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Handout # 5

THE BASIC MONETARY MODEL: MONEY AS A VEIL

This handout – which corresponds to Chapter 5 in the book – develops a "pure" monetary model in the sense that there will be no interaction between the monetary and real sectors of the economy. In other words, money will be just a "veil".

Consider a small open economy inhabited by a large number of identical, infinitely-lived consumers who are endowed with perfect foresight. The economy is perfectly integrated with the rest of the world in both goods and capital markets. There is only one (tradable and non-storable) good, whose price is given by the law of one price. The economy receives a constant endowment of the good (y). The international real interest rate (r) is given and constant over time.

1 Derivation of budget constraint

- Consumers hold two assets: domestic money (M) and foreign bonds (B). Nominal assets (A) are given by:

$$A_t = M_t + E_t B_t, \tag{1}$$

  where E_t is the nominal exchange rate.

- Law of one price:

$$P_t = E_t P_t^*,$$

  where P_t^* is the foreign price of the good (i.e., the dollar price) and P_t is the domestic price of the good.

- Dividing (1) by P_t:

$$a_t = m_t + b_t, \tag{2}$$

  where $a(\equiv A/P)$, $m(\equiv M/P)$, and $b(\equiv B/P^*)$.

- Consumer's flow constraint in nominal terms:

$$\dot{A}_t = \underbrace{E_t i_t^* B_t}_{\text{interest payments}} + \underbrace{\dot{E}_t B_t}_{\text{capital gains}} + P_t y_t + P_t \tau_t - P_t c_t, \tag{3}$$

where i_t^* is the foreign nominal interest rate, τ are real lump-sum transfers from the government, and c is consumption.

- Divide this last expression by P_t:

$$\frac{\dot{A}_t}{P_t} = (i_t^* + \varepsilon_t)b_t + y_t + \tau_t - c_t, \tag{4}$$

where $\varepsilon_t (\equiv \dot{E}_t / E_t)$ is the devaluation rate.

- By definition, $a_t = A_t / E_t P^*$. Hence:

$$\dot{a}_t = \frac{\dot{A}_t}{P_t} - (\varepsilon_t + \pi_t^*) a_t. \tag{5}$$

- Substituting (4) into (5):

$$\dot{a}_t = (i_t^* - \pi_t^*) a_t + y + \tau_t - c_t - (i_t^* + \varepsilon_t) m_t. \tag{6}$$

- Assuming that the Fisher equation holds in the rest of the world (i.e., $i_t^* = r + \pi_t^*$) and taking into account that perfect capital mobility implies that interest parity holds (i.e., $i_t = i_t^* + \varepsilon_t$), rewrite (6) as:

$$\dot{a}_t = r a_t + y + \tau_t - c_t - i_t m_t. \tag{7}$$

- Integrating forward equation (7) and imposing the transversality condition $\lim_{t \to \infty} a_t e^{-rt} = 0$ (for reasons discussed in Chapter 1), we get:

$$a_0 + \int_0^\infty (y_t + \tau_t) e^{-rt} dt = \int_0^\infty (c_t + i_t m_t) e^{-rt} dt. \tag{8}$$

2 Consumer's problem

- Utility function:

$$\int_0^\infty [u(c_t) + v(m_t)]e^{-\beta t} dt. \tag{9}$$

- Lagrangian:

$$L = \int_0^\infty [u(c_t) + v(m_t)]e^{-\beta t} + \lambda \left\{ a_0 + \int_0^\infty (y_t + \tau_t - c_t - i_t m_t)e^{-rt} dt \right\} \tag{10}$$

- First-order conditions with respect to c and m (assuming $\beta = r$):

$$u'(c_t) = \lambda, \tag{11}$$

$$v'(m_t) = \lambda i_t. \tag{12}$$

- Combining the first-order conditions implicitly defines a money demand with standard properties:

$$\frac{u'(c_t)}{v'(m_t)} = \frac{1}{i_t} \Longrightarrow m_t = L(\underset{+}{c_t}, \underset{-}{i_t}).$$

3 Government

- Let H^* be the amount of foreign bonds (measured in terms of foreign currency) that the government holds and $H (\equiv EH^*)$ denote the domestic currency value of these bonds. The government's flow constraint in nominal terms is given by

$$\dot{H}_t = i_t^* E_t H^* + \dot{E}_t H_t^* + \dot{M}_t - P_t \tau_t. \tag{13}$$

