Chapter 9
Interest Rate Policy *

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1 Introduction

As discussed in detail in previous chapters, an open economy can choose either the nominal exchange rate or the money supply as its main nominal anchor. In fact, the analysis of monetary policy in small open economies has traditionally been cast almost exclusively in those terms. There is, however, a third nominal variable that could act as a nominal anchor: the nominal interest rate. In recent years, this "third" nominal anchor has received increased attention in the academic literature, inspired particularly by policy controversies surrounding IMF-sponsored high interest rates policies in Asian countries in the aftermath of the 1997-1998 crises. This chapter will thus be entirely devoted to the analysis of the use of nominal interest rates as a nominal anchor in small open economies.

While analyzing the traditional nominal anchors (i.e., the exchange rate and the money supply) poses no particular challenges in terms of how to formulate such policies in a theoretical model, this is not the case when it comes to modelling interest rates as a nominal anchor. In fact, Sargent and Wallace (1975) were the first to point out that, at an analytical level, "interest rate targeting" (as controlling nominal interest rates is often referred to) in a flexible prices model leads to a price level indeterminacy. Understanding such a theoretical problem should be the logical first step in any analysis of interest rates as a nominal anchor and is thus examined in detail in Section 2. The main conclusion to come out of Section 2, however, is that the Sargent-Wallace indeterminacy results from a theoretical failure to fully specify monetary policy. We will clearly see that the price level indeterminacy arises because targeting the nominal interest rate essentially amounts to setting the rate of growth of the nominal exchange rate but *not* its initial level (or setting the rate of growth

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of the money supply but not its initial level). It is thus hardly surprising that there is nothing in the model to tie down the initial price level.

Having clarified the source of the price level indeterminacy, the chapter proceeds to discuss several ways in which the specification of monetary policy can be completed to generate a well-defined model. Section 3 discusses the so-called "fiscal theory of the price level" (as formulated by Auernheimer and Contreras (1992)). The key assumptions behind this approach are that money must be introduced into the economy through open market operations (as opposed to "helicopter drops") and that government transfers are exogenously-given. This has the implication that the price level – instead of being determined in the money market as in standard models – will be determined by the requirement that the fiscal constraint holds. While this approach offers an elegant solution to the problem of price level indeterminacy and allows us to use the model to ask useful policy questions (i.e., how does an increase in interest rates affect the nominal exchange rate?), it has the problem that if we used this set-up to analyze the two traditional anchors, the system would now be overdetermined. In any event, we show that, under certain restrictions, an increase in interest rates leads to a nominal appreciation of the domestic currency.

Following Calvo and Végh (1995), Section 4 pursues quite a different route in dealing with the Sargent-Wallace indeterminacy. This formulation essentially sidesteps the problem by assuming that the interest rate controlled by the monetary authority is the interest rate borne by some liquid asset. Thus, in this formulation, this policy-controlled nominal interest rate is an *additional* policy instrument. In other words, the monetary authority can control both this interest rate and either the money supply or the exchange rate. The model captures what is perhaps the most common channel alluded to by policymakers and practitioners alike: a rise in interest rates makes domestic-currency denominated assets more attractive to hold, thus leading to an appreciation of the domestic currency as investors dispose of foreign-currency denominated assets.

Finally, Section 5 introduces sticky prices into the model. While one might think that this is perhaps the most obvious solution to the price level indeterminacy (clearly, if prices are sticky, there cannot be price level indeterminacy!), such is unfortunately not the case. This section shows that all that sticky prices accomplish is to push the indeterminacy problem to another area of the model. In particular, we show (following Calvo (1983)) that interest rate targeting in a sticky-prices model leads to a higher-order indeterminacy (i.e., the rate of inflation is undetermined). We then solve this problem by introducing Taylor-type interest rate rules.

2 Price level indeterminacy

This section will illustrate how interest rate targeting leads to a price level indeterminacy. We will show this in the context of the model studied in Chapter 5 (except that we will abstract from non-tradable goods and introduce money in the utility function).

Consider a small open economy inhabited by a large number of identical, infinitely-lived consumers who are endowed with perfect foresight. The economy is perfectly integrated with the rest of the world in both goods and capital markets. There is only one (tradable and non-storable) good, whose price is given by the law of one price. The economy receives a constant endowment of the good (y). The international real interest rate (r) is constant over time.

2.1 Consumer's problem

2.1.1 Budget constraints

The consumer holds two assets: domestic money (M) and an internationally-traded bond denominated in the foreign currency (B). Nominal asset holdings are therefore:

$$A_t = M_t + E_t B_t, \tag{1}$$

where E is the nominal exchange rate (units of domestic currency per unit of foreign currency). By the law of one price,

$$P_t = E_t P_t^*,$$

where P_t is the domestic currency price of the good and P_t^* is the foreign currency price. The numeraire of this economy will be the tradable good. Hence, "real" variables will be defined in terms of tradable goods. Dividing (1) by P_t, we obtain:

$$a_t = m_t + b_t, \tag{2}$$

where $a(\equiv A/P)$, $m(\equiv M/P)$, and $b(\equiv B/P^*)$ denote real financial assets, real money balances, and real foreign bonds, respectively.

The consumer's flow constraint in nominal terms is given by:

$$\dot{A}_t = E_t i_t^* B_t + \dot{E}_t B_t + P_t y + P_t \tau_t - P_t c_t, \tag{3}$$

where τ denotes real lump-sum transfers from the government and c denotes consumption. The term $E_t i_t^* B_t$ captures interest payments on the foreign bonds (in terms of domestic currency), while the term $\dot{E}_t B_t$ denotes capital gains on the foreign bonds.

To express the flow constraint in real terms, divide (3) by P_t (taking into account the law of on price) to obtain:

$$\frac{\dot{A}_t}{P_t} = (i_t^* + \varepsilon_t) b_t + y + \tau_t - c_t, \tag{4}$$

where $\varepsilon(\equiv \dot{E}/E)$ denotes the rate of depreciation/devaluation. Given that, by definition, $a_t = A_t / E_t P^*$, it follows that:

$$\dot{a}_t = \frac{\dot{A}_t}{P_t} - (\varepsilon_t + \pi_t^*) a_t. \tag{5}$$

Substituting (4) into (5) and rearranging terms:

$$\dot{a}_t = (i_t^* - \pi_t^*)a_t + y + \tau_t - c_t - (i_t^* + \varepsilon_t)m_t. \tag{6}$$

Assuming that the Fischer equation holds in the rest of the world (i.e., $i_t^* = r + \pi_t^*$) and taking into account that perfect capital mobility implies that interest parity will hold (i.e., $i_t = i_t^* + \varepsilon_t$), we can rewrite (6) as:

$$\dot{a}_t = ra_t + y + \tau_t - c_t - i_t m_t. \tag{7}$$

Integrating forward equation (7) and imposing the transversality condition $\lim_{t\to\infty} a_t e^{-rt} = 0$ (for the reasons discussed in Chapter 1), we finally obtain:

$$a_0 + \frac{y}{r} + \int_0^\infty \tau_t e^{-rt} dt = \int_0^\infty (c_t + i_t m_t) e^{-rt} dt. \tag{8}$$

This lifetime constraint makes perfect sense as it says that the present discounted value of "total expenditures" (given by the RHS, and which include the opportunity cost of holding real money balances) must be equal to the consumer's wealth (LHS), which comprises his/her initial real assets and the present discounted value of his/her income and government transfers.

2.1.2 Utility maximization

The consumer's lifetime utility is given by

$$\int_0^\infty [u(c_t) + v(m_t)] e^{-\beta t} dt, \tag{9}$$

where $\beta (> 0)$ is the subjective discount rate, and the functions $u(.)$ and $v(.)$ are strictly increasing and strictly concave in their arguments.

The consumer's problem consists in choosing $\{c_t, m_t\}$ for all $t \in [0, \infty)$ to maximize lifetime utility (9) subject to the lifetime constraint (8), for a given path of τ_t and i_t and given values of r, y, and a_0.

Assuming, as usual, that $\beta = r$, the first order conditions imply

$$u'(c_t) = \lambda, \tag{10}$$
$$v'(m_t) = \lambda i_t. \tag{11}$$

Equations (10) and (11) implicitly define a real money demand with standard properties (where a sign under an argument indicates the sign of the corresponding partial derivative):

$$m_t = L(\underset{+}{c}, \underset{-}{i_t}). \tag{12}$$

2.2 Government

The government comprises the fiscal authority and the monetary authority. Let H^* denote the value of international reserves in nominal dollars held by the central bank, $h(\equiv H^*/P^*)$ denote the value of international reserves in real dollars, and $H(= EH^*)$ denote the domestic currency value of international reserves. The government's flow budget constraint in nominal terms is given by

$$\dot{H}_t = E_t i_t^* H^* + \dot{E}_t H^* + \dot{M}_t - P_t \tau_t. \tag{13}$$

To obtain the government constraint in real terms, we proceed in the same way as we did above for the consumer. Since $h = H/EP^*$, it follows that (using the law of one price, $P = EP^*$):

$$\dot{h}_t = \frac{\dot{H}_t}{P_t} - (\varepsilon_t + \pi_t^*) h_t. \tag{14}$$

Dividing (13) by P_t, substituting the resulting expression into (14) (and using Fischer equation for the rest of the world), we obtain:

$$\dot{h}_t = r h_t + \frac{\dot{M}_t}{P_t} - \tau_t. \tag{15}$$

Integrating forward equation (15) and imposing the transversality condition $\lim_{t \to \infty} h_t e^{-rt} = 0$, we obtain the government's intertemporal constraint:

$$h_0 + \int_0^\infty \frac{\dot{M}_t}{P_t} e^{-rt} dt = \int_0^\infty \tau_t e^{-rt} dt. \tag{16}$$

2.3 Equilibrium conditions

The assumption of perfect capital mobility implies that interest parity holds:

$$i_t = i_t^* + \varepsilon_t. \tag{17}$$

Let $k(\equiv b + h)$ denote the economy's stock of net foreign assets. Combining the consumer's flow constraint (equation (7)) with the government's (equation (15)) yields the economy's current account:

$$\dot{k}_t = r k_t + y - c_t. \tag{18}$$

Combining the consumer's and government's intertemporal constraints (equations (8) and (16)), respectively, yields the economy's resource constraint:

$$k_0 + \frac{y}{r} = \int_0^\infty c_t e^{-rt} dt. \tag{19}$$

2.4 Perfect foresight equilibrium

We now characterize the perfect foresight equilibrium path (PFEP) of the model for a constant value of the foreign nominal interest rate, i^*. It follows from first-order condition (10) that consumption will be constant along a PFEP. Hence, from the resource constraint,

$$\bar{c} = rk_0 + y.$$

From (12), real money demand along a PFEP will be given by:

$$m_t = L(\bar{c}, i_t). \tag{20}$$

Hence, real money demand will be constant if the nominal interest rate is constant over time.

