Fitting Observed Inflation Expectations

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VERY PRELIMINARY AND INCOMPLETE

Abstract

We compare the fit of two variants of a standard Christiano et al. (2005)/ Smets and Wouters (2003) -type DSGE model, one where agents have perfect information about the value of the policymaker's inflation target, and one where they need to infer this value from changes in interest rates as in Erceg and Levin (2003). We find that a standard set of macro variables is unable to discriminate among the two models. Observed inflation expectations provide strong evidence as to which model fits the data best: the perfect information.

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1 Introduction

This paper uses inflation expectations as an observable in the estimation of a DSGE model, along with a standard set of macro variables. Observed inflation expectations are rarely included among the observables in estimating DSGE model, but arguably contain information that is valuable in discriminating across models. We compare the fit of two variants of a standard DSGE model with with several nominal and real rigidities along the lines of Christiano et al. (2005), Smets and Wouters (2003), and Smets and Wouters (2007), where the different lies in the agents' information set. In the first model (Perfect Information) agents have perfect information about the Central Bank's inflation target, while in the second model (Imperfect Information) agents need to infer the target from the behavior of interest rates, as in (Erceg and Levin (2003)). We find that a standard set of macro variables over a standard estimation period (the post-Volcker disinflation period: 1982Q2-2008Q2) is unable to discriminate among the two models. Observed inflation expectations instead provide strong evidence as to which model fits the data best. This is perhaps the least plausible of the two models: the Perfect Information one. We provide evidence that the relative failure of the Imperfect Information model to fit observed inflation expectations is due to the fact that this model imposes much more stringent cross-equation restriction on the law of motion of the perceived inflation target than the Perfect Information model.

There are several reasons for including measured inflation expectations among the set of observables in the estimation of DSGE models. First, as our study shows, inflation expectations help discriminate across models, especially when these models differ in the way agents form expectations. Yet observed expectations are rarely formally used in previous literature, even when comparing rational expectations with learning models (e.g., Milani (2007); a recent paper by Ormeno (2009) is an exception). In fact, we know very little on the extent to which DSGE models can accurately describe the behavior of observed inflation expectations. Second, inflation expectations are allegedly important in determining the term structure of interest rates. While this paper makes no attempt to explain the term structure directly since we use a linear model, the model comparison exercise done here can be helpful indirectly in investigating how one should formulate expectation formation.[1] Models that have a hard time generating observed inflation expectations may not be too helpful in understanding the term structure of interest rates. A third reasons to add observed expectations (for inflation as well as other variables) to the econometrician's information set is that agents in the real economy have a richer information set than the econometrician using a standard set of macro variables. Including measured expectations among the observables is a way to exploit such information set.[2] This information can be exploited for both forecasting and estimating latent variables, such as shocks. We show for instance that the estimated process for the inflation target changes whether we include or not inflation expectations among the observables.

There are several issues with using measured expectations as observables in DSGE models, which we discuss in section 4: data revisions, timing, choice of the expectation measures. This paper shows that the results are robust to different choices of measurement and timing assumptions, but does not really address many of these difficult issues. By pointing out the information content from measured expectations, we hope we have shown that it is worthwhile for future research to address these issues more thoroughly than we have. Also, there are several other mechanism of expectation formations, notably learning, that we do not consider in this paper. It is interesting to ask whether learning models provide a better description of observed inflation expectations than rational expectation models (Ormeno (2009) contains some preliminary results on this question).

Our results, while negative for the Imperfect Information model, are not necessarily in contrast with Erceg and Levin (2003)'s. Erceg and Levin (2003) focus

[1] There are attempts to use DSGE models to explain the terms structure, e.g. Rudebusch and Swanson (2008).

[2] Following the FAVAR methodology (Bernanke et al. (2005)) there are some attempts to combine factor and DSGE models with the goal of incorporating as much of the available data as possible (Boivin and Giannoni (2006), Giannone et al. (2008)). We take a different route and incorporate this information by adding agent's expectations to the list of observables.

on the Great Deflation ($81-85$), while we are interesting in assessing which model best describes the evolution of inflation expectation in the post deflation period – a period where allegedly the policy regime has not changed. Investigating the Great Deflation, while very interesting, involves issues of changes in policy regimes that we do not address at this stage.

The next section briefly discusses the econometric framework for evaluating how a model estimated to fit a baseline set of time series – here, the standard macro variables – fares in fitting an additional time series – here, inflation expectations. This is a straightforward application of Bayesian updating, which is routinely done in the DSGE estimation literature in the time series dimension, to the cross-sectional dimension. Section 3 describes the model, with particular emphasis on the difference between perfect and imperfect information. Section 5 discusses our findings.

2 Training-Sample Priors in the Cross-Section

A natural question in the DSGE model estimation literature is the following: How does a model that is estimated to fit time series $y^{1,T}$ through $y^{J,T}$ fare in fitting time series $y^{J+1,T}$ through $y^{J+K,T}$ (where $y^{i,T} = \{y_t^i\}_{t=1}^T$)? In this paper, for instance, we ask how the Christiano et al. (2005)/Smets and Wouters (2003) model, which allegedly fits standard macro time series well, fare in describing observed inflation expectations. The same question can be posed for asset prices, the yield curve, and several other time series.

One can of course compute the marginal likelihood for the model at hand (which we call $\mathcal{M}_i$) using series $y^{1,T}$ though $y^{K+J,T}$, which we denote by $p(Y^{0,T}, Y^{1,T}|\mathcal{M}_i)$ where $Y^{0,T}$ and $Y^{1,T}$ are shorthand notations for $\{y^{1,T}, \dots, y^{J,T}\}$ and $\{y^{J+1,T}, \dots, y^{J+K,T}\}$, respectively. While the quantity $p(Y^{0,T}, Y^{1,T}|\mathcal{M}_i)$ is certainly of interest, it may not necessarily address the researcher's question. This is for two reasons. First, by construction, the marginal likelihood depends on the prior chosen:

$$p(Y^{0,T}, Y^{1,T}|\mathcal{M}_i) = \int p(Y^{0,T}, Y^{1,T}|\theta, \mathcal{M}_i)p(\theta|\mathcal{M}_i)d\theta, \quad (1)$$

where $p(Y^{0,T}, Y^{1,T}|\mathcal{M}_i)$ denotes the likelihood function for model $\mathcal{M}_i$, θ the vector of DSGE model parameters, and $p(\theta|\mathcal{M}_i)$ the prior chosen for θ. Prior elicitation for some of the DSGE model parameters can be challenging, and the choice of prior – not surprisingly given the above definition – affects the marginal likelihood computation and therefore the outcome of model comparisons (see Del Negro and Schorfheide (2008)). The researcher who is interested in knowing how well the model fits the time series $y^{J+1,T}$ through $y^{J+K,T}$ may want to use as a prior the posterior obtained from estimating the model on time series $y^{1,T}$ through $y^{J,T}$. This posterior – $p(\theta|Y^{0,T}, \mathcal{M}_i)$ – will be far less dependent on the initial prior $p(\theta|\mathcal{M}_i)$ chosen. In our case, the exercise would be to use the posterior obtained from fitting standard macro time series in order to evaluate the model's ability to fit expectations. The object of interest would then be:

$$p(Y^{1,T}|Y^{0,T}, \mathcal{M}_i) = \int p(Y^{1,T}|\theta, Y^{0,T}, \mathcal{M}_i)p(\theta|Y^{0,T}, \mathcal{M}_i)d\theta. \tag{2}$$

In expression (2) the set of time series $Y^{0,T}$ represents the training sample in Bayesian parlance, and $p(\theta|Y^{0,T}, \mathcal{M}_i)$ is the training sample prior, whence the title of the section. While training sample priors are often used in Bayesian macroeconometrics along the time series dimension (that is, using $\{y^i_t\}^0_{t=-P}$ as a training sample and then estimating the model over $y^{i,T} = \{y^i_t\}^T_{t=1}$ for the same set of time series $i = 1, .., J$), here we apply the approach to the cross sectional dimension. The second reason why we may be interested in $p(Y^{1,T}|Y^{0,T}, \mathcal{M}_i)$, rather than in $p(Y^T|\mathcal{M}_i)$, is that $p(Y^T|\mathcal{M}_i)$ provides information on how well model $\mathcal{M}_i$ fits both $Y^{0,T}$ and $Y^{1,T}$, while the researcher may want to disentangle the goodness of fit of one set of time series versus the other. The quantity $p(Y^{1,T}|Y^{0,T}, \mathcal{M}_i)$ tells us how well model $\mathcal{M}_i$ fits $Y^{1,T}$ only, conditional on the parameter distribution delivering the best possible fit for $Y^{0,T}$. This quantity easily obtains as the ratio of two objects we know how to compute, $p(Y^{0,T}, Y^{1,T}|\mathcal{M}_i)$ and $p(Y^{0,T}|\mathcal{M}_i)$, since:

$$p(Y^{1,T}|Y^{0,T}, \mathcal{M}_i) = \frac{p(Y^{0,T}, Y^{1,T}|\mathcal{M}_i)}{p(Y^{0,T}|\mathcal{M}_i)}. \tag{3}$$

3 Model

The economy is described by a medium-scale New Keynesian model with price and wage rigidities, capital accumulation, investment adjustment costs, variable capital utilization, and habit formation. The model is based on work of Smets and Wouters (2003), Smets and Wouters (2007), and Christiano et al. (2005). The specific version is taken from Del Negro et al. (2007). For brevity we only present the log-linearized equilibrium conditions and refer the reader to the above referenced papers for the derivation of these conditions from assumptions on preferences and technologies.