- Let h denote international reserves in terms of real dollars; that is, $h \equiv H^*/P^* = H/P$. Hence:

$$\dot{h}_t = \frac{\dot{H}_t}{P_t} - (\varepsilon_t + \pi_t^*) h_t. \tag{14}$$

- Dividing (13) by P_t, substituting the resulting expression into (14) (and imposing the Fisher equation for the rest of the word):

$$\dot{h}_t = rh_t + \frac{\dot{M}_t}{P_t} - \tau_t. \tag{15}$$

- Rewrite the last equation as:

$$\dot{h}_t = rh_t + \dot{m}_t + (\varepsilon_t + \pi_t^*)m_t - \tau_t. \tag{16}$$

- Integrating forward equation (15) and imposing the transversality condition $\lim_{t\to\infty} h_t e^{-rt} = 0$:

$$h_0 + \int_0^\infty \frac{\dot{M}_t}{P_t} e^{-rt} dt = \int_0^\infty \tau_t e^{-rt} dt. \tag{17}$$

4 Equilibrium conditions

- Perfect capital mobility:

$$i_t = i_t^* + \varepsilon_t. \tag{18}$$

- Combining the consumer's flow constraint (equation (7)) with the government's (equation (16)) yields the economy's flow constraint:

$$\dot{k}_t = rk_t + y - c_t, \tag{19}$$

where $k(\equiv b + h)$ denote the economy's stock of net foreign assets.

- Rewrite the last equation as:

$$\underbrace{\dot{h}_t}_{\Delta h} = \underbrace{-\dot{b}_t}_{KA} + \underbrace{\underbrace{r(b_t + h_t)}_{IB} + \underbrace{y - c_t}_{TB}}_{CA}, \tag{20}$$

where Δh is the increase in international reserves, KA is the capital account, IB is the income balance, TB is the trade balance, and CA is the current account.

- Integrating forward and imposing the appropriate transversality condition:

$$k_0 + \int_0^\infty y_t e^{-rt} dt = \int_0^\infty c_t e^{-rt} dt \tag{21}$$

5 Solving the model: Real side

- From (11), c_t is constant over time (denoted by $\bar{c}$) Hence, from (21),

$$\bar{c} = r\left(k_0 + \int_0^\infty y_t e^{-rt} dt\right). \tag{22}$$

- Real money demand:

$$m_t = L(\bar{c}, i_t). \tag{23}$$

6 Predetermined exchange rates

(In what follows, we assume that foreign inflation is constant over time and equal to $\overline{\pi^*}$.) Under predetermined exchange rates, the monetary authority sets the path of domestic credit and the path of the nominal exchange rate. The path of the nominal exchange rate is given by an initial level $\overline{E_0}$ and a constant rate of devaluation, $\bar{\varepsilon}$. The path of domestic credit is given by an initial level, $\overline{D_0}$, and a constant rate of growth of domestic credit, $\bar{\theta}$. The path of the nominal money supply is endogenously determined.

- Nominal interest rate:

$$\bar{\imath} = r + \overline{\pi^*} + \bar{\varepsilon}. \tag{24}$$

- Money demand:

$$\bar{m} = L(\bar{c}, \bar{\imath}).$$

- Since $m \equiv M/EP^*$ and m is constant over time:

$$\bar{\mu} = \overline{\pi^*} + \bar{\varepsilon}.$$

- Initial money supply:

$$M_0 = \overline{E_0} P_0^* L(\bar{c}, \bar{\imath}).$$

- Path of international reserves. From central bank's balance sheet:

$$\dot{h}_t + \dot{d}_t = \dot{m}_t.$$

By definition, $d \equiv D/EP^*$. Hence:

$$\dot{d}_t = d_t(\bar{\theta} - \bar{\varepsilon} - \overline{\pi^*}). \tag{25}$$

Combining the last two equations (and noticing that $\dot{m}_t = 0$):

$$\dot{h}_t = -d_t(\bar{\theta} - \bar{\varepsilon} - \overline{\pi^*}).$$

Assume $\varepsilon = \theta + \pi^*$. Then $\dot{h}_t = 0$.
Initial level of reserves:

$$h_0 = L(\bar{c}, \bar{\imath}) - \frac{\overline{D_0}}{\overline{E_0}}. \tag{26}$$

