Competing with Loyalty Discounts

by

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Abstract

Loyalty discounts, offered to customers that meet purchase thresholds, can shift share from rival firms. In a differentiated product duopoly, only one firm employs a program that customers adopt in equilibrium. Whenever consumers strongly prefer the product of said firm, such discounts increase producer surplus. When a firm requires customers to meet thresholds in multiple rivalrous markets, the loyalty discount and consumer surplus may fall. Finally, when a "branded" product monopolist faces competition for an unrelated "generic" product, the monopolist raises the branded spot price, and offers a discount to customers that purchase generic product only from the monopolist.

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1. Introduction

When consumers make multiple purchases, sellers have an incentive to employ non-linear pricing strategies in order to increase sales volume while simultaneously extracting greater surplus on inframarginal units. Like volume discounts and two-part tariffs, loyalty discounts can be considered another instrument in the same pricing toolbox: a reward in the form of lower prices to customers who meet a purchase threshold.

Loyalty discounts can refer to something as simple as a coffee shop offering a free cup of coffee for every ten cups purchased. The term “loyalty discount,” however, typically applies to pricing strategies that condition a discount on satisfying a minimum purchase requirement in each period. The target threshold may be based on purchase volume, growth over the previous period, or share of purchase requirements. While this definition does not clearly differentiate loyalty discounts from volume discounts or other forms of non-linear pricing, several elements distinguish loyalty discounts. First, loyalty discounts typically require allocating a substantial share of total purchases to a single supplier. The term “market share discount” is often employed to describe this pricing strategy. Second, loyalty programs separate customers into two distinct groups: those who qualify, and those who do not, unlike the more gradual gradations possible with other pricing strategies. In addition, because the target is often not based solely on current purchases from a given seller, two customers that purchase the same volume from a seller may pay different prices.[1]

Even in the simple case of an occasional free cup of coffee for regular patrons, the implications of a loyalty discount scheme may not be as straightforward as at first glance: do regular customers really enjoy a discount, or do other customers pay a non-loyalty surcharge? In either case, this pricing scheme can be classified as a form of price discrimination, with firms competing more aggressively for loyal customers. Combining the discount with a minimum purchase requirement in each

[1] This discriminatory aspect has contributed to the abuse of dominance scrutiny that loyalty rebate schemes have attracted, particularly in the European Community; see for example, Case No. IV/D-2/34.780, Virgin/British Airways (1999).

period complicates the analysis. While the same elements of aggressive pricing and price discrimination are present, there is now an additional question about whether these purchase requirements, coupled with a loyalty discount for buyers who comply with the purchase terms, can function as exclusionary behavior to the detriment of rival firms and competition. This is of particular concern when the firm offering loyalty discounts is much larger than its rivals. While the discount on each unit sold may be small, the aggregate loyalty discount across all products may generate a substantial rebate. A smaller competitor, able only to offer a larger discount on its smaller sales volume, may find it unprofitable to match the overall discount. Thus such discounts can be a powerful tool in shifting sales to the larger firm.

Several recent antitrust cases have brought the potential exclusionary role of loyalty discounts to the forefront. In *Concord Boat*, the dominant manufacturer of stern drive marine engines, Brunswick, offered discounts of several percent to buyers that bought a minimum share of their engines from Brunswick.[2] A number of boat builders jointly won a jury verdict in a suit against Brunswick, but lost on appeal. In *Virgin Atlantic*, British Airways offered loyalty discounts to travel agents and corporate customers that satisfied minimum purchase requirements across a number of routes served by British Airways out of Heathrow Airport.[3] Virgin Atlantic sued, claiming that these discounts contributed to "predatory foreclosure" of Virgin from several Heathrow routes. British Airways won a summary judgment motion, which was upheld on appeal, on the grounds that Virgin had not offered adequate proof of the predatory claim. While British Airways successfully defended its discount policies in American courts, the European Commission found that its commission programs violated Article 82. In a third case, 3M was accused of using loyalty discounts across a number of different products to unlawfully exclude *LePage's* in the market for transparent tape.[4] The decision has gone back in forth in the courts, with the most recent *en banc* opinion of the Third Circuit going against 3M. In June 2004, the Supreme Court declined to hear the case.

[2] Concord Boat Corp. v. Brunswick Corp., 207 F.3d 1039 (8th Cir. 2000).

[3] Virgin Atlantic Airways Ltd. V. British Airways PLC, 257 F.3d 256 (3rd Cir. 2001); Case No. IV/D-2/34.780, Virgin/British Airways (1999).

[4] LePage's Inc. v. 3M Co. 324 F.3d 141 (3rd Cir 2003), *cert. denied*, 124 S.Ct. 2932 (2004).

The shifting outcomes of these cases and the legal commentary that they have spawned suggest that whether loyalty discounts are anti-competitive depends greatly on the legal framework under which they are analyzed. Is the issue whether the discounts effectively foreclose competitors from a sufficient share of the market? Is it whether the discounts lead to sales below cost, as was alleged by the plaintiff in *Virgin Atlantic*, and was the decisive issue in *Concord Boat*, and in the decision by the Third Circuit panel in *LePage's*? Does it matter whether the intent of the dominant firm was exclusionary, or whether there is harm to consumers?[5]

One element absent from the debate is an understanding of when firms would use loyalty programs as a competitive strategy, even in the absence of exclusionary motives. This paper contributes by studying the equilibrium use of loyalty programs, and how they affect pricing, market shares and welfare. In addition to providing a better backdrop against which to observe exclusionary or predatory behavior, our analysis lays the groundwork for tests that can distinguish such behavior.

The incentive to employ loyalty discounts as a competitive strategy in a differentiated product market is similar to the incentive to use non-linear pricing (*e.g.* frequent flyer programs) more generally. Stole (1995), Hamilton and Thisse (1997), Gasmi, Moreaux, and Sharkey (2000), Mark and Vickers (2001), Min *et al.* (2002), and Page and Montiero (2003), among others, have shown that non-linear pricing typically emerges as an equilibrium strategy in a price setting oligopoly.[6] Loyalty programs make marginal purchases less price sensitive, which is similar to imposing switching costs.[7] A portion of the literature treats loyalty discounts as a form of partial exclusive dealing that can generate pro-competitive and anti-competitive effects. On the pro-competitive side, Mills (2004) shows how market share discounts can lessen the incentive to free ride on manufacturer supplied promotions. In contrast, Tom, Balto, and Averitt (2000) discuss potential exclusionary effects of market share discounts, while Cairns and Galbraith (1990) and Larsen and Storm (2002) identify circumstances under which frequent flyer

[5] O'Donoghue (2002), Keyte (2003), and Hewitt (2003) discuss these different perspectives.

[6] Kolay, Shaffer, and Ordover (2004) study a monopolist's use of similar all-units discounts.

[7] See, for example, Caminal and Matutes (1990) and the survey by Klemperer (1995).

discounts can create entry barriers. Finally, some research addresses multi-product loyalty programs as a form of tying. Greenlee, Reitman, and Sibley (2004), and Nalebuff (2004a,b) examine how a monopolist's loyalty program can leverage market power into competitive markets.

As the variety of legal cases discussed above suggests, no single model covers all market structures that raise concern about the anti-competitive use of loyalty discounts. In cases like *Concord Boat*, the concern can stem from the use of loyalty discounts by a large firm in a single market. The potential for abuse may be greater if the dominant firm offers a loyalty program that links multiple markets, as in *Virgin Atlantic*. And finally, a monopolist in one market may use loyalty discounts to extend its market power to other markets, as was alleged in *LePage's*. This paper presents a set of models that covers this range of market structures.

This paper is organized in the following manner. Section 2 introduces a duopoly model of a single market, like *Concord Boat*, in which firms offer loyalty contracts based on sales in that market. Firms have identical costs but may differ in the value customers place on their products. In this setting, firms always choose to use a loyalty program in equilibrium, aside from any exclusionary motive, but only one firm's program is adopted by consumers. Typically, the firm selling the preferred product offers the accepted loyalty discount, and such discounts increase producer surplus whenever preferences for said firm's product are sufficiently strong.

Section 3 extends the model to settings, like *Virgin Atlantic*, in which a large firm can link the discount across a number of related markets. When some rivals cannot offer loyalty programs, linking several rivalrous markets in a loyalty program allows the large firm to reduce the discount. The analysis is followed by a discussion that distinguishes the equilibrium strategy in the model from intentional exclusionary behavior, and offers a test to distinguish between the two.

Section 4 introduces a second market in which one firm is a monopolist, and asks whether the monopolist has an incentive to use loyalty discounts to link sales between the monopolized market and a rivalrous market. Again, even without an exclusionary motive, the answer typically is yes. As in Section 3, the analysis is followed by a discussion of when loyalty discounts across monopolized and rivalrous markets may be exclusionary. Section 5 offers a brief conclusion. An Appendix contains the proofs of our first and final propositions.

2. Loyalty discounts in a single market

These models incorporate some of the important common elements of loyalty programs. While loyalty discounts may apply only to incremental sales, one common feature of the cases discussed above is that the discounts applied to every unit sold.[8] Our models assume that firms offer discounts on all units sold whenever a customer satisfies the loyalty requirement. A spot price pertains to customers who, perhaps due to size or an inability to be readily monitored, are not candidates for a loyalty program. In addition to spot prices, firms select the prices and restrictions that characterize a loyalty program. Customers then choose among the available prices and programs. By assumption there is foresight on both sides: customers know at the outset whether they will fulfill the terms of each program, and firms correctly anticipate product choices and customers' success in meeting program requirements.

Consider two firms, A and B, that each sell a differentiated product and compete by setting prices. Both firms have constant marginal costs c. The firms sell to two groups of customers. Large customers buy multiple products, and for each purchase the customer has a different evaluation of the relative value of firm A's and firm B's products. Large customers, for example, may be travel agents, and each purchase may reflect the idiosyncratic preferences of an individual client. Small customers make individual purchases at the spot price, and are not candidates for a loyalty program. Let $q > 0$ be the ratio of purchases made by small customers to purchases made by large customers, so that the fraction of all purchases that are made by large customers is $1/(1+q)$.

An important feature of these models is that they incorporate two forms of customer heterogeneity. One, as mentioned above, is that some customers are not candidates for the loyalty program and purchase only at the spot price. Without these customers, firms could use the spot price as an out-of-equilibrium threat: join

[8] Such price reductions are often referred to as "dollar-one," "all-unit," or "rollback" discounts.

my loyalty program or pay an exorbitant price.[9] The first model also assumes that the products of different firms are imperfect substitutes. Customers make multiple purchases, but each purchase is idiosyncratic in the extent to which it is better suited for one or the other firm's product. This assumption makes it costly, both to the firm and to social welfare, to shift many purchases to one firm via a loyalty program. The pecuniary benefits to firms, however, typically exceed this loss.