2.5 Nominal anchors

The determination of the nominal interest rate and the paths of both the nominal exchange rate and the nominal money supply will depend on the specific monetary regime adopted by the monetary authority. We will now study the determination of these variables under three different nominal anchors: the exchange rate (predetermined exchange rates), the money supply (flexible exchange rates) and interest rate targeting. While we have already studied the first two regimes in Chapters 5 through 7, we will revisit the determination of nominal variables under both predetermined and flexible exchange rates as it provides a useful benchmark to understand the source of the price level indeterminacy that arises under interest rate targeting.

2.5.1 Predetermined exchange rates

Under predetermined exchange rates, the monetary authority sets the path of the nominal exchange rate. In particular, the monetary authority sets the initial level, $\bar{E}_0$, and a constant rate of growth, $\bar{\varepsilon}$, of the nominal exchange rate. Given $\bar{\varepsilon}$, the interest parity condition (17) determines a constant level of the nominal interest rate:

$$\bar{\imath} = i^* + \bar{\varepsilon}.$$

The constancy of the nominal interest rate implies, from (20), that real money balances will be constant over time at a level given by:

$$\bar{m} = L(\bar{c}, \bar{\imath}). \tag{21}$$

Hence, $\dot{m}_t = 0$ for all $t \in [0, \infty)$. Since, by definition, $m = M/EP^*$, it follows that the rate of money growth will also be constant over time:

$$\bar{\mu} = \bar{\varepsilon} + \pi^*.$$

The only nominal variable yet to be determined is the initial level of the nominal money supply, M_0. Since equation (20) holds at time 0, we can write:

$$\frac{M_0}{\bar{E}_0} = L(\bar{c}, \bar{\imath}).$$

Solving for M_0:

$$M_0 = \bar{E}_0 L(\bar{c}, \bar{\imath}).$$

We thus see that all nominal variables are perfectly well-defined.

Finally, we turn our attention to the level of reserves and transfers. How does the level of international reserves get determined? As explained in Chapter 5, the monetary authority also sets the path of nominal domestic credit (i.e., the initial level, D_0, and rate of growth, θ). We assume that the domestic credit policy is consistent with the rate of devaluation (i.e., $\theta = \varepsilon$).[1] Hence, the path of real domestic credit will be constant over time; that is, $d_t = d_0$ for $t \geq 0$. Hence, from the central bank's balance sheet, international reserves will also be constant over time at a level given by:

$$\bar{h} = \bar{m} - d_0.$$

Finally, notice that the variable that adjusts to make the government constraint hold is the level of transfers. From (15), and taking into account that h_t, ε_t, and m_t are all constant over time, it follows that:

$$\tau_t = r\bar{h} + (\pi^* + \bar{\varepsilon})\bar{m}.$$

2.5.2 Flexible exchange rates

Under flexible exchange rates, the monetary authority sets the path of the nominal money supply. In particular, the monetary authority sets the initial level, $\bar{M}_0$, and a constant rate of growth, $\bar{\mu}$, of the nominal money supply. We first show that real money balances will be constant over time. To see this, notice that $\dot{m}_t/m_t = \bar{\mu} - \varepsilon_t - \pi^*$, and use (11) and (17) to obtain:

$$\dot{m}_t = m_t \left[r + \bar{\mu} - \frac{v'(m_t)}{\lambda} \right].$$

Linearizing this equation around a stationary value for real money balances (given by $r + \bar{\mu} = v'(m_t)/\lambda$), we see that this is an unstable differential equation. Intuitively, if m increases the nominal interest rate must fall to accommodate this increase. By interest parity, this implies a fall in the rate of depreciation (inflation), which in turn implies that real money supply will grow faster which requires a further fall in the nominal interest rate and so forth. Hence, the

[1] As explained in Chapter 5, this ensures that the predetermined exchange rate regime is sustainable over time. This assumption will be relaxed in Chapter 16 when we deal with balance of payments crises.

only convergent equilibrium path is for real money balances to be constant and (implicitly) given by

$$r + \bar{\mu} = v'(\bar{m})/\lambda,$$

for all $t \geq 0$. Given this value for real money balances, equation (20) determines a unique nominal interest rate, $\bar{\imath}$. Further, since $\dot{m}_t/m_t = \bar{\mu} - \varepsilon_t - \pi^* = 0$, the (constant) rate of depreciation will be given by:

$$\bar{\varepsilon} = \bar{\mu} - \pi^*.$$

The only nominal variable yet to be determined is the initial level of the nominal exchange rate, E_0 (i.e., the initial price level). Since equation (20) holds at time 0, we can write:

$$\frac{\bar{M}_0}{E_0} = L(\bar{c}, \bar{\imath}).$$

Solving for E_0:

$$E_0 = \frac{\bar{M}_0}{L(\bar{c}, \bar{\imath})}.$$

We have thus shown that, as under predetermined exchange rates, the value of all nominal variables is perfectly well-defined under flexible exchange rates.

How does the level of transfers get determined? Notice that, by construction, the level of international reserves is constant under flexible exchange rates because the monetary authority is assumed not to intervene in foreign exchange markets. For simplicity, we assume that this constant level of international reserves is equal to zero. Hence, from (15):

$$\tau_t = (\bar{\varepsilon} + \pi^*)\bar{m}.$$

2.5.3 Interest rate targeting

Under interest rate targeting, the monetary authority is assumed to set a constant level of the nominal interest rate, $\bar{\imath}$. Given this value of the nominal interest rate, the interest parity condition (17) determines a constant rate of depreciation given by:

$$\bar{\varepsilon} = \bar{\imath} - i^*.$$

The given value of the nominal interest rate also determines a constant level of real money demand, given by:

$$\bar{m} = L(\bar{c}, \bar{\imath}). \tag{22}$$

Since real money balances are constant over time, the constant rate of growth of money is given by:

$$\bar{\mu} = \bar{\varepsilon} + \pi^*.$$

Two nominal variables remain to be determined: M_0 and E_0. However, there is nothing in the model capable of tying down these variables. To see why, notice that, from (22), it follows that:

$$\frac{M_0}{E_0} = \bar{m}.$$

There are clearly infinitely many values of M_0 and E_0 that satisfy this equation. For example, if $\bar{M}_0$ and $\bar{E}_0$ satisfy this equation, so do $\alpha\bar{M}_0$ and $\alpha\bar{E}_0$ for any $\alpha > 0$. Hence, the price level (i.e., the nominal exchange rate) is indeterminate. Interest rate targeting thus leads to a price level indeterminacy, which is Sargent and Wallace's (1975) celebrated result.

Of course, there is nothing surprising about this. To put it in the simplest possible way, think of the following two equations:

$$\begin{aligned} \frac{M_0}{E_0} &= L(i), \\ i &= i^* + \varepsilon. \end{aligned}$$

There are four variables to be determined (M_0, E_0, i, and ε) but only two equations. Hence, monetary policy needs to pin down two of the four variables for the equilibrium to be determined. Predetermined exchange rates pin down E_0 and ε; flexible exchange rates pin down M_0 and ε (since ε will be determined by μ). But interest rate targeting pins down only i. It should therefore come as no surprise that there is a nominal indeterminacy. We thus conclude that the reason why there is a price level indeterminacy under interest rate targeting is simply because monetary policy has not been completely specified by the model. It is a failure of theory, not of the real world!

2.5.4 Completing the specification of monetary policy

Since our goal is to have an operational model of interest rate policy (i.e., a model in which we can ask policy questions), we need to complete the specification of monetary policy to have a fully-determined model. How can we do this? The literature has come up with four (quite different) ways of dealing with this issue.

The first solution was proposed by McCallum (1981). He suggested perhaps the more obvious solution, which is (in terms of our formulation) to have the monetary authority also choose the initial level of the nominal money supply, M_0.[2] While this is certainly a perfectly fine solution, it is not particularly appealing because interest rate targeting becomes identical to flexible exchange rates (i.e., money supply targeting).

[2] In McCallum's formulation, implementing this policy is more complicated due to the stochastic nature of the model but the essence is the same: fix the nominal money supply at the beginning of each period.

The second solution, originally proposed by Auernheimer and Contreras (1992) and Woodford (1994), amounts to the so-called "fiscal theory of the price level".[3] The basic idea is to assume that the monetary authority cannot introduce money discretely into the economy via "helicopter drops" but instead can alter the nominal money supply only via open-market operations. We will explore this set-up in detail in Section 3 below.

The third way of dealing with the price level indeterminacy – proposed by Calvo and Végh (1995) – essentially sidesteps the problem by assuming that the interest rate controlled by the monetary authority is the interest rate borne by an interesting-bearing liquid asset (think of it as interest-bearing money). In this set-up, this interest rate becomes an additional instrument of monetary policy. We analyze this setup in Section 4.

Lastly, in a world of sticky prices, we can resort to specifying interest rules à la Taylor to solve the indeterminacy problem. This is dealt with in Section 5.

3 The fiscal theory of the price level

This section shows how price level determinacy can be recovered by using the government's budget constraint and ruling out the possibility of endogenous transfers. Having obtained an operational model, we will study the effect of a higher nominal interest rate on the nominal exchange rate. We will follow Auernheimer and Contreras' (1992) model.

There are two important differences with respect to the standard monetary model that we used in the previous section:

i) the fiscal authority is no longer passive in the sense that government transfers are now exogenously set,

ii) money is introduced through open market operations.

3.1 Consumer's problem

3.1.1 Budget constraints

As far as the consumer is concerned, the only change with respect to the model of Section 2 is that the consumer can hold either foreign bonds or domestic bonds. The two assets, however, are perfect substitutes and will thus bear the same return in equilibrium. Real financial assets are thus given by:

$$a_t = b_t + b_t^g + m_t,$$

where b^g denotes the consumer's holding of government bonds.

The flow and intertemporal constraints remain the same as before (given by equations (7) and (8)).

[3] See also Auernheimer (2008).

3.1.2 Utility maximization

The consumer's lifetime utility is given by

$$\int_0^\infty \left(\log c_t + \frac{m_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right) e^{-\beta t} dt, \tag{23}$$

where $\beta(>0)$ is the subjective discount rate, and $\sigma < 1$.[4]

The consumer's problem is to choose $\{c_t, m_t\}$ for all $t \in [0, \infty)$ to maximize lifetime utility (23), subject to (8), and for a given path of τ_t and i_t and given values of y and a_0.

Assuming $\beta = r$ (to guarantee the existence of a steady state), the first order conditions imply

$$\frac{1}{c_t} = \lambda, \tag{24}$$

$$m_t^{-1/\sigma} = \lambda i_t. \tag{25}$$

It follows from (24) that consumption will be constant along a perfect foresight equilibrium path. Equations (24) and (25) implicitly define the real money demand:

$$m_t = \left(\frac{c_t}{i_t} \right)^\sigma . \tag{26}$$

Notice, for further reference, that the interest rate elasticity of money demand is σ which, by assumption, is less than one. This ensures that, for given consumption, the revenues from the inflation tax, im, are an increasing function of i:

$$\frac{\partial im}{\partial i} = (1-\sigma) i^{-\sigma} c^\sigma > 0. \tag{27}$$

As will become clear below, this implication will play a critical role in obtaining the result that a higher interest rate leads to a nominal appreciation of the domestic currency.

