a)^\upsilon$ will be constant along the new perfect foresight path. Further, as shown in Appendix 8.2, $c_t - \phi(\ell_t^a)^\upsilon$ falls on impact. Hence, real money demand will fall on impact as well and remain at that level thereafter.

Finally, we turn to the issue of welfare. It should be clear that the economy's adjustment under sticky wages is costlier than under flexible wages. In fact, since there are no distortions of any kind in the flexible wages case, the economy's adjustment constitutes the first-best response. In other words, given that the economy is poorer, it is optimal to adjust immediately to the new reality. It follows that the adjustment under sticky nominal wages – which deviates from the first-best adjustment – is costlier.

It is, in fact, easy to verify that welfare falls by more in the sticky wage than in the flexible wage case. As shown in Appendix 8.2, $c_t - \phi(\ell_t^a)^\upsilon$ – and hence real money balances – fall by more in the sticky wage case than in the flexible wage case. Hence, welfare will also be lower.

6.8 Optimal devaluation

As illustrated in Figure 10, we have concluded from our analysis that sticky nominal wages interfere with the optimal adjustment of the economy to a negative shock. Since the economy has become poorer as the result of a permanent fall in productivity, the optimal adjustment consists in an immediate reduction in the real wage (from point A to point B in Figure 9), which ensures that full employment continues to prevail (albeit at a lower level given that the economy is now less productive). Under fixed exchange rates, the adjustment in the real wage should take place through a fall in the nominal wage. By preventing the real wage from adjusting, wage stickiness causes the economy to undergo a costlier adjustment. In fact, the economy must go through a protracted period of unemployment before it finally reaches the long-run equilibrium, As a result, welfare is lower than in the flexible wage equilibrium.

Is there anything policymakers can do to ease the economy's adjustment under sticky wages? They certainly can. In fact, by devaluing when the negative shock hits, policymakers can reduce the real wage from w^f to $\left(w^f\right)'$ in Figure 8 thus managing to take the economy from point A to point B in spite of sticky wages. The reduction in the real wage is achieved through a devaluation rather than through a fall in the nominal wage. Clearly, the devaluation is the first-best response to this negative shock as it reproduces the outcome that would prevail under flexible wages.[22] [23]

Even if policymakers do not devalue as soon as the negative shock hits, it would still be optimal to devalue at any point in time as the economy travels from point C to point B in Figure 9 and take the economy immediately to point B, rather than letting the adjustment take its natural course. In fact, the adjustment along the segment CB in Figure 9 is characterized by low output and (initial) trade deficits, symptoms that are often seen as requiring a discrete devaluation. Needless to say, however, to make the best-case scenario for a devaluation, we have ignored many other important aspects of reality, such as credibility problems, that may play an important role in practice (see Box 3).

7 Conclusions

This chapter has removed the veil from our monetary model in Chapter 5 by incorporating sticky prices, by far the most popular friction in open economy models. In such a context, permanent changes in monetary policy (both in the level and in the rate of growth of the money supply) are expansionary as they lead to higher aggregate demand and output. Sticky prices also allowed us to rationalize the high volatility of nominal exchange rates (i.e., the overshooting phenomenon). In a similar vein, a devaluation leads to higher

[22]For the same reasons, it is easy to see that, under flexible exchange rates, the economy would adjust instantaneously from point A to point B *in spite* of sticky wages! This suggests that in a world with nominal rigidities – and in response to real shocks – flexible rates are better than fixed rates. The opposite, however, will be true for monetary shocks. We will study these issues in detail in Chapter 12 on optimal exchange rate regimes.

[23]Notice, however, that a change in the nominal exchange rate (either a devaluation or a revaluation) starting from a steady-state would always lead to a fall in labor and output, as analyzed in Exercise 3 at the end of this chapter.

output and consumption, in contrast to the model of Chapter 6, in which a devaluation reduces consumption. We have also studied a disequilibrium model of sticky wages, a slightly more complicated theoretical set-up but arguably a more insightful representation of a world with nominal rigidities. In particular, we saw how such a model can rationalize the need for a devaluation of the domestic currency in response to a negative shock.

This chapter concludes our first incursion into monetary models. We studied the basic monetary model in Chapter 5 – in which money is a veil – and then removed the veil by abstracting from interest-bearing bonds (Chapter 6), introducing a link between nominal interest rates and consumption (Chapter 7) and sticky prices (Chapter 8). We now move to Part II of the book in which we will put all these tools to work in our quest for understanding important macroeconomic policy issues.

8 Appendices

8.1 Calvo's (1983) staggered prices

Calvo (1983) developed an extremely useful continuous-time version of the staggered-prices models à la Taylor (1979, 1980) and Fischer (1977). Suppose that there is a large number (technically, a continuum) of firms in the $[0,1]$ interval. Total number of firms is therefore one. Each firm produces a non-storable good at zero variable cost, the quantity of which is demand-determined. Each firm may change its price only when it receives a random price signal. The probability of receiving a signal follows an exponential distribution. When a firm changes its price, it takes into account the expected average price and the level of excess aggregate demand (A) expected to prevail in the future.

The probability of receiving a price signal j periods from now is $\delta e^{-\delta j}$, where $\delta > 0$. The firm's price setting rule is assumed to be given by:

$$\log(V_t) = \delta \int_t^{\infty} [\log(P_s) + \omega A_s] e^{-\delta(s-t)} ds, \qquad \omega > 0, \tag{72}$$

where V_t is the price quotation set at t, P_s is the price level (to be defined below) and A_s denotes excess aggregate demand. Note that V may jump if an unexpected change in, say, A takes place.

If price changes are stochastically independent across firms, the proportion of prices set at time s that have not been modified as of time t is given by $\delta e^{-\delta(t-s)}$. The (logarithm of) the price level is defined as the weighted average of prices currently quoted. Hence:

$$\log(P_t) = \delta \int_{-\infty}^{t} \log(V_s) e^{-\delta(t-s)} ds. \tag{73}$$

An important observation is that, unlike V_t, P_t is a *predetermined* variable because it is given by past price quotations. Along paths where P_t and A_t are uniquely determined, however, V_t is a continuous function of time. Differentiating equation (73) with respect to time yields (using Leibnitz's rule):

$$\pi_t = \delta \left[\log(V_t) - \log(P_t) \right], \tag{74}$$

where $\pi \equiv \dot{P}_t / P_t$.[24] Notice that anticipated changes in A_t will not affect π. In other words, along a perfect foresight path, π_t will be a continuous function of time.

At points in time at which A_t is continuous, we can differentiate equation (72) to obtain (again, using Leibnitz's rule)

$$\frac{\dot{V}_t}{V_t} = \delta \left[\log(V_t) - \log(P_t) - \omega A_t \right]. \tag{75}$$

It follows from (74) and (75) that (at points in time at which A_t is continuous)

$$\dot{\pi}_t = -\theta A_t, \tag{76}$$

where $\theta \equiv \delta^2 \omega > 0$. Equation (76) is thus a "higher" order inverse Phillips curve which indicates that the *change* in the inflation rate is *negatively* related to excess demand.

[24] In its more general formulation, the Leibnitz's rule states that if we have a function, $F(t)$, defined as

$$F(t) = \int_{g(t)}^{x(t)} f(s,t) ds,$$

its derivative is given by

$$F'(t) = f(s, x(t)) x'(t) - f(s, g(t)) g'(t) + \int_{g(t)}^{x(t)} \frac{\partial f(s,t)}{\partial_t} ds.$$

8.2 Sticky wage model

This appendix analyzes the behavior of $c_t - \phi(\ell_t^a)^{\upsilon}$ in response to the permanent fall in the productivity parameter, ψ. First-order condition (50) – with ℓ_t^a in lieu of ℓ_t^s – indicates that $c_t - \phi(\ell_t^a)^{\upsilon}$ will be constant along the new perfect foresight path. To pin down the level, we need to compute the present discounted value of $c_t - \phi(\ell_t^a)^{\upsilon}$. Using the economy's resource constraint, we can write it as:

$$PDV \equiv \int_0^{\infty} \left[c_t - \phi(\ell_t^a)^{\upsilon}\right] e^{-rt} dt = k_0 + \int_0^{\infty} \left[\psi \left(\ell_t^a\right)^{\alpha} - \phi(\ell_t^a)^{\upsilon}\right] e^{-rt} dt.$$

We now compute the change in the present discounted value as a result of a small change in ψ:

$$\frac{dPDV}{d\psi} = \int_0^{\infty} \left(\ell_t^a\right)^{\alpha} + \underbrace{\left[\psi\alpha \left(\ell_t^a\right)^{\alpha-1} - \phi\upsilon(\ell_t^a)^{\upsilon-1}\right]}_{+} \underbrace{\frac{d\ell_t^a}{d\psi}}_{+} e^{-rt} dt > 0. \qquad (77)$$

As indicated, the term in curve brackets is positive because it is the difference between the marginal productivity of labor and the marginal disutility of labor. (Notice that this term is zero in the flexible wages case.) Equation (77) thus indicates that in response to a fall in ψ, the PDV of $c_t - \phi(\ell_t^a)^{\upsilon}$ also falls and, in fact, falls by more than in does in the flexible wages case.