Monetary Policy: Perfect versus Imperfect Information The central bank follows a standard feedback rule:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left(\psi_1 \pi_t - \psi_1 \pi_t^* + \psi_2 \hat{y}_t\right) + \sigma_r \epsilon_{R,t}, \tag{4}$$

where $\hat{y}_t$ captures some measure of economic activity in log-deviations from its steady state (in the baseline specification $\hat{y}_t$ coincides with the growth rate of output), and $\epsilon_{R,t}$ is an i.i.d. shock. The inflation target π_t^*, defined in log-deviations from its non-stochastic steady state π^*, evolves according to

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \sigma_P \epsilon_{P,t}, \tag{5}$$

where $0 < \rho_{\pi^*} < 1$ and $\epsilon_{P,t}$ is an i.i.d. shock. Under perfect information, agents observe π_t^*. Under imperfect information they need to infer the inflation target from the observed interest rate behavior (see Erceg and Levin (2003)). Call $\tilde{\pi}_t$ the residual in the feedback rule, defined as:

$$\tilde{\pi}_t = (\rho_r R_{t-1} + (1-\rho_r)(\psi_1 \pi_t + \psi_2 \widehat{y}_t) - R_t)/(1-\rho_r)\psi_1. \tag{6}$$

Agents solve a signal extraction problem using

$$\tilde{\pi}_t = \pi_t^* + \sigma_T \epsilon_{R,t} \tag{7}$$

as the measurement equation (where $\sigma_T = \frac{\sigma_r}{(1-\rho_R)\psi_1}$) and (5) as the transition equation. The law of motion of $\pi_{t+1|t}^*$ is obtained using the steady state Kalman filter

$$\pi_{t+1|t}^* = \rho_{\pi^*} \pi_{t|t-1}^* + \rho_{\pi^*} K \left(\tilde{\pi}_t - \pi_{t|t-1}^*\right), \tag{8}$$

where $K = \frac{V(\frac{\sigma_P}{\sigma_T}, \rho_{\pi^*})}{1+V(\frac{\sigma_P}{\sigma_T}, \rho_{\pi^*})}$ is the steady state Kalman gain coefficient and $\sigma_T^2 V(\frac{\sigma_P}{\sigma_T}, \rho_{\pi^*})$ is the steady state uncertainty regarding the inflation target. V solves:

$$V = \rho_{\pi_*}^2 \left[V - V\,(V+1)^{-1}\, V \right] + (\frac{\sigma_P}{\sigma_T})^2.$$

We also consider the alternative law of motion for inflation target π_t^* proposed in Gurkaynak et al. (2005):

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \chi \pi_{t-1} + \sigma_P \epsilon_{P,t}. \tag{9}$$

As above agents know the policy rule and the evolution of the unobserved inflation target. The forecast of the unobserved inflation target $\pi_{t+1|t}^*$ (10) now becomes: steady state Kalman filter

$$\pi_{t+1|t}^* = \rho_{\pi_*} \pi_{t|t-1}^* + \rho_{\pi_*} K \left(\tilde{\pi}_t - \pi_{t|t-1}^* \right) + \chi \pi_t \tag{10}$$

where K is defined as before.

Firms. The economy is populated by a continuum of firms that combine capital and labor to produce differentiated intermediate goods. These firms have access to the same Cobb-Douglas production function with capital elasticity α and total factor productivity Z_t. Total factor productivity is assumed to be non-stationary, and its growth rate $z_t = \ln(Z_t/Z_{t-1})$ follows the autoregressive process:

$$z_t = (1 - \rho_z)\gamma + \rho_z z_{t-1} + \sigma_z \epsilon_{z,t}. \tag{11}$$

Output, consumption, investment, capital, and the real wage can be detrended by Z_t. In terms of the detrended variables the model has a well-defined steady state. All variables that appear subsequently are expressed as log-deviations from this steady state.

The intermediate goods producers hire labor and rent capital in competitive markets and face identical real wages, w_t, and rental rates for capital, r_t^k. Cost minimization implies that all firms produce with the same capital-labor ratio

$$k_t - L_t = w_t - r_t^k \tag{12}$$

and have marginal costs

$$mc_t = (1-\alpha)w_t + \alpha r_t^k. \tag{13}$$

The intermediate goods producers sell their output to perfectly competitive final good producers, which aggregate the inputs according to a CES function. Profit maximization of the final good producers implies the following demand curve

$$\widehat{y}_t(j) - \widehat{y}_t = -\left(1 + \frac{1}{\lambda_f e^{\widetilde{\lambda}_{f,t}}}\right)(p_t(j) - p_t). \tag{14}$$

Here $\widehat{y}_t(j) - \widehat{y}_t$ and $p_t(j) - p_t$ are quantity and price for good j relative to quantity and price of the final good. The price p_t of the final good is determined from a zero-profit condition for the final good producers. We assume that the price elasticity of the intermediate goods is time-varying. Since this price elasticity affects the mark-up that intermediate goods producers can charge over marginal costs, we refer to $\widetilde{\lambda}_{f,t}$ as mark-up shock. Following Calvo (1983), we assume that in every period a fraction of the intermediate goods producers ζ_p is unable to re-optimize their prices. A fraction ι_p of these firms adjust their prices mechanically according to lagged inflation, while the remaining fraction $1 - \iota_p$ adjusts to steady state inflation π^*. All other firms choose prices to maximize the expected discounted sum of future profits, which leads to the Phillips curve:

$$\pi_t = \frac{\beta}{1+\iota_p\beta}I\!E_t[\pi_{t+1}] + \frac{\iota_p}{1+\iota_p\beta}\pi_{t-1} + \frac{(1-\zeta_p\beta)(1-\zeta_p)}{\zeta_p(1+\iota_p\beta)}mc_t + \frac{1}{\zeta_p}\lambda_{f,t}, \tag{15}$$

where π_t is inflation and β is the discount rate.[3] Our assumption on the behavior of firms that are unable to re-optimize their prices implies the absence of price dispersion in the steady state. As a consequence, we obtain a log-linearized aggregate production function of the form

$$\widehat{y}_t = (1-\alpha)L_t + \alpha k_t. \tag{16}$$

Equations (13), (12), and (16) imply that the labor share lsh_t equals marginal costs in terms of log-deviations: $lsh_t = mc_t$.

[3] We used the following re-parameterization: $\lambda_{f,t} = [(1-\zeta_p\beta)(1-\zeta_p)\lambda_f/(1+\lambda_f)(1+\iota_p\beta)]\widetilde{\lambda}_{f,t}$.

Households. There is a continuum of households with identical preferences, which are separable in consumption, leisure, and real money balances. Households' preferences display (internal) habit formation in consumption, that is, period t utility is a function of $\ln(C_t - hC_{t-1})$. Households supply monopolistically differentiated labor services. These services are aggregated according to a CES function that leads to a demand elasticity $1 + 1/\lambda_w$ (see Equation (14)). The composite labor services are then supplied to the intermediate goods producers at real wage w_t. To introduce nominal wage rigidity, we assume that in each period a fraction ζ_w of households is unable to re-optimize their wages. A fraction ι_w of these households adjust their $t-1$ nominal wage by $\pi_{t-1}e^{\gamma}$, where γ represents the average growth rate of the economy, while the remaining fraction $1 - \iota_p$ adjusts to steady state wage growth $\pi^* e^{\gamma}$. All other households re-optimize their wages. First-order conditions imply that

$$\begin{aligned} \tilde{w}_t &= \zeta_w \beta I\!E_t \Big[\tilde{w}_{t+1} + \Delta w_{t+1} + \pi_{t+1} + z_{t+1} - \iota_w \pi_{t-1} \Big] \\ &\quad + \frac{1 - \zeta_w \beta}{1 + \nu_l (1 + \lambda_w)/\lambda_w} \left(\nu_l L_t - w_t - \xi_t + \frac{1}{1 - \zeta_w \beta} \phi_t \right), \end{aligned} \tag{17}$$

where $\tilde{w}_t$ is the optimal real wage relative to the real wage for aggregate labor services, w_t, and ν_l would be the inverse Frisch labor supply elasticity in a model without wage rigidity ($\zeta_w = 0$) and differentiated labor. Moreover, ξ_t denotes the marginal marginal utility of consumption defined below and ϕ_t is a preference shock that affects the intratemporal substitution between consumption and leisure. The real wage paid by intermediate goods producers evolves according to

$$w_t = w_{t-1} - \pi_t - z_t + \iota_w \pi_{t-1} + \frac{1 - \zeta_w}{\zeta_w} \tilde{w}_t. \tag{18}$$

Households are able to insure the idiosyncratic wage adjustment shocks with state contingent claims. As a consequence they all share the same marginal utility of consumption ξ_t, which is given by the expression:

$$(e^{\gamma} - h\beta)(e^{\gamma} - h)\xi_t = -(e^{2\gamma} + \beta h^2) c_t + \beta h e^{\gamma} I\!E_t[c_{t+1} + z_{t+1}] + h e^{\gamma}(c_{t-1} - z_t), \tag{19}$$

where c_t is consumption. In addition to state-contingent claims households accumulate three types of assets: one-period nominal bonds that yield the return R_t, capital $\bar{k}_t$, and real money balances.[4]

The first order condition with respect to bond holdings delivers the standard Euler equation:

$$\xi_t = \mathbb{E}_t[\xi_{t+1}] + R_t - \mathbb{E}_t[\pi_{t+1}] - \mathbb{E}_t[z_{t+1}]. \tag{20}$$

Capital accumulates according to the following law of motion:

$$\bar{k}_t = (2 - e^{\gamma} - \delta)\left[\bar{k}_{t-1} - z_t\right] + (e^{\gamma} + \delta - 1)i_t, \tag{21}$$

where i_t is investment, δ is the depreciation rate of capital. Investment in our model is subject to adjustment costs, and S'' denotes the second derivative of the investment adjustment cost function at steady state. Optimal investment satisfies the following first-order condition:

$$i_t = \frac{1}{1+\beta}\left[i_{t-1} - z_t\right] + \frac{\beta}{1+\beta}\mathbb{E}_t[i_{t+1} + z_{t+1}] + \frac{1}{(1+\beta)S''e^{2\gamma}}(\xi_t^k - \xi_t), \tag{22}$$

where ξ_t^k is the value of installed capital and evolves according to:

$$\xi_t^k - \xi_t = \beta e^{-\gamma}(1-\delta)\mathbb{E}_t\left[\xi_{t+1}^k - \xi_{t+1}\right] + \mathbb{E}_t\left[(1-(1-\delta)\beta e^{-\gamma})r_{t+1}^k - (R_t - \pi_{t+1})\right]. \tag{23}$$

Capital utilization u_t in our model is variable and r_t^k in the previous equation represents the rental rate of effective capital $k_t = u_t + \bar{k}_{t-1}$. The optimal degree of utilization is determined by

$$u_t = \frac{r_*^k}{a''}r_t^k. \tag{24}$$

Here a'' is the derivative of the per-unit-of-capital cost function $a(u_t)$ evaluated at the steady state utilization rate. The aggregate resource constraint is given by:

$$\widehat{y}_t = (1+g_*)\left[\frac{c_*}{y_*}c_t + \frac{i_*}{y_*}\left(i_t + \frac{r_*^k}{e^{\gamma} - 1 + \delta}u_t\right)\right] + g_t. \tag{25}$$

Here c_*/y_* and i_*/y_* are the steady state consumption-output and investment-output ratios, respectively, and $g_*/(1+g_*)$ corresponds to the government share of aggregate output. The process g_t can be interpreted as exogenous government

[4] Since preferences for real money balances are assumed to be additively separable and monetary policy is conducted through a nominal interest rate feedback rule, money is block exogenous and we will not use the households' money demand equation in our empirical analysis.

spending shock. It is assumed that fiscal policy is passive in the sense that the government uses lump-sum taxes to satisfy its period budget constraint. Finally, all stochastic processes described above are assumed to be AR(1) processes with normally distributed errors.