Small and large customers have the same distribution of reservation prices across purchases, though they may have a different total number of purchases. Purchases of firm B's product have reservation price, $R + x/2$, while the reservation price for firm A's product is $R - x/2$, where x is distributed according to $F(x)$ on $x \in [L, H]$. We assume the corresponding density function $f(x)$ is strictly positive on $[L, H]$. The mix of purchases is assumed to be deterministic, so that large customers can ensure that they purchase a s share of their purchases from firm A by allocating all purchases with $x \le F^{-1}(s)$ to firm A.[10] We maintain the following monotone hazard assumption in order to guarantee that both firms have the usual shaped differentiated product Bertrand reaction functions, with slope less than one, so that this equilibrium exists and is unique.

ASSUMPTION 1: For all $x \in [L, H]$, $\frac{d}{dx}\left[\frac{F(x)}{f(x)}\right] > 0 > \frac{d}{dx}\left[\frac{1-F(x)}{f(x)}\right]$.

In general, demand uncertainty may prevent customers from perfectly predicting their own aggregate demand or how that demand breaks out among available brands. Nevertheless, customers often take steps to ensure that they hit a target by shifting purchases across accounting periods, by steering purchases to a particular brand, or by buying for inventory. Such steps provide greater assurance that purchasers will clear program hurdles. Accordingly, the models in the present

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[9] The effect of this type of out-of-equilibrium threat can still be determined by considering the limiting case where the number of non-participating customers is vanishingly small.

[10] Alternatively, one can assume that individual purchases are atomistic, so that, by the law of large numbers, the program is satisfied with probability one using the same decision rule.

analysis downplay demand uncertainty. We assume throughout that customers achieve the program target for a supplier with certainty by allocating sufficient sales to that supplier's product. Given the absence of demand uncertainty, we downplay the type of sales target employed. Customers may prefer share targets to volume targets, for example, when the distribution of demand across brands is more predictable than the overall level of demand. Conversely, a volume target may be better when demand for the dominant firm's product is more predictable than demand for competing products. The choice of levels or growth rates depends in part on whether demand shocks are correlated across time. The models here assume that firms offer a discount on all purchases if customers meet a designated market share target, but an equivalent result holds when firms use volume or growth targets.[11]

Firms simultaneously set a spot price, p_i, and a loyalty program, $\{z_i, q_i\}$, that offers a discounted price z_i to large customers that satisfy firm i's share requirement. Firm A charges z_A for each unit whenever the customer buys at least an $F(q_A)$ share of its purchases from firm A. Similarly, firm B charges z_B for each unit whenever the customer buys at least a $1-F(q_B)$ share from firm B. After prices and loyalty programs have been announced, customers decide whether to satisfy either program and allocate purchases accordingly.

For small customers and large customers that fail to meet either program's terms, the optimal decision allocates purchases to the firm offering the higher surplus, given the announced spot prices. For each purchase, customers receive a surplus of $R - x/2 - p_A$ whenever they buy from firm A and $R + x/2 - p_B$ whenever they buy from firm B. Thus these customers buy from firm A whenever $x < p_B - p_A$ and from firm B whenever $x > p_B - p_A$.

If customers satisfy firm A's loyalty program but not firm B's, and if firm A's contract is a binding constraint for customers that satisfy it, then firm profits are:

[11] For the same reason, we do not distinguish between the terms loyalty discounts, loyalty rebates, fidelity rebates, and market share discounts, which refer to similar pricing schemes.

$$\begin{aligned}&\Pi_A(\{z_A, q_A, p_A\}, p_B) = (z_A - c)\, F(q_A) + q\,(p_A - c)\, F(p_B - p_A) \quad \text{and} \\ &\Pi_B(\{z_A, q_A, p_A\}, p_B) = (p_B - c)[1 - F(q_A) + q - q\, F(p_B - p_A)].\end{aligned} \tag{1}$$

Alternatively, if customers only satisfy firm B's program and this program is a binding constraint for customers that satisfy it, then firm profits are:

$$\begin{aligned}&\Pi_A(p_A, \{z_B, q_B, p_B\}) = (p_A - c)[F(q_B) + q\, F(p_B - p_A)] \quad \text{and} \\ &\Pi_B(p_A, \{z_B, q_B, p_B\}) = (z_B - c)[1 - F(q_B)] + q\ (p_B - c)[1 - F(p_B - p_A)].\end{aligned} \tag{2}$$

Finally, if customers opt for neither program, then profits are

$$\begin{aligned}&\Pi_A(p_A, p_B) = (p_A - c)(1 + q)\ F(p_B - p_A) \quad \text{and} \\ &\Pi_B(p_A, p_B) = (p_B - c)(1 + q)[1 - F(p_B - p_A)].\end{aligned} \tag{3}$$

For this latter case, the equilibrium spot prices are the prices in the differentiated product Bertrand equilibrium of the game without loyalty programs. Those prices satisfy the first order conditions:

$$p_A^* = c + \frac{F(p_B^* - p_A^*)}{f(p_B^* - p_A^*)} \quad \text{and} \quad p_B^* = c + \frac{1 - F(p_B^* - p_A^*)}{f(p_B^* - p_A^*)}. \tag{4}$$

For example, the uniform distribution on $[L, H]$ satisfies Assumption 1 and generates $p_A^* = (H - 2L)/3$ and $p_B^* = (2H - L)/3$.

One final possibility is that $q_A \le q_B$ and large customers satisfy both loyalty programs. In that case, large customers pay a discounted price for every purchase, and only small customers buy at the spot prices, so the same first order conditions apply: the equilibrium spot prices are p_A^* and p_B^*.

Before presenting our first results, we introduce some additional notation. Define $CS(\{z_A, q_A\}, p_B)$ to be the expected consumer surplus per purchase received by large customers when they buy from firm A if and only if $x \le q_A$, and let $CS(p_A, p_B)$ be the consumer surplus per purchase received by customers that satisfy neither program and buy from firm A whenever $x \le p_B - p_A$. We have

$$CS(\{z_A, q_A\}, p_B) = \int_L^{q_A} (R - x/2 - z_A)\, f(x)\, dx + \int_{q_A}^{H} (R + x/2 - p_B)\, f(x)\, dx \tag{5}$$

$$CS(p_A, p_B) \quad = \quad \int_L^{p_B - p_A} (R - x/2 - p_A)\ f(x)\, dx \ + \ \int_{p_B - p_A}^{H} (R + x/2 - p_B)\ f(x)\, dx \tag{6}$$

Our first proposition establishes that in equilibrium, exactly one firm offers a non-trivial loyalty discount that customers accept. This is shown in two steps. First, we establish that if both firms offer loyalty discounts that customers satisfy, then at least one of them is trivial, meaning that $z_i = p_i^*$ for some $i \in \{A, B\}$. Second, we show that whenever customers satisfy neither firm's loyalty program, each firm can offer a profit-improving loyalty program that customers will accept.

PROPOSITION 1: *For the single market case,*

(i) *If customers satisfy both loyalty programs in equilibrium, then at least one program offers no actual discount, that is* $z_A = p_A = p_A^*$ *or* $z_B = p_B = p_B^*$.

(ii) *In equilibrium, at least one firm offers a non-trivial loyalty program that customers accept.*

(iii) *Therefore, exactly one firm offers a non-trivial loyalty program that consumers accept in equilibrium.*

PROOF: See Appendix.

Trivial loyalty programs in which firms offer no discount have no effect on the market, and can be ignored. Thus Proposition 1.i effectively rules out equilibria in which customers adopt both loyalty programs. Proposition 1.ii establishes that any combination of pricing strategies in which customers choose to satisfy neither program will always be undercut by one of the firms with a profit-improving program that customers will adopt. Therefore, in equilibrium firms offer prices and programs such that customers choose to satisfy (only) one loyalty program. We next characterize the equilibrium, focusing in turn on the loyalty requirements (q_A, q_B), the spot prices (p_A, p_B), and the loyalty discounts (z_A, z_B).

PROPOSITION 2: *Define* $h = \min\{H, p_B - c\}$ *and* $l = \max\{L, c - p_A\}$. *In equilibrium,* $q_A = h$ *and* $q_B = l$.

PROOF: We establish something slightly stronger. Namely, firm A's best response to any strategy of firm B always satisfies $q_A = h$, and similarly for firm B. Proposition 1 implies that only one firm's loyalty program is adopted in equilibrium. We first establish the result for each firm when consumers satisfy its loyalty program in equilibrium, and then turn to the case when the other firm's program is adopted.

Suppose customers adopt firm A's program in equilibrium, and let $\overline{CS}$ be a large customer's surplus under an initial profile of prices and loyalty programs. The Lagrangian for firm A's maximization problem is:

$$\begin{aligned} L^A &= (z_A - c)F(q_A) + q(p_A - c)F(p_B - p_A) \\ &\quad + l\left(\int_L^{q_A}(R - \tfrac{1}{2}x - z_A)\, f(x)\,dx \;+\; \int_{q_A}^{H}(R + \tfrac{1}{2}x - p_B)\, f(x)\,dx \;-\; \overline{CS}\right) \end{aligned} \tag{7}$$

The first order conditions for z_A and q_A are

$$\frac{dL^A}{dz_A} \;=\; (1 - l)F(q_A) \;=\; 0 \tag{8}$$

$$\frac{dL^A}{dq_A} \;=\; \left[z_A - c + l\,(p_B - z_A - q_A)\right] f(q_A) \;=\; 0 \tag{9}$$

The solutions to (8) are $q_A = L$ and $l = 1$. We ignore the former solution because it corresponds to a trivial loyalty program. Note that the latter solution ensures that the consumer surplus from the loyalty program equals $\overline{CS}$. Substituting $l = 1$ into (9), and recalling that $f(x) > 0$ for all $x \in [L, H]$, establishes that $q_A = p_B - c$ solves (9). If $p_B - c > H$, then the objective is increasing in q_A so $q_A = H$ is optimal. Thus in equilibrium, $q_A = h$ when consumers adopt firm A's loyalty program. A similar calculation establishes that $q_B = l$ when consumers satisfy firm B's program.

