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International Finance (Econ 741)
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PROBLEM SET # 2

This problem set complements Handout # 2 and # 3 (corresponding to Chapters 2 and 3 in the book). It will be due in class on Wednesday, October 7.

You are asked to solve the exercises at the end of Chapter 2 and Chapter 3 (attached, for your convenience, to this cover page).

Exercises²⁰

1. Reduced forms for consumption and welfare under CARA utility

Consider the incomplete markets case analyzed in subsection 3.1 with the following two modifications. First, assume that preferences are given by the constant absolute risk aversion function:

$$u(c) = -\frac{1}{\alpha}e^{-\alpha c}. \quad (85)$$

Second, instead of the Bernoulli distribution for period 2's output specified in (16), suppose that

$$y_2 = y_1 + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2).$$

In this context:

- (a) Derive reduced-form solutions for c_1 and c_2 . In particular, how does a higher σ^2 affect c_1 and c_2 ? [Hint: Recall that if $x \sim N(E\{x\}, \sigma_x^2)$, then $E\{e^x\} = e^{E\{x\} + \sigma_x^2/2}$.]
- (b) Compute the correlation coefficient between c_2 and y_2 .
- (c) Show that welfare is a decreasing function of σ^2 .

In all cases, discuss the intuition behind the results.

2. An ad-hoc upward sloping supply of funds

Let preferences be given by:

$$W = \log(c_1) + \beta \log(c_2), \quad (86)$$

where $\beta (\equiv 1/(1 + \delta))$ is the discount factor and δ is the discount rate. Assume that $\beta(1 + r) = 1$, where r is the world real interest rate.

The flow constraints are given by:

²⁰An answer key is available from the author upon request.

$$c_1 = d_1, \tag{87}$$

$$c_2 = y_2 - (1 + r^s)d_1, \tag{88}$$

where d_1 is net external debt and $y_2 > 0$ is second period's output (notice that, for simplicity, we have assumed that output in the first period is zero).

The economy faces an upward sloping supply of funds of the form:

$$r^s = r + f(d_1), \quad f(0) = 0, \quad f'(d_1) > 0, \tag{89}$$

where r^s is the real interest rate charged to the country.

In this context:

- (a) Solve the planner's problem. Show that the planner will choose not to smooth consumption over time.
- (b) Solve the consumer's problem (i.e., the free market solution). Show that, relative to the planner's solution, the market solution implies that consumption in the first period is too high relative to the second period.
- (c) Consider a linear version of this model (i.e., linear preferences and linear supply of funds):

$$W = c_1 + \frac{c_2}{1 + \delta}, \tag{90}$$

$$f(d_1) = \alpha d_1. \tag{91}$$

Assume $\delta > r$. In this context:

- i. Derive a reduced-form solution for the equilibrium values of d_1 and r^s for both the planner's problem and the market problem.
- ii. Provide a graphical illustration of how the equilibrium values of d_1 and r^s are determined and interpret the results intuitively. [Hint: Think of the country as a monopsonist in world capital markets and proceed as in the textbook analysis of a monopsony in factor markets. To this end, you may want to review the analysis of a monopsonist in your favorite undergraduate microeconomics textbook.]

- iii. Show that by imposing a borrowing tax rate (which increases linearly with the amount of borrowing), the government can implement the planner's solution.

3. Numerical example of incomplete markets model with CRRA preferences

Consider the incomplete markets model analyzed in Subsection 3 with CRRA preferences of the form

$$u(c) = \frac{c^{1-\theta} - 1}{1-\theta},$$

where θ is the coefficient of relative risk aversion. In this context:

1. Plot c_1 , c_2^H , c_2^L , current account, and expected utility as a function of the coefficient of risk aversion. Explain the intuition behind the results
2. Plot the same variables as a function of a mean-preserving spread in the output distribution. Explain the intuition behind the results.