We thus conclude that $c_t - \phi(\ell_t^a)^{\upsilon}$ falls on impact and remains at that level thereafter. From (71), it follows that m falls and, in fact, falls by more than in the flexible wage case. It follows that welfare follows by more in the sticky wage case than in the flexible wages case because both $c_t - \phi(\ell_t^a)^{\upsilon}$ and m fall by more.

Exercises[25]

1. ### Temporary reduction in money growth rate

   The purpose of this exercise is to show that the sticky-prices model developed in the text is capable of explaining situations of "stagflation" (i.e., the co-existence of high inflation and output below the full-employment level).

   In the context of the model developed in Section 2:

   (a) Analyze the effects of a temporary reduction in the money growth rate.

   (b) Explain the intuition behind the results.

2. ### Fiscal policy in a sticky prices model

   This exercise incorporates fiscal policy into the sticky prices model analyzed in this chapter and studies the effects of a permanent increase in government spending on non-tradable goods under both flexible and predetermined exchange rates.

   Specifically, suppose preferences are given by

$$\int_0^\infty \left[\log(c_t^T) + \log(c_t^N) + \log(m_t)\right] e^{-\beta t} dt,$$

   The consumer's intertemporal constraint is given by

$$a_0 + \int_0^\infty \left(y_t^T + \frac{y_t^N}{e_t} + \tau_t \right) e^{-rt} dt = \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t} + i_t m_t \right) e^{-rt} dt.$$

   The government's flow budget constraint is given by

$$\dot{h}_t = r h_t + \dot{m}_t + \varepsilon_t m_t - \tau_t - \frac{g_t^N}{e_t}.$$

   The corresponding intertemporal constraint is given by

$$h_0 + \int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt = \int_0^\infty \left(\frac{g_t^N}{e_t} + \tau_t \right) e^{-rt} dt.$$

[25] An answer key is available from the author upon request.

Equilibrium in the non-tradable goods market dictates that

$$y_t^N = c_t^N + g_t^N.$$

The rest of the model remains the same as in the text.

In the context of this model:

(a) Analyze the effects of a permanent and unanticipated increase in government spending on non-tradable goods under flexible exchange rates.

(b) Analyze the effects of a permanent and unanticipated increase in government spending on non-tradable goods under predetermined exchange rates.

(c) Explain intuitively why the response is the same. Do you think that it would continue to be the same under non-logarithmic preferences? [26]

3. Devaluation/revaluation in a sticky wages model

In the context of the sticky wages model of Section 6, show that both a devaluation and a revaluation of the currency (i.e., an increase and a decrease in the nominal exchange rate) lead to a fall in actual labor and output. Explain the intuition behind the results. (Lahiri and Vegh (2002) use this feature of the sticky-wages model to think about the costs of a fluctuating nominal exchange rate.)

[26] In fact, the data suggest that output responds differently under predetermined and flexible exchange rates (see Ilzetzki, Mendoza, and Vegh (2010)).

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Box 1. Are devaluations expansionary or contractionary?

As argued in the text, the real effects of a currency devaluation are theoretically ambiguous. In the model laid out in Chapter 6 – with no interest-bearing bonds – a devaluation is contractionary, as private agents reduce consumption to rebuild their stock of real money balances. In this chapter's model with sticky prices, a devaluation is expansionary since it reduces on impact the relative price of non-traded goods, thus leading to an increase in aggregate demand and output. In addition to these two channels, the literature has explored other, mostly contractionary, channels. Among the most relevant ones:[27]

- *Effects on imported inputs/investment.* In models with imported inputs, a devaluation will rise the domestic price of such inputs and thus be contractionary (see, for example, Gylfason and Schmid (1983), van Wijnbergen (1986) and Edwards (1986)). In a similar vein, Buffie and Wong (2001) show how a devaluation is likely to reduce aggregate investment and output in a model in the spirit of Chapter 6 (i.e., consumers hold no foreign bonds) but with a fully-fledged two-sector supply side. In their model, the resulting real devaluation increases the relative price of capital in the non-tradable sector but decreases it in the tradable sector. Furthermore, the marginal productivity of capital increases in the tradable sector and falls in the non-tradable sector. Based on these two effects, investment would rise in the tradable sector and fall in the non-tradable sector. A third effect – the real money balances effect examined in Chapter 6 – tends to reduce investment in both sectors as households switch from capital to money to rebuild their real money balances. Calibrations of the model suggest that overall investment will fall.

- *Income redistribution.* Profits will be boosted in export and import competing industries as devaluations lead to higher relative prices for traded goods. When this increased price level leads to lower real wages, national spending is likely to shrink since the marginal propensity to

[27] The following list is far from exhausive. The reader is referred to Lizondo and Montiel (1989) and Agenor and Montiel (1999, Chapter 8) for a more detailed discussion.

save from profits exceeds that from wages (see, in particular, Diaz-Alejandro (1963), Cooper (1971), and Krugman and Taylor (1978)).

- *Balance sheet effects.* The risk premium typically increases if public and private sector's net worth is affected by real exchange rate movements (see, for instance, Cespedes, Chang and Velasco (2004)).

The theory thus identifies several contractionary channels that could, in principle, more than outweigh the traditional expansionary effects emphasized by sticky-prices model. The overall effect thus becomes an empirical question.

What do the data say? Table 1 summarizes the results of some notable contributions on the subject. An early study by Gylfason and Shmid (1983) reported a mostly positive effect of a devaluation on output. Subsequent studies, however, tended to find a weak negative impact, if any, of devaluations on output. A recent study by Gupta, Mishra, and Sahay (2007), which relies a on large dataset comprising 91 developing countries and spanning almost 30 years, documents that, on average, output has fallen when currency crises have taken place[28] Still, a significant number of episodes (43 percent) have been associated with output increases.

[Table 1]

What could explain different responses of output to a devaluation? According to some authors, the output effects may be conditional on other relevant variables. Gupta, Mishra, and Sahay (2007), for instance, find that the output effect is positively associated with commercial integration with the rest of the world but negatively associated with financial integration and previous periods of capital inflows. They also report that large emerging economies are more likely to suffer contractionary devaluations than small ones. On the other hand, Cavallo *et al* (2004) stress the empirical importance of balance sheet mismatches. They claim that, among developing economies, output contraction has been greater in relatively large and more developed economies than in smaller and less developed economies.

[28] It should be noted that the focus of this paper is on currency crises (the definition of which typically includes increases in the exchange rate but also includes changes in reserves). Ideally, one would like to see a similarly large dataset used to study the effects of devaluations only. The overlapping, of course, would be large.

In sum, the jury is still out on whether, in practice, devaluations are expansionary or contractionary. If anything, the often-mixed evidence suggests that the real effects of devaluation likely depend on the circumstances surrounding it. For instance, it stands to reason that a devaluation carried out in the middle of a full-fledged balance of payments and financial crisis and possibly accompanied by tighter fiscal and monetary policies is much more likely to be contractionary than a devaluation undertaken as part of an orderly adjustment to, say, a perceived exchange rate "misalignment". Unfortunately, controlling for these factors is not a trivial empirical task.

Box 2. How sticky are prices?

As mentioned in the Introduction, price stickiness is, by far, the most common friction used in models that study the real effects of monetary and exchange rate policies. Many closed and open economy models assume the existence of a large number of differentiated goods and introduce price stickiness a la Calvo (1983) so that, at every point in time, there is a fraction of firms that are unable to change their price. But how sticky are prices in practice? Empirically, price stickiness is typically measured by analyzing time series for prices of different goods, estimating the frequency of price changes for each series, and aggregating the results by appropriately weighting each price in order to obtain an estimate of the average time between price changes for different categories of goods.