3.2 Government

The government holds no international reserves (i.e., no foreign assets).[5] The government issues a domestic bond, b^g. Its flow constraint (in real terms) is given by:

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[4] We specify the utility function just for algebraic convenience. The only important assumption (in addition to separability) is that $\sigma < 1$, which (as we shall see) ensures that we are always on the "correct" side of the Laffer curve. Exercise 1 at the end of this chapter asks you to solve the case in which the utility function is non-separable in consumption and real money balances.

[5] Since the government allows the exchange rate to float freely, it does not need to hold international reserves for intervention purposes.

$$\dot{b}_t^g = rb_t^g + \tau_t - \dot{m}_t - (\varepsilon_t + \pi_t^*)m_t. \tag{28}$$

Integrating forward constraint (28) and imposing the transversality condition $\lim_{t\to\infty} b_t^g e^{-rt} = 0$, we obtain:

$$b_0^g = \int_0^\infty \left[\dot{m}_t + (\varepsilon_t + \pi^*)m_t - \tau_t\right] e^{-rt}dt. \tag{29}$$

Let $d_t(\equiv m_t + b_t^g)$ denote government's total liabilities. Using the interest parity condition, we can rewrite the government's flow constraint (28) as

$$\dot{d}_t = rd_t + \tau_t - i_t m_t. \tag{30}$$

Up to now (i.e., in Chapters 5 through 7), we have assumed that money is introduced into the economy via "helicopter drops".[6] In other words, the central bank simply prints out money and hands it out to the public. Since money (or, more precisely, the monetary base) is a government liability, the presence of "helicopters drops" implies that government's liabilities can change discretely at any instant in time. Instead, we will assume that money may only be introduced into the economy through open market operations. In other words, when the government wishes to increase (reduce) the nominal money supply, it buys (sells) government bonds. Formally, we assume that:

$$\Delta M_t = -\Delta B_t^g,$$

or equivalently,

$$\Delta D_t = 0,$$

where $D_t = M_t + B_t^g$. In other words, the government's liabilities, D, are a predetermined variable and cannot change discretely at any point in time.

In addition, we will assume that transfers are exogenously set at the level $\bar{\tau}$:

$$\tau_t = \bar{\tau}.$$

This assumption implies that the monetary authority must accommodate the fiscal authority and not viceversa. Up to this point in the book, we have always assumed the opposite (i.e., τ_t is endogenously determined), which implies that the fiscal authority accommodates the monetary authority.

3.3 Equilibrium conditions

Interest parity, given by condition (17), continues to hold. As before, combining the consumer's and the government's flow constraints, given by equations (7) and (28), respectively, we obtain the economy's flow constraint:

$$\dot{b}_t = rb_t + y - c_t. \tag{31}$$

[6] To withdraw money from circulation, imagine "giant vacuum cleaners".

By the same token, combining (8) and (29), we obtain the economy's resource constraint:

$$b_0 + \int_0^\infty y e^{-rt} dt = \int_0^\infty c_t e^{-rt} dt. \tag{32}$$

3.4 Solution of the model

Let the foreign nominal interest rate be constant over time and equal to i^*. Suppose that the monetary authority sets the nominal interest rate at a constant value given by $\bar{\imath}$. We solve for the corresponding perfect foresight equilibrium path (PFEP).

Given the interest parity condition (17), the rate of depreciation will be constant over time and given by:

$$\bar{\varepsilon} = \bar{\imath} - i^*. \tag{33}$$

Along a PFEP, consumption is constant (by (24)) and given by (from (32)):

$$\bar{c} = r b_0 + y. \tag{34}$$

Since consumption and the nominal interest rate are constant over time, real money balances will also be constant along a PFEP and given by (from (26)):

$$\bar{m} = \left(\frac{\bar{c}}{\bar{\imath}} \right)^\sigma. \tag{35}$$

Since real money balances are constant along a PFEP, it follows that $\dot{m}_t / m_t = \mu_t - (\bar{\varepsilon} + \pi^*) = 0$, which implies a constant rate of money growth:

$$\bar{\mu} = \bar{\varepsilon} + \pi^*.$$

We have arrived at a critical juncture. The rate of growth of both the nominal exchange rate and the nominal money supply has been determined. But how about the initial levels of nominal money balances and the nominal exchange rate? In other words, since the government is not setting the initial level of the nominal money supply, M_0, how will the initial exchange rate be determined?

Given the constant values of τ_t, i_t, and m_t, we can rewrite the government's flow constraint (30) as:

$$\dot{d}_t = r d_t + \bar{\tau} - \bar{\imath}\bar{m}.$$

This is an unstable differential equation in d (i.e., the only root is $r > 0$). It follows that, along a PFEP, $\dot{d}_t = 0$ and hence $d_t = d_0$ for all $t \geq 0$. Hence,

$$d_0 = \frac{\bar{\imath}\bar{m} - \bar{\tau}}{r}. \tag{36}$$

Since $d_0 = D_0 / E_0$, it follows that:

$$E_0 = \frac{rD_0}{\bar{\imath}\bar{m} - \bar{\tau}}. \tag{37}$$

In other words, the initial level of the nominal exchange rate (i.e., of the price level) is such that the real value of government liabilities satisfies the government flow constraint. The fact that the initial price level is determined by the fiscal constraint explains why this type of model is referred to as "the fiscal theory of the price level."

3.5 Permanent increase in the interest rate

Suppose now that an instant before time 0 the economy is in a stationary equilibrium as the one we have just characterized. At time $t = 0$, there is an unanticipated and permanent increase in the nominal interest rate from $\bar{\imath}$ to i^H, where $\bar{\imath} < i^H$ (see Figure 1, Panel A). The consumer reoptimizes immediately. The economy will thus jump to a new PFEP characterized by the same equations as above, with the nominal interest rate given by i^H.

The rate of devaluation jumps up to a higher level (recall equation (33)), as depicted in Figure 1, Panel B. Since the economy's resources have not changed, consumption remains unchanged and given by equation (34) (see Figure 1, Panel C).[7] From (35), we see that real money demand will fall because the opportunity cost of holding money has increased (see Figure 1, Panel D). Since, for a constant level of consumption, im is an increasing function of i (recall (27)), equation (36) makes clear that real liabilities (d_0) will increase (see Figure 1, Panel E). Finally, notice from (37), that at $t = 0$ the nominal exchange rate must fall (Figure 1, Panel F) to generate the higher level of real liabilities (Figure 1, Panel E).

[Figure 1 here]

The key result is thus that an increase in the nominal interest rate leads to a nominal appreciation of the domestic currency. Intuitively, an increase in the interest rate increases revenues from the inflation tax (given that the interest rate elasticity of money demand is less than one). This implies that government revenues increase. Therefore, at the pre-shock value of real government liabilities, there would be an operational surplus. The exchange rate (i.e., the price level) needs to fall to increase the government's real liabilities and restore fiscal balance.

[7] Due to the assumption of separability between consumption and real money balances in the utility function, even a *temporary* increase in the interest rate would have no effect on consumption. Exercise 1 at the end of this chapter shows that if the utility function is non-separable and the cross derivative between consumption and real money balances is positive, then a temporary increase will lead to a temporary fall in consumption.

3.6 A final comment

The fiscal theory of the price level offers an interesting and plausible way of recovering price level determinacy under interest rate targeting. Intuitively, however, the channel through which a higher interest rate leads to an appreciation of the domestic currency is not particularly appealing (coming, as it does, through the fiscal constraint as opposed to being a money market phenomenon).

Formally, a more important drawback of this specification is that if we analyze predetermined or flexible exchange rates, we will have an overdetermined system. Consider first the case of predetermined exchange rates, in which the monetary authority sets the initial level and the rate of change of the exchange rate. Since the government's flow constraint would also determine an initial price level (i.e., an initial exchange rate), the system would be overdetermined. In the case of flexible exchange rates, the monetary authority sets the initial level of the money supply which determines an initial price level through the money market equilibrium. Again, this price level need not be consistent with the one determined by the fiscal constraint.

4 Interest rates as an additional policy instrument

The second way of dealing with the price level indeterminacy that arises under interest rate targeting – which follows Calvo and Végh (1995) – involves a sharp departure from the previous model. The reason is that in this set-up the monetary authority is assumed to control the interest rate on a liquid asset (which can be thought of as interest-bearing money).

The original motivation behind this way of thinking about interest rate policy was that, in many high inflation countries (i.e., Argentina, Brazil, and Uruguay during the 1980s) commercial banks lent heavily to the government (compulsively and voluntarily) as high interest rates on public debt and scarce profitable opportunities in the private sector made this the best financial strategy (see Box 1). Commercial banks, in turn, issued time deposits of very short maturity (i.e., highly liquid deposits) to the public. The interest rate set by the government was thus indirectly determining the interest rate paid by banks to depositors. In fact, at some point in Brazil, the entire monetary base was interest-bearing. While perhaps in a less extreme form, this state of affairs is in generally typical of developing countries where commercial banks hold a large fraction of their assets in the form of public debt.

4.1 Consumer's problem

4.1.1 Budget constraints

When it comes to the consumer's problem, the only difference with the case analyzed in Section 2 is that money now bears interest.[8] Hence, the consumer's flow constraint corresponding to (7) now reads as:

$$\dot{a}_t = ra_t + y + \tau_t - c_t - (i_t - i_t^m)m_t, \tag{38}$$

where i_t^m denotes the interest rate borne by money. Notice that, since money is interest-bearing, the opportunity cost of holding money is given by $i_t - i_t^m$. As will become clear below, these two nominal interest rates will be related by an arbitrage condition that will incorporate the liquidity services provided by money. Hence, in equilibrium, $i_t > i_t^m$. The corresponding lifetime budget constraint is thus:

$$a_0 + \frac{y}{r} + \int_0^\infty \tau_t e^{-rt} dt = \int_0^\infty [c_t + (i_t - i_t^m)m_t] e^{-rt} dt. \tag{39}$$

4.1.2 Utility maximization

Let preferences take the logarithmic form:[9]

$$\int_0^\infty [\log(c_t) + \log(m_t)]\, e^{-\beta t} dt. \tag{40}$$

The consumer's problem thus consists in choosing c_t and m_t to maximize lifetime utility (40) subject to the lifetime budget constraint (39), for given paths of τ_t, i_t, and i_t^m and a given value of y and a_0.

Assuming $\beta = r$, the first order conditions imply

$$\frac{1}{c_t} = \lambda, \tag{41}$$

$$\frac{1}{m_t} = \lambda(i_t - i_t^m). \tag{42}$$

Equations (41) and (42) implicitly define the real demand for money:

$$m_t = \frac{c_t}{i_t - i_t^m}. \tag{43}$$

Notice that, for a given value of i_t, an increase in i_t^m reduces the opportunity cost of holding money and leads to higher real money demand. This will be the key channel through which interest rate policy will operate in this model.