Now consider the firm whose program is not adopted in equilibrium. The alternative level of consumer surplus that firm A must provide to large customers in order to have them participate in firm A's program ($\overline{CS}$) is obtained either from adopting firm B's loyalty program (large customers pay p_A and z_B), or from satisfying neither program (large customers pay p_A and p_B). In equilibrium, the alternative cannot be satisfying neither program, because if it were, firm B could offer a profit-improving program that large customers would accept. That is, firm B initially earns $\Pi'_B = (p_B - c)[1 - F(q_A) + q(1 - F(p_B - p_A))]$ and large customers enjoy the consumer surplus as if they faced p_A and p_B. If all customers faced p_A and p_B (loyalty programs not feasible), firm B would earn $\Pi_B = (p_B - c)[1 - F(p_B - p_A) + q(1 - F(p_B - p_A))]$ which, using $q_A = p_B - c$ and $p_A > c$, weakly exceeds Π'_B. Proposition 1.ii implies that firm B can raise its profit above Π_B (and hence above Π'_B) by offering a program that large customers will adopt (because it provides at least the same consumer surplus as paying p_A and p_B). Thus in equilibrium, the $\overline{CS}$ constraint that firm A faces must be based on an alternative in which large customers adopt firm B's loyalty program. If customers adopt firm A's program in equilibrium, can firm B's program be anything other than a best response to firm A? No, because if B's loyalty program is not a best response in the candidate equilibrium, then firm B can reduce z_B so that large customers adopt it instead of firm A's program, and Π_B increases. Thus, in equilibrium, firm B must offer a best response loyalty program, even if customers do not adopt it. From above, this implies that $q_B = l$. A similar argument establishes that $q_A = h$ (even) when large customers adopt firm B's program in equilibrium. □

Note that h and l determine whether any loyal purchasers always buy from one firm. Specifically, h demarcates those purchases for which the customer prefers firm B's product at p_B even if firm A's product is sold at marginal cost, while l demarcates the analogous set of purchases that always go to firm A at p_A.[12]

As the proof of Proposition 2 establishes, the binding consumer surplus constraint in an equilibrium in which large customers adopt firm A's program is the surplus they would obtain from firm B's program. Using Proposition 2, the consumer surplus from just satisfying firm B's loyalty program is:

$$\overline{CS} = \int_L^l (R - \tfrac{1}{2}x - p_A)\, f(x)\,dx \; + \; \int_l^H (R + \tfrac{1}{2}x - z_B)\, f(x)\,dx \tag{10}$$

Substituting this term into firm A's maximization problem (7), differentiating with respect to p_A, and substituting $l = 1$ yields

$$\frac{dL^A}{dp_A} = \; 0 \; = \; q\; F(p_B - p_A) - q\;(p_A - c)\; f(p_B - p_A) \; + \; F(l) \tag{11}$$

In a candidate equilibrium in which consumers satisfy firm A's loyalty program, firm B sets p_B to maximize:

$$\Pi_B = (p_B - c)[1 - F(q_A)] + q(p_B - c)[1 - F(p_B - p_A)] \tag{12}$$

Differentiating with respect to p_B and substituting $q_A = h$ yields

$$\frac{d\Pi_B}{dp_B} = 1 \; - \; F(h) \; + \; q\,[1 \; - \; F(p_B - p_A) \; - \; (p_B - c)\; f(p_B - p_A)] \; = \; 0 \tag{13}$$

One can rewrite (11) and (13) in the following fashion:

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[12] Recall that consumers prefer A whenever $x < p_B - p_A$, and thus $x > p_B - c$ demarcates the region in which consumers prefer B when A is priced at marginal cost. The definition of h accounts for the possibility that $p_B - c$ may not lie in $[L, H]$, or in words, that all consumer types prefer B at p_B, even when $p_A = c$. A similar explanation holds for l.

$$
\begin{aligned}
p_A &= c \ + \ \frac{F(p_B - p_A)}{f(p_B - p_A)} \ + \ \frac{F(l)}{q\ f(p_B - p_A)} \\
p_B &= c \ + \ \frac{1 - F(p_B - p_A)}{f(p_B - p_A)} \ + \ \frac{1 - F(h)}{q\ f(p_B - p_A)}
\end{aligned}
\tag{14}
$$

Since $p_A > c$ and Proposition 2 together imply that $q_A > p_B - p_A$, the loyalty contract is strictly binding. Our next proposition establishes that both firms price above the Bertrand reaction function that arises when loyalty programs are not feasible. Before presenting the proof, we explain the different reason each firm has for abandoning the Bertrand reaction function. If large customers fulfill firm A's program in equilibrium, then firm B sets $p_B > p_B^*$ because it earns additional profits from the remaining large customer purchases going to firm B. Since firm A's loyalty program is binding, all large customer purchases still made at firm B are for clients that strictly prefer firm B's product given prevailing prices. Thus, sufficiently small increases in p_B above p_B^* do not generate any lost large customer sales, and hence are profitable for firm B. Turning to firm A, since large customers adopt firm A's loyalty program, no large customer sales are made at p_A. Nevertheless, a small increase in p_A above p_A^* relaxes the consumer surplus constraint on its loyalty program price without having a first order effect on profits from small customers.[13]

When large customers adopt firm B's loyalty program, firm B's Lagrangian is

[13] An analogy can be made to network competition. Satisfying a loyalty program is like joining a network, buying a good at a loyalty program price is like making an on-net call, and buying at a spot price is like making an off-net call. Raising the cost of access to network A causes network B to charge its customers higher prices for off-net calls (that terminate on network A). This reduces the surplus that network B provides to customers, so either network B reduces other prices, or network A can profitably raise its prices. See, for example, Laffont, Rey, and Tirole (1998).

$$\begin{aligned}L^B = &(z_B - c)[1 - F(q_B)] + q(p_B - c)[1 - F(p_B - p_A)]\\&+ l\left(\int_L^{q_B}(R - \tfrac{1}{2}x - p_A)\, f(x)\,dx \;+\; \int_{q_B}^{H}(R + \tfrac{1}{2}x - z_B)\, f(x)\,dx - \right.\\&\left.\int_L^{q_A}(R - \tfrac{1}{2}x - z_A)\, f(x)\,dx \;-\; \int_{q_A}^{H}(R + \tfrac{1}{2}x - p_B)\, f(x)\,dx\right)\end{aligned} \tag{15}$$

and firm A profits are:

$$\Pi_A = (p_A - c)F(q_B) \;+\; q\,(p_A - c)F(p_B - p_A) \tag{16}$$

Differentiating these with respect to own spot price, substituting $l = 1$, and using Proposition 2, the corresponding first order conditions $d\Pi_A / dp_A = 0$ and $dL^B / dp_B = 0$ can be expressed as (14). Thus the equilibrium spot prices do not depend on which loyalty program consumers adopt. Only the reasons each firm raises its price are reversed. Since spot prices (weakly) increase with the introduction of loyalty programs, small customers are harmed.

PROPOSITION 3: *If* $p_B^* - c = H$ *and* $c - p_A^* = L$, *then* $(p_A, p_B) = (p_A^*, p_B^*)$ *in equilibrium. Otherwise* $p_A > p_A^*$ *and* $p_B > p_B^*$.

PROOF: When $p_B^* - c = H$ and $c - p_A^* = L$, the system of equations (14) matches the system of equations (3), and therefore the equilibrium spot prices when loyalty programs are feasible match the spot prices when loyalty programs are not feasible, *i.e.* $(p_A, p_B) = (p_A^*, p_B^*)$. Let $r_i(\cdot)$ and $\hat{r}_i(\cdot)$ respectively denote firm i's reaction functions when loyalty programs are not feasible, and when they are feasible. Claim $\hat{r}_A(p_B) \geq r_A(p_B)$. Suppose not, so there exists a p for which $\hat{r}_A(p) < r_A(p)$. Using (3) and (14), this implies that

$$\frac{F(p - r_A(p))}{f(p - r_A(p))} > \frac{F(p - \hat{r}_A(p))}{f(p - \hat{r}_A(p))} + \frac{F(c - \hat{r}_A(p))}{q\; f(p - \hat{r}_A(p))} \geq \frac{F(p - \hat{r}_A(p))}{f(p - \hat{r}_A(p))}.$$

Assumption 1 then implies that $p - r_A(p) > p - \hat{r}_A(p)$ which contradicts $\hat{r}_A(p) < r_A(p)$. Therefore $\hat{r}_A(p_B) \geq r_A(p_B)$. A similar calculation establishes that $\hat{r}_B(p_A) \geq r_B(p_A)$. These inequalities are strict when $p_B^* - c < H$ or $c - p_A^* > L$.

Let (p_A, p_B) be part of a candidate equilibrium when loyalty programs are feasible, and assume that $p_A \leq p_A^*$. If $p_A = \hat{r}_A(p_B)$, then $p_A > r_A(p_B)$. Claim that $p_A \geq p_A^* > r_A(p_B)$ implies that $p_B < r_B(p_A)$. Suppose not, so $p_A > r_A(p_B)$ and $p_B \geq r_B(p_A)$. If reaction curves intersect at (p_A^*, p_B^*), then for some $p_A \leq p \leq p_A^*$, we must have $r_B'(p) > [r_A^{-1}(p)]' = \dfrac{1}{r_A'(p)}$. The proof of Proposition 1, however, establishes that $r_i'(\cdot) < 1$ for $i = A, B$, so we have a contradiction. Thus (p_A, p_B) cannot be an equilibrium, or said differently, $p_A > p_A^*$. Reversing the firm identifiers and repeating the same argument establishes $p_B > p_B^*$. □

Proposition 2 determines the equilibrium loyalty requirements—firm A requires that customers obtain a $F(h)$ share of their purchases from firm A, while firm B requires customers to purchase a $1 - F(l)$ share. Bertrand competition determines the prices in these contracts. Suppose that, in the proposed equilibrium, customers satisfy firm A's program. If firm B can offer an alternative contract that guarantees large customers the consumer surplus that they obtain from firm A's loyalty program while increasing its own overall level of profits, it will do so. The proof of Proposition 2 shows that firm B's optimal deviation generates the same large customer surplus as under the proposed equilibrium. That determines firm B's discount to large customers when it deviates from the proposed equilibrium.[14] In equilibrium, firm A sets z_A so that large customers enjoy the same surplus from adopting firm A's program as they would from switching to firm B's program, and firm B earns the same profit as it would from its optimal deviation. Note that at a

[14] At the same time, firm B can change its spot price to obtain additional profits from small customers, since large customers no longer buy at p_B. However, as noted above, the optimal spot price for firm B does not depend on whose requirements contract large customers adopt.

fixed p_B, firm B would sell more if large customers abandoned both loyalty programs because firm A's program requirement (q_A) is strictly binding. Thus this constraint ensures that at prevailing prices customers prefer firm A's loyalty program to no loyalty program at all. The equilibrium satisfies:

$$CS(\{z_A, h\}, p_B) = CS(p_A, \{z_B, l\}) \text{ , and} \tag{17}$$

$$\Pi_B(\{z_A, h, p_A\}, p_B) = \Pi_B(p_A, \{z_B, l, p_B\}) \ . \tag{18}$$

Equations (17) and (18) determine the equilibrium values of z_A and z_B. These can be written as

$$z_B[1-F(l)] - p_B[1-F(h)] = z_A F(h) - p_A F(l) + \int_l^h x\ f(x)dx \tag{19}$$

$$(p_B - c)\,[1-F(h)] = (z_B - c)\,[1-F(l)], \tag{20}$$

and solving them yields

$$z_A = c + (p_A - c)\frac{F(l)}{F(h)} - \frac{1}{F(h)}\int_l^h x\ f(x)dx \tag{21}$$

$$z_B = c + (p_B - c)\frac{1-F(h)}{1-F(l)} \ . \tag{22}$$

Equations (14), (21) and (22) identify the equilibrium prices when large customers adopt firm A's program in equilibrium. If customers adopt firm B's program instead of firm A's, the equilibrium spot prices (p_A, p_B) and loyalty requirements (q_A, q_B) do not change. The equilibrium loyalty program prices are

$$z_A = c + (p_A - c)\frac{F(l)}{F(h)} \text{ , and} \tag{23}$$

$$z_B = c + (p_B - c)\frac{1-F(h)}{1-F(l)} + \frac{1}{1-F(l)}\int_l^h x\ f(x)dx \,. \tag{24}$$

For "short-tailed" distributions, in which $p_B^* - c \geq H$ or $c - p_A^* \leq L$ so that $F(h) = 1$ or $F(l) = 0$,[15] no large customers buy at the spot prices in equilibrium. For such distributions, the equilibrium contract prices, when firm A's loyalty program is adopted in equilibrium, are

$$z_A = c + (p_A - c)F(l) - \int_l^h x\, f(x)dx, \quad \text{and} \tag{21a}$$

$$z_B = c. \tag{22a}$$

For short-tailed distributions, when firm A offers the chosen loyalty program, firm B makes no profit from large customers. Thus it undercuts firm A's program by setting its discount price z_B at marginal cost. If, in addition, F is symmetric with mean zero, then both firms set their loyalty discount prices at marginal cost, and neither firm profits from large customers. If $F(\cdot)$ is symmetric with a negative mean, implying that customers on average favor firm A's product, then only firm A's loyalty program is accepted in equilibrium. Conversely, when the mean of $F(\cdot)$ is positive, the only equilibrium has customers satisfying firm B's program.