Early empirical work measuring the frequency of price changes in retail and wholesale prices established that many prices often go unchanged for many months. However, these estimates were usually based on relatively narrow sample of goods.[29] In recent years, the availability of richer datasets has allowed researchers to come up with a much clearer picture of the behavior of individual prices. In an influential paper, Bils and Klenow (2004) examined the frequency of price changes for 350 categories of goods and services covering about 70 percent of the U.S. Consumer Price Index (CPI). Surprisingly, they found a median time between price changes between 4.3 to 5.5 months, well below previous estimates. They also found a very large degree of heterogeneity in the behavior of price changes across goods.

More recently, Nakamura and Steinsson (2008), Klenow and Kryvtsov (2008), and Klenow and Malin (2010) have also studied the U.S. CPI but using datasets that allowed them to differentiate between regular price changes and those related to temporary price sales, finding that the inclusion or exclusion of price sales has sizable effects on the estimated frequency of price changes. For example, Nakamura and Steinsson (2008) report a median duration of price changes between 4.4 to 4.6 months when sale-related prices are included in the data, and a median duration between 8 to 11 months when sale-related price changes are excluded from the data. By now, many studies have applied approaches similar to Bils and Klenow's to the analysis of the frequency of price changes in other countries, as reported in Table 2.

[29] See, for example, Carlton (1986), Cecchetti (1986), Kashyap (1995), Levy *et al* (1997) and Blinder *et al* (1999).

[Table 2]

The availability of scanner data has allowed researchers to further probe into the issue of price stickiness. For example, using scanner data from a large grocery store chain in Chicago, Midrigan (2008) reports the presence of many small and short lived price changes and uses this as a motivation to construct a menu cost model where firms face economies of scale when adjusting their prices. Eichenbaum et. al. (forthcoming), on the other hand, use scanner data from a large U.S. retailer and find that nominal rigidities take the form of inertia in reference prices, with weekly prices fluctuating around reference values that tend to remain constant over extended periods of time.[30] A newer and equally promising source of information on high frequency price movements is scraped online data (data extracted from internet websites)[31].

Finally, on a line of work more related to open economy issues, Burstein *et al* (2005) use their own survey data for Argentina and examine the effects of large devaluations on the real exchange rate. These authors present evidence that suggests that the large decline in the real exchange rate is caused by the sluggish adjustment of the prices of non-tradable goods and not from deviations from the law of one price, thus providing support for set-up used in this paper that posits fully flexible tradable goods prices and sticky non-tradable goods prices.

[30] A reference price is defined as the most common price in a given time window. Eichenbaum *et al* (2009) use a quarter.

[31] Using scrapped data for Argentina, Brazil. Chile and Colombia, Cavallo (2010) finds that the distributions of the size of price changes in these countries are bimodal, that hazard functions are upward-sloping, and that there is strong daily price synchronization within narrow categories of goods, suggesting that strategic complementarities play an important role in price-setting decisions.

Box 3. To devalue or not to devalue? That is the question

In December 1987, annual inflation in Mexico had reached 160 percent. In response, the Mexican government instituted an exchange-rate based stabilization program initially based on a fixed exchange rate and income and wages policies. As expected – and analyzed in detail in Chapter 13 – the exchange rate-based stabilization program led to a significant real appreciation of the currency (Figure 11). By April 1994, the continuing real appreciation accompanied by slow growth and a widening current account deficit had called into question the stabilization strategy. Further, a speculative attack on the peso and a sharp upturn in interest rates were taken as a ominous sign of a future crisis.

Two sharply contrasting assessments of Mexico's situation at the time became the subject of intense public debate. On the one hand, some observes and, in particular, the authorities at the time put forward the view that the real appreciation simply reflected an equilibrium phenomenon resulting from the reforms that had been undertaken, including budget, trade, and wealth effects resulting from the exchange rate based stabilization plan. The logical corollary of this view was that no policy remedies were needed. On the other hand, there was a disequilibrium view which, even if it accepted the potential benefits of reforms, liberalization, and the presence of NAFTA, argued that the overvaluation of the domestic currency was a policy mistake that could and should be remedied. Rudy Dornbush, particularly in a joint paper with Alejandro Werner, was the most famous proponent of this view. Their idea was that the interaction of the exchange rate-based stabilization and incomes policy had been responsible for the overvaluation. Using a model very similar to the sticky-inflation model of Chapter 13, they argued that fixing the nominal interest rate immediately reduced the nominal interest rate. Inertia in the inflation process, however, would imply that inflation would fall only slowly over time, thus leading to a real appreciation. Further, the resulting fall in real interest rates would push up aggregate demand and reinforce the process of real appreciation. In Dornbusch and Werner's view of the world, this real appreciation would slow growth and increase unemployment. In their minds, therefore, the policy remedy was clear: a once-and-for-all devaluation of about 20 percent, which would take care of the real appreciation.

In his discussion of the Dornbush-Werner piece, Guillermo Calvo vehemently disagreed. "In my opinion, this is not the time to implement a Dornbusch-Werner devaluation. The forces that have held together the "good" equilibrium may dissipate overnight." He pointed out that the biggest problem for Mexico is credibility. He argues that it is the lack of credibility of authorities' policies what caused the boom-bust cycle (as in Chapter 7). The intertemporal substitution caused by an intertemporal distortion (imperfect credibility) gives rise to a socially costly consumption boom. Individuals are forced to cut consumption in the future to satisfy budget constraint. Thus, future real wages will fall and real exchange rate will rise. In this context, downward price-wage inflexibility will result in higher unemployment and excess capacity. A devaluation a la Dornbusch and Werner may solve the overappreciation problem in the short run, but it will also cause a more pronounced appreciation and inflation in the future. "Authorities could have revealed their taste for discretionary policy, and people may come to believe that it could happen again. Therefore, the same mechanism that provoked the present misalignment will be set in motion again."

In December 1994, Mexico devalued the peso by 15 percent. The devaluation set off a firestorm: since reserves were low before the devaluation, there was an immediate attack on the Mexican peso setting off a more substantial fall in reserves. Almost immediately, the government was forced to allow the peso to float. From late 1994 to early 1995 the peso depreciated by almost 80 percent and the yield on CETES (Mexican T-bills) more than tripled (Figure 12). The inescapable conclusion is that Calvo was right. Models such as the one in Section 6 that ignore credibility problem may yield the wrong policy prescription!