[8] Alternatively, these may be thought of as interest-bearing demand deposits.

[9] We adopt this log specification for simplicity. Any separable utility function would lead to the same results.

4.2 Government

Except for the fact that the government pays interest on money, the government's flow constraint remains the same as in Section 2. Hence, the constraint corresponding to (15) now reads:

$$\dot{h}_t = rh_t + \dot{m}_t + (\varepsilon_t + \pi^* - i_t^m)m - \tau_t. \tag{44}$$

Integrating forward and imposing the usual transversality condition, we obtain the government's intertemporal constraint:

$$h_0 + \int_0^\infty [\dot{m}_t + (\varepsilon_t + \pi_t^* - i_t^m)m_t]e^{-rt}dt = \int_0^\infty \tau_t e^{-rt}dt \tag{45}$$

4.3 Equilibrium conditions

As before, perfect capital mobility implies that the interest parity condition (17) holds. It is also easy to verify that combining the consumer's and the government's flow constraints (given by equations (38) and (44), respectively) yields the economy's current account, given by equation (18). By the same token, combining the consumer's and the government's intertemporal budget constraints (given by equations (39) and (45), respectively) yields the economy's resource constraint, given by equation (19).

4.4 Perfect foresight equilibrium

In this model, the government can choose two policy instruments: the exchange rate or the money supply and i^m.[10] Let us assume that the government chooses the path of the money supply (i.e., its initial level, $\bar{M}_0$, and the rate of money growth, $\bar{\mu}$) and the level of i^m.

Notice first that, from first-order condition (41), consumption will be constant along a PFEP. From (19), it follows that

$$\bar{c} = rk_0 + y. \tag{46}$$

We now show that i_t is governed by an unstable differential equation and therefore will need to be constant along a PFEP. To see this, differentiate first-order condition (42), taking into account the interest parity condition (17) and the fact that $\dot{m}_t/m_t = \bar{\mu} - \varepsilon_t - \pi^*$, to obtain:

$$\dot{i}_t = (i_t - i^m)(i_t - r - \bar{\mu}). \tag{47}$$

Since this is an unstable differential equation, i_t must be constant along a PFEP:

[10] Notice that the government could also set the path of the exchange rate and the initial level of nominal money supply (or the path of nominal money supply and the initial level of the exchange rate) and let i^m be determined endogenously.

$$\bar{\imath} = r + \bar{\mu}.$$

Since both consumption and the nominal interest are constant along a PFEP, real money balances will be constant as well. From (43), it follows that:

$$\bar{m} = \frac{rk_0 + y}{r + \bar{\mu} - i^m}. \tag{48}$$

Since real money balances are constant, then $\dot{m}_t = 0$, which implies that:

$$\varepsilon_t = \bar{\mu} - \pi^*.$$

Finally, the initial price level (i.e., the initial value of the exchange rate) will be determined by money market equilibrium at time 0:

$$E_0 = \bar{M}_0 \frac{r + \bar{\mu} - i^m}{rk_0 + y}. \tag{49}$$

4.5 Permanent increase in interest rates

Suppose now that an instant before time 0 the economy is in the stationary equilibrium we have just characterized. At time 0, there is an unanticipated and permanent increase in the policy-controlled interest rate, i^m (Figure 2, Panel A)

[Figure 2 here]

Since the shock is unanticipated, the consumer reoptimizes immediately and a new perfect foresight path as the one just described emerges, with the policy-controlled interest rate now at a higher level.

From (46), we see that consumption will not change (Figure 2, Panel B). From (47), it is clear that the nominal interest rate cannot jump because, if it did, it would follow a divergent path. Hence, the nominal interest rate must remain put at its pre-shock level (as illustrated in Figure 2, Panel C). Since i does not change at time 0 but i^m has gone up, the opportunity cost of holding real money balances falls (Figure 2, Panel D). Hence, real money demand goes up, as (48) makes clear (see Figure 2, Panel E). For real money balances to increase, the nominal exchange rate must fall, as follows from (49) (see Figure 2, Panel F).

The punchline of this exercise is thus that an increase in the policy-controlled interest rate leads to a nominal appreciation of the domestic currency. This is, of course, the same result that we obtained in the previous section. The intuition, however, is very different. In this case, the higher interest rate on money reduces the opportunity cost of holding money, which increases real money demand and thus leads to a fall in the exchange rate.

4.6 Fiscal effects of higher interest rates

A common concern of policymakers when it comes to using higher interest rates to defend the currency is the negative effect on the fiscal accounts. Paying higher interest rates on the public debt will worsen the fiscal deficit. A case in point is Brazil during the Real Plan (the stabilization program implemented in July 1994). Commenting on Brazil (*Financial Times*, January 22, 1999), Jeffrey Sachs argued at the time that

> ... at that point [when the Asian crises hit], an urgent re-assessment of monetary exchange rate policy was due. And yet the IMF defended the Brazilian decision in October 1997 to put up interest rates to 50 percent per year precisely in order to hold the currency. This decision was fateful. It cemented the end of Brazilian economic growth, and built in a fiscal time bomb. When the misguided defence of the currency began, the deficit was about 4 percent of GDP. A fiscal adjustment, supposedly of 2 percent of GDP was announced, and praised by the IMF. But instead of reducing the deficit to 2 percent of GDP, the 1998 budget deficit in fact jumped to 8 percent of GDP, in large part the result of the self-induced economic slowdown (which reduced tax collection) and the rapid build-up of interest payments on public debt.

How can we incorporate a fiscal effect in our model? This is easily done by assuming that government transfers are exogenously given at some level $\bar{\tau}$. Since the government needs to finance an exogenously-given level of transfers, paying a higher interest rate on the central bank's liabilities will require an increase in the inflation rate (which will in turn increase the nominal interest rate). This inflationary effect of a higher interest rate will thus tend to undo some the direct effect of a higher interest rate on money. In fact, Exercise 2 at the end of this chapter shows a case in which this fiscal effect exactly offsets the higher interest rate on money, thus leaving the opportunity cost of holding money $(i - i^m)$ unchanged. As result, the higher interest rate on money has no impact effect on the nominal exchange rate. This exercise thus illustrates the fiscal perils of higher interest rates.

4.7 Output effects of higher interest rates

As the above quote by Jeffrey Sachs also illustrates, a second concern of policymakers on the use of higher interest rates to defend a currency is a possible negative output effect. This output effect can be introduced into our model by assuming a sticky-prices formulation in which output of non-tradable goods is demand-determined, as in Exercise 3 at the end of the chapter. In this set-up, an increase in i^m appreciates the domestic currency but at the cost of fall in output of non-tradable goods.

4.8 A final comment

The approach to interest rate policy followed by Calvo and Végh (1995) has two advantages. First, it avoids indeterminacy problems. Second, it allows us to study interest rate policy independently of changes in money supply (or in the exchange rate). However, it should be emphasized that the results obtained in this framework are not directly comparable to those of the interest rate targeting literature because the interest rate controlled by policymakers is not the same.

5 Interest rate rules under sticky prices

This section deals with interest rate targeting in a model with sticky prices. At first glance, it may seem that sticky prices will solve the indeterminacy problem. After all, if prices are sticky (i.e., they are a predetermined variable at each point in time), they surely cannot be undetermined! While this is certainly true, we will show that, in the presence of sticky prices, interest rate targeting will lead to a "higher order indeterminacy". In other words, interest rate targeting will lead to a multiplicity of equilibrium paths for the inflation rate.

Consider a small open economy inhabited by a large number of identical, infinitely lived consumers, who are endowed with perfect foresight. The economy is perfectly integrated with world markets in both goods and capital markets. There are two (non-storable) goods: tradables and non-tradables. The supply of the tradable good is fixed. The supply of the non-tradable good is demand-determined.

5.1 Consumer's problem

5.1.1 Budget constraints

As far as the consumer is concerned, the only difference with the model of Section 2 is that there are now non-tradable goods. The flow constraint corresponding to equation (7) is thus now given by (using the tradable good as the numeraire):

$$\dot{a}_t = ra_t + y_t^T + \frac{y_t^N}{e_t} + \tau_t - c_t^T - \frac{c_t^N}{e_t} - i_t m_t, \tag{50}$$

where c^T and c^N denote consumption of tradables and non-tradables, respectively, y_t^T and y_t^N denote output of tradables and non-tradables, respectively, and e is the relative price of tradable goods in term of non-tradable goods.

Integrating the flow constraint (50) forward and imposing the transversality condition $\lim_{t\to\infty} a_t e^{-rt} dt = 0$, we obtain:

$$a_0 + \int_0^\infty \left(y^T + \frac{y_t^N}{e_t} + \tau_t \right) e^{-rt} dt = \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t} + i_t m_t \right) e^{-rt} dt. \tag{51}$$

5.1.2 Utility maximization

The consumer's lifetime utility takes the following logarithmic form:

$$\int_0^\infty (\log c_t^T + \log c_t^N + \log m_t) e^{-\beta t} dt, \tag{52}$$

where $\beta(> 0)$ is the subjective discount rate. None of the results obtained below changes with general separable preferences.

The consumer's problem consists in choosing $\{c_t^T, c_t^N, m_t\}$ for all $t \in [0, \infty)$ to maximize lifetime utility (52), subject to (51), for given paths of τ_t, e_t, y_t^T, y_t^N and i_t and a given value of a_0.

Assuming $\beta = r$, the first order conditions imply

$$\frac{1}{c_t^T} = \lambda, \tag{53}$$

$$\frac{1}{c_t^N} = \frac{\lambda}{e_t}, \tag{54}$$

$$\frac{1}{m_t} = \lambda i_t. \tag{55}$$

First-order conditions (53) and (55) define a standard real money demand:

$$m_t = \frac{c_t^T}{i_t}. \tag{56}$$

5.2 Supply side

We now turn to the supply side of the model. The supply of the tradable good will be assumed to be constant over time and equal to y^T. The non-tradables sector operates under staggered price setting and output is demand determined. Following Calvo (1983), we postulate:[11]

$$\dot{\pi}_t = -\theta(y_t^N - y_f^N), \tag{57}$$

where π_t is the inflation rate of home goods, θ is a positive parameter and y_f^N is the "full employment" level of output of home goods. As discussed in detail in Chapter 8, equation (57) can be derived from a set-up in which firms set prices in an asynchronous manner, taking into account the expected future path of the average price of home goods and the path of excess demand in that market.