State-Space Representation of the DSGE Model. We use the method in Sims (2002) to solve the log-linear approximation of the DSGE model. We collect all the DSGE model parameters in the vector θ, stack the structural shocks in the vector ϵ_t, and derive a state-space representation for our vector of observables y_t, which is composed of the transition equation:

$$s_t = \mathcal{T}(\theta)s_{t-1} + \mathcal{R}(\theta)\epsilon_t, \tag{26}$$

which summarizes the evolution of the states s_t, and of the measurement equations:

Real output growth (%, annualized)	$400(\ln Y_t - \ln Y_{t-1})$	$=$	$400(\widehat{y}_t - \widehat{y}_{t-1} + z_t)$
Hours (%)	$100 \ln L_t$	$=$	$100(L_t + \ln L^{adj})$
Labor Share (%)	$100 \ln lsh_t$	$=$	$100(L_t + w_t - \widehat{y}_t + \ln lsh_*)$
Inflation (%,annualized)	$400(\ln P_t - \ln P_{t-1})$	$=$	$400(\pi_t + \ln \pi_*)$
Interest Rates (%,annualized)	$400 \ln R_t$	$=$	$400(R_t + \ln R_*)$,
Inflation Expectations (%,annualized)	$\pi_t^{O,t+k}$	$=$	$400(\mathbb{E}_t^{dsge}[\pi_{t+k}] + \ln \pi_*)$

where LS_*, π_*, and R_* are the steady states of the labor share, the inflation rate, and the nominal interest rate, respectively, and where in the last equation $\pi_t^{O,t+k}$ represents the observed k periods ahead (in the benchmark specification $k = 4$) inflation expectations and $\mathbb{E}_t^{dsge}[.]$ are the expectations obtained from the DSGE model. The parameter L^{adj} captures the units of measured hours. It can be viewed as a re-parameterization of the steady state associated with the time-varying preference parameter ϕ_t that appears in the households' utility function.

4 Measurement and Issues with Modeling Inflation Expectations

Several issues arise in using inflation expectations as observables in the estimation of DSGE models. First, there are several measures of inflation expectations available, for different inflation measures, and at different horizons. Our measurement choice of inflation expectations for the benchmark specification coincides with that of Erceg and Levin (2003): we use four-quarter ahead expectations for the GDP deflator obtained from the Survey of Professional Forecasters. We check for the robustness of the results using different sources of expectations (Blue Chip versus SPF), and different inflation measures (CPI versus GDP deflator). An alternative source of inflation expectations is the Michigan Survey of households, which are available at the one and ten years horizons. However in that Survey households are asked about inflation in general, as opposed to any specific measure, and that may create measurement error when matched with specific measures of inflation, especially at the one year horizon. In terms of forecast horizons, we choose the longest forecast horizon for which data are available since the 1980s, since arguably longer forecast horizons are more informative on agents' views about the plicymakers' inflation target.[5]

Second, forecasters (SPF or Blue Chip) have only the latest vintage of data available, while the econometrician often uses the final vintage. This is potentially a large issue, especially for revision in the inflation measure itself, which will heavily

[5] In principle we could use shorter horizons forecasts along with 4-quarter ahead expectations, but we have not done that yet. Measures of inflation expectations for forecasting horizon longer than 4 quarters ahead are available but with limitations in terms of sample length and frequency. SPF provides 10-years ahead CPI inflation forecasts but the sample starts in 1990Q4. Bluechip and the Philadelphia Fed's Livingston survey also provide 10-years CPI inflation forecast staring 1979Q4 but the forecasts are taken only twice a year. Concerning the 5-years horizon, Bluechip includes forecasts which are also taken twice a year, while SPF produces quarterly forecast starting only in 2005Q3. SPF also provides quarterly 5 and 10 years forecast for PCE inflation but those start in 2007Q1. Finally, SPF produces 2 year forecasts for CPI (core and total) and PCE (core and total) inflation but they are available since 2007Q1 (CPI is available since 2005Q3).

condition the forecasts. We do not fully address this issue, and we believe this may be a worthwhile exercise for future research. We do however show the robustness of the results when we use CPI as a measure of inflation, as opposed to the more heavily revised GDP deflator. Non-seasonally CPI is never revised. Seasonally adjusted CPI adjusted has revisions, but these are fairly small compared to those for the GDP deflator. See Figure 1. Of course, measured expectations are also function of measures of economic activity. In this sense our results, even those obtained with CPI, are still open to the issue of data revisions.

Third, there is an issue of information synchronization. SPF forecasters provide their forecasts in the middle of the quarter, and hence have partial information about the state of the economy in the current quarter. We deal with this issue by checking the robustness of the results to different assumptions regarding the timing of the agents' information set. The benchmark results are obtained assuming that observed expectations are formed using current quarter information. The alternative assumption, which we call "Lagged Information" specification, is that the forecasters are only endowed with information up to the previous quarter. Last, forecasts are heterogeneous, and our model cannot account for such heterogeneity (sticky information models can produce heterogeneous expectations, see Mankiw et al. (2003)). Again, this is an interesting avenue for future research.

Finally, the standard set of macro data used in the estimation includes the following variables: Output growth (log differences, quarter-to-quarter, in %); hours worked (log, in %); labor share (log, in %); inflation (annualized, in %, we use either GDP deflator and CPI, depending on the the corresponding inflation expectation measure); nominal interest rate (annualized, in %). See Appendix A for details. We use 97 quarters of data spanning the Volcker-Greenspan period: 1984Q2 to 2008Q2.

5 Comparing Perfect and Imperfect Information Models of Time-Varying Inflation Target

5.1 Prior Choice and Prior Predictive Checks

Table 1 shows the priors for the parameters of the policy rule (4) and the associated law of motion for the inflation target π_t^* (5), which are the key parameters for the exercise conducted here. Priors for the responses to inflation (ψ_1) and the measure of economic activity (ψ_2) – output growth in the baseline specification – in the policy rule, persistence (ρ_r), and steady state inflation target (π^*) are as chosen as follows. In particular, The prior on π_* is centered using pre-sample information on inflation, as in Del Negro and Schorfheide (2008). The prior on ψ_1 and ψ_2 are centered at 2 and .2 respectively, and imply a fairly strong response to inflation and a much moderate response to output. Priors on variance of i.i.d. policy shocks σ_r is centered at .15. In general the priors on the standard deviations of the shocks are chosen so that overall variance of endogenous variables is roughly close to that observed in the pre-sample 1959Q3-1984Q1, informally following the approach in Del Negro and Schorfheide (2008). Key priors are those on persistence and standard deviation of the innovation to π_t^* process, as they determine, together with the prior on σ_r, the agents' Kalman gain in the Imperfect Information model. We follow Erceg and Levin (2003) and make the process followed by π_t^* very persistent: The prior for ρ_{π^*} is centered at .95 and the 90% bands range from about .91 to .99.

In the Benchmark prior the prior on σ_{π^*}, centered at .05, is independent from all other parameters, and is fairly loose.[6] An alternative prior ("Signal-to-Noise Ratio Prior") places a prior directly on the Signal-to-Noise ratio (and hence induces dependence between σ_{π^*} and σ_r) and is centered at the value that delivers a Kalman gain of approximately .13, the value calibrated by Erceg and Levin (2003).

Priors on nominal rigidities parameters are shown in the top panel of Table 2). To check robustness to the degree of nominal rigidities in the economy we consider

[6] In this and all other tables the standard deviations σ_{π^*} and σ_r are not annualized.

two priors, as in Del Negro and Schorfheide (2008): "Low Rigidities" (loosely calibrated at Bils and Klenow (2004) values of average duration less than 2 quarters), and "High Rigidities" (duration about 4 quarters).

Priors on remaining parameters are shown in the bottom panel of Table 2). The priors on "Endogenous Propagation and Steady State" are all chosen as in Del Negro and Schorfheide (2008). Specifically, the prior for the habit persistence parameter h is centered at 0.7, which is the value used by Boldrin et al. (2001). The prior for a' implies that in response to a 1% increase in the return to capital, utilization rates rise by 0.1 to 0.3%. These numbers are considerably smaller than the one used by Christiano et al. (2005). The 90% interval for the prior distribution on ν_l implies that the Frisch labor supply elasticity lies between 0.3 and 1.3, reflecting the micro-level estimates at the lower end, and the estimates of Kimball and Shapiro (2003) and Chang and Kim (2006) at the upper end. We use a pre-sample of observations from 1959Q3-1984Q1 to choose the prior means for the parameters that determine steady states.

The priors on standard deviations and autocorrelations are chosen so that overall variance and autocorrelations of endogenous variables is roughly close to that observed in the pre-sample 1959Q3-1984Q1 (see Table 3). Table 3 also shows that although we use the same prior for both the models under consideration – the Imperfect and Perfect Information models – the prior predictive statistics are fairly similar across models.

5.2 Model Comparison Results

Table 4 shows the log marginal likelihood for three models: Imperfect Information, Perfect Information, and the model with constant inflation target (Fixed-π^*). For all models we use the Benchmark prior. The Dataset with Expectations uses the SPF 4-quarters ahead median forecast for the GDP deflator. For these results we assume that the expectations are generated using current quarter information. In the remainder of the paper we condition on two lags of the variables included in Y^0 when computing both marginal likelihoods and posteriors.

