Department of Economics, University of Maryland
International Finance (Econ 741)
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Handout # 6B

**THE MONETARY APPROACH TO THE BALANCE OF PAYMENTS II:
DEVALUATION IN A TWO-GOOD WORLD**

This handout – which corresponds to Chapter 6 of the book – adds non-tradable goods and endogenous production to the model of Handout # 6A and focuses on the adjustment of the economy to a devaluation. Unless otherwise noticed, we continue to use the same notation as before.

1 Consumer's problem

- Utility function:

$$\int_0^\infty [\gamma \log(c_t^T) + (1-\gamma)\log(c_t^N) + \log(m_t)]e^{-\beta t}dt, \tag{1}$$

where c^T and c^N denote consumption of tradables and non-tradables, respectively, and $m_t(\equiv M_t/E_t)$ denotes real money balances.

- Consumer's flow constraint:

$$\dot{m}_t = y_t^T + \frac{y_t^N}{e_t} + \tau_t - c_t^T - \frac{c_t^N}{e_t} - \varepsilon_t m_t, \tag{2}$$

where y^T and y^N denote production of tradable and non-tradables, respectively, e is the relative price of tradable goods in terms of non-tradable goods, ε_t is the rate of devaluation, and τ_t are lump-sum taxes from the government.

- Production:

$$y_t^T = Z^T \left(n_t^T\right)^\alpha, \tag{3a}$$
$$y_t^N = Z^N n_t^N, \tag{3b}$$

where Z^T and Z^N are positive productivity parameters, n^T and n^N denote labor employed in the tradable and non-tradable sector, respectively, and $0 < \alpha < 1$.

- Labor supply constraint:

$$\bar{n} = n_t^T + n_t^N, \tag{4}$$

where $\bar{n}$ is the exogenous labor endowment.

- Substituting (3a), (3b) and (4) into the flow constraint (2), we can set up the Hamiltonian:

$$\begin{aligned} H &= \gamma \log(c_t^T) + (1-\gamma)\log(c_t^N) + \log(m_t) \\ &\quad + \lambda_t \left[Z^T \left(n_t^T\right)^\alpha + \frac{Z^N\left(\bar{n} - n_t^T\right)}{e_t} + \tau_t - c_t^T - \frac{c_t^N}{e_t} - \varepsilon_t m_t \right]. \end{aligned}$$

- First-order conditions:

$$\frac{\gamma}{c_t^T} = \lambda_t, \tag{5a}$$

$$\frac{1-\gamma}{c_t^N} = \frac{\lambda_t}{e_t}, \tag{5b}$$

$$\alpha Z^T \left(n_t^T\right)^{\alpha-1} = \frac{Z^N}{e_t}, \tag{5c}$$

$$\dot{\lambda}_t = \left(\beta + \varepsilon_t\right)\lambda - \frac{1}{m_t}. \tag{5d}$$

2 Government

The government's constraints remain unchanged relative to Handout # 2. (We just repeat them here for convenience.) The government holds non-interest bearing international reserves (say, dollars). Its flow constraint is

$$\dot{h}_t = \frac{\dot{M}_t}{E_t} - \tau_t. \tag{6}$$

Proceeding as in Handout # 1, we get (recall that $\pi^* = 0$):

$$\dot{h}_t = \dot{m}_t - d_t(\bar{\theta} - \bar{\varepsilon}).$$

To ensure that the predetermined exchange rate regime is sustainable over time, we assume $\bar{\varepsilon} = \bar{\theta}$. Hence:

$$\dot{h}_t = \dot{m}_t. \tag{7}$$

Changes in international reserves will thus reflect changes in real money demand.

3 Equilibrium conditions

- Equilibrium in the non-tradable goods market:

$$c_t^N = y_t^N. \tag{8}$$

- Substituting the government's flow constraint (6) into the households' flow constraint (2) and imposing equilibrium in the non-tradable goods market, given by (8), yields:

$$\dot{h}_t = y_t^T - c^T.$$

- Substituting (7) into the last equation:

$$\dot{m}_t = y_t^T - c^T. \tag{9}$$

4 Dynamic system

We will set up a differential equation system in c^T and m.