5.3 Government

The government's flow constraint is given by:

[11] Think of this formulation as a continuous-time version of the overlapping-contracts models à la Fischer (1977) and Taylor (1979, 1980).

$$\dot{h}_t = rh_t + \dot{m}_t + \varepsilon_t m_t - \tau_t. \tag{58}$$

Integrating forward and imposing the transversality condition $\lim_{t\to\infty} h_t e^{-rt} dt = 0$, we obtain the government's intertemporal budget constraint:

$$\int_0^\infty \tau_t e^{-rt} dt = h_0 + \int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt. \tag{59}$$

5.4 Equilibrium conditions

The assumption of perfect capital mobility (coupled with the assumption that foreign inflation is zero) implies the interest parity condition

$$i_t = r + \varepsilon_t. \tag{60}$$

Since the output of non-tradable goods is demand-determined, it must be the case that:

$$y_t^N = c_t^N. \tag{61}$$

Combining (50), (58) and (61), we obtain the flow constraint of the economy (i.e., the current account),

$$\dot{k}_t = rk_t + y^T - c_t. \tag{62}$$

where $k_t \equiv h_t + b_t$.

Finally, by combining (51), (59), and (61), we obtain the economy's resource constraint:

$$k_0 + \int_0^\infty y^T e^{-rt} dt = \int_0^\infty c_t^T e^{-rt} dt. \tag{63}$$

5.5 Higher order indeterminacy

Suppose that the government sets the nominal interest rate at the level $\bar{\imath}$. We will now solve for the corresponding PFEP and show that such a monetary rule leads to a multiplicity of equilibrium paths.

For starters, note that first-order condition (53) implies that consumption of tradables is constant along a PFEP. Hence, from the economy's resource constraint (63), it follows that:

$$\overline{c^T} = rk_0 + y^T. \tag{64}$$

Given the nominal interest rate set by the monetary authority and (64), real money demand will be constant and given by:

$$\bar{m} = \frac{rk_0 + y^T}{\bar{\imath}}.$$

From the interest parity condition (60), it follows that the rate of depreciation is given by:

$$\varepsilon_t = \bar{\imath} - r.$$

To study the dynamic behavior of the rest of the endogenous variables of the model, we will construct a dynamic system of two differential equations in π and e. From (53) and (54), it follows that $c_t^N = e_t c_t^T$. Substituting this into (57):

$$\dot{\pi}_t = \theta(y_f^N - e_t c_t^T). \tag{65}$$

Since, by definition, $e_t = E_t P^*/P_t^N$, using (60) it follows that

$$\dot{e}_t = e_t(\bar{\imath} - r - \pi_t). \tag{66}$$

Since c^T is given to this system by (64) and $\bar{\imath}$ is the policy instrument, equations (65) and (66) constitute a dynamic system in π and e. The steady state of this dynamic system is given by (point A in Figure 3):

$$\begin{aligned} e_{ss} &= \frac{y_f^N}{\overline{c^T}}, \\ \pi_{ss} &= \bar{\imath} - r. \end{aligned}$$

[Figure 3 here]

Linearizing the system around the steady-state:

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{e}_t \end{bmatrix} = \begin{bmatrix} 0 & -\theta\overline{c^T} \\ -e_{ss} & 0 \end{bmatrix} \begin{bmatrix} \pi_t - \pi_{ss} \\ e_t - e_{ss} \end{bmatrix}. \tag{67}$$

The determinant associated with the linear approximation is therefore:

$$\Delta = -e_{ss}\theta\overline{c^T} < 0.$$

Since there is one positive and one negative root, the system exhibits saddle path stability. There is thus a unique converging path (see Figure 3). However, neither π nor e are predetermined variables and are therefore free to take any value at time 0.[12] This implies that any initial pair (π_0,e_0) lying on the saddle path (including the steady-state itself) will converge to the steady-state. Figure 3 illustrates three such points: A, B, and C. There are thus a multiplicity of equilibrium paths. This is what we mean by a "higher order indeterminacy".

[12] Recall the rule that the number of predetermined variables has to be equal to the number of negative roots in order to have a unique solution. In this case, there is one negative root and no predetermined variable and the system is thus undetermined (i.e., there are infinite solutions).

5.6 An interest rate rule

We now show how a policy rule whereby the interest rate is varied according to some observable variable leads to a unique equilibrium path. Specifically, suppose that policymakers set an inflation target, $\bar{\pi}$, and vary the nominal interest rate according to the following rule:

$$\dot{i}_t = \alpha(\pi_t - \bar{\pi}). \tag{68}$$

Since the rule is specified in terms of rates of change, it also implies that the nominal interest rate is a predetermined variable at each point in time. In other words, by appropriately varying the nominal money supply, the monetary authority will not let the interest rate jump at any point in time. In practice, interest rate rules of this type have become the hallmark of "inflation targeting" regimes, as discussed in Box 2.

Differentiating first-order condition (54), taking into account that $\dot{e}_t = e_t(\varepsilon_t - \pi_t)$, and using (60), it follows that:

$$\dot{c}_t^N = c_t^N(i_t - \pi_t - r). \tag{69}$$

Equations (57), (68), and (69) constitute a three-equation differential equation system in π, i, and c^N. We will solve this system by linearizing it around the steady-state.

The system's steady state is given by

$$\begin{aligned} i_{ss} &= r + \bar{\pi}, & (70) \\ \pi_{ss} &= \bar{\pi}, & (71) \\ c_{ss}^N &= y_f^N. & (72) \end{aligned}$$

The linear approximation of the system around the steady state is given by:

$$\begin{bmatrix} \dot{i}_t \\ \dot{c}_t^N \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & \alpha \\ y_f^N & 0 & -y_f^N \\ 0 & -\theta & 0 \end{bmatrix} \begin{bmatrix} i_t - i_{ss} \\ c_t^N - y_f^N \\ \pi_t - \bar{\pi} \end{bmatrix},$$

which implies that the trace and the determinant of the matrix associated with the linear approximation are given by, respectively:

$$\begin{aligned} Trace &= 0, \\ \Delta &= -\alpha\theta y_f^N < 0. \end{aligned}$$

The fact that $\Delta < 0$ implies that there are either three negative roots or one negative and two positive roots. However, since the trace (which equals the sum of the roots) is zero, it must be the case that the system has one negative and two positive roots. The system is thus saddle path stable; that is, there is a single line converging to the steady state in R^3. Since there is only one predetermined variable (i), the system has a unique solution: for a given value of i_0, π_0 and c_0^N will adjust so as to position the system along the saddle path.

Let δ_1 be the negative root. To obtain the eigenvector associated with this root we should solve

$$\begin{bmatrix} -\delta_1 & 0 & \alpha \\ y_f^N & -\delta_1 & -y_f^N \\ 0 & -\theta & -\delta_1 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

where h_{11}, h_{12} and h_{13} are the elements of the eigenvector associated with the negative eigenvalue.

Therefore,

$$\begin{aligned} \frac{h_{11}}{h_{13}} &= \frac{\alpha}{\delta_1} < 0, \\ \frac{h_{12}}{h_{13}} &= -\frac{\delta_1}{\theta} > 0. \end{aligned}$$

As will become clear below, this will provide a crucial piece of information when it comes to deriving the dynamic behavior of the system.

Setting to zero the constants corresponding to the unstable roots (i.e., the two positive roots), the solution to the linear approximation of the dynamic system takes the form:

$$\begin{aligned} i_t - i_{ss} &= A_1 h_{11} e^{\delta_1 t}, \\ c_t^N - y_f^N &= A_1 h_{12} e^{\delta_1 t}, \\ \pi_t - \overline{\pi} &= A_1 h_{13} e^{\delta_1 t}. \end{aligned} \tag{73}$$

It follows that

$$\frac{i_t - i_{ss}}{\pi_t - \pi_{ss}} = \frac{h_{11}}{h_{13}} < 0, \tag{74}$$

$$\frac{c_t^N - y_f^N}{\pi_t - \overline{\pi}} = \frac{h_{12}}{h_{13}} > 0. \tag{75}$$

Since there is only one negative root, the adjustment of all three variables towards the steady-state will be monotonic. Hence, equation (74) indicates that i_t and π_t move in opposite directions, while equation (75) says that c_t^N and π move in the same direction.

To pin down the arbitrary constant A_1, notice that i_t is a predetermined variable. Further, since the eigenvector is determined up to a scalar, we can set $h_{11} = 1$. Hence, evaluating (73) at $t = 0$ and solving for A_1 yields:

$$A_1 = i_0 - i_{ss}. \tag{76}$$

5.7 Permanent reduction in the inflation target

To understand the dynamic adjustment of this economy operating under a nominal interest rate rule, let us now analyze the response of the economy to an unanticipated and permanent reduction in the inflation target (see Figure 4, Panel A).

[Figure 4 here]

As is clear from (70), the nominal interest rate will be lower in the new steady-state. Hence, from (76), $A_1 > 0$. Differentiating (73) with respect to time (and recalling that $h_{11} = 1$), it follows that i_t falls over time (Figure 4, Panel B). How does the inflation rate behave? From (71), it is clear that the inflation will be lower in the new steady-state. Further, we know from (74) that π_t will move in the opposite direction as i_t. Hence, π_t will increase over time. For this to happen, it must be the case that, at $t = 0$, π falls below its new steady-state value and then gradually increase towards it (Figure 4, Panel C). Finally, notice from (72) that consumption of home goods does not change across steady-states. Further, from (75), c^N and π move in the same direction. Hence, c^N will be increasing over time. For this to be the case, c^N must fall on impact and increase gradually over time towards its unchanged steady-state (Figure 4, Panel D). Given first-order condition (54), the real exchange rate will follow a path identical to c^N, following on impact (real appreciation) and then gradually increasing over time towards its unchanged steady-state (Figure 4, Panel E).

Finally, let us look at the behavior of real money balances in terms of home goods. As in chapter 8, define real money balances in terms of home goods as:

$$n_t \equiv \frac{M_t}{P_t^N}. \tag{77}$$

Using (54) and (55), we obtain the demand for real money balances in terms of home goods:

$$n_t = \frac{c_t^N}{i_t}. \tag{78}$$

In the new steady-state, n will be higher reflecting the lower nominal interest rate. On impact, n falls since c^N falls and i remains unchanged. Hence, n will increase over time towards its higher steady-state, as illustrated in Figure 4, Panel E. Since P^N is sticky, it follows from (77) that the nominal money supply falls at time 0. In fact, it is precisely this reduction in the nominal money supply at time 0 that prevents the nominal interest rate from falling.

6 Real interest rate rules

While interest rate rules based on *nominal* interest rate rules are by far the most common in practice, some countries have occasionally experienced with *real* interest rate rules. Most notably, Chile conducted monetary policy for more than a decade using as the main policy instrument a 90-day real interest rate on Central Bank liabilities (see Box 3 in Chapter 12). This, of course, raises the obvious question of whether nominal indeterminacies will arise under real interest rate targeting. Not surprisingly – and as illustrated in Exercise 4 at the end of this Chapter – a real interest rate targeting whereby the real interest rate (defined in terms of home goods) is set at a constant level results in a undetermined inflation rate. The intuition is clear enough: since the monetary authority is not setting the rate of growth of any nominal magnitude, there is nothing tying down the rate of inflation. However, we also show that, when combined with an inflation target (in the same vein as rule (68)), a real interest rate rule leads to a well-behaved dynamic system.[13] The key is that the inflation target (which is assumed to be fully credible) provides the nominal anchor to the economy. In practice, of course, this should be viewed as a dangerous practice since any credibility problems on the part of the public regarding the announced inflation target will lead to a highly unstable system.