Table 4 shows that for the dataset without expectations (column *(1)*) all three models perform about the same, with the Fixed-π^* model performing slightly worse. The difference in $lnp(Y^{0,T}|\mathcal{M}_i)$ for the Imperfect and Perfect Information models is .69, which implies a posterior odd of roughly 2 in favor of the Imperfect Information model. The difference in $ln\ p(Y^{0,T}|\mathcal{M}_i)$ for the Fixed-π^* is larger, about 5. Although this difference implies that the posterior odds are heavily against the Fixed-π^* model, Del Negro and Schorfheide (2008) show that for marginal likelihoods for DSGE models are quite sensitive to the choice of priors, so that a difference of 5 can in principle be overturned by choosing a slightly different prior.

When SPF inflation expectations are included among the observables, the Perfect Information model with time-varying π^* performs significantly better than both the Fixed-π^* and, most importantly, the Imperfect Information model. The difference in the log marginal likelihoods $ln\ p(Y^{0,T}, Y^{1,T}|\mathcal{M}_i)$ between the Perfect and Imperfect Information models is about 25 in favor of the latter. The data disfavors the Fixed-π^* even more strongly. Since the marginal likelihoods $ln\ p(Y^{0,T}|\mathcal{M}_i)$ are similar across models, these differences translate into differences in $ln\ p(Y^{1,T}|Y^{0,T}, \mathcal{M}_i)$. They imply that the Perfect Information model fits observed inflation expectations much better than either the Imperfect Information or the Fixed-π^* model. The fact that the differences are large indicates that the extra observable included in $Y^{1,T}$ contains quite a lot of information as to which model describes it best. In the remainder of the section we will provide additional evidence that the Imperfect Information model has a much harder time at explaining observed inflation expectations than the Perfect Information one. Next, we will provide the intuition as to why this is the case.

Table 5 shows the median in-sample forecast errors for the Imperfect and Perfect Information models computed using the Kalman filter. In the top panel we compute the RMSEs using for each model the respective parameter values that maximize the posterior for the dataset without expectations (that is, the value of θ that maximizes $p(\theta|Y^{0,T}, \mathcal{M}_i)$). Columns (1) and (2) show the errors for the two models computed without providing the econometrician with the information about observed inflation expectations. Specifically, for each variable x_t we show the average value of $(x_t - \mathbb{E}[x_t|Y^{0,t-1}, \mathcal{M}_i])^2$. The next column shows the ratio of the RMSEs for the two models. These figures are all in the neighborhood of one, indicating that the forecasting performance of the models is roughly equal. In fact, the log likelihood $ln\ p(Y^{0,T}|\mathcal{M}_i, \theta)$ is quite similar for the two models. Interestingly, the ratio of RMSEs is about one also for observed inflation expectations, which are not part of the econometrician's information set (the numbers for inflation expectations are in parenthesis to emphasize that the corresponding forecast errors are computed without including this variable in the information set).

For the same set of parameters the forecast performance of the two models for the variables in $Y^{0,T}$ worsens considerably when inflation expectations are included into the econometrician's information set, and this is particularly the case for the Imperfect Information model. This is apparent from columns (3) and (4), which show $(x_t - \mathbb{E}[x_t|Y^{0,t-1}, Y^{1,t-1}, \mathcal{M}_i])^2$ for the two models. The last two columns of Table 5 show the ratio of the RMSEs with and without including inflation expectations among the observables for the Imperfect and Perfect Information models, respectively. All these figures are larger than one for both models for all the variables included in $Y^{0,T}$ (of course, for inflation expectations the RMSEs decrease). The worsening of in-sample forecasting performance is particularly large for the Imperfect Information model, where the increase in RMSEs range from 7% to 46%. As a consequence, when inflation expectations are included in the set of observables the Perfect Information model performs better than the Imperfect Information one: The ratios between the figures in column (3) and (4) are all larger than one (and the log likelihood $ln\ p(Y^{0,T}, Y^{1,T}|\mathcal{M}_i, \theta)$ is much larger for the Perfect Information model).

The values of θ that maximizes $p(\theta|Y^{0,T}, \mathcal{M}_i)$ for the two models are of particular interest because it is the mode of the prior in formula (2). Nonetheless, such value may overemphasize the effect of including inflation expectations among the observables, since it maximizes the model's fit (adjusting for the prior) when this variable is excluded. Therefore the bottom panel of Table 5 repeats the exercise using the value of θ that maximizes $p(\theta|Y^{0,T}, Y^{1,T}, \mathcal{M}_i))$ for each of the two models, respectively. For these parameters it is still the case that the ratios of the RMSEs in column (3) and (4) are all larger than one, except for inflation expectations were the in-sample forecasting performance is the same. Moreover, for the Imperfect Information model it is also still the case that forecasting performance worsens for all variables when inflation expectations are included into the econometrician's information set. For the Perfect Information model this is the case only for some of the variables, notably for inflation.

Including inflation expectations among the observables worsens the fit of Imperfect Information model relative to that of the Perfect Information, consistently with the marginal likelihood results in Table 4. In order to understand this result we ask what kind of inflation expectations the two models generate whenever actual inflation expectations are not among the observables. Figure 3 plots the projections for the 4-quarter ahead inflation forecasts generated by the Imperfect (black solid) and Perfect (gray solid) Information models. This exercise is performed using the value of θ that maximizes $p(\theta|Y^{0,T}, \mathcal{M}_i)$ for the two models – the mode of the prior in formula (2).

To the extent that the inflation expectations generated by the model are roughly in line with the observed data, including measured expectations among the observables is unlikely to change the estimates of the states, and hence the forecasts of the other variables. However, if there is a large discrepancy between a model's forecasts of inflation expectations and what we observe in the data, we expect both the estimates of the states and the forecasts of the other variables to change substantially following the addition of measured expectations to econometrician's information set. Figure 3 also plots the actual inflation expectation data – namely, the SPF 4-quarters ahead median forecast for the GDP deflator (red dashed-and-dotted) – along with the projections. It is clear that the inflation forecasts generated by the both models are at odds with the data. They are too low in the early part of the sample, and too high in the later part. Interestingly, the inflation expectations generated by the two models are very similar.

The top panel of Figure 4 plots the mean estimate of the latent variable $\pi^*_{t|t}$ for the Imperfect Information model for the dataset without (black line) and with (gray line) inflation expectations. Similarly, the middle panel shows the mean estimate of the latent variable π^*_t for the Perfect Information model for the dataset without (black line) and with (gray line) inflation expectations. Since in the Imperfect Information model agents do not observe the actual π^*_t, these two latent variables are conceptually equivalent in that in each model they drive the agents' beliefs about the inflation target.[7] In both panels these figures are computed using the value of θ that maximizes $p(\theta|Y^{0,T}, \mathcal{M}_i)$. Both panels also show observed inflation expectations (dashed-and-dotted line).

The time series for $\pi^*_{t|t}$ and π^*_t look very similar across the two models when the econometrician does not have information about inflation expectations (black lines in top and middle panels). Not surprisingly, for both models the movement in these time series mirrors that of the model-generated inflation expectations in Figure 3. When inflation expectations are included among the observables, the path for π^*_t in the Perfect Information model moves closer to that of observed inflation expectations. Very loosely speaking, the filtering procedure realizes that the model is failing to match the new observable, and adjusts the latent state π^*_t accordingly. For the Imperfect Information model the path for $\pi^*_{t|t}$ barely move, and only at the very beginning. The law of motion of the agents' perception of the inflation target $\pi^*_{t|t}$ is given by:

$$\pi^*_{t|t} = (1 - K)\rho_{\pi^*}\pi^*_{t-1|t-1} + K\tilde{\pi}_t, \tag{27}$$

which obtains rearranging equation (10). As we iterate this law of motion forward starting from the initial condition $\pi^*_{0|0}$, we realize that the econometricians only degree of freedom lies in the choice of this initial condition. After that, the path for $\pi^*_{t|t}$ is pinned down by that of the interest feedback rule residual $\tilde{\pi}_t$, defined in equation (6). In the baseline model where the interest rate responds to inflation and output growth this residual is pinned down by the data, for given parameters

[7] In the Imperfect Information model all the econometrician can infer from the data is the agents' belief about π^*_t.

(the bottom panel of Figure 4 plots $\tilde{\pi}_t$, and shows that its fluctuations are consistent with the evolution of $\pi^*_{t|t}$). Hence the filtering procedure cannot adjust $\pi^*_{t|t}$ to match inflation expectations, and needs to rely on large, and likely persistent, shocks to fill the gap between $\pi^*_{t|t}$ and observed expectations. These large shocks negatively affect the fit for the other observables. In conclusion, the Imperfect Information model imposes tighter cross-equation restrictions than the Perfect Information model, in the sense that it cannot rely on adjusting the latent variable π^*_t to fit the data.

5.3 Robustness to the Choice of Priors, Datasets, Timing Conventions, and Policy Rules

This section investigates the robustness of the model comparison results to the choice of priors, datasets, timing conventions, and policy rules. Lines (1) and (2) of Table 6 report the model comparison results under the "'High Nominal Rigidities" prior and "Signal-to-Noise Ratio" prior described in section 5.1, respectively. We find that the "High Nominal Rigidities" prior favors the Perfect Information relative to the Imperfect Information model, in that the difference in $ln\ p(Y^{1,T}|Y^{0,T}|\mathcal{M}_i)$ is larger in favor of the Perfect Information model (we use the "Low Nominal Rigidities" prior precisely because it gives the Imperfect Information model the best shot). Using the "Signal-to-Noise Ratio" prior makes little difference.

Lines (3) through (8) show the log marginal likelihoods for the two models under different timing assumptions ("Lagged Information" specification), source for inflation expectations ("Blue Chip" versus SPF), and inflation measure (CPI versus GDP deflator), and measures of the short term interest rate (3 Month T-Bill versus the Fed Funds rate). Under the "Lagged Information" specification the forecasters in the SPF Survey are only endowed with information up to the previous quarter. Results are robust to both timing assumptions and measurement choices. The gap in $ln\ p(Y^{1,T}|Y^{0,T}|\mathcal{M}_i)$ between the Perfect and Imperfect Information models varies among the different specifications, but is always larger than 20. The gap widens substantially whenever we use CPI (which is less subject to revisions) as opposed to the GDP Deflator.