With "long-tailed" distributions, the firm whose loyalty program is rejected still earns positive profit from large customers in equilibrium, so it does not set its unfulfilled loyalty program price at marginal cost. This affords some pricing cushion to the firm whose program is adopted. This implies that there are two distinct pure strategy equilibria when the mean of the preference distribution is close to zero, and consumers adopt each firm's program in one of the equilibria. As the mean of the distribution diverges from zero, the firm with the on average less preferred product must make deeper and deeper price cuts to induce customers to adopt its program. With sufficiently strong preferences for one product, the required price falls below marginal cost, and the only equilibrium has large customers adopting the program of the firm selling the preferred product. The next proposition establishes that profits fall when loyalty programs become feasible for symmetric firms.

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[15] The uniform distribution, for example, is a short-tailed distribution.

PROPOSITION 4: *When the distribution of reservation prices is symmetric with mean zero, firms earn at least as much in equilibrium when loyalty programs are not feasible as in the equilibrium when loyalty programs are feasible.*

PROOF: When F is symmetric with mean zero and loyalty programs are infeasible, the equilibrium prices from (3) become $p_A^* = p_B^* = p^* = c + F(0)/f(0) = c + [2f(0)]^{-1}$ and each firm earns $p^* = (1+q)(p^* - c)F(0) = (1+q)/[4f(0)]$. When symmetric firms can employ loyalty programs, they offer the same prices p and z, and earn the same profits p, even though consumers satisfy only one program in equilibrium. In the short-tailed case, $p = p^*$ and $z = c$, so firms earn the same profits as in the no loyalty program equilibrium from small customers, while earning zero profits from large customers. Thus $p < p^*$. In the long-tailed case, (14) implies that $p = p^* + F(c-p)/(q\,f(0)) > p^*$, while (21) and (22) imply that $z = c + (p-c)F(c-p)/F(p-c)$. Profits are greater from small customers, but that is countered by aggressive competition for large customers. Overall, $p = (p-c)F(c-p) + \frac{1}{2}q(p-c)$. One interpretation is that this is firm A's profits when firm B's program is adopted by large customers. For this case $q_B = c - p$ so the first term is the profit from selling to large customers (who purchase a $1 - F(q_B)$ share from firm B). The second term is the profit earned from selling to one half of the small customers. Using the equilibrium values of p and p^*, this can be rewritten as $p = [F(c-p) + \frac{1}{2}q]^2/(q\ f(0))$. Thus $p > p^*$ only if $[F(c-p) + \frac{1}{2}q]^2/(q\ f(0)) > (1+q)/[4f(0)]$. Simplifying, this can be expressed as

$$4F(c-p)^2 + 4q\,F(c-p) > q\,. \tag{25}$$

Suppose loyalty programs are not feasible, firm A charges $p^* + p - c$, and firm B charges p^*. With these prices firm A makes sales whenever $x \le p_B - p_A = c - p$, and thus earns $(1+q)\,F(c-p)[p^* + p - 2c]$. By definition of equilibrium, these profits must be weakly less than $(1+q)/[4f(0)]$, firm A's profits when it charges

p^*. Using the definitions of p and p^* above, this profit inequality can be written as $4F(c-p)^2 + 4qF(c-p) \leq q$, which contradicts (25). Thus $p \leq p^*$. □

When F has mean sufficiently close to zero, firms generally earn less profit in the loyalty program equilibrium than they do when neither firm can offer such programs. When preferences for one brand are sufficiently strong, the preferred firm does better using a loyalty program, and in some cases the less preferred firm also benefits due to higher spot prices paid by small customers.

The consumer surplus implications are ambiguous. Spot prices increase in the loyalty program equilibrium, so small customers are definitely harmed. While loyalty programs reduce profits when firms are symmetric, large customer consumer surplus does not necessarily increase due to the substantial number of purchases shifted from what would be the preferred product in the absence of a share requirement. Generally, when preferences for one brand are sufficiently strong, allowing loyalty programs lowers overall consumer surplus.

3. Bundling across parallel markets

Suppose the market conditions described in Section 2 occur in a number of parallel markets. That is, assume that customers make multiple purchases in several related markets (such as different destinations from a hub airport) and a firm can use a loyalty program that links purchases in all markets. Large customers enjoy a discount only if they meet the share requirement in every market. A customer that misses the target share in any one market loses the discount in all markets. How does this potential to link across markets change the equilibrium?

It is helpful at first to make some simplifying assumptions (these can be relaxed, as will be discussed shortly). Assume that there are N markets symmetric in all characteristics, and that large customers buy products in all markets. A key assumption is that firm A competes in all N markets, but that rival firm B_i

competes just in market i.[16] Of firm A's rivals, we assume that $B_1,...,B_k \;\; (k \le N)$ have the capability to offer loyalty discounts, and we study how equilibria depend on k/N. We focus on equilibria in which customers adopt firm A's program.

Let $p_A^i,\; p_B^i,\; q_A^i,\; q_B^i,\; z_A^i,\; z_B^i,\; h^i$, and l^i be the market i analogues to the variables introduced in the previous section. The Lagrangian for firm A, corresponding to (7), is

$$L^A = \sum_{i=1}^{N} \left[(z_A^i - c) F(q_A^i) \;+\; q\,(p_A^i - c) F(p_B^i - p_A^i) \right] \;+\; l\left[\sum_{j=1}^{N} \left(\int_L^{q_A^j} (R - \tfrac{1}{2}x - z_A^j)\, f(x)\,dx \;+\; \int_{q_A^j}^{H} (R + \tfrac{1}{2}x - p_B^j)\, f(x)\,dx \right) \;-\; \overline{CS} \right] \tag{26}$$

The first order conditions $dL^A / dz_A^i = 0$ and $dL^A / dq_A^i = 0$ match (8) and (9), so as in Proposition 2, $l = 1$ and $q_A^i = h^i$. The argument in the second part of the proof of Proposition 2 can be applied to each of $B_1,...,B_k$, so $\overline{CS}$ must be the surplus available to large customers that satisfy all the loyalty programs of $B_1,...,B_k$. Thus, as in Proposition 2, $q_B^i = l^i$ for such rivals. The first order conditions $dL^A / dp_A^i = 0$ and $d\Pi_B^i / dp_B^i = 0$ match (11) and (13), so the spot prices are determined by (14), and we can drop the superscripts on $p_A^i,\; p_B^i,\; q_A^i,\; q_B^i,\; h^i$, and l^i. In words, linking parallel rivalrous markets together into a single loyalty program has no effect on spot prices or loyalty requirements.

We now turn to the loyalty program prices. The procedure for finding z_A^i and z_B^i is the same: firm A sets prices in a manner that prevents firm $B_j,\; j \le k$, from profitably offering a loyalty program that provides large consumers the same consumer surplus as in the proposed equilibrium. Equation (18), which compares a rival firm's equilibrium profits to the profits from its optimal deviation, does not

[16] For example, firm A may be a dominant airline at a hub airport while each rival serves only a single spoke from the hub.

change. This implies that z_B^i remains the same (and we can drop the superscript). The consumer surplus constraint (17), however, does change. If rival firms $B_1,...,B_k$ collectively offer a sufficiently attractive alternative to the proposed equilibrium, customers will abandon firm A in order to satisfy the program terms of $B_1,...,B_k$. But by assumption, customers that do not meet firm A's loyalty requirement in any market lose the discount in all N markets. Thus the lower prices offered by $B_1,...,B_k$ must compensate not only for the lost discount in those k markets, but also for the lost surplus in the remaining $N-k$ markets. Condition (17) becomes

$$\sum_{i=1}^{N} CS(\{z_A^i, h\}, p_B) \quad = \quad k\, CS(p_A, \{z_B, l\}) \;+\; (N-k)\, CS(p_A, p_B) \tag{27}$$

Letting $\bar{z}_A = \frac{1}{N}\sum z_A^i$ and dividing through by N, this constraint can be expressed as

$$CS(\{\bar{z}_A, h\}, p_B) \quad = \quad \frac{k}{N} CS(p_A, \{z_B, l\}) \;+\; \left(1-\frac{k}{N}\right) CS(p_A, p_B)\,. \tag{28}$$

Equation (28) shows that the average large consumer surplus from firm A's loyalty program is the weighted average of the consumer surplus from a rival program (offered by B_j, $j \le k$) and the lower consumer surplus from the higher spot prices in markets that cannot have a rival program (markets $k+1,...,N$). Observe that (28) matches (17) when $k = N$. In such cases, equilibria of the multiple market setting are simply N-fold copies of a corresponding equilibrium of the single market case. In general, as N increases or k decreases, additional weight is placed on the last term in (28) and firm A responds by raising its loyalty program prices. This is captured in our next result.