- To obtain the first differential equation, time-differentiate (5a) and use (5d) to get:

$$\dot{c}_t^T = c_t^T \left(\frac{c^T}{\gamma m_t} - \beta - \varepsilon_t \right). \tag{10}$$

- To obtain the second differential equation, substitute (3a) into (9) to get:

$$\dot{m}_t = Z^T \left(n_t^T\right)^\alpha - c_t^T. \tag{11}$$

We now derive an expression for n^T in terms of c^T. Using (3b), (4), (5a), (5b), (5c), and (8), it follows that

$$c_t^T = \left(\frac{\gamma}{1-\gamma}\right) \frac{\alpha Z^T (\bar{n} - n_t^T)}{\left(n_t^T\right)^{1-\alpha}}, \tag{12}$$

which implicitly defines n^T as a decreasing function of c^T:

$$n_t^T = \widetilde{n^T}(c_t^T), \tag{13}$$

where

$$\widetilde{n^T}'(c_t^T) = -\frac{\left(\frac{1-\gamma}{\gamma}\right)}{\alpha Z^T \left[\left(n_t^T\right)^{\alpha-1} + \frac{(1-\alpha)(\bar{n}-n_t^T)}{\left(n_t^T\right)^{2-\alpha}}\right]} < 0$$

Substituting (13) into (11) yields our second differential equation:

$$\dot{m}_t = Z^T \left[\widetilde{n^T}(c_t^T)\right]^\alpha - c_t^T. \tag{14}$$

Equations (10) and (14) constitute a dynamic system in c^T and m, for a given and constant value of the rate of devaluation, $\bar{\varepsilon}$.

- Steady-state:

$$\begin{aligned} m_{ss} &= \frac{c_{ss}^T}{\gamma\left(\beta + \bar{\varepsilon}\right)}, & (15) \\ c_{ss}^T &= Z^T \left[\widetilde{n^T}(c_{ss}^T)\right]^\alpha. & (16) \end{aligned}$$

- Linearized system

Linearizing the system and using (15) and (16), we obtain:

$$\begin{bmatrix} \dot{c}_t \\ \dot{m}_t \end{bmatrix} = \begin{bmatrix} \beta + \bar{\varepsilon} & -\gamma\left(\beta + \bar{\varepsilon}\right)^2 \\ \alpha Z^T \left(\widetilde{n^T}\right)^{\alpha-1} \widetilde{n^T}' - 1 & 0 \end{bmatrix} \begin{bmatrix} c_t - y \\ m_t - m_{ss} \end{bmatrix}.$$

The determinant of the matrix associated with the linear approximation is given by

$$\Delta = \gamma\left(\beta + \bar{\varepsilon}\right)^2 \left[\alpha Z^T \left(\widetilde{n^T}\right)^{\alpha-1} \widetilde{n^T}' - 1\right] < 0,$$

which indicates that the system has one negative and one positive root and is thus saddle-path stable. Figure 1 illustrates the corresponding phase diagram. (Notice that, qualitatively, the phase diagram remains the same as in Handout # 2.)

5 Initial steady-state

The initial steady-state is characterized by:

$$\begin{aligned} m_{ss} &= \frac{c_{ss}^T}{\gamma\beta}, \\ c_{ss}^T &= Z^T \left[\tilde{n}^T(c_{ss}^T)\right]^{\alpha}, \end{aligned}$$

$$c_{ss}^T = \left(\frac{\gamma}{1-\gamma}\right) \frac{\alpha Z^T (\bar{n} - n_{ss}^T)}{(n_{ss}^T)^{1-\alpha}},$$

$$n_{ss}^N = \bar{n} - n_{ss}^T,$$

$$\begin{aligned} y_{ss}^T &= Z^T \left(n_{ss}^T\right)^{\alpha}, \\ y_{ss}^N &= Z^N n_{ss}^N, \end{aligned}$$

$$c_{ss}^N = Z^N n_{ss}^N,$$

$$e_{ss} = \left(\frac{\gamma}{1-\gamma}\right) \frac{c_{ss}^N}{c_{ss}^T},$$

$$TB_{ss} \equiv y_{ss}^T - c_{ss}^T = 0.$$

6 Devaluation

Suppose that the economy is initially in the steady state given by point A in Figure 1. There is then an unexpected and permanent devaluation (i.e., E is increased). As a result, the system jumps to a point like B in Figure 1 and then travels back to the initial steady state along the saddle path. Figure 2 illustrates the path of the different variables.

- Path of n^T (and hence of y^T) follows from the fact that c^T and n^T are negatively related (equation (13)).
- Path of n^N (and hence of y^N and c^N) follows from the path of n^T and the labor supply constraint (4).
- Path of e follows from the path of n^T and the production efficiency condition, given by (5c).
- Path of P^N follows from the path of e.