Exercises¹⁶

1. Effects of terms of trade shocks on the current account

The importable/exportable model developed in this chapter provides a convenient framework to answer an important question in development macroeconomics: what will be the effect of a change in the terms of trade on the current account?¹⁷ The traditional answer to this question – known in the literature as the Harberger-Laursen-Metzler (HLM) effect after the contributions of Harberger (1950) and Laursen and Metzler (1950) – has been that a deterioration in the terms of trade will worsen the current account. The rationale behind their analysis was essentially that a worsening in the terms of trade would result in lower “real income” because of the smaller purchasing power of exports. Within a typical Keynesian model with a marginal propensity to consume less than one, a fall in real income lowers savings and results in a current account deterioration. This exercise will illustrate the point that, in the context of our intertemporal model, whether a (temporary) deterioration in the terms of trade worsens the current account or not depends critically on the value of the intertemporal elasticity of substitution.¹⁸

To study this question, consider the model introduced in Section 2 of this chapter (using the CES preferences specified in (17)) with two modifications: (i) we abstract from the government (and hence tariffs) and (ii) the *international* terms of trade are now given by p_t . In this context, the consumer’s flow constraint becomes:

$$\dot{b}_t = rb_t + y_t - p_t c_t, \quad (37)$$

with the corresponding lifetime budget constraint given by:

$$\int_0^{\infty} p_t c_t e^{-rt} dt = b_0 + \frac{y}{r}. \quad (38)$$

In the context of this model:

- (a) Analyze the effects of an unanticipated and permanent reduction in the terms of trade (i.e., an increase in p).
- (b) Analyze the effects of an unanticipated and temporary reduction in the terms of trade. In particular, show that the effect on the current account critically depends on the value of the intertemporal elasticity of substitution (with the HLM effects holding when the intertemporal elasticity of substitution is less than one).

¹⁶An answer key is available from the author upon request.

¹⁷We will revisit this question in Chapter 4 in the context of a model with non-tradable goods.

¹⁸See Svensson and Razin (1983) for a more general analysis in a similar intertemporal framework.

2. Consumption versus expenditure considerations

Consider the temporary liberalization discussed in Subsection 3.2. Let $q \equiv p^L/p^H < 1$. Assume that preferences are given by:

$$u(c_t) = \log c_t.$$

- (a) Obtain a reduced form for c^1 and c^2 as a function of q and T .
- (b) Using the reduced forms for c^1 and c^2 , verify that

$$PVC \equiv \int_0^T c^1 e^{-rt} dt + \int_T^\infty c^2 e^{-rt} dt = b_0 + \frac{y}{r}.$$

- (c) Obtain a reduced form for the present value of expenditure (as a function of q and p^H):

$$PVE \equiv \int_0^T p^L c^1 e^{-rt} dt + \int_T^\infty p^H c^2 e^{-rt} dt.$$

- (d) Obtain a reduced form (as a function of q and p^H) for the present value of expenditure associated with a flat path of consumption:

$$PVE^* \equiv \int_0^T p^L (rb_0 + y) e^{-rt} dt + \int_T^\infty p^H (rb_0 + y) e^{-rt} dt.$$

- (e) Show that, for a given p^H , the function $Z(q) \equiv PVE^* - PVE$ reaches its minimum when $q = 1$ (i.e., when there is no intertemporal distortion). Interpret this result.

3. Effects of changes in the liberalization period and the intertemporal elasticity of substitution

Consider again the temporary liberalization discussed in Subsection 3.2 with $q \equiv p^L/p^H < 1$. Assume that preferences take the iso-elastic form given by equation (17) where σ is the intertemporal elasticity of substitution. (Notice that $\sigma = 1$ corresponds to the logarithmic case and $\sigma = 0$ to the Leontief case).

In this context:

- (a) Obtain a reduced form for c^1 and c^2 as a function of q , T , and σ .
- (b) Show how c^1 and c^2 change as the liberalization period is shortened (i.e., as T becomes smaller).
- (c) Show how c^1 and c^2 change as the intertemporal elasticity of substitution (σ) becomes larger.

- (d) Consider the logarithmic case (i.e., $\sigma = 1$). Derive the indirect life-time utility as a function of q and T . Show how it varies with T .
- (e) Show how welfare changes in the case of (a) a one instant liberalization (i.e., $T \rightarrow 0$) and (b) an arbitrarily long liberalization ($T \rightarrow \infty$).
- (f) Plot welfare as a function of T (for reasonable parameter values) and verify that welfare has a U-shaped form.

4. Durable goods and intertemporal price speculation.

This exercise follows Calvo (1988). Consider the case of an individual who has Leontief preferences over time and therefore chooses a flat path of consumption (\bar{c}) independently of the path of p . Suppose also that the importable good can be stored at no cost and that there is no depreciation. Based on arbitrage considerations, it should be clear that, if p is constant over time, there are no incentives to accumulate stocks of the importable good because the good is dominated in rate of return by the foreign bond. Suppose instead that there is a “one-instant liberalization”:

$$\begin{aligned} p_0 &= 1, \\ p_t &= p > 1, \quad t > 0. \end{aligned}$$

Assume that the proceeds from the tariff are given back to consumers in a lump-sum fashion.

