Discussion of “Risk Aversion and the Labor Margin in Dynamic Equilibrium Models ” by Eric Swanson

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What does this paper do?

- From a well-known paper

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t^\tau \tag{1}$$

where β is the discount factor and the instantaneous utility function is separable in consumption and labor (leisure):[7]

$$U_t^\tau = \varepsilon_t^b \left(\frac{1}{1-\sigma_c} (C_t^\tau - H_t)^{1-\sigma_c} - \frac{\varepsilon_t^L}{1+\sigma_l} (\ell_t^\tau)^{1+\sigma_l} \right) \tag{2}$$

Utility depends positively on the consumption of goods, C_t^τ, relative to an external habit variable, H_t, and negatively on labor supply ℓ_t^τ. σ_c is the coefficient of relative risk aversion of households or the inverse of the intertemporal elasticity of substitution; σ_l represents the inverse of the elasticity of work effort with respect to the real wage.

- This paper points out that σ_c (or γ in its notation) is **not** the "coefficient of relative risk aversion for the households".

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- **Contribution of the paper**: At the steady state $RRA \neq \gamma$ for various specifications of utility.

$$u(c,h) = \frac{c^{1-\gamma}}{1-\gamma} - \eta \frac{h^{1+\chi}}{1+\chi} \Longrightarrow RRA \approx \left(\frac{1}{\gamma} + \frac{1}{\chi}\right)^{-1}$$

  - For quasi-linear preferences [Hansen-Rogerson, Lagos and Wright (2005)] $RRA = 0$!
- The idea that risk aversion is not equal to γ when labor is endogenous is not well-known.
- No labor:
- Imply that with labor $RRA < \gamma$.

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- Separate the decision problem household into two: assets (a) and intratemporal choices and rewrite the choice problem using an indirect utility function $V(a)$.

- Compute how much the households would be willing to pay to avoid lotteries in assets. (like Arrow-Pratt)

- **Key :** Period utility is now implicitly defined in terms a composite good combining c and h.
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General Comments to Think About

- Extend to some key models:
  - Labor-search : Typically assume consumption insurance.
  - Monetary models : Either have QL preferences (Lagos-Wright) or CRRA utility for real money balances.
- Reconciliation of macro / micro results ?
  - Typical micro estimate for RRA: 1. (Chetty, 2006)
  - Typical micro estimate for Frisch elasticity : 0.5. (Chetty, 2010)
  - $1 = \left(\frac{1}{\gamma} + 0.5\right)^{-1} \Longrightarrow \gamma = 2, \chi = 2$
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- How about dynamics?
  - Is $RRA \approx \gamma$ still a good approximation?
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Prototype RBC Model with Separable Preferences

- $V(a,k) = \max_{c,h,a',k'} u(c,h) + \beta E V(a',k')$
- $c + a' + k' = (1+r)a + (r^k + 1 - \delta)k + wh$
- Model with capital $\Longrightarrow$ real return $r_t = r_t^k - \delta$.
- Relative risk aversion : $RRA_t = \dfrac{A_t E_t V_1(t+1)}{E_t V_{11}(t+1)}$.
- A_t : lifetime wealth. Here I set $A_t = A = c/r$.

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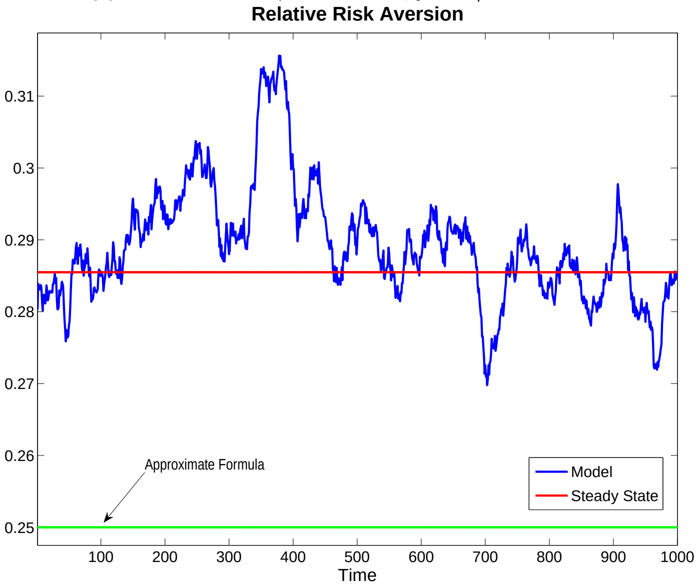
Second-order approximation. $\gamma = 1$ and $\chi = 1/3$



