Optimal Fiscal and Monetary Policy When Money is Essential*

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Abstract

We study optimal fiscal and monetary policy in an environment where explicit frictions give rise to valued money, making money essential in the sense that it expands the set of feasible trades. The two main results are that the Friedman Rule is typically not optimal, and the long-run capital income tax is not zero. Neither of these results is due to any incompleteness of the tax system, as can sometimes occur in standard Ramsey analysis. Rather, by developing a precise notion of margins of adjustment using standard concepts of MRS and MRT, we show that the tax system in our model is complete. The need to distort cash-intensive activity in some sense *causes* a nonzero capital tax in our model. This deep connection between monetary issues and fiscal policy is in contrast to existing models of jointly-optimal fiscal and monetary policy, in which the monetary aspects of the economic environment have little to do with capital taxation prescriptions. Taken together, these findings reframe some conventional wisdom from baseline Ramsey models.

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1 Introduction

Monetary theory has made important advances of late, ones that enable researchers interested in applied policy questions to consider explicit frictions that give rise to valued money. In this paper, we build on the work of [31] and [9] to study optimal fiscal and monetary policy, in the tradition of [32] and [15]. Two main results emerge from our work: the Friedman Rule is typically not optimal, and the optimal long-run capital income tax is not zero. The first result is opposite that of standard flexible-price Ramsey monetary models. The second result, although also obtainable in standard flexible-price Ramsey models, is driven by a unique connection between monetary policy and fiscal policy present in our model that is absent in reduced-form models of money demand. Taken together, these results reframe conventional wisdom from baseline Ramsey monetary analyses.

The contribution of [31] and [9] — hereafter, LW and AWW, respectively — was to integrate search-based monetary theory, in the spirit of [26] and [27], with standard dynamic general equilibrium macroeconomics. This integration makes the study of policy questions much easier and more relevant than in earlier search-based models. However, these models have been criticized on two grounds. First, they superficially resemble standard cash-in-advance (CIA) or money-in-the-utility-function (MIU) models, making some question whether they really are any deeper than reduced-form models of money. This point has been raised by, among others, [23]. Second, until now, the policy questions addressed in these new models have been largely confined to the long-run welfare costs of inflation. When parameterized to seem as close as possible to standard CIA and MIU models, the quantitative answers they have yielded to this question are similar to those obtained with CIA and MIU models, further adding to the sense that these new models simply re-invent CIA or MIU. In this paper, we ask a different policy-relevant question, the jointly-optimal fiscal and monetary policy, and even when we parameterize the model to look very similar to standard reduced-form models of money, we reach conclusions very different from those reached by [15] — hereafter, CCK — and others. Our results thus show that the answers to policy questions may indeed be very different once monetary frictions are treated seriously.

Our first main result is that the nominal interest rate is typically positive because it is optimal to tax activities that require cash.[1] The reason behind this result is that, because all final goods should be taxed to some degree as part of an optimal tax system, taxation of cash activities is naturally part of the second-best allocation. This prescription is simply standard Ramsey theory. In the LW and AWW environments, the explicit spatial and informational frictions that make

[1] In a different context, one that abstracts from public finance considerations, [35] show that a positive nominal interest rate may be optimal because it can correct inefficiencies along the extensive margin of bilateral trading by influencing the relative number of traders on each side of the market. In other micro-founded models of money that also abstract from public finance considerations, [38], [12] and [22] also find that deviations from the Friedman Rule can be optimal.

money essential (in the sense of [28] that it expands the set of feasible trades) render inflation the most natural way of taxing activities that require money. As we discuss, our results can be reconciled both technically and conceptually with those of CCK. Interestingly, [29] conjectured that the Friedman Rule may not be optimal in a Ramsey problem in search-based models. Our results show his conjecture is correct.

Our second main result is that capital income subsidies are optimal in the long run, for two distinct reasons. First, the holdup problems in capital investment that AWW show arise in this environment (that is, the fact that some parties must make unilateral capital investment decisions but then share the fruits of the capital via production in bilateral trades) lead to inefficiently-low capital accumulation, which in turn calls for, perhaps not surprisingly, optimal capital-income subsidies. The second rationale highlights the idea that capital-tax policy can depend on the primitive reasons for money demand. In a version of our model where the capital holdup problem is eliminated, the optimal *monetary* policy, which entails a deviation from the Friedman Rule, in turn distorts private-sector capital accumulation. Inflation thus acts as an indirect tax on capital accumulation, which a capital-income subsidy (either partially or wholly) offsets.