The consumer’s intertemporal budget constraint can be written as

$$Z + p\bar{c} \int_{\phi}^{\infty} e^{-rt} dt = b_0 + \int_0^{\infty} ye^{-rt} dt + T,$$

where Z denotes the stock of importables accumulated at $t = 0$, T denotes the present discounted value of government transfers, and ϕ denotes the time at which the stock of importables, Z , is depleted. Since there is no depreciation, it follows that

$$\phi\bar{c} = Z.$$

In this context:

- (a) Show that the intertemporal budget constraint can be written as

$$\bar{c}(r\phi + pe^{-r\phi}) = r \left(b_0 + \int_0^{\infty} ye^{-rt} dt + T \right).$$

- (b) Find the optimal ϕ . Discuss the intuition behind the results.

- (c) Find a reduced form for \bar{c} . (Hint: take into account that, in equilibrium, $T = (p - 1)\bar{c} \int_{\phi}^{\infty} e^{-rt} dt$.)
- (d) Discuss the welfare implications of a one-instant liberalization.

5. Lack of credibility

This exercise, which follows Engel and Kletzer (1991), deals with a formalization of the idea of lack of credibility that we discussed in Section 4. Consider a two-period endowment economy. The economy is endowed with a constant endowment, y , of an exportable good. It consumes an importable good, c . The international terms of trade are equal to one. The domestic price of importables, however, may be greater than one if a tariff is imposed. The economy can borrow/lend at a fixed rate, r . In the first period there is no tariff (i.e., the domestic relative price of importables is equal to one). In the second period a tariff $(p - 1)$ may be imposed with probability π .

The consumer's problem is to maximize expected utility

$$EU = \log c_1 + \beta(1 - \pi) \log c_2 + \beta\pi \log c_2^*,$$

where $\beta(1 + r) = 1$, c_1 is consumption in period 1, c_2 is consumption in period 2 if the tariff is not imposed and c_2^* is consumption in period 2 if a tariff is imposed.

Using the exportable good as the numeraire, the consumer's flow constraints are given by (initial net assets are assumed to be zero; that is, $b_0 = 0$):

$$\begin{aligned} c_1 &= y - b_1, \\ c_2 &= y + (1 + r)b_1, \\ pc_2^* &= y + (1 + r)b_1 + \tau^*, \end{aligned}$$

where b_1 denotes end of period 1 (beginning of period 2) net foreign assets and τ^* denotes lump-sum transfers in case the tariff is imposed.¹⁹ (In general equilibrium, $\tau^* = (p - 1)c_2^*$.)

In this context:

- (a) Compute reduced forms for c_1 , c_2 and c_2^* . Discuss the intuition behind the results.
- (b) Compute consumer's welfare as a function of π . Interpret the results.

¹⁹Note that we are implicitly assuming that consumers cannot insure against uncertain trade policy in period 2 (which is, of course, the natural assumption). In other words, there are incomplete markets (as defined in Chapter 2).

- (c) Compare consumer's welfare when the tariff is $(p - 1)$ in both periods with certainty.
- (d) What would you conclude about the desirability of trade reforms if you took the model at face value?
- (e) How could you modify the model to yield a more sensible policy prescription?

6. Effects of changes in the liberalization period with wealth effect

Let preferences be given by

$$u(c_t) = \log c_t.$$

Assume that a fraction ϕ of tariff revenues is spent on unproductive government spending, g , while a fraction $1 - \phi$ is returned to consumers as lump-sum transfers:

$$\begin{aligned} g_t &= \phi(p_t - 1)c_t, \\ \tau_t &= (1 - \phi)(p_t - 1)c_t. \end{aligned}$$

Notice that the two cases analyzed in the text are particular cases of this more general formulation: $\phi = 0$ corresponds to the full rebate case (Section 2) while $\phi = 1$ corresponds to the no rebate case (Section 5).

In this context:

- (a) Compute the reduced form for c^1 and c^2 as a function of q , T , and ϕ .
- (b) Derive the indirect utility function as a function of q , T , and ϕ .
- (c) Plot the consumer's indirect utility function as a function of T for different values of ϕ . In particular, show that for low values of ϕ there is a welfare loss for values of T below some critical value and a welfare gain for higher values (as illustrated in Figure 6), whereas for higher values of ϕ the temporary liberalization will always be welfare improving.