PROPOSITION 5: *As N increases or k decreases, the average loyalty program price* $\bar{z}_A$ *increases. This reduces large consumer surplus and overall consumer surplus.*

PROOF: As noted above, p_A, p_B, q_A, q_B, z_B, h, and l do not depend on k or N. Let $g = 1 - \frac{k}{N}$. To establish that increasing N or decreasing k reduces large customer surplus, it suffices to show that $(1-g)\,CS(p_A,\{z_B,l\})+g\;CS(p_A,p_B)$ is a declining function of g, or that $CS(p_A,p_B)-CS(p_A,\{z_B,l\})<0.$ Using (6) and

$$CS(p_A,\{z_B,l\})=\int_L^l (R-\tfrac{1}{2}x-p_A)\,f(x)\,dx \;+\; \int_l^H (R+\tfrac{1}{2}x-z_B)\,f(x)\,dx\,, \qquad (29)$$

we have

$$\begin{aligned}
&CS(p_A,p_B)-CS(p_A,\{z_B,l\})\\
&\qquad = \int_l^{p_B-p_A}(p_B-p_A-x)\,f(x)\,dx \;-\; (p_B-z_B)[1-F(h)]\\
&\qquad < (p_B-p_A-l)[F(p_B-p_A)-F(l)] \;-\; (p_B-z_B)[1-F(h)]\\
&\qquad = (p_B-p_A-l)[F(p_B-p_A)-F(l)] \;-\; (p_B-c)\left[\frac{F(h)-F(l)}{1-F(l)}\right]\\
&\qquad \le (p_B-c)[F(p_B-p_A)-F(l)] \;-\; (p_B-c)\left[\frac{F(h)-F(l)}{1-F(l)}\right]\\
&\qquad = (p_B-c)\left[F(p_B-p_A)-F(l)\;-\frac{F(h)-F(l)}{1-F(l)}\right]\\
&\qquad \le -(p_B-c)[F(h)-F(l)]F(l)/[1-F(l)]\le 0
\end{aligned}$$

The first inequality above follows from the integrand being bounded above by $(p_B-p_A-l)\,f(x).$ The next equality uses (22). The first weak inequality uses $l\ge c-p_A$ and $F(p_B-p_A)\ge F(l),$ and the last line uses $F(h)\ge F(p_B-p_A).$ Thus $CS(p_A,p_B)<CS(p_A,\{z_B,l\})$, and increasing N or decreasing k diminishes large customer surplus. Since small customer surplus is invariant, overall consumer surplus must also decline. Given that h and p_B are invariant to k and N, a fall in $CS(\{\bar{z}_A,h\},p_B)$ implies that $\bar{z}_A$ must have increased. □

Although this discussion presumes symmetric markets, all that is really needed is a lack of large customer heterogeneity. When elements like a large customer's preference distribution and purchase volume differ across markets, spot prices and loyalty requirements will match the case when firms compete in each market separately, though they may differ across markets. Adding a market that can offer a loyalty discount tightens the consumer surplus constraint in (17) by increasing the surplus available to large customers when they adopt the loyalty programs of firms $B_1,...,B_k$ instead of firm A. In contrast, adding a market that cannot offer a loyalty discount relaxes the consumer surplus constraint in (17) by increasing the surplus that customers forego when they abandon the dominant firm's program. The latter scenario allows the dominant firm to offer less attractive loyalty program discounts.

3.1 Exclusion: A First Look

Proposition 1 establishes that (short-term) profit-maximizing firms employ loyalty programs in equilibrium. While the overall surplus effects depend in part on brand preferences, one cannot conclude that such pricing strategies are necessarily anti-competitive. Dominant firms, however, may use loyalty programs with an eye toward weakening or excluding rivals.

For a fixed mass of large customers, a firm can increase its share from current loyal customers by raising the loyalty requirement. If the initial program has not left money on the table, then the firm must offer an improved loyalty discount if it hopes to retain customers while demanding increased loyalty. As long as the firm with the adopted loyalty program sells the favored product, its discounted price exceeds cost and there is room to cut price. That means that, at least for small changes, the new price is not predatory using an (Areeda-Turner) price versus marginal cost test. If the incremental discount is included in the incremental cost of serving the additional customer, however, then it is possible that the firm would fail a price versus incremental cost test. Specifically, for each additional purchase the firm generates by raising the loyalty program target, it must compensate customers with an incremental discount of $z_A + q_A - p_B$, otherwise customers will switch to firm B.[17] Including this incremental discount as a cost, the cost per purchase from an increase in the program target $(c + z_A + q_A - p_B)$ exceeds the price (z_A) whenever

[17] Switching one purchase from B (at p_B) to A (at z_A) reduces consumer surplus by $R + \frac{1}{2}q_A - p_B - R + \frac{1}{2}q_A + z_A = z_A + q_A - p_B$. Since firm A's equilibrium loyalty program generates the same consumer surplus as firm B's program, Firm A must compensate the consumer by this amount to retain the customer as a loyalty program participant.

$q_A > p_B - c$. This, according to Proposition 2, is the equilibrium share requirement when firms employ loyalty contracts to maximize short-term profits apart from an exclusionary motive. Therefore, this test is the proper tool to distinguish legitimate from excessive loyalty program sales targets.

This incremental cost test, with incremental discounts counted as costs, is essentially what Douglas Bernheim used as expert for the plaintiff in *Virgin Atlantic*. Thus our analysis provides an analytical justification for a tool that has already been introduced in antitrust litigation. The District Court in *Virgin Atlantic*, as affirmed by the Second Circuit, ultimately rejected the implementation of Bernheim's test, but did not reject the test itself.[18]

Virgin Atlantic, of course, arises in a multiple market context. Our analysis gives some sharp welfare results when loyalty contracts are extended from one market to multiple markets. If rivals in the added markets cannot offer loyalty programs, then in equilibrium customers receive less attractive loyalty discounts and consumer surplus drops. From that perspective, there does not appear to be any societal benefit from having loyalty contracts that link multiple markets. But the anti-competitive effect of linked loyalty contracts is limited to this increase in market power. Specifically, as the number of linked markets increases, there is neither an increase in the dominant firm's market shares nor any decrease in rival profits or market share. Loyalty contracts are not any more exclusionary when linked across multiple markets than when implemented in markets separately. Thus the Bernheim test would be the appropriate standard in the multiple market context as well: if a firm sets the target level so high that it loses money on incremental

[18] As an expert for the plaintiff in Ortho Diagnostic Sys. v. Abbott Lab., F. Supp. 145 (1993), Janusz Ordover advocated a similar test that includes rebates (or loyalty discounts) as an incremental cost. See Ordover and Willig (1981) for an introduction to their concept of "compensatory pricing", and Greenlee, Reitman, and Sibley (2004) and Nalebuff (2004b) for discussions of this price test applied to bundled rebates and several cases including *Ortho*.

sales, then there is a valid inference of exclusionary intent.[19] Otherwise the use of loyalty programs in single markets or multiple linked markets is consistent with short-term profit maximizing behavior.

4. Bundling Monopolists

Since the dominant firm benefits from linking its loyalty program across several markets, the next logical step is to consider whether linking a loyalty program to an unrelated market is beneficial as well. In the previous model the linked markets need not be for similar products or have common cost or production components, but they had two common features: (1) common (large) customers who participate in every market, and (2) the dominant firm faces a rival in each market. As long as competition in each market induces the dominant firm to offer a loyalty program in that market, it is (weakly) profitable to link the markets to common customers. Suppose now that the dominant firm sells one or more products as a monopolist and would not offer a loyalty discount unless it were linking together several products. Does the monopolist profit from offering a loyalty program that links monopolized and rivalrous markets?

Before characterizing loyalty programs that benefit the dominant firm, it is worthwhile briefly to examine a bundled discount that does not. Consider a firm that competes in a market like the one in Section 2, and is a monopolist in a second market. Suppose this firm offers a rebate in the form of a lump sum deduction off the bill for the monopolized product whenever the customer meets a loyalty target in the rivalrous market. This rebate augments the consumer surplus that customers can expect from meeting the loyalty program in (17). If each customer's demand in the rivalrous market is completely inelastic, then the other elements of the optimal contract are unaffected, so the net result is that the loyalty program price in the rivalrous market increases by an amount that exactly offsets the rebate. The fixed deduction simply transfers money paid in the monopoly market to the rivalrous market, with no effect on customer choices or effective prices. If market demand is

[19] Investing in technology to track sales to more customers is another way to increase loyalty program sales. Price/cost tests may not be useful to distinguish exclusionary from non-exclusionary motives for such strategies.

not completely inelastic, such a transfer harms the monopolist because customers' reduced purchases generate deadweight losses that cannot be recovered as profit.

The conclusion is quite different, however, when the monopolist offers customers a per unit discount on purchases of the monopoly good. In order to focus on this role for loyalty discounts, it helps to change several features of the model. We assume the rivalrous market has limited or no perceived product differentiation, so that firms would not profit from using a loyalty program apart from linking this product to the monopolized market. We assume now that the market demand curve is downward-sloping so that the disadvantage to bundling discussed above comes into play. Finally, when two of its products are substitutes, the dominant firm may have an additional incentive to link sales in order to rationalize the allocation of purchases across products.[20] To eliminate this potential advantage from bundling, we assume the products are in separate product markets.[21] Nevertheless, in keeping with recent case history, we refer to the monopolized product as the branded product, and the rivalrous market as the generic market.

The monopolist can try to limit competition in the generic market by linking generic sales to branded sales. Specifically, the monopolist can offer a program that provides a discount on branded purchases to customers that satisfy a loyalty requirement in the generic market. The results of Section 2 suggest that when there is limited product differentiation, the equilibrium loyalty program entails supplying the entire market. Thus, we start by assuming that the monopolist makes an all-or-nothing offer in the generic market, and later discuss what happens when the monopolist only requires customers to buy a predetermined share of their generic purchases from the dominant firm. The question is whether, or in what circumstances, the additional generic profits exceed the branded market discount.

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[20] In several of the legal cases that address loyalty programs that link monopolized and rivalrous products, the rivalrous product is a substitute for at least one of the monopolized products. For example, transparent tape in *LePage's* and cephalosporin antibiotics in *SmithKline*. SmithKline Corp. v. Eli Lilly & Co., 575 F.2d 1056 (3rd Cir. 1978).

[21] In *LePage's*, for example, 3M linked private label tape sales to discounts offered across a number of different product categories, including healthcare and retail automotive products.

Assume that demand in the branded market has two segments: large customers have demand $L(p)$, also buy the generic product, and are candidates for the loyalty program. Small customers, with demand $S(p)$, either do not buy generic products or are not candidates for a program. Generic market demand is given by $G(k)$, and is supplied by the monopolist and one or more competitors.[22] Define $e_I(y)$ to be the price elasticity of demand in market (segment) I at price y, and assume that each of these demand curves has the usual property that $e_I'(\cdot)<0$, *i.e.* the point elasticity of demand has greater magnitude at higher prices. Demand in each market segment is comprised of customers who each buy multiple units. To simplify the analysis, assume that the demand profile of individual customers across their various purchases is similar, so that aggregate demand can be treated as if it is supplied to a representative consumer. The monopolist produces branded products at marginal cost c, and all firms produce the generic product at marginal cost c_g.

The monopolist's profits in the generic market depend on such things as the number of competitors in that market, the nature of competition, and the perceived degree of product similarity. The monopolist can alter competition in the generic market by offering to supply all customer requirements at a discounted price without linking to the branded market. Like an auction for the entire market, competing all-or-nothing offers effectively transform the market into pure Bertrand competition. If customers do not prefer one supplier's generic product to another, then the equilibrium all-or-nothing price in the generic market is c_g. Absent scale economies, all-or-nothing offers for an undifferentiated product are not profitable.