Figure 1. Phase diagram

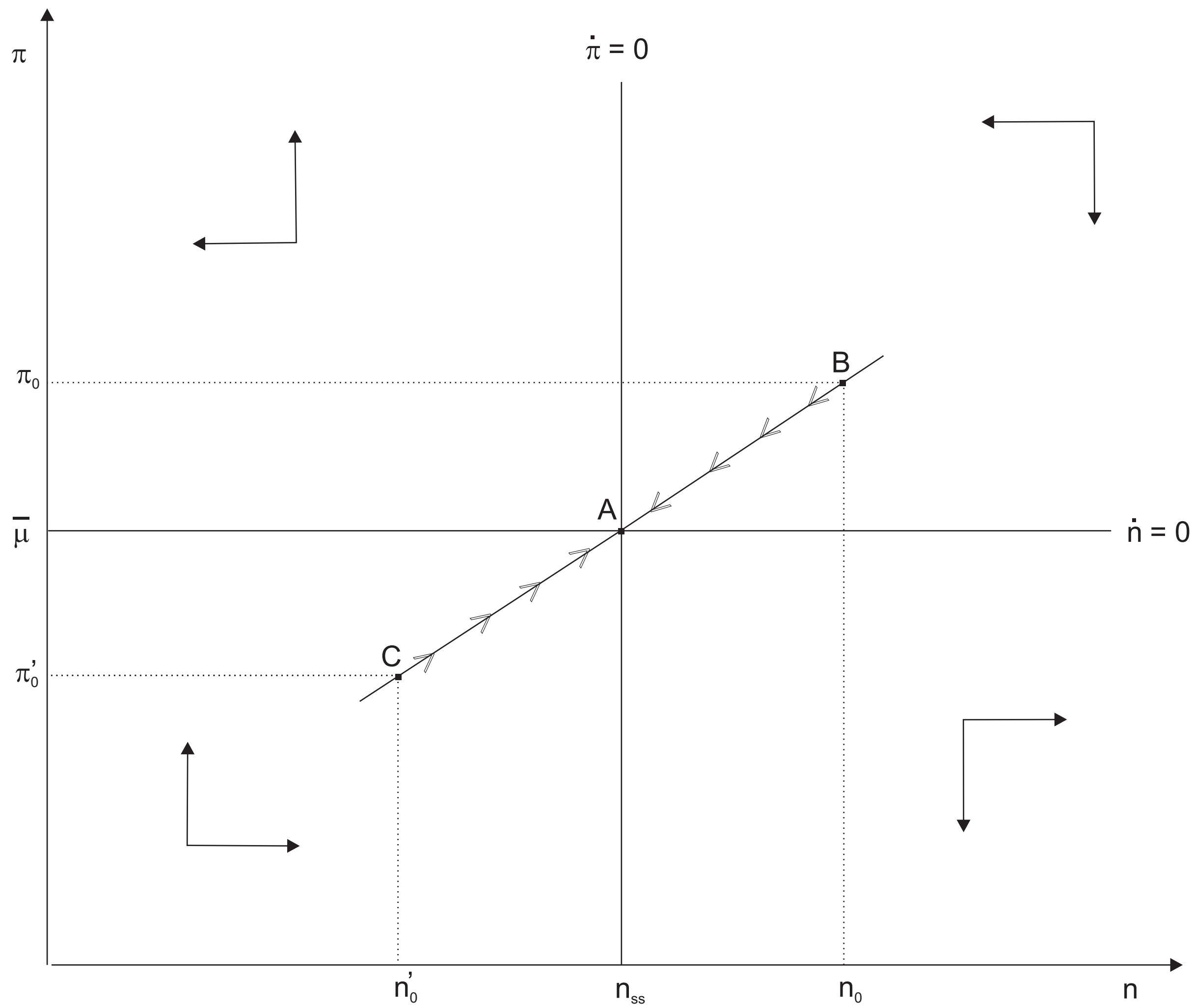


Figure 2. Permanent increase in money supply

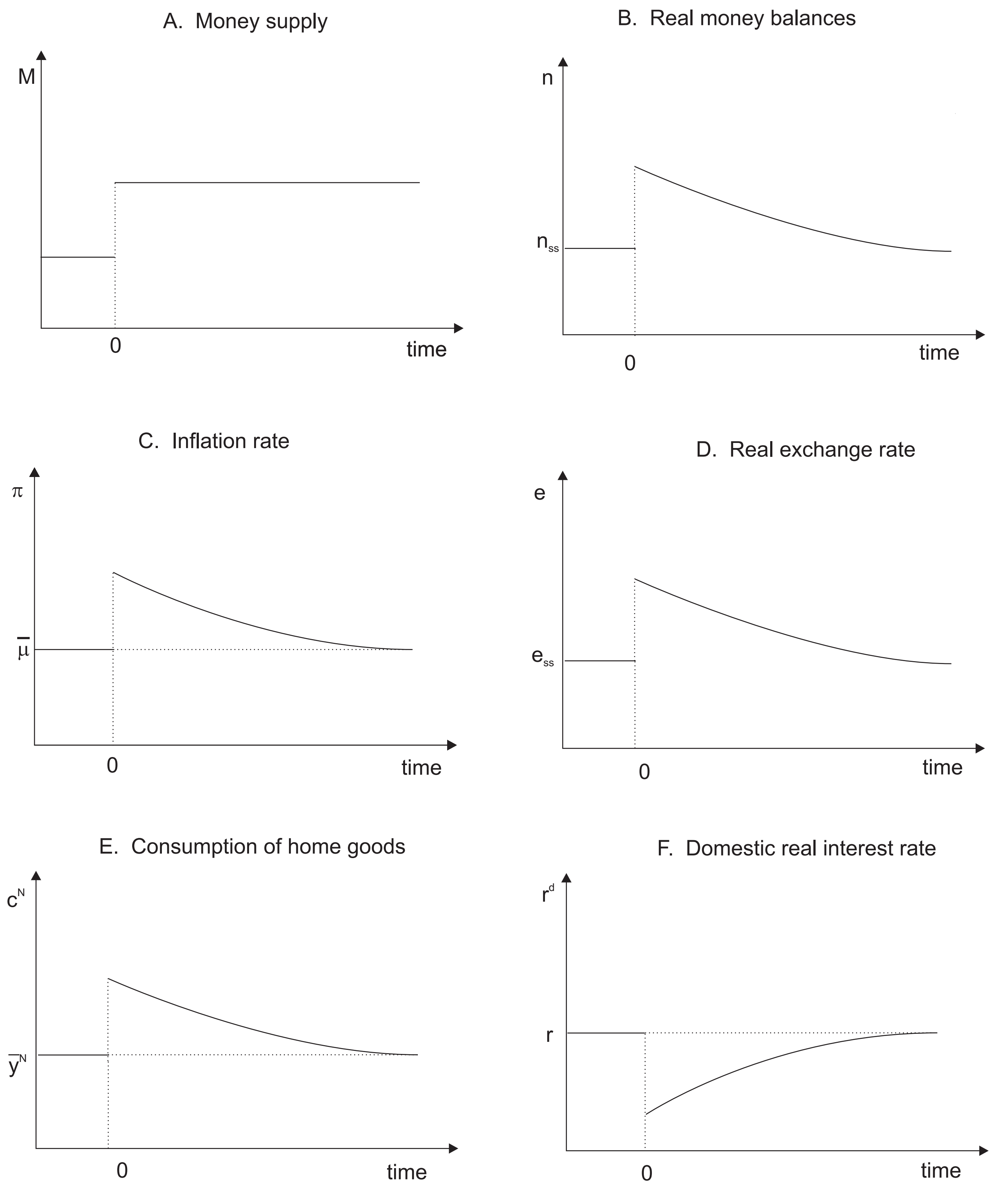


Figure 3. Permanent reduction in rate of money growth