Lines (9) through (11) report the model comparison results under different specifications of the policy rule, where the policy makers target output growth as opposed to the output gap ("Output Growth"), a four-quarter moving average of inflation as opposed to current inflation ("4Q Inflation"), or where the the law of motion for the inflation target follows the rule suggested by Gurkaynak et al. (2005) ("GSS"). Under this rule the marginal likelihood gap between the Imperfect and Perfect Information models stays roughly constant or increases. Under the rule proposed by Gurkaynak et al. (2005) the gap narrows, but it is still larger than 17.[8] The last two rows of Table 6 show the marginal likelihoods for the models where we allow for measurement error in expectations. We discuss this case in detail in section 6.

5.4 Posterior Estimates and Variance Decomposition

Table 7 shows the posterior mean and standard deviation (in parenthesis) of the parameters. The differences in parameter estimates between the posterior without ($p(\theta|Y^{0,T}, \mathcal{M}_i)$) and with inflation expectations ($p(\theta|Y^{0,T}, Y^{1,T}, \mathcal{M}_i)$) are not particularly noticeable for the Imperfect Information model. The ratio of σ_{π^*} to σ_r decreases from .13 to .11 between columns 1 ($p(\theta|Y^{0,T}, \mathcal{M}_i)$) and 2 ($p(\theta|Y^{0,T}, Y^{1,T}, \mathcal{M}_i)$) , and the estimates of ρ_{π_*} and $\rho_r$decrease as well. The importance of nominal rigidities decreases, consistently with the results in line (1) of Table 6. The importance of investment adjustment increases by about 60%, which implies that investment specific shocks become much more powerful when inflation expectations are used in the estimation. The persistence of shocks all increase, except for productivity shocks, and the increase is particularly noticeable for preference shocks to leisure ϕ_t (recall that hours is the variable for which the RMSE in Table 5 worsens the most when inflation expectations become part of the econometrician's information set). The shocks standard deviations generally rise, and particularly that of government spending shocks g_t.

Changes in parameters for the Perfect Information model are even less dramatic.

[8]In the estimation of the GSS model we used the value of $\chi = .02$ in expression (9), which is the value used by Gurkaynak et al. (2005).

The curvature of the dis-utility from working nu_l decreases between columns 1 ($p(\theta|Y^{0,T}, \mathcal{M}_i)$) and 2 ($p(\theta|Y^{0,T}, Y^{1,T}, \mathcal{M}_i)$), thereby making hours more elastic, and the persistence of ϕ_t shock decreases (with a more elastic labor supply the reliance on ϕ_t shocks to explain movements in hours decreases). Movements in the inflation target become larger and more persistent (both σ_{π_*} and ρ_{π_*} increase).

Table 8 shows the (unconditional) variance decomposition computed using the posterior distribution for the Imperfect and Perfect Information models obtained using the dataset that includes observed inflation expectations. The time -varying inflation target π_t^* is the main driver of inflation expectations in the Perfect Information model, while it explains very little under Imperfect Information, consistently with the intuition discussed in section 5.2.

6 Introducing Measurement Error in Observed Inflation Expectations

Rows (9) and (10) of Table 6 show the marginal likelihoods for the models where we allow for measurement error in expectations. The measurement error is either i.i.d. ("i.i.d. Meas. Error") or follows and AR(1) process ("AR(1) Meas. Error"). The Perfect Information model is still superior to the specification with Imperfect Information when the measurement error is i.i.d.. The difference in $ln\ p(Y^{1,T}|Y^{0,T}|\mathcal{M}_i)$ is about 16, which is smaller than in Table 4 but still substantial. The fit of the two models are essentially the same under AR(1) measurement error.

We conjecture that the autoregressive measurement error largely "takes care" of the misspecification in the Imperfect (and to some extent also in the Perfect) Information model, so we revert to the original result that when the dataset does not include inflation expectations the fit of the two models is about the same. We substantiate this conjecture using the variance decomposition for observed inflation expectations – both unconditional and 10-quarters ahead – shown in Table 9. We find that i.i.d. measurement error is not all that important for both the Imperfect and Perfect Information models. Its contribution is small for the unconditional variance, and between 30 and 45% at the 10-quarters ahead horizon. The AR(1) measurement error is the most important source of variation for observed expectations in both models, however. Measurement error explains about 60 and 40-45 percent of the variance for the Imperfect and Perfect Information models, respectively. While issues of data revisions and data synchronization are likely to introduce a mismatch between measured and model-generated inflation expectations, our prior would be that this mismatch is relatively short-lived. The results for the AR(1) measurement error show otherwise. We certainly do not claim that the measurement error results fully address the issues mentioned above (we assume that measurement error is independent from the state of the economy, while there is evidence that data revisions are not). But these results, together with the intuition developed in section 5.2, suggest that there may be more to the discrepancy between measured and model-generated inflation expectations than just issues of data revisions, especially for the Imperfect Information model.

7 Conclusions

TBW

References

Bernanke, Ben, Jean Boivin, and Piotr S. Eliasz, "Measuring the Effects of Monetary Policy: A Factor-augmented Vector Autoregressive (FAVAR) Approach," *The Quarterly Journal of Economics*, January 2005, *120* (1), 387–422.

Bils, Mark and Peter Klenow, "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 2004, *112*, 947–985.

Boivin, Jean and Marc Giannoni, "DSGE Models in a Data-Rich Environment," NBER Technical Working Papers 0332, National Bureau of Economic Research, Inc December 2006.

Boldrin, Michele, Lawrence Christiano, and Jonas Fisher, "Habit Persistence, Asset Returns, and the Business Cycle," *American Economic Review*, 2001, *91*, 149–166.

Calvo, Guillermo, "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 1983, *12*, 383–398.

Chang, Youngsun and Sun-Bin Kim, "Individual to Aggregate Labor Supply: A Quantitative Analysis based on Heterogeneous Agent Macroeconomy," *International Economic Review*, 2006, *47*, 1–27.

Christiano, Lawrence, Martin Eichenbaum, and Charles Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 2005, *113*, 1–45.

Erceg, Christopher J. and Andrew T. Levin, "Imperfect Credibility and Inflation Persistence," *Journal of Monetary Economics*, 2003, *50*, 915–944.

Giannone, Domenico, Francesca Monti, and Lucrezia Reichlin, "Incorporating Conjunctural Analysis in Structural Models," *in "The Science and Practice of Monetary Policy Today", Volker Wieland editor, Springer*, 2008.

Gurkaynak, Refet S., Brian Sack, and Eric Swanson, "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models," *American Economic Revie*, 2005, *95* (1), 425–36.

Kimball, Miles and Matthew Shapiro, "Labor Supply: Are the Income and Substitution Effects Both Large or Both Small?," *Manuscript, University of Michigan*, 2003.

Mankiw, N. Gregory, Ricardo Reis, and Justin Wolfers, "Disagreement about Inflation Expectations," in "NBER Macroeconomics Annual 2003, Volume 18" 2003.

Milani, Fabio, "Expectations, Learning, and Macroeconomic Persistance," *Journal of Monetary Economics*, 2007, *54* (7), 2065 – 2082.

Negro, Marco Del and Frank Schorfheide, "Forming priors for DSGE models (and how it affects the assessment of nominal rigidities)," *Journal of Monetary Economics*, 2008, *55* (7), 1191 – 1208.

_ , _ , Frank Smets, and Raphael Wouters, "On the Fit of New Keynesian Models," *Journal of Business and Economic Statistics*, 2007, *25* (2), 123 – 162.

Ormeno, Arturo, "Inflation Expectations, Learning, and DSGE Model Estimation," *mimeo UPF*, 2009.

Rudebusch, Glenn and Eric Swanson, "The bond premium in a DSGE model with long-run real and nominal risks," Working Paper Series 2008-31, Federal Reserve Bank of San Francisco 2008.

Sims, Christopher A., "Solving Rational Expectations Models," *Computational Economics*, 2002, *20*, 1–20.

Smets, Frank and Raf Wouters, "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 2003, *1* (5), 1123 – 1175.

_ **and** _ , "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 2007, *97* (3), 586 – 606.

A Data

The data set is obtained from Haver Analytics (Haver mnemonics are in italics). We compile observations for the variables that appear in the measurement equation (27). Real output is obtained by dividing the nominal series (*GDP*) by population 16 years and older (*LN16N*), and deflating using the chained-price GDP deflator (*JGDP*). We compute quarter-to-quarter output growth as log difference of real GDP per capita and multiply the growth rates by 100 to convert them into percentages. Our measure of hours worked is computed by taking total hours worked reported in the National Income and Product Accounts (NIPA), which is at annual frequency, and interpolating it using growth rates computed from hours of all persons in the non-farm business sector (*LXNFH*). We divide hours worked by *LN16N* to convert them into per capita terms. We then take the log of the series multiplied by 100 so that all figures can be interpreted as percentage changes in hours worked. The labor share is computed by dividing total compensation of employees (*YCOMP*) obtained from the NIPA by nominal GDP. We then take the log of the labor share multiplied by 100. Inflation rates are defined as log differences of the GDP deflator and converted into annualized percentages. The nominal rate corresponds to the effective Federal Funds Rate (*FFED*), also in percent. As an alternative measure of the nominal rate we use the three months Tbill (*FTBS3*),

We use Survey of Professional Forecasters (SPF) quarterly measures of expected inflation. We consider both expectations for GDP deflator[9] and for CPI inflation. In particular, we use the median four -quarters-ahead forecast of inflation in annualized terms. Concerning the information available to the forecasters, the survey is sent out at the end of the first month of each quarter and responses deadlines occur in the middle month of each quarter. Therefore, respondents have knowledge about the BEA advance report of the National Income and Product Accounts. We also compute the revisions in GDP deflator and CPI occurred since 1982 using the real time dataset available from the Federal Reserve Bank of Philadelphia.

[9] In more detail, the forecast are for the GDP price index, seasonally adjusted (base year varies). Prior to 1996, the forecast variable was the GDP implicit deflator. Prior to 1992, the GNP deflator.

