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International Finance (Econ 741)
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Handout # 9

INTEREST RATES AS A POLICY INSTRUMENT: A SIMPLE MODEL

This handout sets up a simple optimizing model (along the lines of Auernheimer and Contreras (1992)), which illustrates some of the basic issues regarding the use of interest rates as policy instruments.

Consider a small open economy perfectly integrated with the rest of the world in both goods and capital markets. Agents have perfect foresight (i.e., there is no uncertainty in the model). The foreign currency price of the good is taken to be unity. Hence, the exchange rate is identically equal to the price level.

1 Consumer

- Consumer's utility function:

$$\int_0^\infty \left[\log(c_t) + \frac{m_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right] \exp(-\beta t) dt, \tag{1}$$

where c denotes consumption of the only (tradable and non-storable) good, m are real money balances, $\beta(> 0)$ is the rate of time preference, and $\sigma < 1$.[1]

- Financial wealth:

$$a_t = b_t + b_t^g + m_t,$$

where b denotes the consumer's stock of net foreign bonds and $b^g (= B^g/E)$ is a domestic bond issued by the government. Since the foreign bond and the domestic bond are assumed to be perfect substitutes, nominal returns in domestic currency are the same (i.e., $i = r + \varepsilon$).

[1] We specify the utility function just for algebraic convenience. The only important assumption is that $\sigma < 1$, which (as we shall see) ensures that we are always on the "correct" side of the Laffer curve.

- Consumer's flow constraint:

$$\dot{a}_t = ra_t + y - c_t - i_t m_t - x_t, \tag{2}$$

where r is the world real interest rate, y is the constant flow endowment of the good, and x_t are lump-sum taxes.

- Consumer's lifetime budget constraint:

Integrating (2) and imposing the transversality condition $\lim_{t\to\infty} a_t \exp(-rt) = 0$, we get:

$$a_0 + \frac{y}{r} = \int_0^\infty (c_t + i_t m_t + x_t) \exp(-rt) dt. \tag{3}$$

- Consumer's first-order conditions (maximize (1) subject to (3), assuming that $\beta = r$):

$$\frac{1}{c_t} = \lambda, \tag{4}$$

$$m_t^{-\frac{1}{\sigma}} = \lambda i_{t,} \tag{5}$$

where λ is the (time-invariant) Lagrange multiplier associated with constraint (3).

- By combining (4) and (5), we get the real money demand (whose interest rate elasticity, in absolute value, is $\sigma < 1$):

$$m_t = \left(\frac{c_t}{i_t}\right)^\sigma. \tag{6}$$

2 Government

- The government holds no international reserves (i.e., no foreign assets). (Since it does not intervene in the foreign exchange market, it does not need reserves.) The government's budget constraint is (note that b^g is a government liability):

$$\dot{b}_t^g = rb_t^g + g_t - x_t - \dot{m}_t - \varepsilon_t m_t. \qquad (7)$$

where ε is the rate of depreciation. The primary deficit is $g_t - x_t$, whereas the operational deficit is $rb_t^g + g_t - x_t$.

- Government's intertemporal constraint:

Integrating (7) and imposing the transversality condition $\lim_{t\to\infty} b_t^g \exp(-rt) = 0$, we get:

$$b_0^g = \int_0^\infty (\dot{m}_t + \varepsilon_t m_t + x_t - g_t) \exp(-rt) dt. \qquad (8)$$

- For further reference, note that we can rewrite (7) as (defining the consumer's domestic financial assets by $d(\equiv b^g + m)$):

$$\dot{d}_t = rd_t + g_t - x_t - i_t m_t. \qquad (9)$$

- Unlike previous handouts (where we implicitly assumed that money was introduced into the economy via lump-sum transfers ("helicopter drops")), we now assume away such possibility and assume instead that money can only be introduced into the economy through open market operations. When the government wants to (reduce) increase the nominal money supply, it buys (sells) government bonds. This implies that:

$$\Delta M_t = -\Delta B^g.$$

3 Equilibrium conditions

- As indicated above, bonds are perfect substitutes (and foreign inflation is zero):

$$i_t = r + \varepsilon_t. \tag{10}$$

- Economy's flow constraint (i.e., the current account) follows from (2), (7), and (10):

$$\dot{b}_t = rb_t + y - c_t - g_t.$$

- Economy's resource constraint follows from (3), (8), (10) and the condition $\lim_{t \to \infty} m_t \exp(-rt) = 0$:

$$b_0 + \frac{y}{r} = \int_0^\infty (c_t + g_t) \exp(-rt) dt. \tag{11}$$

4 Initial perfect foresight equilibrium

- The fiscal authority sets g_t and x_t at the constant levels $\bar{g}$ and $\bar{x}$.

- The monetary authority fixes the nominal interest rate at the constant level $\bar{i}$. It does so by standing ready to buy or sell government bonds at that interest rate. (Note that, by (10), setting the nominal interest rate implies setting the rate of devaluation. The government, however, is *not* setting the level of the exchange rate.)

- Along any perfect foresight equilibrium path, it follows from (4) that c_t is constant over time. From (11), it then follows that consumption equals permanent income:

$$c_t = rb_0 + y - g. \tag{12}$$

Substituting (12) into (6), we obtain:

$$m_t = \left(\frac{rb_0 + y - g}{\bar{i}} \right)^\sigma. \tag{13}$$

- How is the constant level of the exchange rate (i.e., the price level) determined? It will be that level which ensures that the initial level of d is such that the operational deficit is fully financed by the inflation tax revenues (and thus $\dot{d}_t = 0$). From (9):

$$\bar{E} = \frac{rD_0}{\bar{x} - \bar{g} + \bar{i}m_t}$$

  where D_0 is the initial and given nominal level of domestic assets and m_t is the constant level of real money balances given by (13).

- Figure 1 illustrates the determination of the initial stationary equilibrium at point A, where the $\dot{d}_t = 0$ locus intersects the constant level of the real money demand. Notice that the $\dot{d}_t = 0$ locus slopes down because inflation tax revenues fall as real money balances increase.

5 Unanticipated and permanent shocks

Suppose that just before time 0, the economy is in the equilibrium just described. Consider the effects of the following two unanticipated and permanent shocks at $t = 0$.

5.1 Unanticipated and permanent increase in $\bar{i}$

Suppose the government increases $\bar{i}$ at $t = 0$ on a permanent basis. The new equilibrium becomes point B in Figure 1. The rise in i reduces real money demand and increases the inflation tax revenues (because the interest rate elasticity is less than one). Since the inflation tax revenues increase, the operational deficit must increase. This is achieved by a fall in the level of the exchange rate. The lower real money balances are achieved by an open market operation.

5.2 Unanticipated and permanent decrease in $\bar{x}$.

Suppose the primary deficit increases at $t = 0$. Specifically, suppose that $\bar{x}$ falls on a permanent basis. The new stationary equilibrium obtains at point C. Intuitively, since the primary deficit increases, the government total debt service must fall for the operational deficit to remain constant.

6 In conclusion

This exercise illustrates how the price level is determined under an interest rate targeting when "helicopter drops" are not allowed. The price level is pinned down by the level of nominal assets ($M + B^g$).

More generally, there is a large (and not particularly illuminating) literature on interest rate targeting. The bottom line is that price level indeterminacy arises because monetary policy is incompletely specified. Setting the nominal interest rate is equivalent to setting the rate of devaluation (or the rate of inflation); some other nominal magnitude must therefore be set by policymakers to pin down the price level.

Figure 1: Equilibrium determination