The first channel above can be viewed as a monetary counterpart to [1], who demonstrate, in a purely real environment, that search and bargaining frictions in labor markets lead to holdup problems in capital investment. While they do not explicitly draw optimal taxation implications, it seems clear from their analysis that capital subsidies would be optimal in their bargaining environment. However, in a version of their model with no holdup problem, there would not be any need to subsidize capital, which is a novel result of our model. Thus the second channel is a nontrivial interaction between monetary and fiscal policy. In this sense, our optimal capital-taxation prescription is driven by fundamentally monetary issues, in contrast to the studies of [37] and [18], in which capital-taxation prescriptions are, qualitatively, independent of the monetary policy prescription or even whether or not money demand is modeled.

Neither the non-zero capital tax nor the deviation from the Friedman Rule in our analysis has anything to do with incompleteness of the tax system. Completeness or incompleteness of a tax system is a concept that can be defined in both monetary models and purely real models. In the literature on optimal capital taxation, the examples of [19], [24], and [5] illustrate that incompleteness of the tax system typically leads to non-zero capital-income taxation. This is because the capital tax ends up imperfectly substituting for the ability to create certain wedges. Similarly, one can easily show that in the monetary models of [16], if the tax system were incomplete, the Friedman Rule would not be optimal because a positive nominal interest rate serves as an imperfect substitute for the ability to create a wedge in the consumption-leisure margin. In our analysis, neither the deviation from the Friedman Rule nor the non-zero capital tax arises due to any inability on the part of the government to create any wedges.

The rest of our work is organized as follows. Section 2 describes the environment, characterizes the private sector equilibrium and describes efficient allocations. Section 3 presents the Ramsey problem and presents our results. Section 4 provides further perspectives on and intuition for our results and proves that the tax system is complete in the sense defined above. Section 5 concludes. We provide the details of most of our analytical results in the Appendix.

2 Model

Our model follows closely the baseline model in AWW, which extends LW to allow capital accumulation. Most of our analysis uses the full AWW model, but we also present some results for the LW version of the model and explain how it is indeed a special case of the full model.

The economy is populated by a measure one of infinitely-lived ex-ante identical households. Aggregate states may evolve over time, but they do so deterministically; that is, there is no aggregate uncertainty in the environment. For our purposes, this is without loss of generality because all of our results are either about static (within-period) outcomes or deterministic steady-states of dynamic outcomes.

In any period t, households first trade in a centralized market. During the centralized market (CM), a household rents its previously-accumulated capital and supplies its labor on spot factor markets; it chooses CM consumption in a spot goods market; and it adjusts its holdings of capital, a one-period nominal government bond, and money. Prices are determined competitively in all trades executed in the CM.

Upon exiting the CM, each household receives an idiosyncratic taste shock that governs its trading status in the second subperiod of period t, the decentralized market (DM).[2] A given household is a buyer in the DM with fixed probability σ, a seller with fixed probability σ, and neither a buyer nor a seller with probability $1 - 2\sigma$.[3] In the DM, buyers and sellers meet randomly, and trade is bilateral. In a given trade, a seller household produces goods for the buyer household using effort and capital, and receives money in return from the buyer. The terms of trade in a bilateral meeting are determined either through bargaining, which has become fairly standard in this class of models, or through Walrasian pricing (price-taking) as proposed by [35]). Considering

[2]Compared to AWW, we have reorganized the timing of markets by assuming that the CM meets before the DM in a given period. We make this change to mimic as closely as possible the timing assumption in standard monetary models — in particular, the cash-in-advance models of [32] and CCK — in which asset trade occurs before trade in those goods markets in which money is the medium of exchange. This change in the timing of markets compared to AWW is inconsequential for all of our results.

[3]This "taste shock" structure is simply a shortcut for the fully-specified environment in LW and the earlier monetary theory literature, in which the environment is specified explicitly in terms of search and double-coincidence problems.

different pricing mechanisms allows us to disentangle competing incentives that affect the Ramsey allocation, including the holdup problems we discussed in the Introduction.

In the rest of this section we provide the primitives of the model, characterize household behavior, and define equilibrium. Further details are available in AWW and in an appendix available from the authors.

2.1 Production

In the CM, production takes place according to a constant-returns technology subject to TFP fluctuations, $Y_t = Z_t F(K_t, H_t)$. The notation is standard: K_t denotes aggregate capital, H_t denotes aggregate labor, and Z_t is the state of aggregate TFP. Profit maximization by perfectly-competitive firms leads to standard factor-price conditions: the wage satisfies $w_t = Z_t F_H(K_t, H_t)$, and the rental price of capital satisfies $r_t = Z_t F_K(K_t, H_t)$.