Now suppose that the monopolist links sales in the branded market to the all-or-nothing offer in the generic market. Customers who accept the offer get a discount d off the regular price p on all branded market purchases, provided they agree to buy generic product exclusively from the monopolist at price k. In order to insure

[22] The model focuses on the generic product demand of customers who also buy the branded product. There may be additional sales to generic-only customers that are not included in the model and are not affected by loyalty discounts offered to customers who buy both products.

that customers buy generic product from the monopolist, customers must enjoy at least as much combined consumer surplus in the branded and generic markets as they would receive from buying branded products at the non-discounted price and obtaining generic product from a rival firm at price c_g. That is,

$$\int_p^\infty L(x)\,dx \;+\; \int_{c_g}^\infty G(x)\,dx \quad\le\quad \int_{p-d}^\infty L(x)\,dx \;+\; \int_k^\infty G(x)\,dx, \quad \text{or}$$

$$\int_{c_g}^k G(x)\,dx \quad\le\quad \int_{p-d}^p L(x)\,dx\,. \tag{30}$$

The monopolist solves

$$\max_{p,d,k}\ \Pi_M(p,d,k) = (p-c)\,S(p) + (p-d-c)\,L(p-d) + (k-c_g)\,G(k) \tag{31}$$

subject to (30). The necessary conditions for a solution are

$$S(p) + (p-c)S'(p) + (1-l)L(p-d) + (p-d-c)L'(p-d) + lL(p) = 0 \tag{32}$$

$$-(1-l)L(p-d) - (p-d-c)L'(p-d) \le 0 \tag{33}$$

$$(1-l)G(k) + (k-c_g)G'(k) = 0 \tag{34}$$

along with (30) and two complementary slackness conditions:

$$l\left\{\int_{c_g}^k G(x)\,dx \;-\; \int_{p-d}^p L(x)\,dx\right\} = 0, \text{ and} \tag{35}$$

$$d\,\{(1-l)L(p-d) + (p-d-c)L'(p-d)\} = 0, \tag{36}$$

where $l \ge 0$ is a Lagrange multiplier.

PROPOSITION 6: *In equilibrium, the monopolist offers an all-or-nothing program that includes a positive discount. Namely, the solution to (31) has* $d > 0$.

PROOF: The monopolist never sets $d<0$, since customers will ignore the "discount" and buy at the posted price, p. Suppose $d=0$. From (30), $k=c_g$, and so $l=1$ from (34). Substituting $l=1$ into (32) implies that $p>c$. But then the left hand side of (33) is positive, which is a contradiction. Therefore $d>0$. □

Intuitively, a small discount below the monopoly price causes only a second order loss in the monopolist's profits from branded sales to large customers, but a first order gain in profits in the generic market, and so is always advantageous.[23] Since $d>0$, (36) requires that (33) holds with equality. Therefore, substituting (33) into (32) yields

$$S(p)+(p-c)S'(p)+l\,L(p)=0. \tag{37}$$

Letting $m_I(y)$ be the markup in market (segment) I at price y, (33) can be written as

$$l-1 \;=\; \frac{(p-d-c)L'(p-d)}{L(p-d)} \;=\; m_L(p-d)\,e_L(p-d), \tag{38}$$

while (34) can be expressed as

$$l-1 \;=\; \frac{(k-c_g)G'(k)}{G(k)} \;=\; m_G(k)\,e_G(k). \tag{39}$$

Combining (38) and (39),

$$m_L(P-d)\,e_L(p-d) \;=\; m_G(k)\,e_G(k). \tag{40}$$

The monopolist's optimal all-or-nothing discount program is characterized by (30), (35), (37), (40), and either (38) or (39).

According to Proposition 6, the monopolist always offers large customers a discount off the branded price charged to small customers. What happens to small customers when the monopolist introduces a loyalty program? Analogous to

[23] Burstein (1960) and Mathewson and Winter (1997) establish a similar result for tying.

Proposition 3, the next result establishes that the monopolist increases the spot price in order to accommodate the discount. Let p^o denote the equilibrium monopoly price in the branded market when there is no discount program.

PROPOSITION 7: *The optimal branded spot price when using an all-or-nothing discount program in the generic market has* $p > p^o$.

PROOF: The branded price with no discount program is determined by the first order condition

$$S(p^o) + (p^o - c)S'(p^o) + L(p^o) + (p^o - c)L'(p) = 0. \tag{41}$$

Substituting (38) into (37), the first order condition for p is

$$S(p) + (p - c)S'(p) + L(p) + L(p)\, m_L(P - d)\, e_L(p - d) = 0. \tag{42}$$

Evaluating the left hand side of (42) at p^o and using (41) gives

$$L(p^o)\left(m_L(p^o - d)\, e_L(p^o - d) - m_L(p^o)\, e_L(p^o)\right). \tag{43}$$

Now by assumption $e_L(p^o) < e_L(p^o - d) < 0$, and $m_L(p^o) > m_L(p^o - d) > 0$, so (43) is positive which means that $d\Pi_M / dp > 0$ at $p = p^o$. Thus $p > p^o$ at the optimum. □

The discount linkage between the branded market and the generic market spills over to small customers even though they only buy branded products. The linkage unambiguously harms small customers because they pay higher prices. Large customers get a discount off the branded price, and may appear to benefit, but that benefit is illusory. Paying more in the generic market offsets the branded market discount. At the optimum, condition (30) typically holds with equality. This ensures that large customer surplus matches what they would obtain from paying the undiscounted price in the branded market and buying the generic product from a rival producer. Thus large customers, like small customers, are harmed because $p > p^o$. Their net benefit relative to the equilibrium with no discount program depends on the competitiveness of the generic market. If $k = c_g$, then large customers are unambiguously harmed.

The preceding discussion asserted that (30) typically holds with equality. The next proposition identifies conditions under which that is true, and further characterizes optimal prices. To do this, it is helpful to define monopoly prices when the monopolist can set prices to each segment separately. Let p_S^o, p_L^o, and k^o denote respectively the monopoly prices in the small branded, large branded, and generic market segments, and define $\Gamma(y) = \int_{p_L^o}^{y} L(x)\,dx - \int_{c_g}^{k^o} G(x)\,dx.$

PROPOSITION 8: *The equilibrium all-or-nothing discount program prices satisfy:*

(i) *If* $p_S^o \le p^o$, *then* $p > p_S^o$, $p - d < p_L^o$, *and* $k < k^o$.

(ii) *If* $p_S^o > p^o$ *and* $\Gamma(p_S^o) < 0$, *then* $p > p_S^o$, $p - d < p_L^o$, *and* $k < k^o$.

(iii) *If* $p_S^o > p^o$ *and* $\Gamma(p_S^o) \ge 0$, *then* $p = p_S^o$, $p - d = p_L^o$, *and* $k = k^o$.

PROOF: (i) By assumption $p_S^o \le p^o$, and so, using Proposition 7, $p > p_S^o$. Since p_S^o maximizes profits in the S market segment and $p > p_S^o$, it follows that $S(p) + (p-c)S'(p) < 0$. Therefore, from (37), $l > 0$, which implies, using (38), that $m_L(P-d)\,e_L(p-d) > -1$. Since $m_L(\cdot)$ is increasing, $e_L(\cdot)$ is negative and decreasing, and $m_L(p_L^o)\,e_L(p_L^o) = -1$, it follows that $p - d < p_L^o$. Similarly, using (39), $m_G(k)\,e_G(k) > -1$, and thus $k < k^o$. (iii) If $l = 0$, then (37), (38), and (39) imply, respectively, that $p = p_S^o$, $p - d = p_L^o$, and $k = k^o$, and (40) is satisfied. If $\Gamma(p_S^o) \ge 0$, then at these prices, (30) and (35) are satisfied as well. Thus all of the conditions for an optimum are met. (ii) Suppose, as in (iii), that $l = 0$, which implies that $p = p_S^o$, $p - d = p_L^o$, and $k = k^o$. But then $\Gamma(p_S^o) < 0$ means that (30) is violated at those prices. Thus we must have $l > 0$, which implies that $p - d < p_L^o$ and $k < k^o$. Moreover, from (37), $S(p) + (p-c)S'(p) < 0$, which implies that $p > p_S^o$. □

According to this proposition, the only circumstance in which (30) does not bind occurs when the monopolist charges the monopoly price in each product segment. In that situation, customers taken together are clearly harmed by the introduction of an all-or-nothing loyalty program. Otherwise condition (30) holds with equality.

Proposition 8 highlights an additional motivation for the monopolist to link the branded and generic markets with a discount program, which is to price discriminate among branded customers. By giving large customers a discount, the monopolist effectively introduces separate prices in the two branded market segments. If small customers have more elastic demand than large customers, as in Proposition 8.i, then the discount works against the optimal price discrimination scheme. Nevertheless, the monopolist still profits from charging a lower price to large customers than to small customers. Recalling that large customers are defined as those who can be identified as buying the generic product, it is arguably more plausible that small customers have less elastic demand than large customers. Generic buyers may be inherently more price sensitive for the product overall, and may also have more elastic demand for the branded product because they have access to the generic alternative. If small customers have more inelastic demand, then the discount program not only allows the monopolist to extract profits from the generic market, but also to gain additional profits by price discriminating among branded customers. A loyalty program can also provide a means to distinguish the various demand segments—customers who enroll in the program are generic buyers and are likely to have more elastic demand.

The next two results examine how prices change as the relative sizes of the large and small customer segments of the branded market change. First suppose that relatively few branded customers also buy the generic good. Specifically, define $P_S = \{y \mid S(y) > 0\}$ and $a = \sup_{y \in P_S} [L(y)/S(y)]$. With this, we have the following result.

PROPOSITION 9: *In the limit as* $a \to 0$, $p = p^o$.

PROOF: Equation (37) can be rewritten as

$$0 \le -1 - m(p)e_S(p) = l\,\frac{L(p)}{S(p)} \le l\,a\,. \tag{44}$$

As $a \to 0$, (44) implies that $m(p)e_S(p) \to -1$, which in turn implies that $p \to p_S^o$. Also, as $a \to 0$, (41) implies that $p^o \to p_S^o$. Thus $p \to p^o$. □

The loyalty program has a negligible effect on the branded price when few branded customers also buy the generic good. The monopolist, however, still offers a branded discount to generic customers, the size of which depends on the relative demand for the two products among the customers that buy both.

Now consider the opposite extreme, in which almost all branded customers also buy the generic product, and define $P_L = \{y \mid L(y) > 0\}$, $b = \sup_{y \in P_L}[S(y)/L(y)]$, and $\overline{p}_S = \inf_{y} \{y \mid S(y) = 0\}$.

PROPOSITION 10: *In the limit as* $b \to 0$,

(i) *If* $\Gamma(\overline{p}_S) < 0$, *then* $p \to \overline{p}_S$, $p - d < p_L^o$, *and* $k < k^o$.

(ii) *If* $\Gamma(\overline{p}_S) \ge 0$ *and* $\Gamma(p_S^o) < 0$, *then* $p \to p^*$ *where* $\Gamma(p^*) = 0$, $p - d \to p_L^o$, *and* $k \to k^o$.

(iii) *If* $\Gamma(p_S^o) \ge 0$, *then* $p \to \overline{p}_S$, $p - d = p_L^o$, *and* $k = k^o$.