(http://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/)

As an alternative measure of inflation expectations, we use Bluechip monthly forecasts of CPI inflation. We choose forecast horizons of 3 and 4 quarters ahead. In order to compare Bluechip and SPF quarterly forecast of CPI inflation, we use the Bluechip forecasts available in the middle month of each quarter. This roughly corresponds to the time period when SPF participants provide their forecasts.

Table 1: Priors on Policy Parameters

Parameter	Domain	Density	Para (1)	Para (2)	5%	95%
ψ_1	$\mathbb{R}^+$	Gamma	2.00	0.25	1.592	2.408
ψ_2	$\mathbb{R}^+$	Gamma	0.20	0.10	0.049	0.349
ρ_r	[0,1)	Beta	0.50	0.200	0.170	0.827
π^*	$\mathbb{R}$	Normal	4.3	2.5	0.520	8.17
σ_r	$\mathbb{R}^+$	InvGamma	0.150	4.00	0.080	0.298
ρ_{π^*}	[0,1)	Beta	0.950	0.025	0.913	0.989
Benchmark Prior						
σ_{π^*}	$\mathbb{R}^+$	InvGamma	0.050	8.000	0.032	0.078
Signal-to-Noise Ratio Prior						
$\sigma_{NR} = \frac{\sigma_P}{\sigma_T}$	$\mathbb{R}^+$	Gamma	0.180	0.150	0.001	0.380

Notes: Para (1) and Para (2) correspond to means and standard deviations for the Beta, Gamma, and Normal distributions and to s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$. The last two columns report the 5^{th} and 95^{th} quintile of the prior distribution.

Table 2: Priors on Non-Policy Parameters

Parameter	Domain	Density	Para (1)	Para (2)	5%	95%
Priors on Nominal Rigidities Parameters						
Low Rigidities (Benchmark)						
ζ_p	[0,1)	Beta	0.450	0.100	0.285	0.614
ζ_w	[0,1)	Beta	0.450	0.100	0.285	0.614
High Rigidities						
ζ_p	[0,1)	Beta	0.750	0.100	0.590	0.913
ζ_w	[0,1)	Beta	0.750	0.100	0.590	0.913
Priors on "Endogenous Propagation and Steady State" Parameters						
α	[0,1)	Beta	0.330	0.020	0.297	0.362
s''	$\mathbb{R}^+$	Gamma	4	1.500	1.614	6.303
h	[0,1)	Beta	0.700	0.050	0.619	0.782
a'	$\mathbb{R}^+$	Gamma	0.200	0.100	0.049	0.349
ν_l	$\mathbb{R}^+$	Gamma	2	0.75	0.787	3.137
r^*	$\mathbb{R}^+$	Gamma	1.5	1	0.106	2.883
γ	$\mathbb{R}^+$	Gamma	1.650	1	0.204	3.073
g^*	$\mathbb{R}^+$	Gamma	0.300	0.100	0.143	0.459
ι_p	[0,1)	Beta	0.5	0.280	0.043	0.922
ι_w	[0,1)	Beta	0.5	0.280	0.049	0.932
Priors on ρs and σs						
ρ_z	[0,1)	Beta	0.400	0.250	0.000	0.764
ρ_ϕ	[0,1)	Beta	0.750	0.150	0.530	0.982
ρ_{λ_f}	[0,1)	Beta	0.750	0.150	0.530	0.982
ρ_μ	[0,1)	Beta	0.750	0.150	0.530	0.982
ρ_g	[0,1)	Beta	0.750	0.150	0.530	0.982
σ_z	$\mathbb{R}^+$	InvGamma	0.200	4.000	0.107	0.395
σ_ϕ	$\mathbb{R}^+$	InvGamma	2.500	4.000	1.326	4.930
σ_{λ_f}	$\mathbb{R}^+$	InvGamma	0.300	4.000	0.161	0.596
σ_μ	$\mathbb{R}^+$	InvGamma	0.500	4.000	0.264	0.99
σ_g	$\mathbb{R}^+$	InvGamma	0.300	4.000	0.159	0.594

Notes: Para (1) and Para (2) correspond to means and standard deviations for the Beta, Gamma, and Normal distributions and to s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu,s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. The last two columns report the 5^{th} and 95^{th} quintile of the prior distribution.

Table 3: Prior Implications for Moments of the Endogenous Variables

Variables	St. Dev.			Autocorr.		
	Imperfect Information	Perfect Information	*Data*	Imperfect Information	Perfect Information	*Data*
$OutputGrowth$	3.48	3.47	*4.33*	0.39	0.39	*0.28*
$LaborSupply$	2.98	2.98	*3.20*	0.93	0.93	*0.96*
$LaborShare$	1.39	1.39	*2.24*	0.86	0.86	*0.95*
$Inflation$	3.13	3.15	*2.77*	0.71	0.72	*0.88*
$InterestRate$	4.34	4.38	*4.30*	0.85	0.85	*0.87*
$Exp.\ Inflation$	1.37	1.40		0.86	0.85	

Notes: II: imperfect information; PI: perfect information. The pre-sample statistics (column *Data*) are in italics. These statistics are computed over the sample 1959Q3-1984Q1. Inflation expectations are not available during most of the pre-sample. The in-sample standard deviation and first-order autocorrelation of inflation expectations are 1.21, and 0.86, respectively.

Table 4: Model Comparison

	$\ln p(Y^{0,T})$ Dataset without Expectations (1)	$\ln p(Y^{0,T}, Y^{1,T})$ Dataset with Expectations (2)	$\ln p(Y^{1,T} \mid Y^{0,T})$ (2) - (1)
Imperfect Information	-703.62	-811.04	-107.42
Perfect Information	-704.31	-786.35	-82.04
Fixed π^*	-709.29	-821.84	-112.55

Notes: The Table shows the log marginal likelihood for three models: Imperfect Information, Perfect Information, and the model with constant inflation target (Fixed-π^*). For all models we use the Benchmark prior. The Dataset with Expectations uses the SPF 4-quarters ahead median forecast for the GDP deflator. We assume that the expectations are generated using current quarter information.

Table 5: In-sample RMSEs

	Dataset without Expectations			Dataset with Expectations			Increase/Decrease in RMSE	
	Imperfect Info.	Perfect Info.		Imperfect Info.	Perfect Info.			
	(1)	(2)	(1)/(2)	(3)	(4)	(3)/(4)	(1)/(3)	(2)/(4)
Posterior mode estimates for the dataset without expectations								
Output Growth	2.221	2.258	*0.984*	3.056	2.374	*1.287*	1.376	1.052
Labor Supply	0.573	0.563	*1.017*	0.838	0.660	*1.271*	1.463	1.171
Labor Share	0.537	0.540	*0.994*	0.575	0.559	*1.029*	1.071	1.036
Inflation	0.869	0.895	*0.971*	1.014	0.996	*1.018*	1.167	1.114
Interest Rate	1.526	1.543	*0.989*	1.758	1.673	*1.051*	1.152	1.084
Exp. Inflation	(0.987)	(.959)	(*1.029*)	0.512	0.487	*1.051*	0.518	0.507
Likelihood	-656.7	-660.3		-1088.4	-791.7			
Posterior mode estimates for the dataset with expectations								
Output Growth	2.271	2.166	*1.048*	2.370	2.119	*1.119*	1.044	0.978
Labor Supply	0.580	0.556	*1.043*	0.654	0.554	*1.182*	1.129	0.996
Labor Share	0.544	0.530	*1.025*	0.563	0.539	*1.044*	1.035	1.016
Inflation	1.010	0.922	*1.096*	1.030	0.977	*1.054*	1.020	1.060
Interest Rate	1.583	1.487	*1.064*	1.635	1.499	*1.090*	1.033	1.008
Exp. Inflation	(0.774)	(0.703)	(*1.101*)	0.477	0.479	*0.996*	0.616	0.682
Likelihood	-685.2	-668.7		-760.9	-732.4			

Notes: The table shows the in-sample Root Mean Square Errors (RMSEs) for the Imperfect and Perfect Information models computed using the Kalman filter. The top panel shows the RMSEs using for each model the respective posterior mode for the dataset without expectations. The bottom panel shows the RMSEs using for each model the respective posterior mode for the dataset with expectations.

Table 6: Robustness of Model Comparison Results

Imperfect Information			Perfect Information		
$\ln p(Y^{0,T})$	$\ln p(Y^{0,T}, Y^{1,T})$	$\ln p(Y^{1,T} \mid Y^{0,T})$	$\ln p(Y^{0,T})$	$\ln p(Y^{0,T}, Y^{1,T})$	$\ln p(Y^{1,T} \mid Y^{0,T})$
Dataset without Expectations	Dataset with Expectations		Dataset without Expectations	Dataset with Expectations	
(1)	(2)	(2) - (1)	(3)	(4)	(4) - (3)
Robustness to Priors					
(1) High Nominal Rigidities Prior					
-701.65	-820.84	-119.19	-705.39	-789.26	-83.87
(2) Signal-to-Noise Ratio Prior					
-703.86	-811.97	-108.11	-709.66	-786.59	-76.93
Robustness to Data Sets and Timing Assumptions					
(3) Lagged Information					
-703.62	-800.74	-97.12	-704.31	-780.53	-76.22
(4) Blue Chip Expectations					
-703.62	-761.68	-58.06	-704.31	-742.11	-37.80
(5) CPI and SPF expectations					
-761.28	-844.98	-83.70	-763.72	-771.38	-7.66
(6) CPI and Blue Chip expectations					
-761.28	-865.04	-103.76	-763.72	-779.31	-15.59
(7) Tbill					
-587.31	-679.80	-92.49	-587.99	-638.89	-50.90
(8) Tbill, CPI					
-643.71	-735.99	-92.28	-644.22	-661.41	-17.19

Notes: The table shows the log marginal likelihood for the Imperfect Information and Perfect Information models under different choices of priors, datasets, timing conventions, and policy rules. Lines (1) and (2) report the results under the "High Nominal Rigidities" prior and "Signal-to-Noise Ratio" prior, respectively. Lines (3) to (6) show the log marginal likelihood for the two models under different timing assumptions ("Lagged Information" specification), measures of inflation and measures of inflation expectations ("Blue Chip Expectations", "CPI and SPF Expectations", "CPI and Blue Chip Expectations"). Lines(7) and (8) report the results under different measures of nominal interest rate and inflation ("Tbill","CPI and Tbill"). Lines (9)-(11) report the results under different specifications of the policy rule, where the policy makers target output growth as opposed to the output gap ("Output Growth"), a four-quarter moving average of inflation as opposed to current inflation ("4Q Inflation"), or where the the law of motion for the inflation target follows the rule suggested by Gurkaynak et al. (2005) ("GSS"). Finally, lines (12) and (13) report the log marginal likelihood for the two models measurement errors are added ("i.i.d. Measurement Error", and "AR(1) Measurement Error").