- Level of transfers: From (15), and taking into account that h_t, ε_t, and m_t are all constant over time, it follows that:

$$\tau_t = r\bar{h} + (\bar{\varepsilon} + \overline{\pi^*})\bar{m}. \tag{27}$$

7 Flexible exchange rates

Under flexible exchange rates, the monetary authority does not intervene in the foreign exchange market and allows the exchange rate to be determined by market forces. In other words, the monetary authority sets the path of domestic credit and the path of international reserves (non-intervention implies a constant level of international reserves, assumed zero for simplicity). If international reserves are zero, it follows from the central bank's balance sheet that:

$$D_t = M_t.$$

Hence, setting the path of domestic credit is equivalent to setting the path of the money supply. The path of the nominal money supply is given by an initial level $\overline{M_0}$ and a constant rate of money growth, $\bar{\mu}$.

- We now derive an unstable differential equation in m_t. Since, by definition, $m_t = M_t/E_tP^*$, it follows that (using (12) and (24)):

$$\dot{m}_t = m_t \left[r + \bar{\mu} - \frac{v'(m_t)}{\lambda} \right].$$

- Linearizing this equation around the stationary value for real money balances (given by $r + \bar{\mu} = v'(m_t)/\lambda$):

$$\left.\frac{\partial \dot{m}_t}{\partial m_t}\right|_{ss} = -\frac{m_{ss}v''(m_{ss})}{\lambda} > 0.$$

- Stability requires:

$$r + \bar{\mu} = v'(\bar{m})/\lambda.$$

- Rate of depreciation:

$$\bar{\varepsilon} = \bar{\mu} - \overline{\pi^*}. \tag{28}$$

- Nominal interest rate:

$$\bar{\imath} = r + \overline{\pi^*} + \bar{\mu}.$$

- Initial level of the nominal exchange rate: Since equation (23) holds at time 0, we can solve for E_0:

$$E_0 = \frac{\bar{M}_0}{P_0^* L(\bar{c}, \bar{\imath})}.$$

  We have thus shown that, as under predetermined exchange rates, the value of all nominal variables is perfectly well-defined under flexible exchange rates.

- Level of transfers:

$$\tau_t = (\bar{\varepsilon} + \pi^*)\bar{m}. \tag{29}$$

8 Neutrality and superneutrality

We now check that in this model money is indeed a veil. In particular, we will check that both exchange rate and monetary policy are both neutral and superneutral. This implies that, under predetermined exchange rates, permanent changes in the level of the nominal exchange rate or in the rate of devaluation have no real effects (i.e., leave consumption unchanged) and that, under flexible exchange rates, permanent changes in the level of the nominal money supply or in the rate of money growth have no real effects.

8.1 Predetermined exchange rates

- A permanent increase in the level of the nominal exchange rate (i.e., *a permanent devaluation*) has no effect on consumption (i.e., exchange rate policy is neutral) or real balances. The price level increases by the same proportion. From the central bank's balance sheet, we can see that international reserves increase by an amount equal to the fall in real domestic credit:

$$\Delta h_0 = -\Delta d_0 > 0.$$

Hence, *a devaluation leads to a gain in international reserves.*

- A *permanent increase in the rate of devaluation* has no effect on consumption (i.e., exchange rate policy is superneutral) and leads to a higher nominal interest rate and lower real money demand. From the central bank's balance sheet we see that international reserves *fall*:

$$\Delta h_0 = \Delta m_0 < 0.$$

- Interestingly, while a permanent devaluation leads to an *increase* in international reserves, a permanent increase in the rate of devaluation leads to a *fall* in international reserves.

8.2 Flexible exchange rates

- A permanent increase in the level of the nominal money supply has no effect on consumption (i.e., monetary policy is neutral) or real money balances. The nominal exchange rate increases in the same proportion.

- A permanent increase in the rate of growth of the money supply has no effect on consumption (i.e., monetary policy is superneutral). The nominal interest rate increases and real money demand falls. The nominal exchange rate increases on impact.