In the period-t DM, sellers produce using their capital carried out of the period-t CM, which, according to our timing and notational conventions, is k_{t+1}. Output in the DM is produced according to the technology $q_t = Z_t f(k_{t+1}, e_t)$, where e_t is the effort exerted by the seller, which creates a disutility given by $v(e_t)$. Solving for the effort necessary for producing q_t units of DM output using capital k_{t+1} and given aggregate technology state Z_t and computing its disutility implies a cost function $c(q_t, k_{t+1}, Z_t)$, which describes the (utility) cost of production. With $f(.)$ strictly increasing and strictly concave in each of its two arguments, and $v(.)$ strictly increasing and strictly convex, it readily follows that $c_q > 0$, $c_k < 0$, $c_z < 0$, $c_{qq} > 0$, $c_{qk} < 0$ and $c_{kk} > 0$.

2.2 Households

A household enters the period-t CM with money holdings m_{t-1}, nominal government bond holdings b_{t-1}, and capital holdings k_t. The exogenous, though deterministic, aggregate CM state is denoted collectively as $S_t = (G_t, Z_t)$. Denoting the household's value function at the beginning of the period-t CM by $W_t(.)$ and the household's value function at the beginning of the period-t DM by $V_t(.)$, the household's CM problem is

$$W_t\left(m_{t-1}, b_{t-1}, k_t, S_t\right) = \max_{x_t, h_t, m_t, b_t, k_{t+1}} \left[U(x_t) - Ah_t + V_t(m_t, b_t, k_{t+1}, S_t)\right]$$

subject to

$$P_t x_t + P_t \left[k_{t+1} - (1-\tau_t^k)(r_t - \delta)k_t\right] + m_t + b_t = P_t w_t (1-\tau_t^h) h_t + m_{t-1} + R_{t-1} b_{t-1}.$$

P_t is the nominal price of the (only) consumption good in the CM, which is x_t; R_{t-1} is the gross nominal (non-state-contingent) return on the one-period government bond purchased in period $t-1$

and redeemed in period t; and τ_t^k and τ_t^h are the tax rates on capital income (net of depreciation) and labor income, respectively. Exploiting the linearity of the value function $W_t(.)$ with respect to m_{t-1} we define $\chi_t \equiv \left[A/\{P_{t+1}w_{t+1}(1-\tau_{t+1}^h)\}\right]$ as the marginal value of entering $t+1$ with one extra unit of money. Another important result from the CM problem is that regardless of their asset (money, capital and bond) holdings entering the CM, all agents will choose the same asset holdings, yielding a degenerate distribution of assets.

Turning to the DM, and exploiting the degeneracy of asset distributions, we can write the problem of a household that enters the DM with portfolio (m_t, b_t, k_{t+1}) as

$$\begin{aligned} V_t(m_t, b_t, k_{t+1}, S_t) &= \sigma\left[u\left(q_t^b\right) + \beta W_{t+1}\left(m_t - d_t^b, b_t, k_{t+1}, S_{t+1}\right)\right] \\ &+ \sigma\left[-c\left(q_t^s, k_{t+1}, Z_t\right) + \beta W_{t+1}\left(m_t + d_t^s, b_t, k_{t+1}, S_{t+1}\right)\right] \\ &+ (1-2\sigma)\beta W_{t+1}\left(m_t, b_t, k_{t+1}, S_{t+1}\right). \end{aligned} \tag{1}$$

The first line describes the payoff if the household is a buyer where the household gets utility from consuming the DM good but has less money to take to the next CM. The second line describes the payoff if the household is a seller where the household exerts effort to produce for the buyer and continues to the next CM with more money. The last line describes the payoff if the household does not participate in the DM. We use (q_t^b, d_t^b) and (q_t^s, d_t^s) to represent the terms of trade from the viewpoints of the buyers and sellers, respectively.[4]

Next, we turn to characterizing how the terms of trade (q, d) are determined under the two pricing schemes we consider.

2.2.1 Household DM Problem - Bargaining

In this class of models, the most commonly-used pricing protocol for DM trades is bargaining — specifically, generalized Nash bargaining, with the the bargaining power of the buyer indexed by $\theta \in [0, 1]$. All of the analytical results we obtain are for the specification $\theta = 1$, which has the interpretation that the buyer makes a take-it-or-leave-it offer to the seller. In Section 4.2.4, we describe how results would change if we consider generalized Nash with $\theta < 1$ as well as an alternative bargaining mechanism, proportional bargaining.

