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International Finance (Econ 741)
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Handout # 1B

ADDING INVESTMENT TO THE BASIC MODEL

This handout adds investment to the model of Handout # 1A and corresponds to the second part of Chapter 1.

Consider a small open economy perfectly integrated with the rest of the world in both capital and goods markets.

1 Household's problem

- Production function

$$y_t = A_t f(k_t),$$

where $A_t\,(> 0)$ is a productivity parameter and $f(k)$ is a strictly increasing and strictly concave function ($f'(k) > 0$, $f''(k) < 0$).

- Investment (no depreciation)

$$I_t \equiv k_{t+1} - k_t. \tag{1}$$

- Utility function:

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \tag{2}$$

where c_t denotes consumption of the only traded good and $\beta(> 0)$ is the discount factor.

- Household's flow constraint:

$$b_{t+1} = (1 + r)b_t + y_t - c_t - (k_{t+1} - k_t). \tag{3}$$

where b_t is the (net) stock of an internationally-traded bond; and r is the (constant and exogenously-given) world real interest rate.

- Household's intertemporal budget constraint:

$$(1+r)b_0 + \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t y_t = \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t \left[c_t + (k_{t+1} - k_t)\right]. \tag{4}$$

- First order condition:

  Consumers maximize (2) subject to (4) for given b_0 and k_0. First-order condition are (assuming $\beta = 1/(1+r)$):

$$u'(c_t) = \lambda, \tag{5}$$

$$A_{t+1}f'(k_{t+1}) = r, \tag{6}$$

  where λ is the multiplier associated with the intertemporal budget constraint.

2 Equilibrium conditions

- Definition of current account

$$CA_t \equiv b_{t+1} - b_t. \tag{7}$$

- Hence:

$$CA_t = \underbrace{rb_t + y_t}_{\text{income (GDP)}} - \underbrace{(c_t + I_t)}_{\text{absorption}}. \tag{8}$$

- Define trade balance (TB) and investment income balance (IB):

$$TB_t \equiv y_t - c_t - I_t, \tag{9}$$

$$IB_t \equiv rb_t. \tag{10}$$

- Hence (from (3))

$$CA_t = IB_t + TB_t. \tag{11}$$

- Define saving as:

$$S_t \equiv rb_t + y_t - c_t. \tag{12}$$

- Hence:

$$CA_t = S_t - I_t. \tag{13}$$

We thus have four different ways of thinking about the current account: (i) as the accumulation of net foreign assets; (ii) as the difference between income and absorption; (iii) as the sum of the (investment) income and trade balances; (iii) as the difference between saving and investment.

3 Initial stationary equilibrium

Suppose $A_t = \bar{A}$ for all $t = 0, 1, ..$

- Capital stock:

$$\bar{y} = \bar{A}f(\bar{k}). \tag{14}$$

Suppose $k_0 = \bar{k}$.

- Consumption:

$$\bar{c} = rb_0 + \bar{A}f(\bar{k}). \tag{15}$$

- Trade balance

$$\overline{TB} = -rb_0.$$

4 Permanent increase in A

Suppose that at time 0, there is an unanticipated and permanent increase in A (from $\bar{A}$ to $\bar{A}^H$) [See Figure 6 at the back of this handout.]

- Capital stock (notice that k_0 is given):

$$\bar{A}^H f'(\bar{k}^H) = r, \qquad t = 1, 2, ... \tag{16}$$

- Output:

$$y_0 = A^H f(\bar{k}) > y_{-1}, \tag{17}$$
$$y_t = A^H f(\bar{k}^H) > y_0, \qquad t = 1, 2, ... \tag{18}$$

- Consumption:

$$\bar{c} = rb_0 + \bar{A}^H f(k_0) + \frac{r}{1+r} \underbrace{\left[\frac{\bar{A}[f(\bar{k}^H) - f(k_0)]}{r} - (\bar{k}^H - k_0) \right]}_{\text{Net present value of investment}}. \tag{19}$$

- Saving:

$$S_0 \equiv -\frac{r}{1+r} \underbrace{\left[\frac{\bar{A}^H[f(\bar{k}) - f(k_0)]}{r} - (\bar{k} - k_0) \right]}_{+} < 0. \tag{20}$$

- Trade balance:

$$TB_0 \equiv -rb_0 - \frac{r}{1+r} \left[\frac{\bar{A}f(\bar{k}) - \bar{A}f(k_0)}{r} - (\bar{k} - k_0) \right] - (\bar{k} - k_0) < -rb_0.$$

- Current account:

$$CA_0 = \underbrace{S_0}_{-} - \underbrace{I_0}_{+} < 0$$

- In sum, a permanent increase in productivity leads to a current account deficit: saving is negative (in anticipation of higher future output) and investment is positive (in response to the increase in productivity).

5 One-period increase in A

Suppose that at time 0, there is an unanticipated and temporary increase in A (from $\bar{A}$ to $\bar{A}^H$) that lasts for only 1 period. [See Figure 7 at the back of this handout.]