7 Final remarks

This chapter has analyzed the use of interest rates as a policy instrument. We have seen how a pure nominal interest rate targeting leads to a price level indeterminacy because monetary policy has not been fully specified. We studied three different ways of completing monetary policy and used these models to understand the effects of changes in the nominal interest rate on the exchange rate. With flexible prices, higher nominal interest rates appreciate the domestic currency. This, however, may come at the cost of lower output and a higher public debt burden. In fact, the inflationary effects of a higher public debt service could render interest rate policy ineffective. Hence, from a policy point of view, we conclude that there are perils associated with raising interest rates to defend the domestic currency which will need to be traded off against the benefits of a stronger currency.

[13] Chapter 12 analyzes further this issue by studying a case in which the supply side is characterized by sticky *inflation* (as opposed to sticky *prices*). A real interest rate that relies on an inflation target also leads to a well-defined dynamic system.

Exercises[14]

1. Real effects of interest rates increases in the Auernheimer-Contreras model

   Consider the same model as in Section 3 with the following modification. Suppose that money now enters the utility function in a non-separable way:

$$\int_0^\infty u(c_t, m_t)e^{-\beta t}dt,$$

   where $u(c, m)$ is increasing, strictly concave, and has positive cross derivative:

$$u_c > 0,\ u_m > 0,\ u_{cc} < 0,\ u_{mm} < 0,\ u_{cm} > 0,\ u_{cc}u_{mm} - u_{cm}^2 > 0.$$

   (a) Solve for the perfect foresight equilibrium path for a given value of the nominal interest rate.

   (b) Analyze the effects of an unanticipated and permanent increase in i at time 0.

   (c) Analyze the effects of an unanticipated and temporary increase in i at time 0 that lasts until T.

2. Fiscal effects of higher interest rates

   This problem analyzes the fiscal effects of a higher interest rate on money in the context of the model of Section 4. The only modification relative to the model of Section 4 is that we now assume that lump-sum transfers from the government are set at a constant level $\bar{\tau}$. This has the important implication that the rate of money growth (μ) now becomes an endogenous variables that will adjust so that the fiscal constraint is satisfied. In other words, policymakers now set i^m and the initial level of the nominal money supply (M_0) but not the rate of growth of the money supply (μ).

   In this context:

   (a) Solve for the perfect foresight equilibrium path corresponding to a constant i^m and a given initial level of M_0.

   (b) Analyze the effects of an unanticipated and permanent increase in i^m at time 0. In particular, focus on what happens to the nominal exchange rate on impact.

3. Interest rates as an additional instrument in a sticky-prices model

[14] An answer key is available from the author upon request.

This problem follows Calvo and Végh (1996). It deals with a two-good, sticky-prices version of the model of Section 4. Let preferences be given by:

$$\int_0^\infty (\log c_t^T + \log c_t^N + \log m_t)e^{-\beta t}dt. \tag{79}$$

The consumer's intertemporal constraint is given by:

$$a_0 + \int_0^\infty \left(y^T + \frac{y_t^N}{e_t} + \tau_t\right) e^{-rt}dt = \int_0^\infty \left[c_t^T + \frac{c_t^N}{e_t} + (i_t - i_t^m)m_t\right] e^{-rt}dt, \tag{80}$$

where y^T is the constant endowment of tradables good and y_t^N is the (demand-determined) level of output of home goods. As in Section 5, let the price of home goods be sticky and the rate of change be given by (57).

In this context:

(a) Suppose that, as in Section 4, the monetary authority sets the path of the money supply and controls i^m. Solve for the perfect foresight equilibrium path for a given μ and a given i^m. [Hint: Show first that i_t follows an unstable differential equation. Then set up a dynamic system in real money balances in terms of home goods (i.e., $n \equiv M/P^N$) and π.]

(b) Analyze the effects of an unanticipated and permanent increase in i^m.

4. Real interest rate rules

This exercise furthers our understanding of interest rate rules by looking at real interest rate rules. Consider the model laid out in Section 5. In this context:

(a) As a benchmark for the rest of this exercise, solve the model for the case of predetermined exchange rates. In particular, analyze the effects on all endogenous variables of an unanticipated and permanent reduction in the rate of devaluation at time $t = 0$.

(b) Consider now the case of "real interest rate targeting." To this effect, define the domestic real interest rate as $r^d \equiv i - \pi$. (Notice that this is the real interest rate relevant for the path of consumption of home goods.) Suppose that policymakers set the domestic real interest rate at the constant level $\bar{r}^d$ (and set no other variable). Show formally that such a policy regime leads to an indeterminacy. (Hint: notice that policymakers will have to set $\bar{r}^d$ equal to r, since this is the only value consistent with equilibrium).

(c) Suppose now that policymakers follow the following real interest rate rule:

$$\dot{r}_t^d = \alpha(\pi_t - \bar{\pi}),$$

where $\bar{\pi}$ is the inflation target chosen by the authorities and α is a positive constant. (Notice that under this policy regime, the domestic real interest rate is a predetermined variable.) Characterize an initial stationary equilibrium for a given and fully credible inflation target. Show what happens if policymakers announce at $t = 0$ an unanticipated and permanent reduction of the inflation target. Discuss the intuition behind the results (in particular, how do they compare with the results obtained for the predetermined exchange rates case and why?).

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Box 1. How much government debt is held by banks and why do they hold it?

Banks' holdings of government debt are generally large. Figure 5 shows, as of the end of 2009, the share of government debt in the hands of domestic financial institutions in the euro area. In France, for example, 40 percent of the French public debt is held by French financial institutions. It is important to note that this measure does not incorporate the debt issued by foreign sovereigns that may be held by financial institutions. As is evident from the figure, there is some variation in the share of government debt that financial institutions held, with Finland having the lowest share (14 percent) and Romania the largest (69 percent). On average, 43 percent of the public debt is held by financial institutions.

[Figure 5 here]

Another way to understand the importance of public debt in the balance sheets of financial institutions is to look at how much public debt these institutions hold as a proportion of total assets. The left panel of Figure 6 shows the share of government debt in the balance sheets of financial institutions, while the right panel presents data on financial institutions' net credit to government as a share of their total assets . As Figure 6 shows, government debt represents a significant proportion of total assets. On average, 8 percent of the financial institutions' assets is in the form of government debt.

[Figure 6 here]

Why do banks hold government debt? In order to answer this question, it is important to make a distinction between voluntary and involuntary holdings of government debt by banks.

During the period of financial repression, which took place roughly between 1945-1980 in both developed and developing economies, governments adopted numerous measures that forced commercial bank to hold government debt in their portfolios. Such policies, combined with artificially low interest rates and relatively high inflation, provided the government with a source of cheap financing. The extent to which this was an effective measure varied across countries and over time but was nonetheless common practice in many countries. Even today, banks are forced to hold government debt in their portfolio as compulsory reserve and liquidity requirements.

But banks have also had reasons to choose to hold government debt voluntary. Rodriguez (1992) and Druck and Garibaldi (2000) point out that, in periods of high inflation volatility, the default risk of firms increases making investing in the private sector more risky. As a result, banks choose to lend more to the public sector in an attempt to reduce their exposure to the private sector's increase in riskiness. In addition, it is not uncommon for governments to resort to high interest rates to entice banks to hold its debt.

Kumhof and Tanner (2005) also point to legal and institutional imperfections as a determinant of how much government debt banks are willing to hold. The idea is that the degree of development determines the incentives of banks to lend to different players in the economy. They construct an index of quality of law and institutions by using data on access to credit, bankruptcy laws, degree of contract enforcement, and cost and time delays of transferring title to real estate, among others. They then explore the connection between this index and the share of government debt held by banks. Figure 7 shows the very strong negative relationship between the quality of law and institutions and the share of government debt held by banks. We can see, on the one hand, countries with high quality institutions such as the United Kingdom, the United States, Norway and New Zealand where banks hold little government debt and, on the other hand, emerging countries with historically more unstable institutions such as Indonesia, India, Mexico and Greece with banks holding a much larger share of government debt.

[Figure 7 here]

Box 2. Inflation targeting

Inflation targeting (IT) was first implemented in New Zealand in December 1989. Since then, numerous advanced and emerging economies have followed suit. Most countries that have adopted inflation targeting as a monetary regime did so after many years (or even decades) of high and/or chronic inflation. In that sense – and as argued by Truman (2003) – inflation targeting may be viewed as an attempt to find a more effective nominal anchor (in the form of an inflation target) given the difficulties encountered with traditional nominal anchors (monetary aggregates and the nominal exchange rate).

There is no widespread agreement as to what defines a country as an inflation targeter. According to Truman (2003), a country is considered an inflation targeter if it has an explicit inflation target and has stated that it has adopted an IT regime. The International Monetary Fund (2010) uses a similar definition; namely, a country is considered to be in the IT group if it has a "medium-term numerical target for inflation". As of 2010, the IMF lists 44 countries in the inflation targeting category.[15]

In general, an IT regime exhibits the following features. First, price stability is the main goal of monetary policy. Second, there is some numerical target or a sequence of numerical targets to be met (for example, during a transition period). Third, countries stipulate a time horizon in which the target must be met. This allows for short term deviations from the target without jeopardizing the credibility of the regime. Finally, the monetary authority is held accountable for meeting the target or, it if is not met, to explain why.

It should be noted that countries that have adopted an IT regime may not have price stability as the only goal of monetary policy. Truman (2003) looks at the central bank's mandate in 22 inflation targeting countries. In only 6 of those countries (or 27 percent) does the central bank pursue price stability as its only objective. In another 8 countries (36 percent), the central bank has other goals but there is a hierarchy in which price stability ranks first. In 2 countries (9 percent), currency stability is the main objective. In the remaining 6 countries, there are multiple objectives with no explicit hierarchy. In no country is there an explicit target for other variables (i.e., output). This distinguishes IT from other monetary regimes in which there may be an output target in addition to the inflation target (in line with the so-called Taylor Rule).

Most of the empirical research compares the performance of adopters relative to non-adopters in terms of average inflation, inflation variability, output variability, and interest rates. The most common estimating strategy uses a difference-in-difference type of approach. Other studies use instrumental variables and propensity score matching techniques. Ball and Sheridan (2005) point out to an endogeneity problem that could arise because countries that have

[15] The coutries are: Albania, Armenia, Australia, Austria, Belgium, Brazil, Canada, Chile, Colombia, Cyprus, Czech Republic, Finland, France, Germany, Ghana, Greece, Guatemala, Hungary, Iceland, Indonesia, Ireland, Israel, Italy, Korea, Luxemburgo, Malta, Mexico, Netherlands, New Zealand, Norway, Peru, Philippines, Poland, Portugal, Romania, Serbia, Slovenia, South Africa, Spain, Sweden, Thailand, Turkey, United Kingdom, and Uruguay.

adopted IT regimes tended to have higher levels of inflation before implementing such a regime. To correct for this endogeneity, they suggest adding the prior level of inflation as a control variable. Table 1 summarizes the results of numerous studies reviewed in Ball (2010).