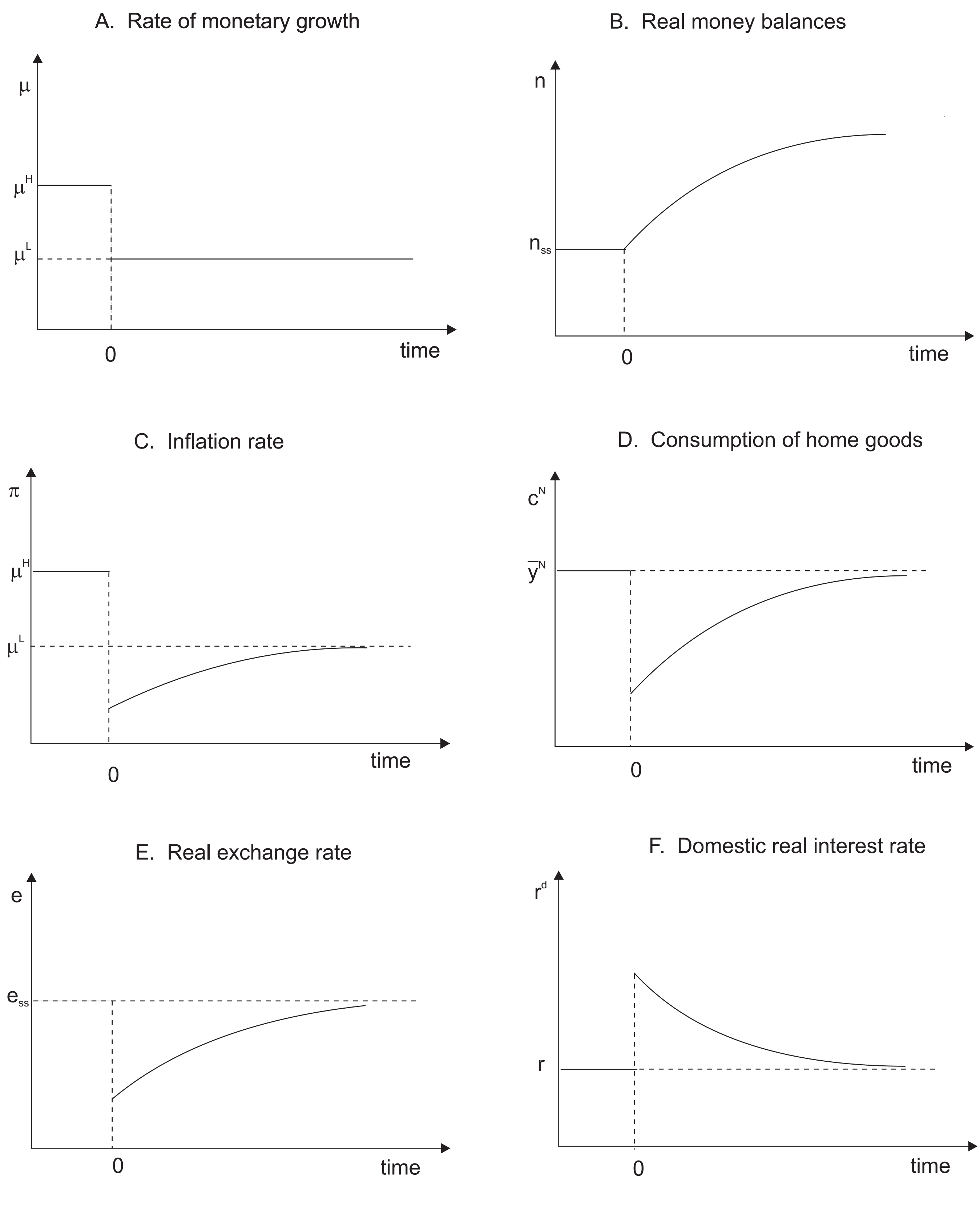


Figure 4. Permanent reduction in money growth rate: Phase diagram

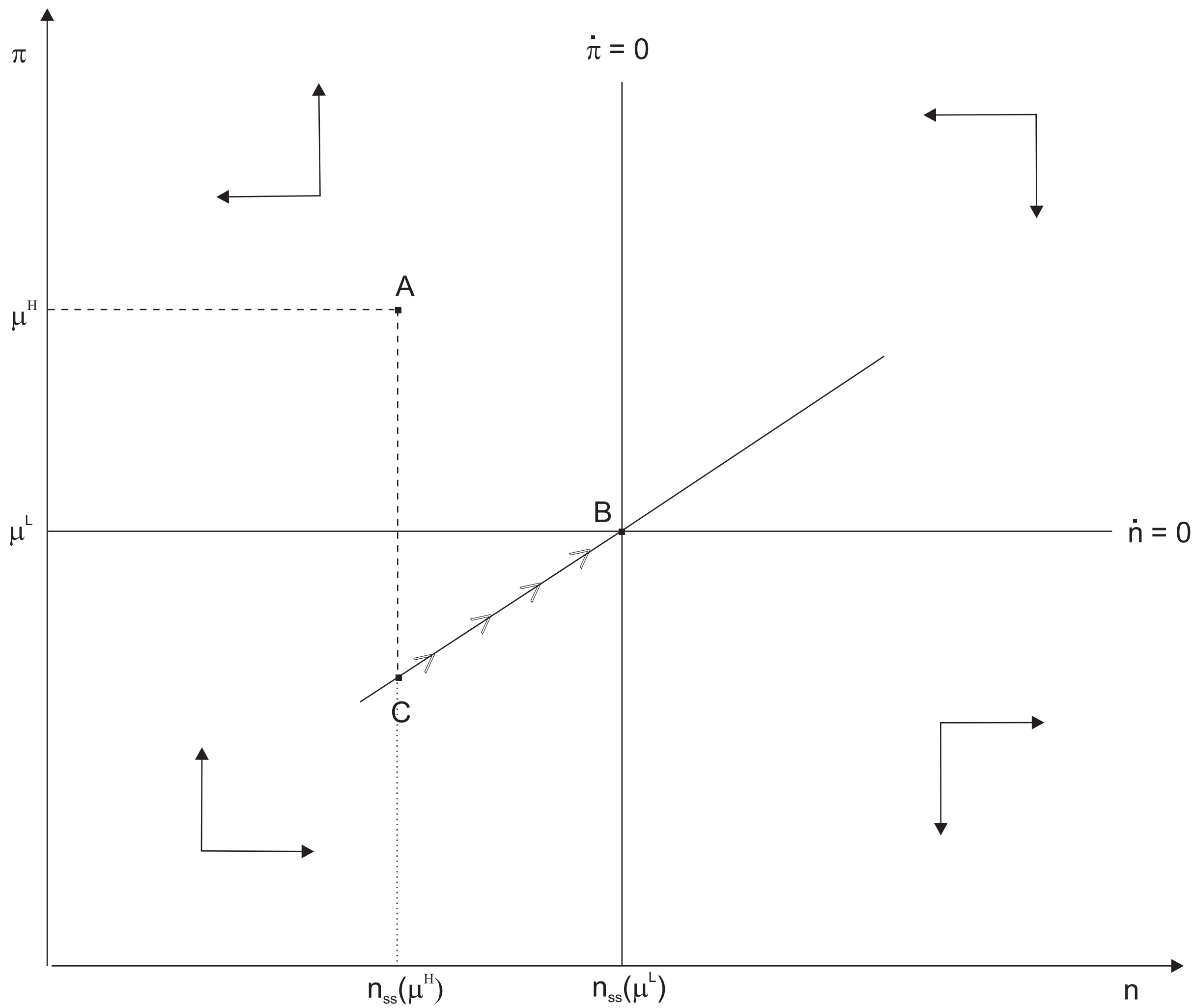


Figure 5. Phase diagram: Predetermined exchange rates