PROOF: Now equation (37) can be rewritten as

$$\frac{S(p)}{L(p)}\left[1 + m_S(p)e_S(p)\right] + l = 0\,. \tag{45}$$

(i) Note that (39), the non-negativity of l, and the increasing elasticity assumption together imply that $k \le k^o$. Since in this case $\Gamma(\overline{p}_S) < 0$, (30) requires that $p - d < p_L^o$, which from (38) means that $l > 0$. Therefore, as $b \to 0$, (45) implies that $e_S(p) \to -\infty$, which in turn implies that $p \to \overline{p}_S$. Also, as in Proposition 8.i, when $l > 0$, $k < k^o$. (ii) With $\Gamma(\overline{p}_S) = 0$, then in the limit with $l \to 0$, $p \to \overline{p}_S$, $p - d \to p_L^o$, and $k \to k^o$, conditions (30) and (45), along with all the other conditions for an optimum, are satisfied. Similarly, if $\Gamma(\overline{p}_S) > 0$, then when $p \to p^*$ all the conditions for optimality are satisfied in the limit. (iii) First note that $p^o \to p_S^o$ as $b \to 0$. With $\Gamma(p_S^o) \geq 0$, all the conditions for an optimum are satisfied with $b \to 0$ as in proposition 9.iii. □

As small customers become less significant, the monopolist raises the spot price in order to make the discount offered to large customers more compelling. In the limiting case, the monopolist uses all the consumer surplus in the branded market to leverage a higher price in the generic market.[24]

The results thus far have addressed the optimal prices in an all-or-nothing discount program. The monopolist's decision to use such a program, however, depends on the price and share it would receive in the generic market when the products are sold separately. Based on the preceding results, we can draw some general conclusions. First, if the generic market were competitive, so that the market price $\overline{k} = c_g$ in the absence of a loyalty program, then the monopolist always benefits from offering an all-or-nothing discount program. In fact, even if the rival firms have a slight cost advantage over the monopolist, the monopolist still finds it profitable to use a loyalty program, and its more efficient rivals are excluded. Second, for any $\overline{k}$, if the monopolist's share at that price is sufficiently low, then the monopolist also profits from introducing a discount program. In some situations when the monopolist would like to price discriminate among branded customers based on their use of the generic product, the monopolist can obtain monopoly profits in both the branded and generic markets by using a loyalty discount program. Finally, the monopolist tends to benefit from using loyalty discounts if (1) branded customers who do not buy the generic product have more inelastic demand for the branded product than customers who buy both generic and branded products, and

[24] For further discussion, see Greenlee, Reitman, and Sibley (2004), and Nalebuff (2004a).

(2) either there are relatively few exclusively branded customers, or customers who buy both products have larger demand for the branded product.

There are two distinct components to the monopolist's profit gain from using a loyalty program to link a rivalrous market to its monopolized market. First, the monopolist can introduce a gap between the monopolized product price for program participants and non-participants in order to extract additional profits from customers who buy both products. Second, the monopolist can offer a discount on the bundled product that is worth more to customers than it costs the monopolist (because of increased sales), and thereby extract additional profits in the linked market. Both components rely on the same mathematical property: the optimal branded product price without loyalty discounts satisfies a first order condition, so small changes in the price have no first order effect on profits, but do have a first order effect on consumer surplus. One can construct variations of this model in which only one of the effects is present. Thus even if customers do not make repeat purchases, the monopolist still profits from introducing a discount program (in effect, tying) that introduces a wedge between the participant price and the non-participant price. Alternatively, even if customers eligible for the loyalty program are of negligible size relative to the total market for the branded product so that, as shown in Proposition 9, the spot price for the branded product does not change, the monopolist still benefits from using a loyalty program.

The model implicitly assumes that the only alternatives available to the dominant firm are to use a loyalty discount that links the branded market to the entire generic market, or else to operate completely separately in the two markets and charge a single price in the branded market. In equilibrium, the dominant firm establishes a loyalty program in large part to extract additional profits in the branded market. Alternative pricing strategies, however, may be available for similar rent extraction. As discussed above, one advantage of the loyalty program is that it operates as a segmenting device that enables price discrimination across branded customers. When exclusively branded customers have more inelastic demand, this tool increases profits. However, if alternative means to distinguish customers exist, then the gains from using a loyalty program are diminished.

When customers make multiple purchases, the monopolist can use a two-part tariff in the branded market. In the extreme case of no uncertainty or heterogeneity in demand, or if a program can be tailored to each customer, the monopolist prices at marginal cost and extracts all consumer surplus via a fixed fee.[25] In such cases, there is no incentive to link a loyalty program to the generic market because any profit gained in the generic market would be sacrificed in the branded market, and the monopolist could not recover the deadweight loss created in the generic market. Moreover, the monopolist cannot use the loyalty program to threaten to charge a higher price in the branded market, because the monopolist already extracts all surplus—at a higher price, the customer makes no branded purchases.

Alternatively, suppose that the monopolist cannot extract all surplus from consumers through a two-part tariff. Specifically, assume that there is demand uncertainty or unobservable heterogeneity among customers so that the monopolist maximizes an average across customer types. The familiar "Mickey Mouse Monopolist" result (Oi 1971) is that, with some restrictions on demand across types, the monopolist charges a per unit price above marginal cost along with a fixed fee. In this setting, our analysis goes through unaltered. Namely, the monopolist still has an incentive to offer a discount to branded customers that make all of their generic purchases from the monopolist. The only difference is that the discount applies to a price that satisfies the first order condition for the monopolist's optimal two-part tariff, rather than the monopoly price.

The model assumes that the monopolist can still offer a branded good discount that restores the customer surplus lost through a higher generic price. This appears to contradict the assumption of uncertainty or unobservable heterogeneity. Implicitly, this interpretation of the model requires the demands in the branded and generic markets to be correlated. As long as sales for the two products move together, the monopolist does not need to know the scale of demand in order to introduce a loyalty program that equates changes in consumer surplus across the branded and generic markets. This suggests that the loyalty program will not be

[25] Incidentally, the fixed fee could be implemented through a discount program similar to a requirements contract, but in which the discount does not apply to every unit purchased.

implemented across products whose demand is completely independent. Even in a case like *LePage's*, in which transparent tape sales were linked to rebates on categories like automotive and healthcare products, it is plausible that the demands were correlated in a dimension unobservable to 3M because the purchases were made at the same stores. That "same store correlation" could provide sufficient incentive to implement at least modest discounts. It also suggests that linking sales across multiple product categories, in order to better distinguish store-specific from product-specific demand shocks, may be profitable for the firm.

4.1 Exclusion Revisited

In equilibrium, rival firms are completely excluded from generic product sales, at least to branded product customers. This equilibrium, however, can be implemented by requiring only a partial share of generic purchases and adjusting the price accordingly. For example, if the equilibrium price k is one dollar over marginal cost, the monopolist could require that large customers make half of their generic purchases from the monopolist at two dollars over marginal cost. The customer's marginal purchase decision is the same, since for every two additional generic units purchased, one must be from the monopolist. In this way, the equilibrium can be implemented with any fixed share requirement, and both the monopolist and the customer are as well off. This holds as long as rival firms do not make profitable sales. If rival firms can price above cost when the monopolist uses a share requirement below 100 percent, then the monopolist benefits from excluding them completely. Alternatively, if rival firms have low marginal costs and would sell at a price below the monopolist's cost, then the monopolist has an incentive to give rival firms a share of the market. Doing so lowers customers' average purchase price for generic products and increases the monopolist's profits.

Instead of using a share requirement, the monopolist could instead require a given volume of generic purchases to qualify for the discount. If rival firms sell the residual at (or close to) marginal cost, then the monopolist actually prefers a volume requirement because it reduces the deadweight loss in the generic market, and increases the consumer surplus that can be captured through the linked volume discount. Of course, if the monopolist can use volume targets, implying that there is little uncertainty about customer types, then it may be able to extract profits using alternative pricing methods, as discussed earlier.

Since complete exclusion is a natural outcome of this model, it is not clear what it means to talk about too much exclusion. Nevertheless, one can still apply the incremental cost test discussed earlier, particularly if the increment can be defined relative to a time period before the monopolist linked the markets together in a loyalty program. The result depends on the implementation of the test. One could look at the incremental generic sales to customers who buy both products, where one of the costs is the branded discount offered to these customers. Letting $\overline{G}$ denote the monopolist's equilibrium output prior to implementing a loyalty program, one could compare $(G(k)-\overline{G})(k-c_g)$ to $d\,L(p-d)$. Observe, however, that

$$(G(k)-\overline{G})(k-c_g) \le \int_{c_g}^{k} G(x)\,dx \le \int_{p-d}^{p} L(x)\,dx \le d\,L(p-d) \tag{46}$$

where the middle inequality is from (30). Thus the monopolist would always fail this implementation of the test. If additional branded sales, to large customers or to both large and small customers, are included in incremental sales then the inequality can be reversed. It is not clear, however, whether any such calculation indicates that the loyalty discount has crossed over from legitimate competition to unlawful exclusion.

The incremental sales test, however, can be used as a safe harbor. That is, suppose that the monopolist's pricing causes the variable profits it earns on generic sales to exceed the total discounts for branded purchases. The next proposition establishes that such prices could not exclude an equally efficient generic rival.[26] Since we prove the result under slightly more general conditions (*e.g.* the branded and generic products can be substitutes) and introduce a fair amount of additional notation, the proof appears in the Appendix.

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[26] For additional discussion of price tests for bundled rebates, including ones based on consumer surplus rather than excluding equally efficient competitors, consult Greenlee, Reitman, and Sibley (2004), and Nalebuff (2004a, 2004b).

PROPOSITION 11: *If the monopolist offers a loyalty program in which its revenues from generic sales exceed the variable cost of generic sales plus the discount offered on branded sales, then there exists a rival price for generic goods r, with* $r > c_g$*, such that large customers would forego the loyalty discount and make all generic purchases from a rival charging r.*

PROOF: See Appendix.

The model necessarily omits other factors that affect whether it is efficient for customers to sole source. For example, scale economies in manufacturing may make dual sourcing undesirable. In addition, costs or network benefits on the customer side may favor buying from a single supplier. Alternatively, customers may have diverse preferences across the product offerings of several competitors. As the model of Section 2 demonstrates, however, such diverse preferences do not prevent firms from offering a loyalty program that covers all but the tails of the distribution.

5. Conclusions

Once loyalty programs are added to the arsenal of competitive tools available to firms in a differentiated-product, price-competitive market, they take on a prominent role. In a single market, loyalty discounts typically displace any equilibrium in which firms compete just by setting spot prices. Multi-product firms from whom customers buy products in several parallel markets have an incentive to offer loyalty discounts that link purchases across markets. And finally, a monopolist in one market that competes in a second market with common customers may have an incentive to offer a loyalty discount across both markets in order (1) to squeeze additional profits out of the monopolized market, and (2) to gain additional sales in the rivalrous market. In each case, the equilibrium is characterized by aggressive pricing to loyalty program participants, higher prices to other customers, and significant share shifts to the firm whose loyalty program is adopted in equilibrium. In the single market and parallel market cases, a simple incremental cost test can distinguish competitive from intentionally exclusionary behavior.