[Table 1]

Two conclusions seem to emerge. First, IT appears to have reduced the mean and variance of inflation, especially in developing countries. Second, there is some evidence that IT may have reduced output variability in emerging economies.

Some studies have also looked at the effect of IT on inflation persistence and inflation expectations. On inflation persistence, the results appear to go in line with what was discussed before: no effect for advanced economies and lower persistence for emerging economies. However, Levin *et al.* (2004) report large effects on inflation persistence for both groups of countries. On the effect of IT on inflation expectations, the results are mixed.

In sum, the empirical evidence seem to suggest that IT has been effective in developing countries, but not necessarily in developed countries.

Figure 1. Permanent increase in the interest rate

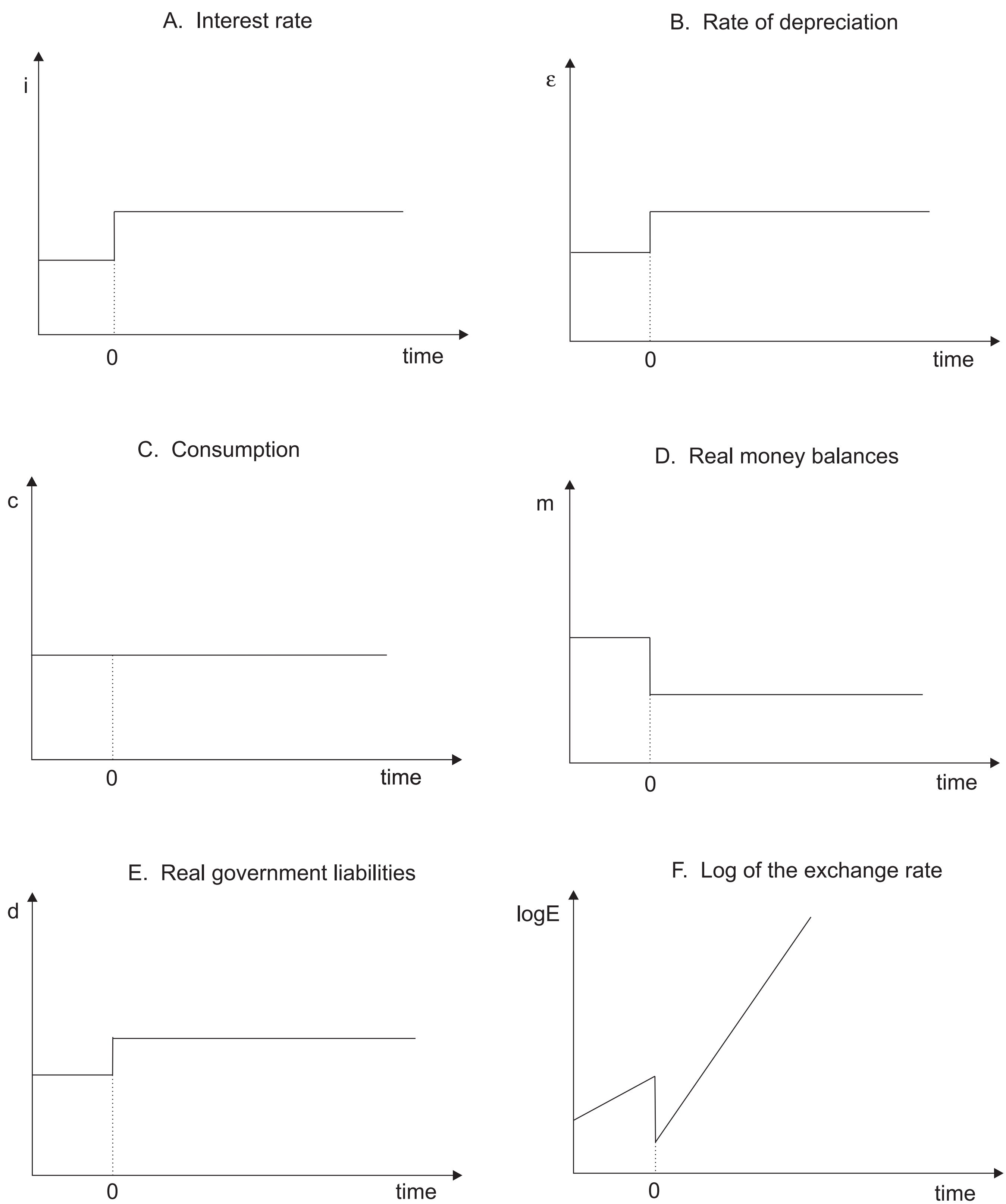


Figure 2. Permanent increase in the policy-controlled interest rate

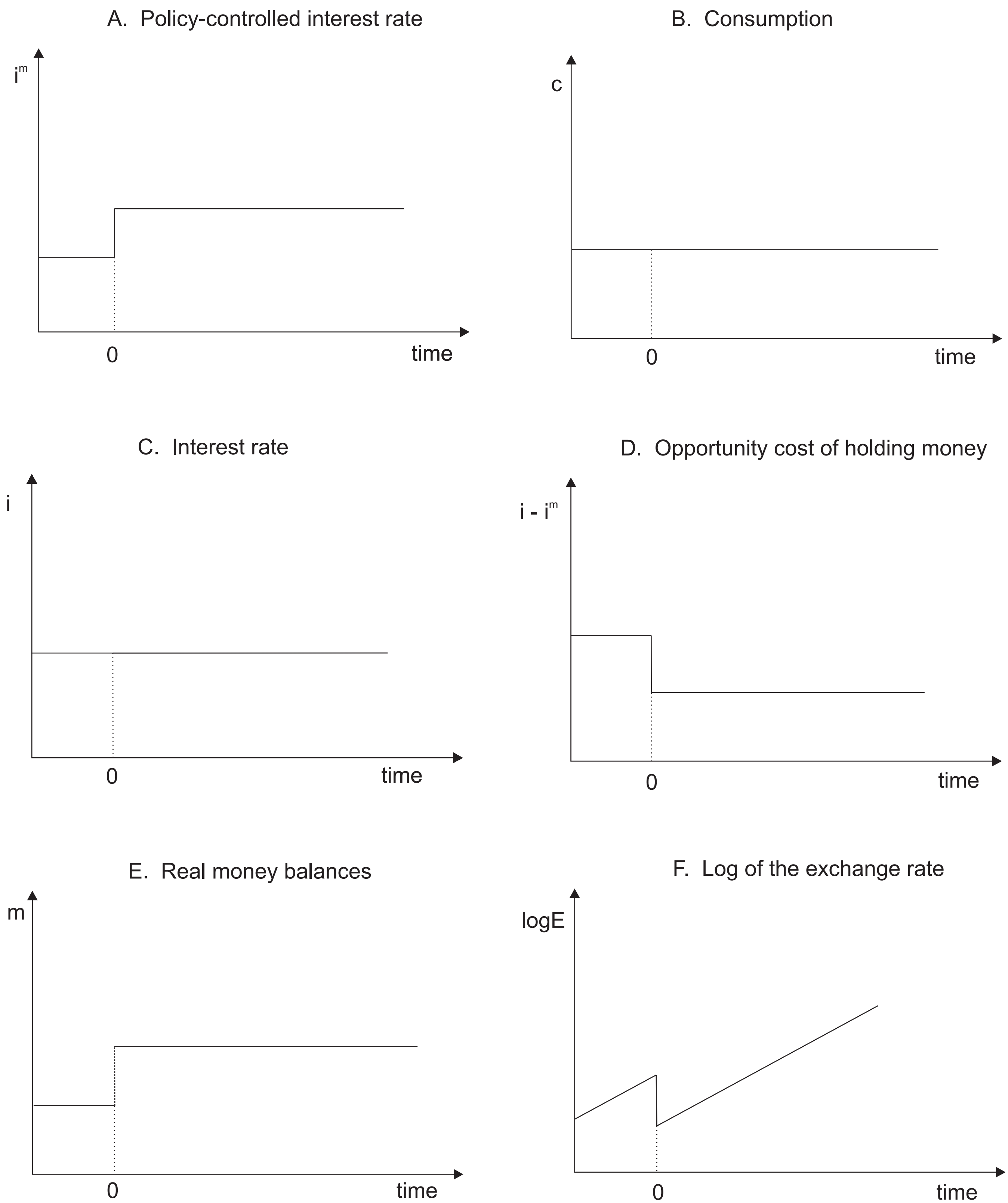


Figure 3. Multiple equilibrium paths

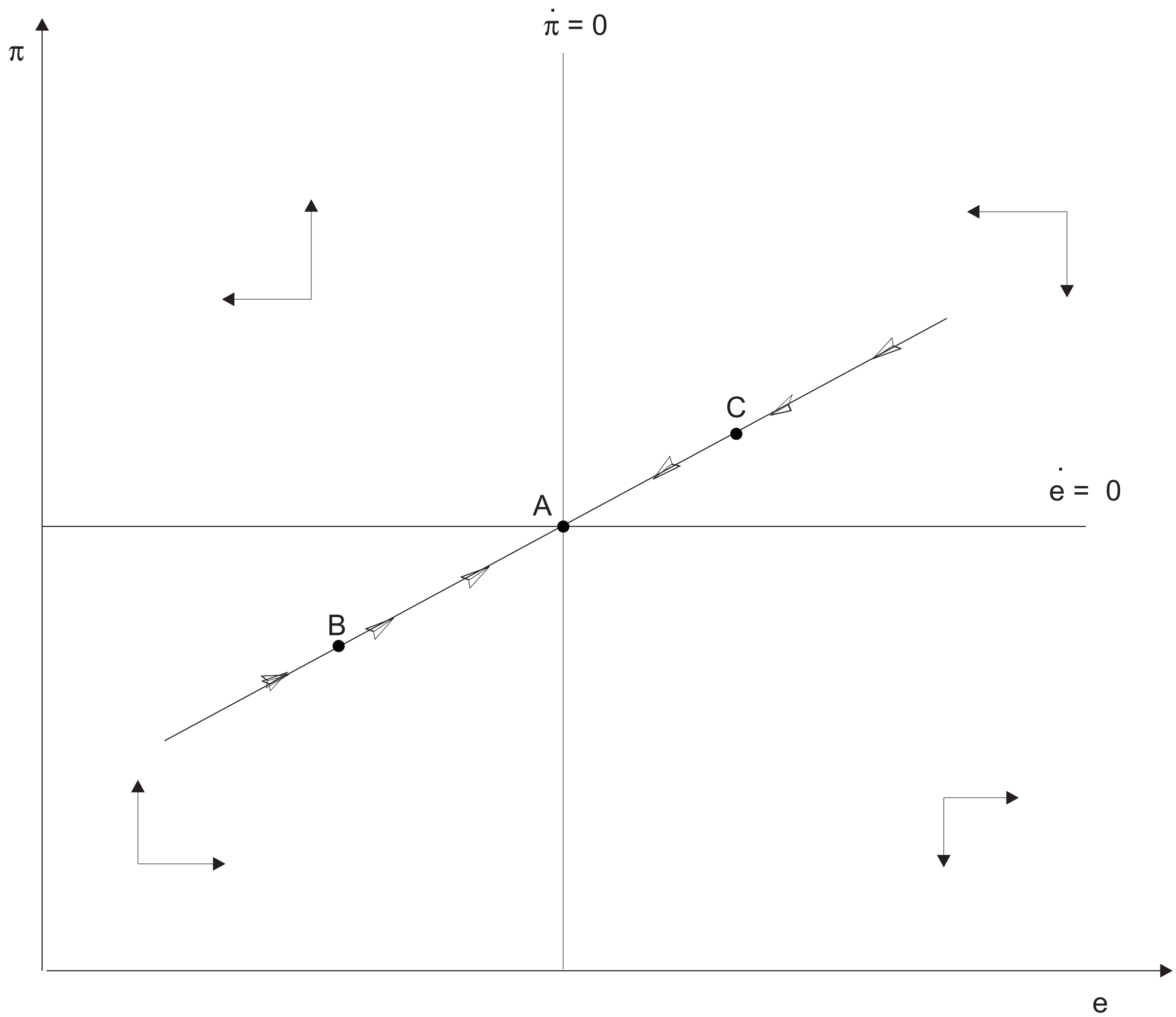


Figure 4. Permanent reduction in the inflation target

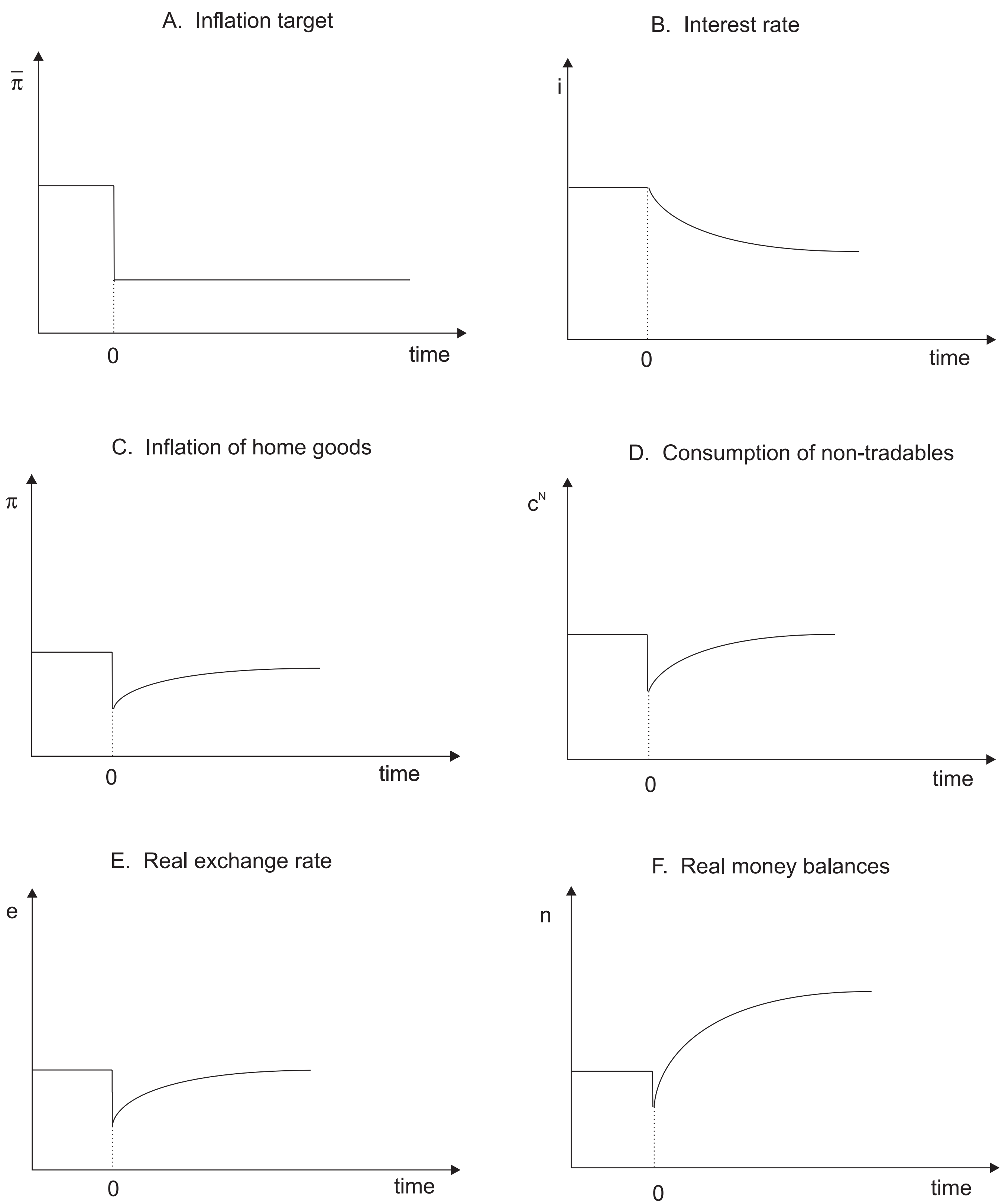
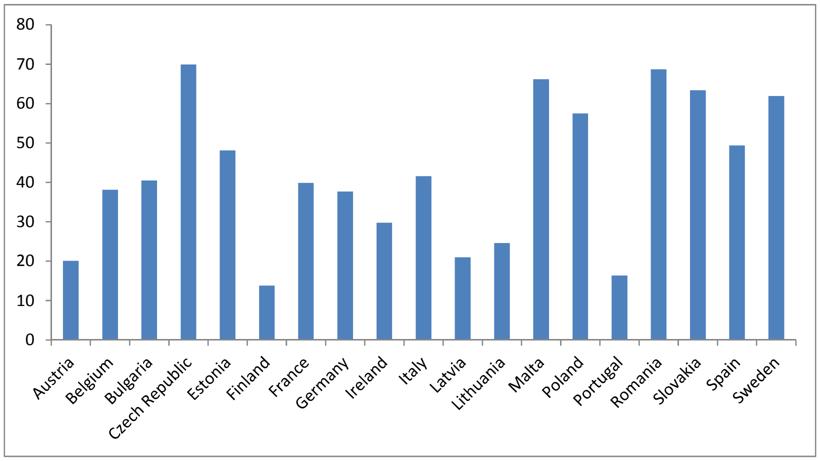