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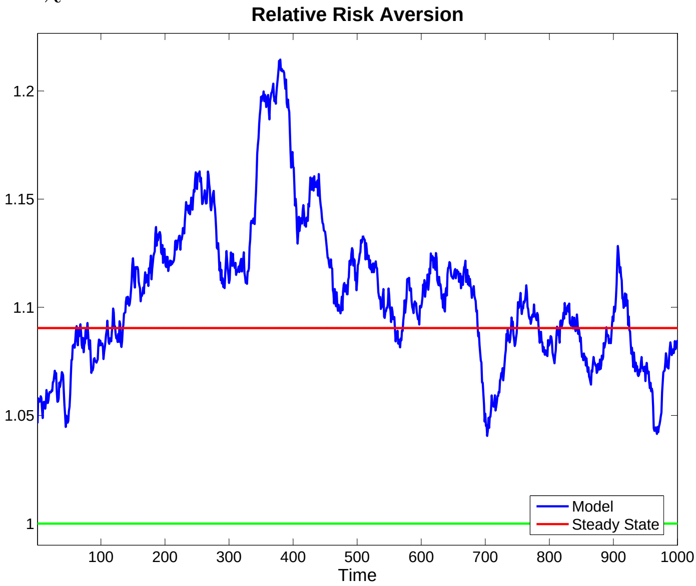
$\gamma = 2$ and $\chi = 2$



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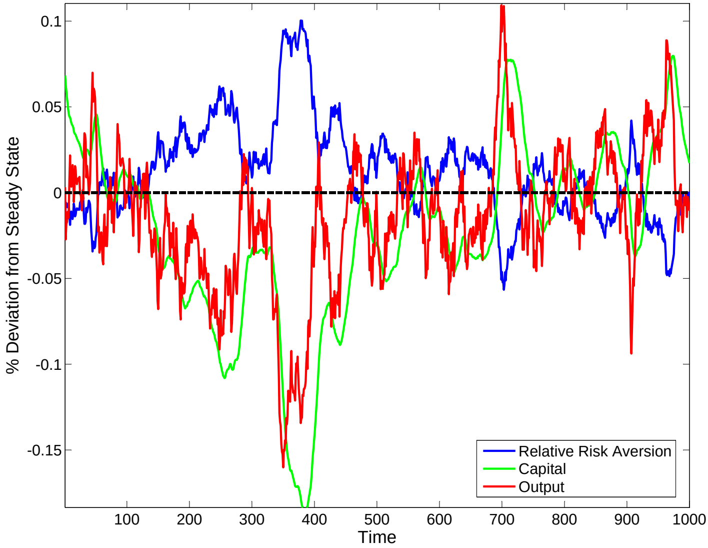

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- First order decision rule : $R\hat{R}A_t = -0.33\hat{K}_t - 0.59\hat{Z}_t$
- $corr(RRA_t, Y_t) = -0.98.$
- Actual and approximate (as in the paper) steady state risk aversions

γ/χ	0.01	0.25	0.5	1	2
1	0.01 / 0.01	0.23 / 0.20	0.37 / 0.33	0.55 / 0.50	0.71 / 0.67
2	0.01 / 0.01	0.26 / 0.22	0.46 / 0.40	0.75 / 0.67	1.09 / 1.00
5	0.01 / 0.01	0.28 / 0.24	0.54 / 0.45	0.97 / 0.83	1.62 / 1.43
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Prototype RBC Model with Separable Preferences