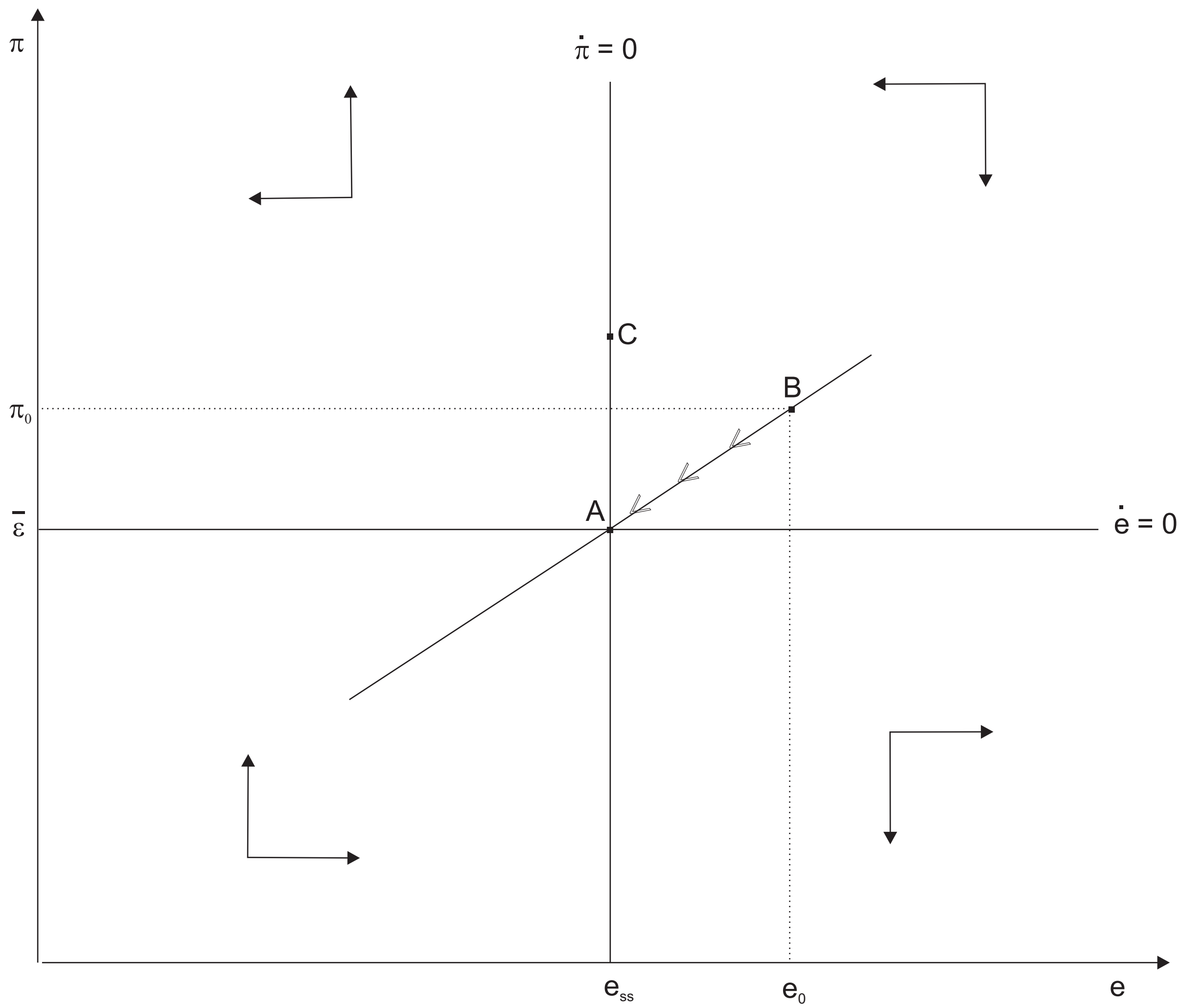


Figure 6. Permanent devaluation

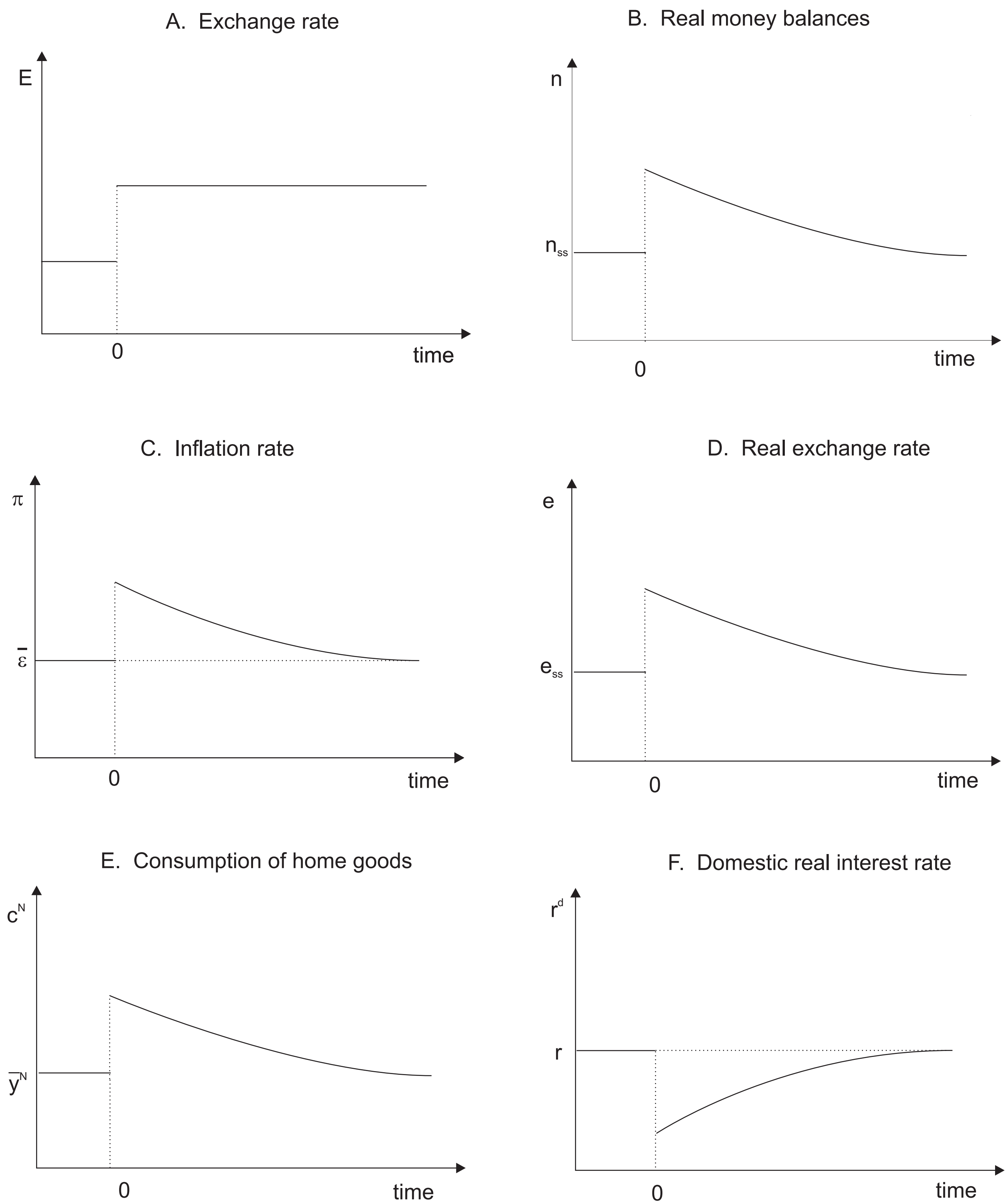


Figure 7. Overshooting

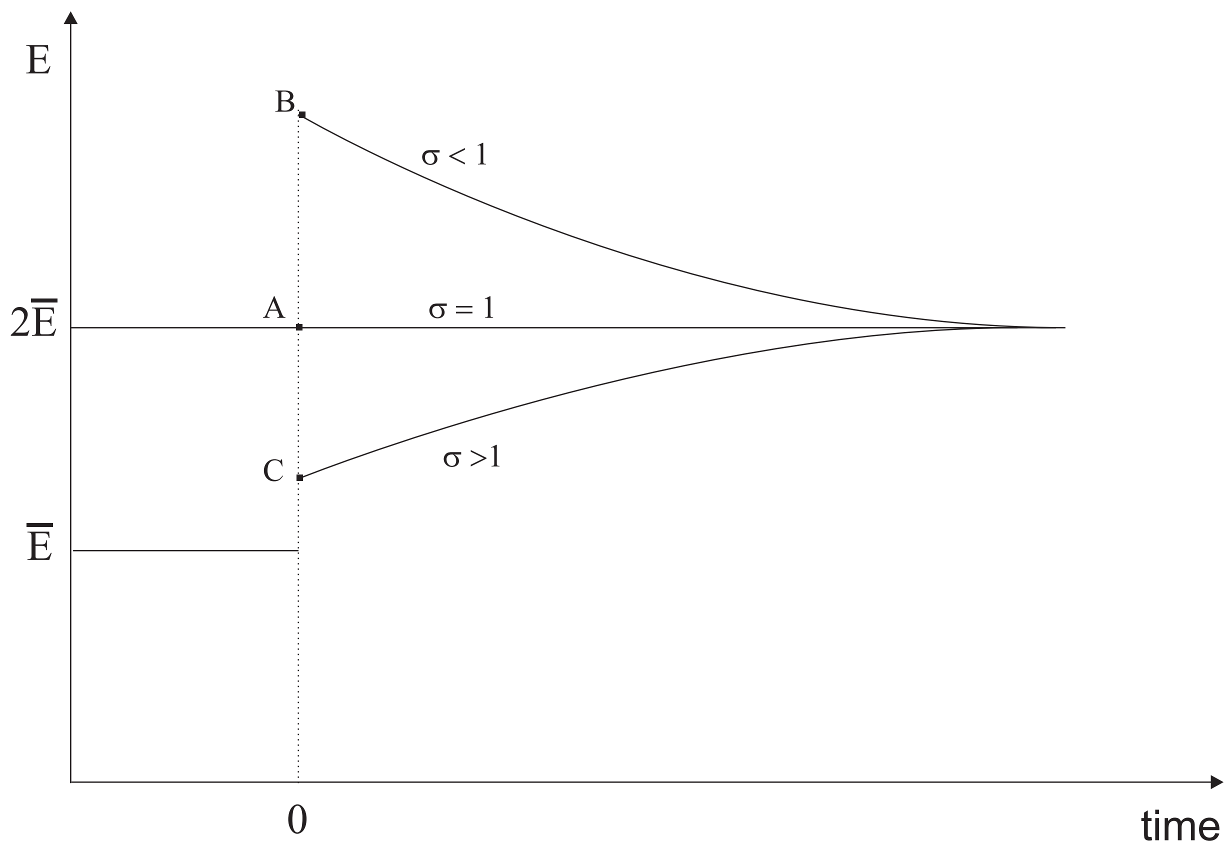


Figure 8. Labor market

