

Discussion of  
“The Aggregate Demand for Treasury Debt”  
by Arvind Krishnamurthy and Annette  
Vissing-Jorgensen

S. Boraĝan Aruoba  
University of Maryland

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# Summary of the Paper

- Start with the reduced-form utility function

$$u \left[ c_t + v \left( \frac{\theta_t^A}{GDP_t}, \xi_t \right) \right]$$

$c_t$ : endowment,  $\theta_t^A$ : “convenience assets”

- “Convenience function” satisfies  $v'(\cdot) > 0$  (with satiation) and  $v''(\cdot) < 0$ .
- Derive asset-pricing conditions comparing treasuries with  
corporate bonds.  
Key implication:  $\log(Debt/GDP)$  should affect various interest rate spreads **negatively**.
- Result : It does!
- Important implications regarding what “risk-free” rate is.

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- Are the empirical results robust with respect to variations?
- It is clear that  $B/PY$  is priced. Is it really because of liquidity and safety?
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- Easier to interpret as

$$E \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{subject to } c_t + \text{cost} \left( \frac{\theta_t^A}{GDP_t}, \xi_t \right) = y_t$$

$$\text{or } c_t = y_t + v \left( \frac{\theta_t^A}{GDP_t}, \xi_t \right)$$

- Direct analogy to transaction-cost models of money

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# Simple Example of Their Result

$$E \sum_{t=0}^{\infty} \beta^t u \left[ c_t + v \left( \frac{b_t}{p_t y_t}, \xi_t \right) \right]$$

$$p_t c_t + b_t + a_t = p_t y_t + R_{t-1}^b b_{t-1} + (1 - \lambda) R_{t-1}^c a_{t-1}$$

## • Euler Equations

$$u'(c_t) = \frac{u'(c_t) v'(c_t)}{y_t} + \beta R_t^b E_t \left[ \frac{u'(c_{t+1})}{\pi_{t+1}} \right]$$

$$u'(c_t) = \beta (1 - \lambda) R_t^c E_t \left[ \frac{u'(c_{t+1})}{\pi_{t+1}} \right]$$

## • Note equation (7) of the paper

$$\frac{P_t^T}{Q_t} u'(C_t) = \frac{P_t^T}{Q_t} u'(C_t) v' \left( \frac{\theta_t^A}{GDP_t} \right) + \beta E_t \left[ \frac{P_{t+1}^T}{Q_{t+1}} u'(C_{t+1}) \right]$$

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- Combining the Euler Equations

$$(1 - \lambda) \left[ 1 - \frac{v'(t)}{y_t} \right] = \frac{R_t^b}{R_t^c}$$

- Taking logs

$$i_t^c - i_t^b = \frac{v' \left( \frac{b_t}{p_t y_t}, \xi_t \right)}{y_t} + \lambda$$

- Since  $v''(\cdot) < 0$ , the implication is

$$\frac{\partial(i_t^c - i_t^b)}{\partial(b_t/p_t y_t)} < 0$$

- Default risk also increases the spread.

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- Micro-foundations matter!

- Isn't it more reasonable to assume  $v$  is a function of  $\left(\frac{b_t}{p_t c_t}\right)$ ?

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- Note that this will bring in many more terms in the FOCs.
- The optimality condition will probably still go through (but not sure).

- Would it make sense to have an analogy to a money-in-the-utility function? (more on this later)

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- As the authors also emphasize, money should have the same liquidity and safety properties, in addition to its medium of exchange property.
- Expand  $v(\cdot)$  to include money, e.g.  $\left(\frac{b_t + m_t}{p_t GDP_t}\right)$
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# Empirical Results 1 : Add Money

Focus on Table I, Panel A (Aaa-Treasury), Column 1

	Benchmark	M1S	M2S-M1S	M2S
Period	1927-2009	1931-2007	1961-2007	1961-2007
$\log(B/GDP)$	-0.69 (0.00)	-0.80 (0.00)	-1.21 (0.00)	-1.44 (0.00)
$\log(M/GDP)$	-	0.03 (0.84)	0.17 (0.76)	-1.07 (0.22)
$R^2$	0.80	0.82	0.76	0.76
N	83	77	47	47
Method	OLS	OLS	OLS	OLS

Notes: All regressions have the Aaa-Treasury spread as the dependent variable and are estimated via OLS, allowing for AR(2) in residuals. Newey-West standard errors. p-values in parentheses.

# Empirical Results 2 : Spread of Bonds and Money

- Money is **also** valued for its medium of exchange role :  $w(m/p)$  in the utility function. (and remove it from  $v(\cdot)$ .)