- Volatility of RRA_t relative to output

γ/χ	0.01	0.25	0.5	1	2
1	0.41	0.41	0.41	0.41	0.41
2	0.45	0.47	0.48	0.49	0.49
5	0.49	0.54	0.58	0.61	0.64
10	0.52	0.59	0.63	0.69	0.75

Prototype RBC Model with KPR Preferences

- King-Plosser-Rebelo (1988) : $u(c,h) = \frac{c^{1-\gamma}(1-h)^{\chi(1-\gamma)}}{1-\gamma}$
- But now $(1-\gamma)$ applies to the composite good $c(1-h)^{\chi}$.
- The approximate formula in the paper $RRA = \frac{\gamma - \chi(1-\gamma)}{1+\chi}$
- This measure remains near γ (more or less) for a "macro" calibration.

γ/χ	0.01	0.25	0.33	0.5	1	2
1	0.99	0.80	0.75	0.67	0.50	0.33
2	1.99	1.80	1.75	1.67	1.5	1.33
5	4.99	4.80	4.75	4.67	4.5	4.33
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Asset Pricing

- r_t^m : real market return, r_t^f : real risk-free return, r_t^e : real excess return

$$E_t \left\{ \left[\beta \frac{u_1(c_{t+1}, h_{t+1})}{u_c(c_t, h_t)} (1 + r_{t+1}^i) - 1 \right] Z_t \right\} = 0 \qquad (1)$$

$i = m, f$ and $Z_t = [1 \;\; \delta log(c_{t-1}) \;\; r_{t-1}^i \;\; ... \;\; \Delta log(c_{t-4}) \;\; r_{t-4}^i]$.

$$E_t \left\{ \left[\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} (1 + r_{t+1}^i) - 1 \right] Z_t \right\} = 0 \qquad (2)$$

Estimate via GMM (Hansen and Singleton, 1982, 1984)

- r^m : $\hat{\beta} = 0.98$ (0.02), $\hat{\gamma} = -0.31$ (2.72), OIR p-value : 0.99
- r^f : $\hat{\beta} = 0.998$ (0.002), $\hat{\gamma} = 0.26$ (0.24), OIR p-value : 0.02

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$$E_t \left\{ \left[\frac{u_1(c_{t+1}, h_{t+1})}{u_c(c_t, h_t)} r^e_{t+1} \right] Z_t \right\} = 0 \tag{3}$$

$$E_t \left\{ \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} r^e_{t+1} \right] Z_t \right\} = 0 \tag{4}$$

- r^e : $\hat{\gamma} = 107$ (49), OIR p-value : 0.74

With non-separable labor:

$$E_t \left\{ \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \left(\frac{1 - h_{t+1}}{1 - h_t} \right)^{\chi(1-\gamma)} r^e_{t+1} \right] Z_t \right\} = 0 \tag{5}$$

- r^e : $\hat{\chi} = 1.27$ (0.05), $\hat{\gamma} = 140$ (67), OIR p-value : 0.42

Intuition

- Without labor : $E_t(r^e_{t+1}) \approx \gamma cov(\Delta log(c_{t+1}), r^e_{t+1})$

- With labor :

$$E_t(r^e_{t+1}) \approx \gamma cov(\Delta log(c_{t+1}), r^e_{t+1}) - \chi(1-\gamma) cov(\Delta log(1-h_{t+1}), r^e_{t+1})$$

- $cov(\Delta log(c_{t+1}), r^e_{t+1}) \approx 2 cov(\Delta log(1 - h_{t+1}), r^e_{t+1})$

- $\chi \approx 1$

- Without labor we need 2γ. With labor we need $\gamma + 1 < 2\gamma$.

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Bottomline

Very interesting paper.

Looking forward to the asset-pricing implications.

We should stop calling γ, the CRRA parameter!