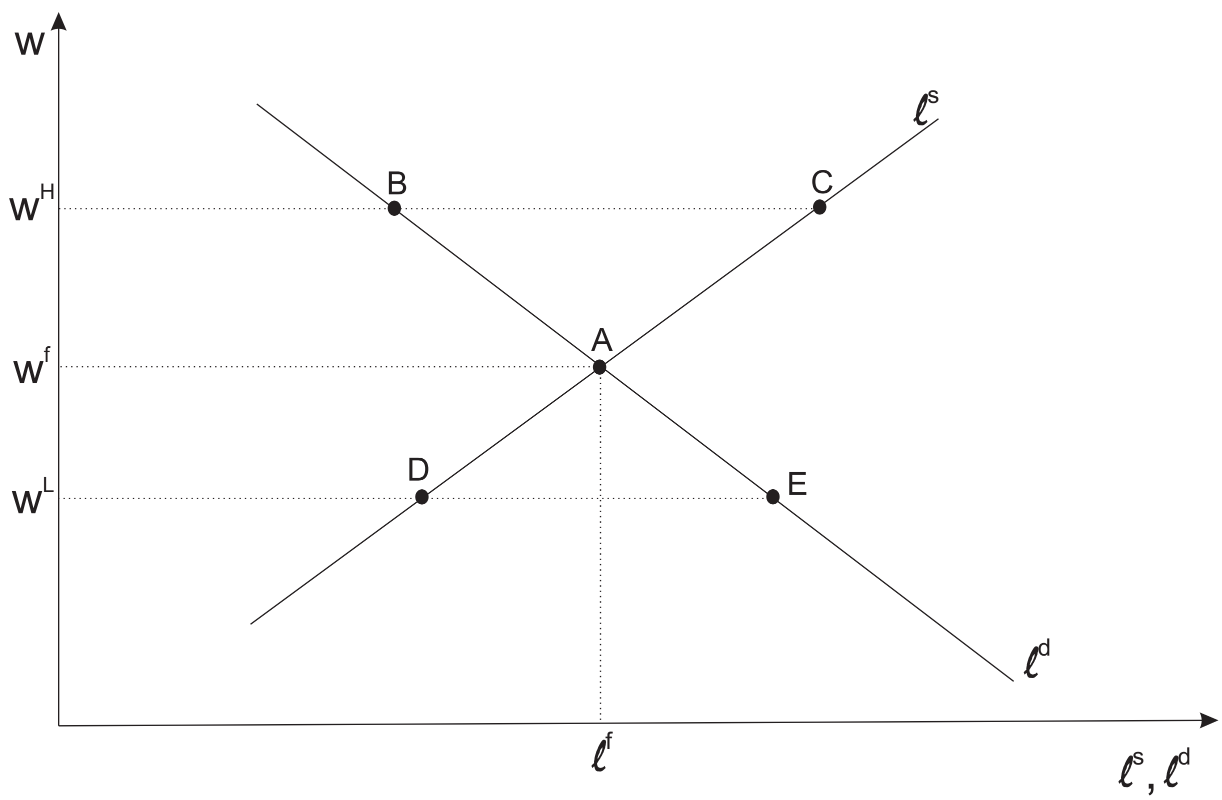


Figure 9. Labor market: Fall in productivity

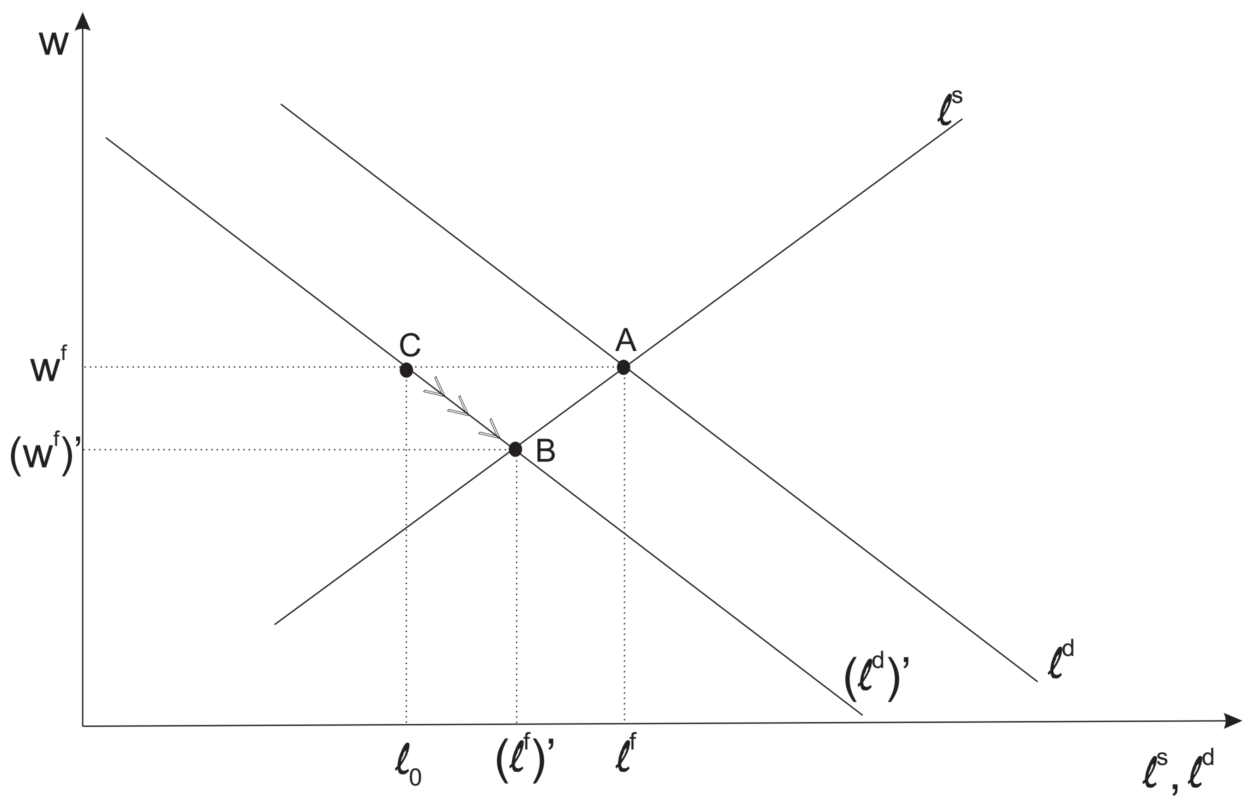
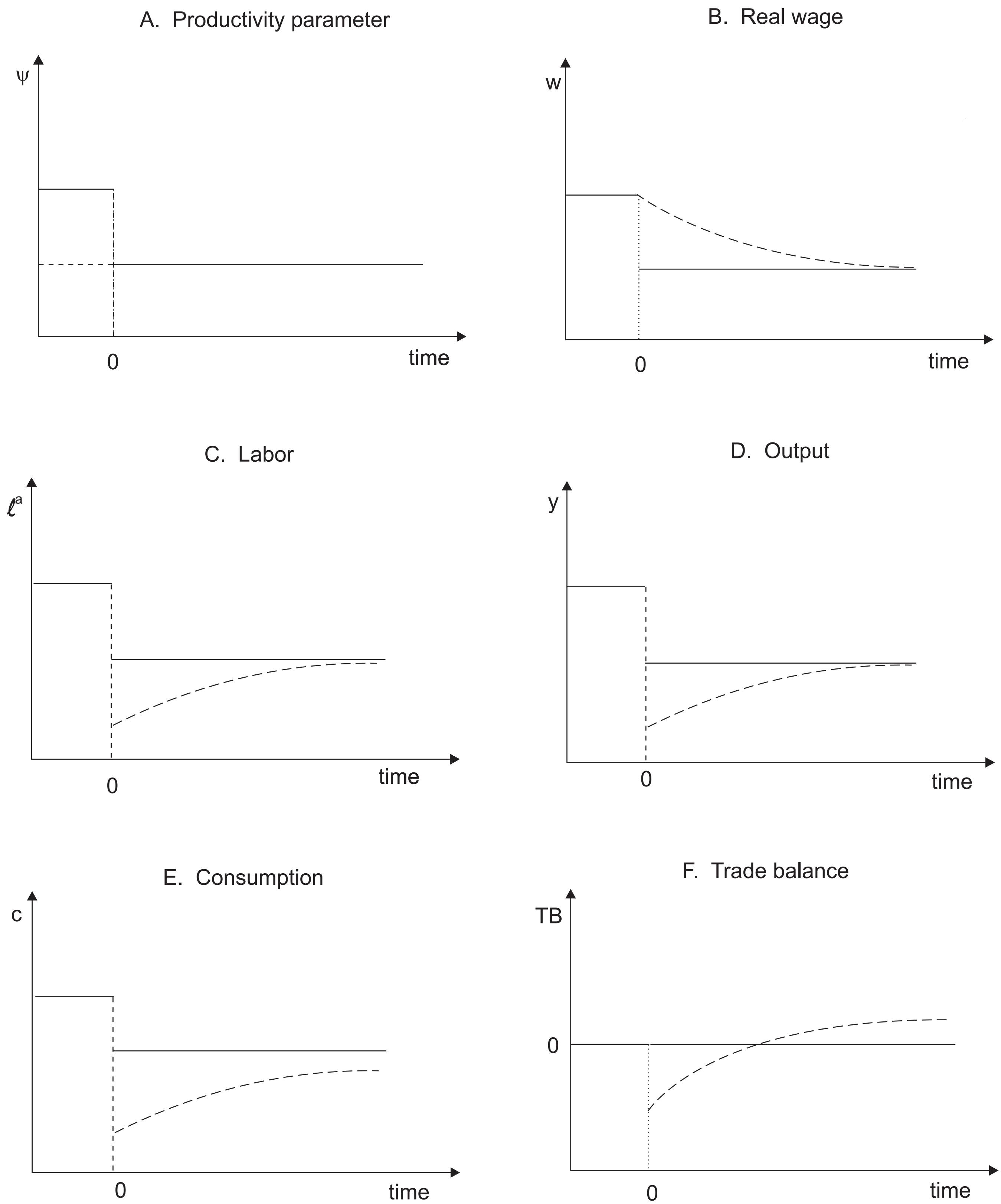
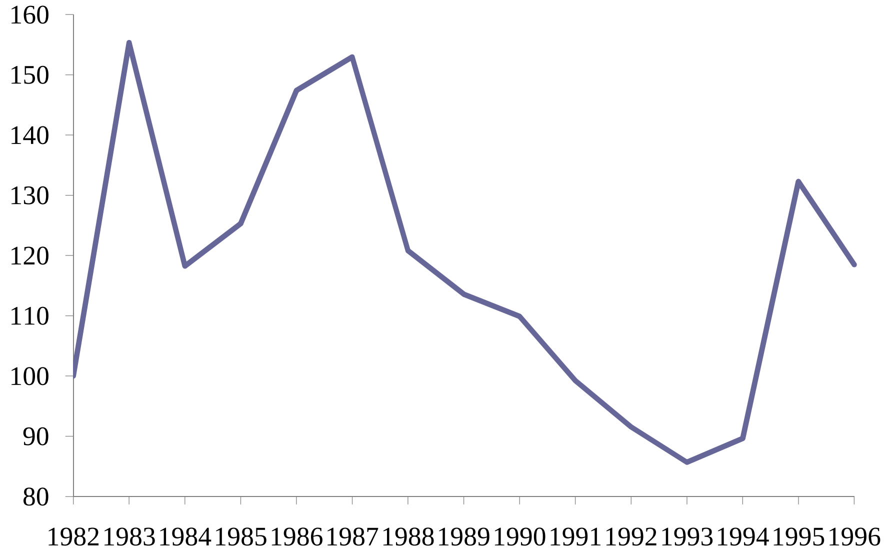