- Capital stock does *not* change

- Consumption:

$$\bar{c} = rb_0 + \bar{A}f(\bar{k}) + \underbrace{\frac{r}{1+r} f(\bar{k})(\bar{A}^H - \bar{A})}_{\text{consumption smoothing effect}}. \tag{21}$$

- Saving:

$$S_0 = \frac{f(\bar{k})(A^H - \bar{A})}{1+r} > 0. \tag{22}$$

- Trade balance:

$$TB_0 \equiv -rb_0 + \frac{f(\bar{k})(A^H - \bar{A})}{1+r} > -rb_0. \tag{23}$$

- Current account:

$$CA_0 = \underbrace{S_0}_{+} - \underbrace{I_0}_{=0} > 0$$

- In sum, a one period increase in productivity leads to a current account surplus due to the consumption smoothing motive analyzed in Handout 1A.

6 Summary of key ideas

- We have seen that:
  - A one-period increase in productivity leads to a current account surplus (this is like a temporary rise in the endowment in Handout 1A)
  - A permanent increase in productivity leads to a current account deficit

It follows that shocks of intermediate duration (i.e., more than one period but not permanent) may lead to either current account surpluses or deficits (see Chapter 1 for a formal analysis of this case). Hence, in the presence of investment, the current account/trade balance may behave countercyclically due to the fact that positive productivity shocks lead to higher investment.

Figure 6. Permanent increase in productivity

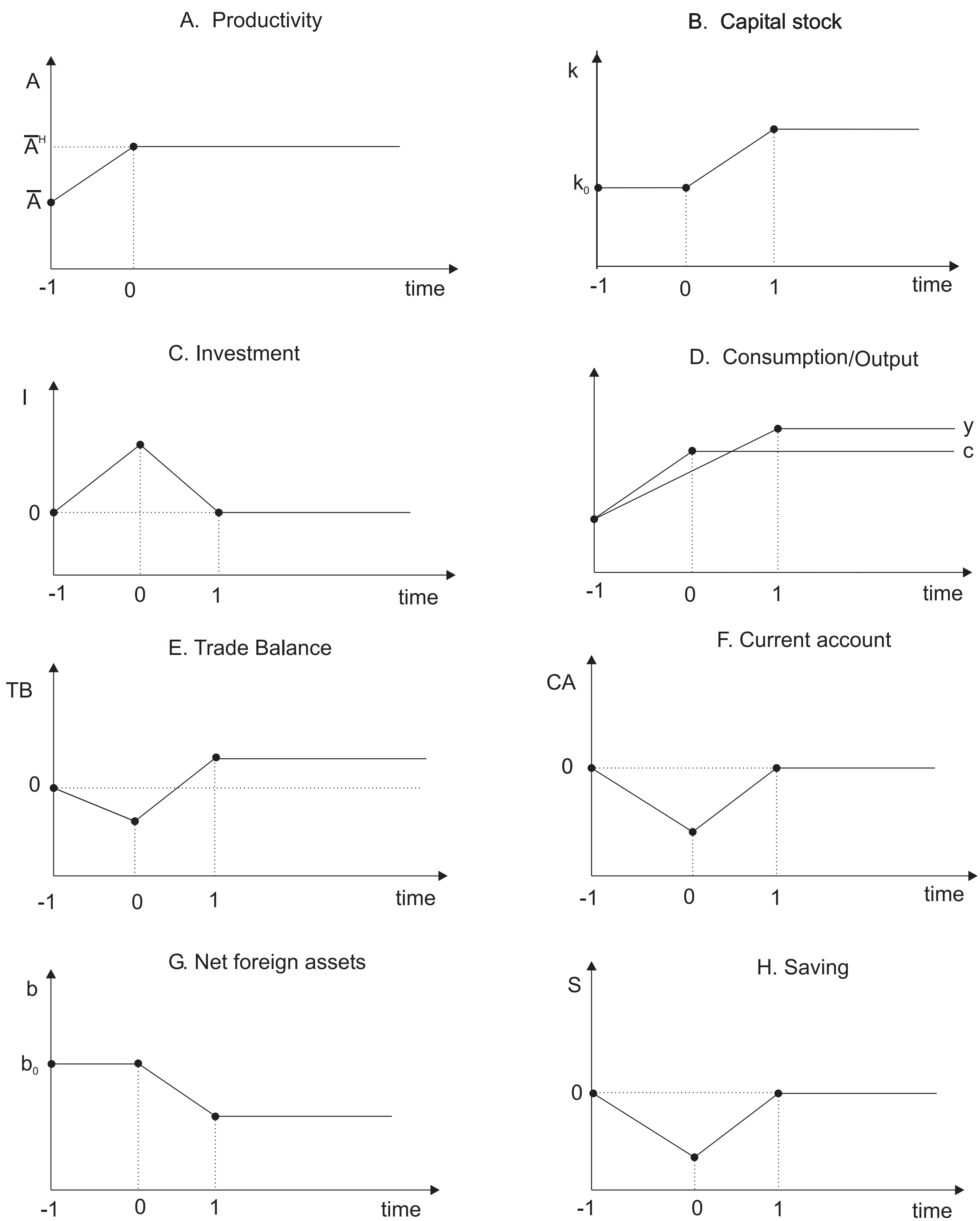

Figure 7. One-period increase in productivity