Table 6: Robustness of Model Comparison Results – Continued

Imperfect Information			Perfect Information		
$\ln p(Y^{0,T})$	$\ln p(Y^{0,T}, Y^{1,T})$	$\ln p(Y^{1,T} \vert Y^{0,T})$	$\ln p(Y^{0,T})$	$\ln p(Y^{0,T}, Y^{1,T})$	$\ln p(Y^{1,T} \vert Y^{0,T})$
Dataset without Expectations	Dataset with Expectations		Dataset without Expectations	Dataset with Expectations	
(1)	(2)	(2) - (1)	(3)	(4)	(4) - (3)
Robustness Policy Rule Specification					
(9) Output Level					
-715.46	-816.23	-100.77	-709.17	-791.74	-82.57
(10) 4Q Inflation					
-703.74	-820.96	-117.22	-698.88	-790.42	-91.5
(11) GSS					
-707.79	-805.64	-97.85	-709.45	-789.99	-80.54
Measurement Error					
(12) i.i.d. Measurement Error					
-703.62	-796.31	-92.69	-704.31	-780.89	-76.58
(13) AR(1) Measurement Error					
-703.62	-775.31	-71.69	-704.31	-775.21	-70.90

Table 7: Posterior Estimates for Selected Parameters

Parameters	**Imperfect Information** Dataset without Expectations (1)	**Imperfect Information** Dataset with Expectations (2)	**Perfect Information** Dataset without Expectations (3)	**Perfect Information** Dataset with Expectations (4)
Policy Parameters				
ψ_1	2.442 (0.225)	1.915 (0.123)	2.497 (0.247)	2.324 (0.191)
ψ_2	0.282 (0.112)	0.255 (0.106)	0.232 (0.093)	0.264 (0.110)
ρ_r	0.407 (0.077)	0.375 (0.065)	0.454 (0.067)	0.592 (0.043)
$\rho_{\pi^\star}$	0.945 (0.025)	0.907 (0.021)	0.943 (0.025)	0.974 (0.011)
σ_r	0.404 (0.037)	0.422 (0.033)	0.389 (0.036)	0.435 (0.035)
σ_{π^*}	0.054 (0.010)	0.048 (0.009)	0.058 (0.012)	0.066 (0.009)
Nominal Rigidities Parameters				
ζ_p	0.579 (0.061)	0.530 (0.057)	0.558 (0.051)	0.580 (0.061)
ι_p	0.285 (0.182)	0.494 (0.202)	0.346 (0.181)	0.317 (0.167)
ζ_w	0.249 (0.069)	0.186 (0.031)	0.238 (0.061)	0.353 (0.098)
ι_w	0.400 (0.251)	0.540 (0.257)	0.375 (0.253)	0.370 (0.236)
Other "Endogenous Propagation and Steady State" Parameters				
α	0.340 (0.003)	0.340 (0.004)	0.340 (0.003)	0.341 (0.003)
s''	2.831 (0.880)	4.529 (1.152)	3.002 (0.902)	3.543 (1.205)
h	0.649 (0.047)	0.636 (0.053)	0.658 (0.049)	0.640 (0.046)
a'	0.291 (0.112)	0.212 (0.097)	0.275 (0.102)	0.274 (0.095)
ν_l	2.153 (0.534)	2.690 (0.649)	2.271 (0.588)	1.327 (0.510)
r^*	1.000 (0.423)	1.424 (0.541)	1.019 (0.452)	1.259 (0.471)
π^*	2.470 (0.996)	3.068 (0.574)	2.106 (0.759)	3.662 (1.134)
γ	1.629 (0.333)	1.511 (0.330)	1.646 (0.362)	1.454 (0.314)
g^*	0.272 (0.090)	0.304 (0.100)	0.287 (0.092)	0.306 (0.107)
ρs and σs				
ρ_z	0.203 (0.094)	0.200 (0.095)	0.247 (0.090)	0.177 (0.098)
ρ_ϕ	0.837 (0.071)	0.980 (0.013)	0.850 (0.062)	0.569 (0.218)
ρ_{λ_f}	0.823 (0.073)	0.838 (0.059)	0.840 (0.058)	0.803 (0.071)
ρ_μ	0.885 (0.050)	0.910 (0.025)	0.897 (0.044)	0.894 (0.051)
ρ_g	0.810 (0.116)	0.824 (0.056)	0.798 (0.140)	0.982 (0.016)
σ_z	0.699 (0.055)	0.693 (0.052)	0.709 (0.055)	0.689 (0.047)
σ_ϕ	3.008 (0.516)	3.327 (0.589)	3.055 (0.638)	2.656 (0.660)
σ_{λ_f}	0.146 (0.031)	0.175 (0.031)	0.156 (0.026)	0.149 (0.023)
σ_μ	0.468 (0.115)	0.410 (0.083)	0.464 (0.111)	0.398 (0.099)
σ_g	0.291 (0.050)	0.426 (0.047)	0.267 (0.050)	0.410 (0.050)

Notes: The table reports the posterior mean and standard deviation (in parenthesis) of the parameters for the Imperfect and Perfect Information models obtained from both the datasets with and without inflation expectations.

Table 8: Variance Decomposition

Variables	Tech	ϕ	μ	g	λ_f	π^*	Money
Imperfect Information							
Output Growth	0.25	0.35	0.11	0.22	0.05	0.00	0.01
Labor Supply	0.00	0.94	0.05	0.01	0.01	0.00	0.00
Labor Share	0.05	0.03	0.00	0.02	0.88	0.00	0.01
Inflation	0.13	0.15	0.44	0.07	0.08	0.03	0.07
Interest Rate	0.08	0.09	0.60	0.08	0.05	0.00	0.00
Exp. Inflation	0.01	0.01	0.90	0.00	0.00	0.05	0.00
Perfect Information							
Output Growth	0.29	0.12	0.17	0.20	0.10	0.00	0.04
Labor Supply	0.03	0.09	0.31	0.3	0.06	0.00	0.01
Labor Share	0.06	0.07	0.00	0.01	0.83	0.00	0.02
Inflation	0.06	0.08	0.11	0.01	0.08	0.59	0.05
Interest Rate	0.05	0.08	0.35	0.01	0.06	0.28	0.14
Exp. Inflation	0.01	0.00	0.14	0.00	0.00	0.84	0.00

Notes: The Table shows the (unconditional) variance decomposition computed using the posterior distribution for the Imperfect and Perfect Information models obtained using the dataset that includes observed inflation expectations.

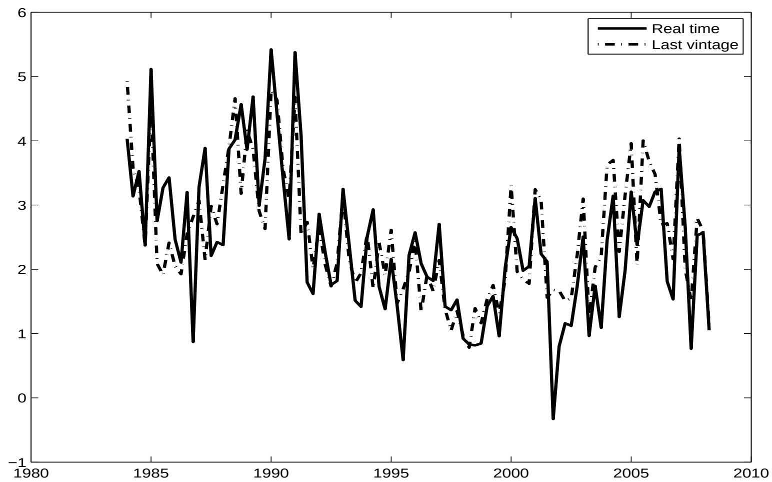
Table 9: Variance Decomposition for Observed Inflation Expectations: Models with Measurement Errors

Variables	Tech	ϕ	μ	g	λ_f	π^*	meas.	Money
Unconditional								
Imperfect Information								
i.i.d. Meas. Error	0.02	0.02	0.63	0.00	0.01	0.16	0.14	0.01
AR(1) Meas. Error	0.01	0.01	0.26	0.00	0.01	0.12	0.57	0.00
Perfect Information								
i.i.d. Meas. Error	0.01	0.01	0.21	0.00	0.01	0.67	0.07	0.00
AR(1) Meas. Error	0.01	0.00	0.25	0.00	0.00	0.27	0.41	0.00
10 Quarters Ahead								
Imperfect Information								
i.i.d. Meas. Error	0.01	0.01	0.39	0.00	0.01	0.12	0.44	0.01
AR(1) Meas. Error	0.01	0.00	0.25	0.00	0.01	0.08	0.63	0.01
Perfect Information								
i.i.d. Meas. Error	0.01	0.02	0.23	0.00	0.02	0.41	0.28	0.00
AR(1) Meas. Error	0.01	0.00	0.26	0.00	0.01	0.24	0.46	0.00

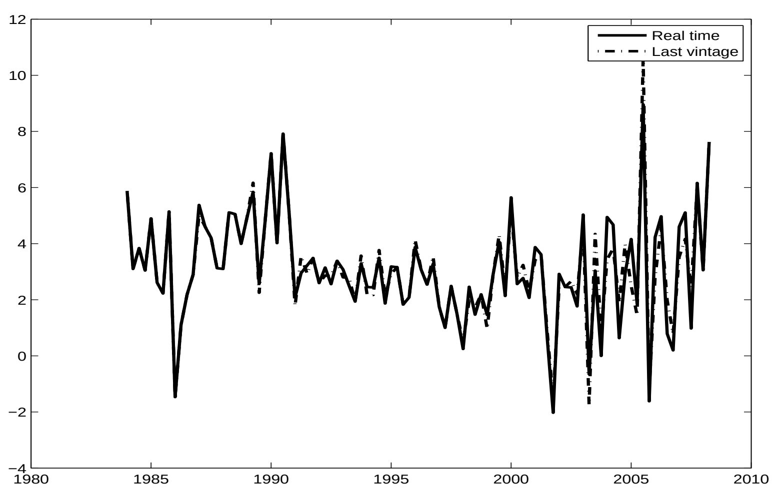
Notes: The Table shows the posterior means of the variance decomposition for observed inflation expectations – both unconditional and 10 quarters ahead – for the Imperfect Information and Perfect Information models with both i.i.d. and AR(1) measurement error. The posteriors are obtained using the dataset that includes observed inflation expectations.