- Euler equations

$$u'(t) = \frac{u'(t)v'(t)}{y_t} + \beta R_t^b E_t \left[ \frac{u'(t+1)}{\pi_{t+1}} \right]$$

$$u'(t) = w'(t) + \beta E_t \left[ \frac{u'(t+1)}{\pi_{t+1}} \right]$$

- Spread between interest rates of treasuries and money (zero)

$$i_t^b \approx -v'(b) + w'(m)$$

- $Treas_t = cons \begin{matrix} -5.56 & \times & \frac{m}{PY} & -0.43 & \times & \frac{b}{PY} \\ (0.00) & & & (0.57) & & \end{matrix}$

- The medium of exchange role of money is priced.  
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## Empirical Results 2 : Spread of Bonds and Money

- Money is **also** valued for its medium of exchange role :  $w(m/p)$  in the utility function. (and remove it from  $v(\cdot)$ .)
- Euler equations

$$u'(t) = \frac{u'(t)v'(t)}{y_t} + \beta R_t^b E_t \left[ \frac{u'(t+1)}{\pi_{t+1}} \right]$$

$$u'(t) = w'(t) + \beta E_t \left[ \frac{u'(t+1)}{\pi_{t+1}} \right]$$

- Spread between interest rates of treasuries and money (zero)

$$i_t^b \approx -v'(b) + w'(m)$$

- $Treas_t = cons \begin{matrix} -5.56 & \times & \frac{m}{PY} & -0.43 & \times & \frac{b}{PY} \\ (0.00) & & & (0.57) & & \end{matrix}$
- The medium of exchange role of money is priced.  
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# Empirical Results 3 : Use $B/P$ instead of $B/PY$

	Benchmark	M1S	M2S-M1S	M2S
Period	1930-2009	1930-2007	1960-2007	1961-2007
$\log(B/P)$	-0.03 (0.80)	-0.73 (0.02)	-1.04 (0.00)	-1.23 (0.00)
$\log(M/P)$	-	1.08 (0.02)	1.26 (0.00)	1.54 (0.00)
$R^2$	0.00	0.16	0.48	0.49
N	80	78	48	48
Method	FMOLS	FMOLS	FMOLS	FMOLS

Notes: All regressions have the Aaa-Treasury spread as the dependent variable and are estimated via FMOLS (fully-modified OLS) to capture the cointegration. Newey-West standard errors. p-values in parentheses.



# Summary So Far

- It is clear that  $B/PY$  is priced.
- If it was **only** due to liquidity / safety they provide, then  $M/PY$  should have been priced as well.
  - Aside: Is it clear that corporate bonds are less liquid?
- It is not!
- It seems  $B/PY$  is priced for a reason that is **not** shared by money.
  - Calvo and Gertler's "liquidity preference"
  - Fed's balance sheet
- Need a structural model to sort these out.
- Using  $B/P$  in the convenience function seems equally reasonable. It doesn't yield the same empirical results.

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  - Cash balance sheet
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# Money Demand Regressions - 1

	M1S	M1S	M2S-M1S	M2S-M1S	M2S	M2S
$i^b$	-0.09 (0.00)	-0.07 (0.00)	0.03 (0.00)	0.02 (0.01)	0.00 (0.55)	-0.00 (0.29)
$\log(y)$	0.83 (0.00)	0.80 (0.00)	0.91 (0.00)	0.94 (0.00)	0.87 (0.00)	0.88 (0.00)
$\log(B/PY)$	-	0.26 (0.00)	-	-0.26 (0.00)	-	-0.12 (0.01)
$R^2$	0.96	0.98	0.96	0.97	0.99	0.99
N	75	75	46	47	46	46

Notes: All regressions have natural logarithm of a measure of real money balances as the dependent variable and are estimated via FMOLS (fully-modified OLS) to capture the cointegration relationship. Newey-West standard errors. p-values in parentheses.

# Money Demand Regressions - 2

## (M1S, before and after 1970)

	Before	Before	After	After
$i^b$	-0.21 (0.00)	-0.08 (0.00)	-0.06 (0.00)	-0.05 (0.00)
$\log(y)$	0.98 (0.00)	0.81 (0.00)	0.93 (0.00)	1.00 (0.00)
$\log(B/PY)$	-	0.28 (0.00)	-	-0.14 (0.09)
$R^2$	0.96	0.99	0.97	0.98
N	40	40	35	35

Notes: All regressions have a natural logarithm of real M1S as the dependent variable and are estimated via FMOLS (fully-modified OLS) to capture the cointegration relationship. Newey-West standard errors. p-values in parentheses.

# Conclusions

- Very interesting paper.
- Document an interesting fact.
  - The stock of debt is priced.
- It is not clear that it is the liquidity property of treasuries that is priced.
- We need more structural work (a model that distinguishes various other roles of treasuries).
- Treasuries seem like
  - complements of very liquid part of money supply. (M1S)
  - substitutes of less liquid part of money supply. (M2S minus M1S)
- Adding stock of treasuries help stability of money demand over time.