7 Conclusion

In sum, a devaluation leads to a fall in consumption of both goods, a switch in production from the non-tradable sector to the tradable goods sector, a trade surplus, and a real depreciation. Notice that the price of non-tradable goods increases by less than the nominal exchange rate (even though there are no sticky prices in this model).

Figure 1. Phase diagram

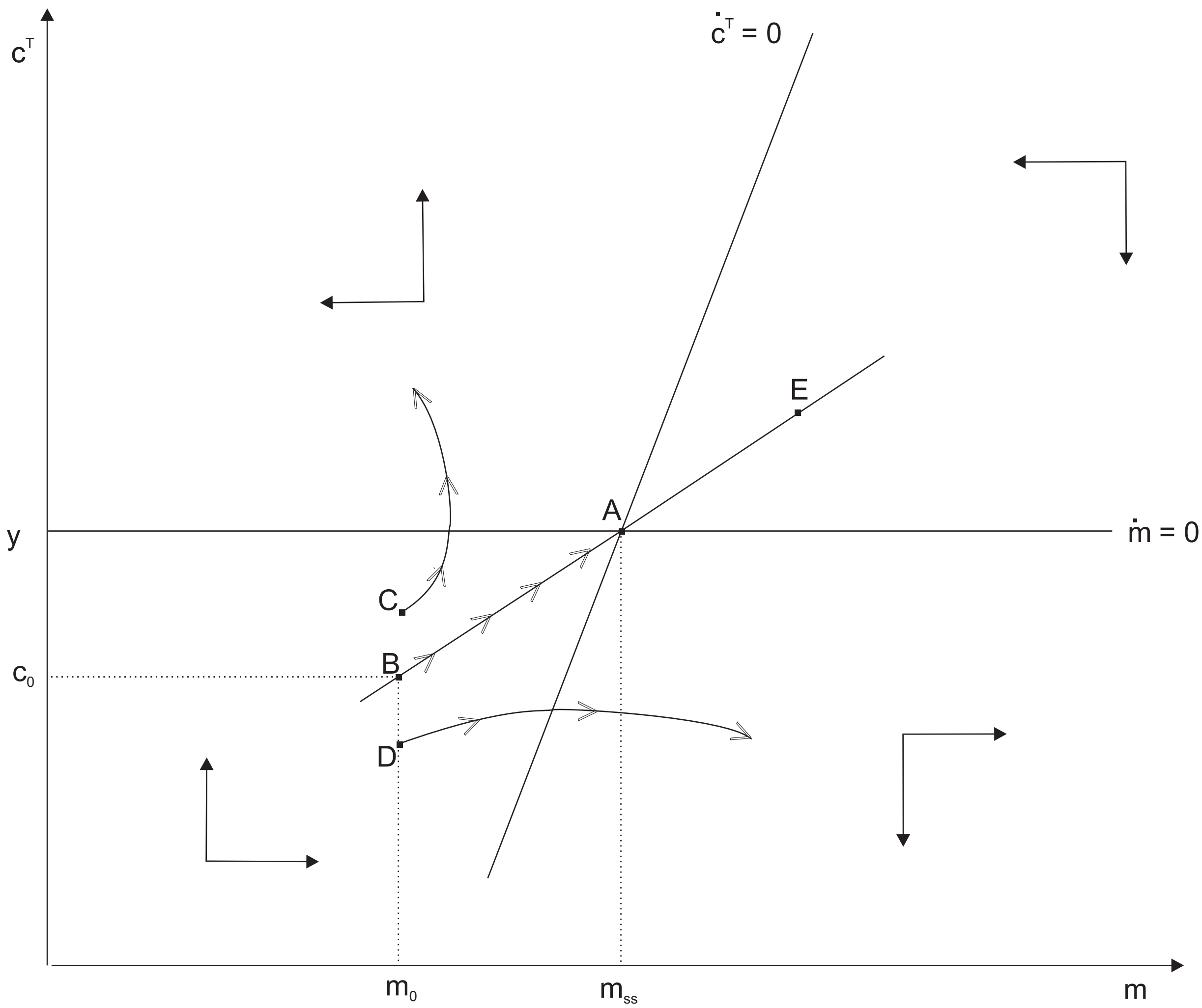

Figure 2. Permanent devaluation in two-good model