6. Appendix

Proof of Proposition 1.

(i) If large customers satisfy both programs, then the equilibrium spot prices satisfy (4), that is $p_A = p_A^*$ and $p_B = p_B^*$. First, we must have $z_A \le p_A^*$ and $z_B \le p_B^*$, or else customers reject the loyalty program and purchase at the spot price. In addition, if both loyalty programs can be satisfied simultaneously, then we must have $q_A \le q_B$ and large customers pay a discounted price for every purchase.

Let $r_i(\cdot)$ denote firm i's reaction curve in the Bertrand game without loyalty programs. We first establish that $0 < r_i'(\cdot) < 1$ for $i \in \{A, B\}$. Let $H_A(x) = F(x)/f(x)$ and $H_B(x) = (1 - F(x))/f(x)$. Using (3), firm A's reaction function satisfies $r_A(p_B) = c + H_A(p_B - r_A(p_B))$. Differentiating with respect to p_B and rearranging yields $r_A'(p_B) = \dfrac{H_A'(p_B - r_A(p_B))}{1 + H_A'(p_B - r_A(p_B))}$. Assumption 1 states that $H_A'(\cdot) > 0$ and this implies that $0 < r_A'(\cdot) < 1$. Similarly, firm B's reaction function satisfies $r_B(p_A) = c + H_B(r_B(p_A) - p_A)$. Differentiating with respect to p_A and rearranging yields $r_B'(p_A) = \dfrac{-H_B'(r_B(p_A) - p_A)}{1 - H_B'(r_B(p_A) - p_A)}$. Assumption 1 states that $H_B'(\cdot) < 0$ and this implies that $0 < r_B'(\cdot) < 1$.

Moving on, $r_i'(\cdot) < 1$ implies that $r_A(p_B^*) - r_A(z_B) < p_B^* - z_B$, or rewriting, $z_B - r_A(z_B) < p_B^* - p_A^*$. Similarly, $r_B'(\cdot) < 1$ implies that $r_B(z_A) - z_A > p_B^* - p_A^*$. We consider in turn the three possible orderings of q_A, q_B, and $p_B^* - p_A^*$. Assume first that $q_A < p_B^* - p_A^* < q_B$. If $z_B - z_A < p_B^* - p_A^*$, then firm B has an incentive to raise z_B because its best response satisfies $r_B(z_A) - z_A > p_B^* - p_A^*$. Similarly, if $z_B - z_A > p_B^* - p_A^*$, then firm A has an incentive to raise z_A because its best response satisfies $z_B - r_A(z_B) < p_B^* - p_A^*$. Thus the only possible equilibrium satisfying $q_A < p_A^* - p_B^* < q_B$ has $z_B - z_A = p_B^* - p_A^*$. Given that the slopes of both reaction functions are less than one, the unique solution is $z_A = p_A^*$ and $z_B = p_B^*$.

Summarizing, if $q_A < p_B^* - p_A^* < q_B$ in equilibrium, then neither firm offers an actual discount. Now assume that $q_A \ge p_B^* - p_A^*$ and consider in turn the possible orderings of q_A and $z_B - z_A$. If $z_B - z_A > q_A$, then $z_B - z_A > p_B^* - p_A^*$ and firm A has an incentive to raise z_A because its best response satisfies $z_B - r_A(z_B) < p_B^* - p_A^*$. If $z_B - z_A = q_A$, then all purchases made at firm A are for customers that strictly prefer firm A's product at z_A rather than firm B's product at z_B. Thus firm A can profitably increase z_A without causing large customers to abandon the program. If $z_B - z_A < q_A$, then firm A's loyalty program is strictly binding. Sales made by firm B are to customers that strictly prefer B to A at prevailing prices (z_A, z_B), so firm B can profitably increase z_B without losing any customers. This establishes that whenever $q_A \ge p_B^* - p_A^*$, one firm always has an incentive to increase its loyalty discount price z_i. Thus for this case we must have $z_A = p_A^*$ or $z_B = p_B^*$. From above, $r_A'(\cdot) < 1$ implies $z_B - z_A < p_B^* - p_A^*$. Together with $z_A \le p_A^*$, this implies that $z_B < p_B^*$ and therefore $z_A = p_A^*$. Summarizing, an equilibrium with both loyalty programs satisfied and $q_A \ge p_B^* - p_A^*$ can only exist if $z_A = p_A^*$. Finally, for $q_B \le p_B^* - p_A^*$, a similar argument establishes that an equilibrium with both programs satisfied exists only if $z_B = p_B^*$.

(ii) Mathematically, we show that whenever neither loyalty program is adopted, firm A has a profitable deviation that includes an accepted loyalty program. (A similar calculation establishes a parallel result for firm B.) Suppose that large customers allocate purchases with $x \le q$ to firm A at price y_A and buy from firm B at y_B when $x > q = y_B - y_A$ with $L < q < H$, $y_A > c$, and $y_B > c$. We show that there exists a loyalty program $\{\hat{z}_A, q_A\}$ with $q < q_A \le \min\{H, y_B - c\}$ such that (a) $CS(\{\hat{z}_A, q_A\}, y_B) = CS(y_A, y_B)$, and (b) $\Pi_A(\{\hat{z}_A, q_A\}, y_B) > \Pi_A(y_A, y_B)$. Condition (a) can be written as:

$$\int_L^q (R - y_A - x/2)\, f(x)dx \;+\; \int_q^H (R - y_B + x/2)\, f(x)dx \;=\; \int_L^{q_A} (R - \hat{z}_A - x/2)\, f(x)dx \;+\; \int_{q_A}^H (R - y_B + x/2)\, f(x)dx, \text{ or}$$

$$(\hat{z}_A - y_B)F(q_A) \ = \ (y_A - y_B)F(q) - \int_q^{q_A} x \ f(x)dx \,. \tag{A1}$$

Let $\hat{z}_A$ satisfy (A1). Now, writing out the expressions for firm A profits,

$$\begin{aligned}\Pi_A(\{\hat{z}_A, q_A, y_A\}, y_B) &= q(p_A - c)\,F(p_B - p_A) + (\hat{z}_A - c)\,F(q_A) \\ &= q(p_A - c)\,F(p_B - p_A) + (y_B - c)\,F(q_A) + (y_A - y_B)\,F(q) - \int_q^{q_A} x\, f(x)\,dx\end{aligned}$$

and $$\Pi_A(y_A, y_B) = q(p_A - c)\,F(p_B - p_A) + (y_A - c)\,F(q)$$

Taking the difference,

$$\begin{aligned}\Pi_A(\{\hat{z}_A, q_A, y_A\}, y_B) - \Pi_A(y_A, y_B) &= (y_B - c)[F(q_A) - F(q)] - \int_q^{q_A} x\, f(x)\,dx \\ &= \int_q^{q_A} (y_B - c - x)\, f(x)\,dx \, > \, 0,\end{aligned}$$

since the integrand is positive between q and q_A. Thus $\{\hat{z}_A, q_A\}$ also satisfies (b).

(iii) Follows directly from (i) and (ii). □

Proof of Proposition 11.

We generalize the demand characterization from the model in the text by assuming that each customer has a multitude of potential purchases characterized by b, the value of buying the branded good for that purchase, and g, the value of buying the generic product for that purchase. Let $F(b, g)$ describe the joint distribution over the consumer's potential purchases. For simplicity, the generic products from the monopolist and from rivals are regarded as perfect substitutes, although this is not critical for the result. The customer allocates potential purchases to the branded product, to the generic product, or to no product. As before, let p denote the branded product spot price, d the loyalty discount on branded sales for customers that purchase the target level of generic product from the monopolist at price k, and c_g the monopolist's marginal cost for the generic product.

Suppose the customer switches from participating in the loyalty program to buying branded products at the spot price and buying all of its generic requirements from a rival at price $r > c_g$. We can partition the set of potential purchases based on the allocation made under the two pricing schemes ("before" refers to purchases made under the loyalty program, "after" refers to purchases made at spot prices):

N = the number of potential purchases for which the customer bought neither product before, and then bought the generic product from the rival afterward.

G = generic purchases made from the monopolist before and from the rival after.

R = generic purchases made from the rival before and after.

S = branded purchases before, and generic purchases after.

B = branded purchases both before and after.

X = branded purchases before, and neither branded or generic after.

Z = neither branded or generic purchase before or after.

We need two additional technical assumptions. First, there are potential purchases in each of these partitions. Second, when the customer gets the same surplus from either buying or not buying the product at given prices, we assume that the customer either always prefers to buy or else always prefers not to buy.[27]

Given this partition, the safe harbor rule can be characterized as

$$(k - c_g) G \geq d\,(S + B + X). \tag{A2}$$

Note that the safe harbor is an *ex post* test. Namely, whether or not it is satisfied depends on the number of purchases made of each product. The main result here is that, if the safe harbor is satisfied, then a rival generic firm that prices sufficiently

[27] Making the lexicographic preference assumption avoids having all customers indifferent between two options, so that the surplus gain is zero even when the rival prices at marginal cost. Alternatively, one can assume that some customers are not on a boundary between partition regions, which would hold if the density function is positive over a range of reservation values.

close to the monopolist's marginal cost can always induce customers to reject the monopolist's loyalty program.

The customer will abandon the loyalty program if buying under the spot prices yields a higher surplus. It is sufficient to look at the surplus change for each of the purchase groups identified above.

N – The average gain in surplus is v, with $0 \le v \le k - r$, so that the total gain in surplus is $v N$.

G – The gain is $k - r$ per purchase.

R – There is no change in surplus.

S – The surplus before was $b - p + d$, and after is $g - r$ with $g - r \ge b - p$. Therefore the average change in surplus per purchase is $g - r - b + p - d = s - d$, with $s \ge 0$.

B – The change in surplus is $-d$ per purchase.

X – The change in surplus is $m - d$, with $0 \le m \le d$.

Z – There is no change in surplus.

Thus the total change in surplus from dropping the loyalty program is

$$\begin{aligned} \Delta CS &= v N + (k - r) G + (s - d) S - d B + (m - d) X \\ &= v N + s S + m X + (k - r) G - d (S + B + X) \end{aligned} \tag{A3}$$

Substituting (A2) into (A3),

$$\Delta CS \ge v N + s S + m X - (r - c_g) G$$

Note that v, s, and m are all non-negative. If customers buy when the surplus is the same, then $m > 0$. Conversely, if customers do not buy when the surplus from buying is zero, then $s > 0$. Together with the assumption that each component of the partition is non-empty, this implies that $\Delta CS > 0$ for r sufficiently close to c_g and thus the customer will abandon the program.

Imposing additional quantity restrictions on the loyalty program, like requiring each customer to buy a minimum quantity of both products, lowers the customer's surplus from participating in the program. Therefore, the customer benefits even more from abandoning such programs, and the result still holds.

7. References

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