Figure 1: Revisions in Inflation Data: Real Time vs Last Vintage

GDP Deflator



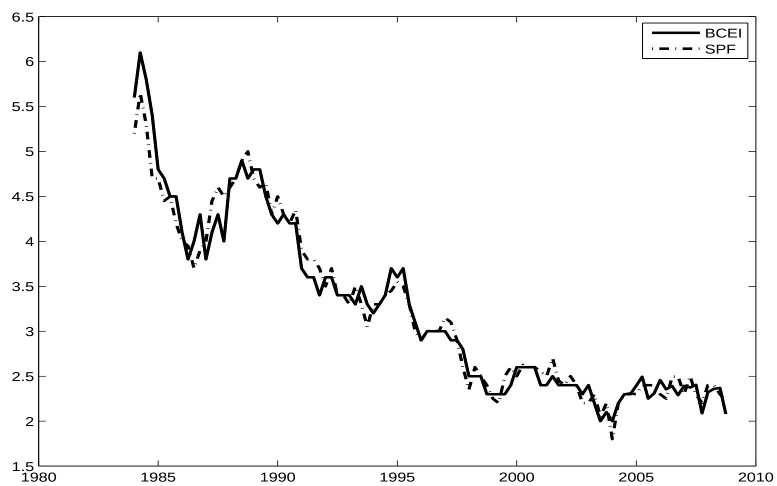
CPI



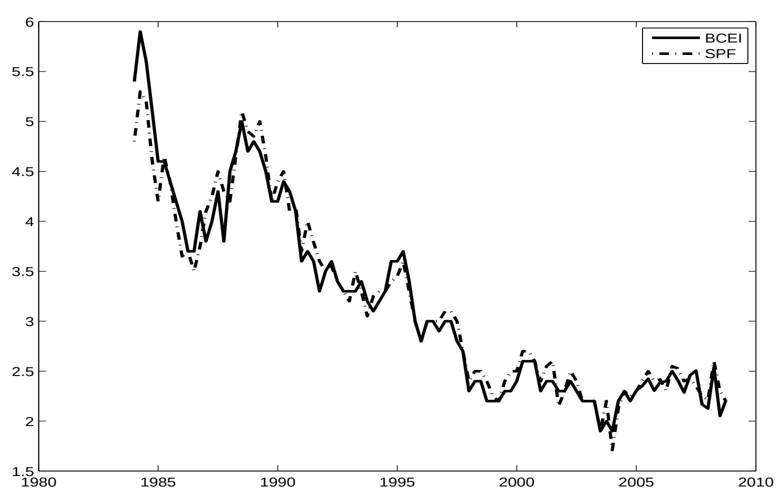
Notes: The figure plots data revisions for two measures of inflation: GDP deflator and CPI. The solid line shows the real time measure (that is, first vintage available) while the dashed-dotted line shows the most recent vintage.

Figure 2: Inflation Expectations: SPF vs Blue Chip

Four quarters ahead

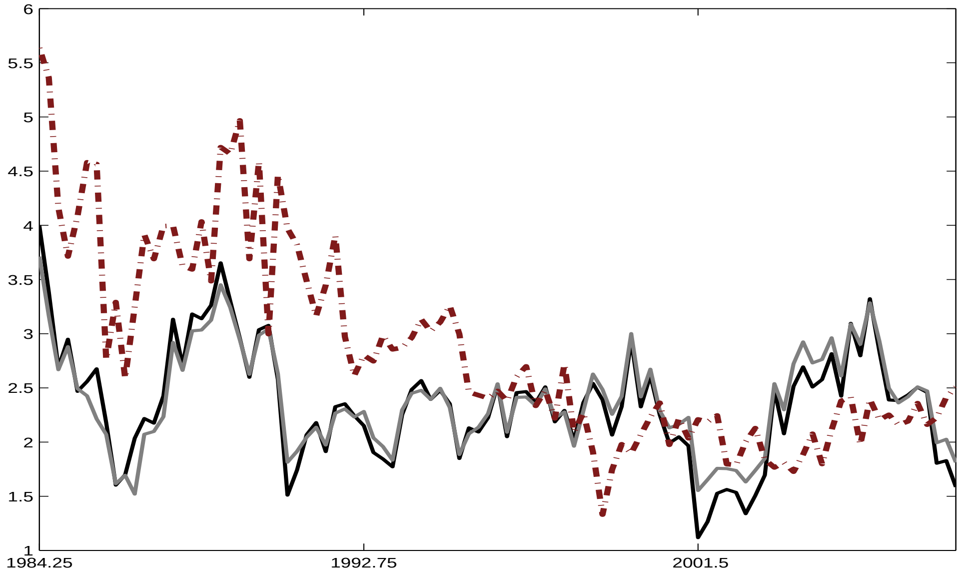


Three quarters ahead



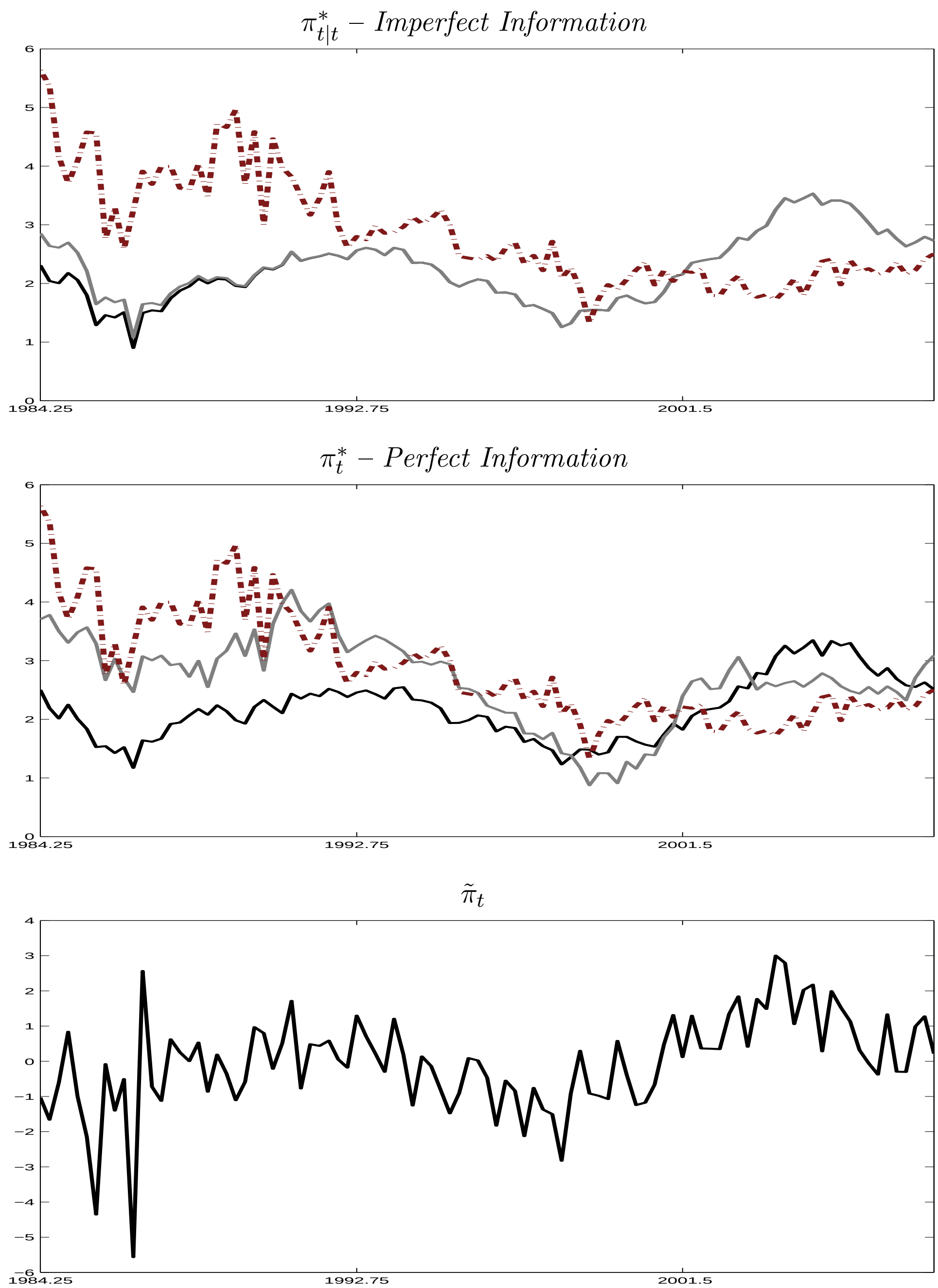
Notes: The figure plots inflation expectations from Blue Chip (solid line) and SPF (dashed line). The top panel shows quarterly (annualized) inflation expectations four quarters ahead, while the bottom panel shows expectations three-quarters ahead.

Figure 3: Inflation Expectations: Data vs Model Prediction



Notes: The figure plots SPF 4-quarters ahead median forecast for the GDP deflator (red dashed-and-dotted), together with the projections for the 4-quarter ahead inflation forecasts generated by the Imperfect (black solid) and the Perfect (gray solid) information models. The projections are computed using for each model the respective posterior mode for the dataset without expectations.

Figure 4: π^*_t



Notes: The top panel of the figure plots the mean estimate of the latent variable $\pi^*_{t|t}$ for the Imperfect Information model for the dataset without (black line) and with (gray line) inflation expectations. The middle panel shows the mean estimate of the latent variable π^*_t for the Perfect Information model for the dataset without (black line) and with (gray line) inflation expectations. The bottom line shows the interest feedback rule residual $\tilde{p}i_t$.