Figure 10. Permanent fall in productivity



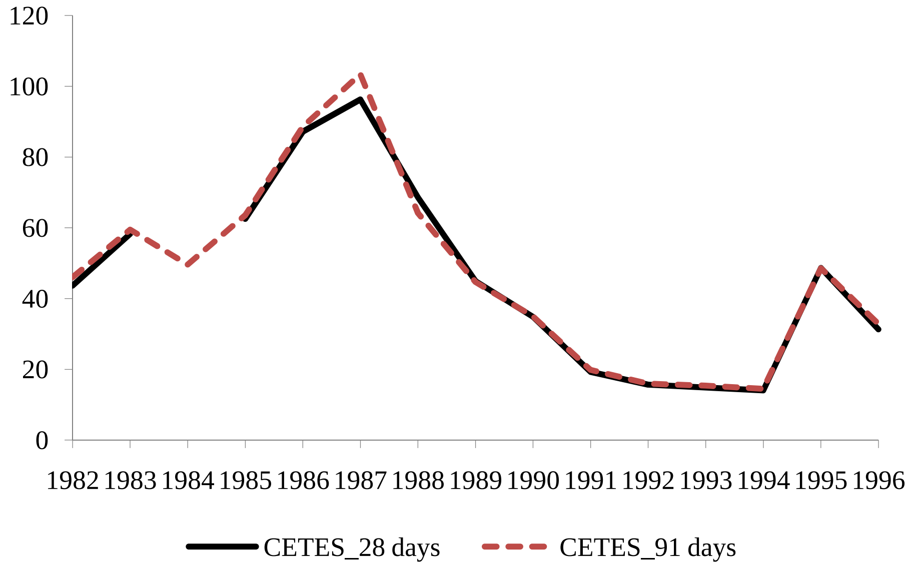
Note: A full line indicates the adjustment under flexible wages; a dashed-line under sticky wages.

Figure 11 - Real Effective Exchange Rate (1982=100)



Source: Banco Central de Mexico

Figure 12 – CETES yields in percent



Source: OECD database

Table 1: Empirical studies on output effect of devaluation

Author/s (year)	Period/ countries	Sign of effect	Methodology and controls
Gylfason and Schmid (1983)	1959-1977 5 industrial and 5 developing countries	Positive in 8 out of the10 countries (effect after two to three years)	Parameter estimation based on dynamic single-equation models
Edwards (1986)	1965-80 12 developing countries	Negative in the short run (effect is offset in the second year)	Panel data (fixed effects) Government spending, money growth
Agenor (1991)	1978-87 23 developing countries	Negative (for anticipated movements of RER) Positive (for unanticipated movements of RER)	Panel data (pooled OLS) Government spending, money supply, foreign income
Morley (1992)	1974-84 28 developing countries	Negative (short run)	Cross section Terms of trade, import growth, the money supply and the fiscal balance
Kamin and Klau (1998)	1970-96 27 countries	Very weakly negative (short run) Insignificant (long run)	Panel data Output gap, short term interest rate, fiscal balance to GDP, terms of trade, capital account to GDP ratio, U.S._interest rate. Two stages least squared
Milesi-Ferreti and Razin (2000)	1970-1996 105 low –and middle –inome countries	Negative (short run) Insignificant (long run)	Panel data. Macroeconomic, external, debt, financial, foreign and regional variables.
Magendzo (2002)	1970-99 155 countries non OECD countries	Insignificant	Matching estimators
Cavallo et al. (2004)	1992-2002 24 countries	Negative (2-year period), the effect is stronger in presence of large foreign debt.	Cross section analysis using OLS, IV and three-stage least square.
Gupta, Mishra, and Sahay (2007)	1970-98 91 developing countries	57 percent of crises contractionary; 43 percent expansionary Outcome influenced by factors such as capital account liberalization and pre-crisis level of economic activity and capital flows	Frequency distribution and OLS Change in external long-term debt, cumulative flow of external private capital, capital controls, currency crises, business cycles, per capita GDP, banking crises, short-term debt to reserves, exchange rate overvaluation, proxies for monetary and fiscal policies, openness, competitive devaluations by others, economic size, external factors

Table 2: Monthly Mean Duration of CPI Price Changes

Country	Paper	Mean Duration*	Excluding Sales
Austria	Baumgartner et al. (2005)	6.1	-
Belgium	Aucremanne and Dhyne (2004)	5.4	-
Brazil	Barros et al. (2009)	2.1	-
	Gouvea (2007)	2.2	-
Chile	Medina et al. (2007)	1.6	-
Denmark	Hansen and Hansen (2006)	5.3	-
Euro Area	Dhyne et al. (2006)	6.1	-
Finland	Vilmunen and Laakkonen (2005)	5.5	-
France	Baudry et al. (2007)	4.8	-
Germany	Hoffmann and Kurz-Kim (2006)	8.3	-
Hungary	Gabriel and Reiff (2010)	6.1	-
Israel	Baharad and Eden (2004)	3.6	-
Italy	Fabiani et al. (2006)	9.5	-
Japan	Saita et al. (2006)	3.8	-
Luxembourg	Lunnemann and Matha (2005)	5.4	-
Mexico	Gagnon (2009)	2.9	-
Netherland	Jonker et al. (2004)	5.5	-
Norway	Wulfsberg (2009)	4.0	4.2
Portugal	Dias et al. (2004)	4.0	-
Sierra LeonE	Kovanen (2006)	1.4	-
Slovakia	Coricelli and Horvath (2010)	2.4	-
South Africa	Creamer and Rankin (2008)	5.7	-
Spain	Álvarez and Hernando (2006)	6.2	-
United Kingdom	Bunn and Ellis (2009)	4.7	6.2
United States	Bils and Klenow (2004)	3.3	-
	Klenow and Kryvtsov (2008)	2.2	2.8
	Nakamura and Steinsson (2008)	3.2	4.2

Source: Klenow and Malin (2010)
*Klenow and Malin report the frequency of price changes. The duration is computed as -1/ln(1-x) where x is the frequency.