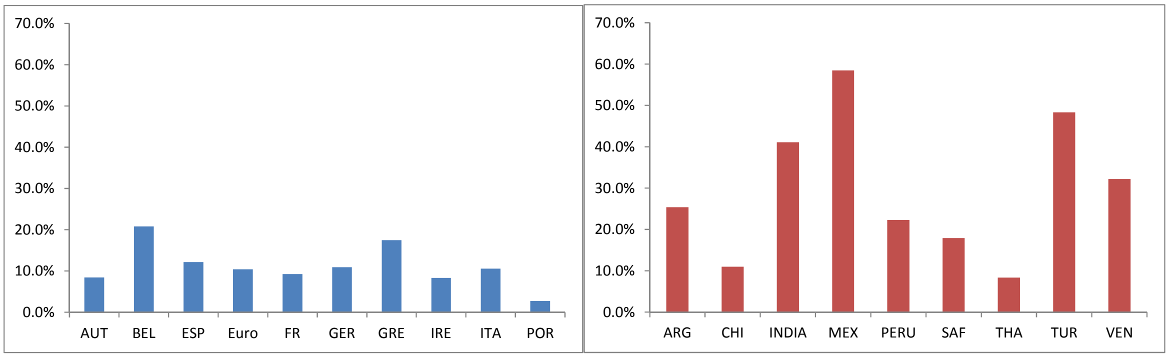


Figure 5: Share of government debt held by financial institutions



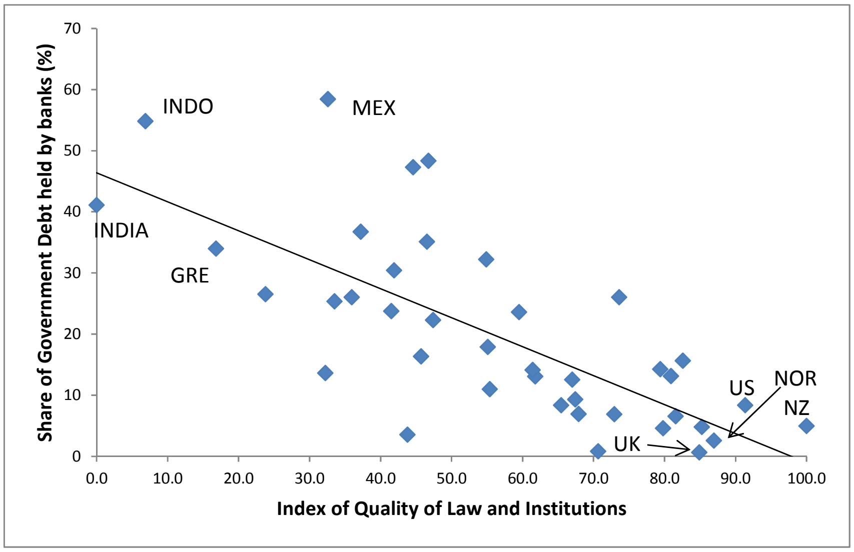
Source: Eurostat (data for 2009).

Figure 6: Holdings of government debt as a proportion of total assets by financial institutions



Source: Left panel; European Central Bank (data as of end of 2003); right panel: Kumhof and Tanner (2005).

Figure 7: Institutional Quality and the Share of Government Debt held by Banks



Source: Kumhof and Tanner (2005)

Table 1. Effects of IT regimes on inflation and output

	No control for initial conditions	Control for initial conditions		
	Difference-in-difference	Difference-in-difference	Instrumental Variables	Propensity Score Matching
Inflation: mean and variance	IT reduces both the mean and variance of inflation.	Advanced economies: weak decrease in average inflation (0.6 p.p.)	Advanced economies: no effect	Advanced economies: no effect
		Emerging economies: large reduction in average inflation of 2.5 p.p.	Emerging economies: large effect in average inflation (7.5 p.p. in the long run)	Emerging economies: reduction in average inflation of 3 p.p.
Output :variance	Mixed results	Advanced economies: no effect		
		Emerging economies: decrease in 1.4 p.p.		

Source: Ball (2010)