Denote by (m_t, b_t, k_{t+1}) the portfolio of the buyer, and by $(\tilde{m}_t, \tilde{b}_t, \tilde{k}_{t+1})$ the portfolio of the

[4] AWW use a slightly more general model where they allow for a fraction $1-\omega$ of trades in the DM take place using credit. We use the version, as does LW, where $\omega = 1$.

seller. The generalized Nash bargaining problem is thus[5]

$$\max_{q_t,d_t} \left[u(q_t) + \beta W_{t+1}\left(m_t - d_t, b_t, k_{t+1}, S_{t+1}\right) - \beta W_{t+1}\left(m_t, b_t, k_{t+1}, S_{t+1}\right)\right]^{\theta}$$
$$\times \left[-c(q_t, \tilde{k}_{t+1}, Z_t) + \beta W_{t+1}\left(\tilde{m}_t + d_t, \tilde{b}_t, \tilde{k}_{t+1}, S_{t+1}\right) - \beta W_{t+1}\left(\tilde{m}_t, \tilde{b}_t, \tilde{k}_{t+1}, S_{t+1}\right)\right]^{1-\theta}$$

subject to

$$d_t \leq m_t. \tag{2}$$

The amount of cash that a buyer turns over to a seller in a DM trade is d_t; the constraint (2) is thus simply a feasibility condition stating the buyer cannot spend more cash than he has on hand before meeting the seller. The threat points in the bargaining problem are the values of continuing on to the $t+1$ CM without consummating a trade.

The solution to this problem will have (2) bind and that the quantity produced solves

$$\beta \chi_t m_t = g(q_t, \tilde{k}_{t+1}, Z_t),$$

where

$$g(q, k, Z) \equiv \frac{\theta c(q, k, Z) u'(q) + (1-\theta) u(q) c_q(q, k, Z)}{\theta u'(q) + (1-\theta) c_q(q, k, Z)}. \tag{3}$$

These last two conditions show that (q, d) depends only on the buyer's money holdings and the seller's capital holdings (along with, of course, the level of TFP), and on neither the seller's money holdings nor the buyer's capital holdings.

2.2.2 Household DM Problem - Price-Taking

An alternative to bargaining is price taking, in which buyers and sellers each take the price of a unit of good in the DM, $\tilde{p}_t$, as given and solve their respective demand and supply problems. The buyer's problem is

$$V^b\left(m_t, b_t, k_{t+1}, S_t\right) = \max_{q_t} \left[u\left(q_t\right) + \beta W_{t+1}\left(m_t - \tilde{p}_t q_t, b_t, k_{t+1}, S_{t+1}\right)\right]$$

subject to $\tilde{p}_t q_t \leq m_t$. In equilibrium, this constraint binds, and we have $q_t = m_t/\tilde{p}_t$.

The seller's problem is

$$V^s\left(m_t, b_t, k_{t+1}, S_t\right) = \max_{q_t} \left[-c\left(q_t, k_{t+1}, Z_t\right) + \beta W_{t+1}\left(m_t + \tilde{p}q, b_t, k_{t+1}, S_{t+1}\right)\right],$$

[5]Implicit in the specification of the problem is the participation of each party, which is ensured by the nonnegativity of each term in the square brackets. When buyers make take-it-or-leave-it offers, the problem simplifies to maximizing the surplus of the buyer – the first square bracket – subject to the participation of the seller – the nonnegativity of the second square bracket.

which yields first-order condition $c_q(q_t, k_{t+1}, Z_t) = \beta \tilde{p}_t \chi_t$.

2.3 Government

Government consumption takes place in the CM and is financed by taxes on capital and labor income as well as money creation and debt issuance. Its CM flow budget constraint is thus

$$M_t + B_t + P_t w_t \tau_t^h H_t + P_t \tau_t^k (r_t - \delta) K_t = P_t G_t + M_{t-1} + R_{t-1} B_{t-1}.$$

2.4 Monetary Equilibrium

Imposing equilibrium ($m_t = M_t$, $k_t = K_t$, etc.), and combining the optimality conditions for firms and households, we now list the equilibrium conditions we use in writing the Ramsey problem.

2.4.1 Bargaining

Given policy processes $\{\tau_t^h, \tau_t^k, R_t\}_{t=0}^{\infty}$, the technology process $\{Z_t\}_{t=0}^{\infty}$, the government spending process $\{G_t\}_{t=0}^{\infty}$, and initial conditions (M_0, B_0, K_0), equilibrium is a collection of sequences $\{q_t, B_t, M_t, K_t, X_t, H_t, P_t\}_{t=0}^{\infty}$ satisfying

$$U'(X_t) = \frac{A}{(1-\tau_t^h) Z_t F_H(K_t, H_t)}, \tag{4}$$

$$\beta M_t \left[\frac{U'(X_{t+1})}{P_{t+1}} \right] = g(q_t, K_{t+1}, Z_t), \tag{5}$$

$$U'(X_t) = \beta U'(X_{t+1}) \left[1 + (1-\tau_{t+1}^k)(Z_{t+1} F_K(K_{t+1}, H_{t+1}) - \delta) \right] + \sigma \gamma(q_t, K_{t+1}, Z_t), \tag{6}$$

$$R_t = \sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma, \tag{7}$$

$$\frac{U'(X_t)}{P_t} = \beta R_t \left[\frac{U'(X_{t+1})}{P_{t+1}} \right], \tag{8}$$

$$X_t + G_t + K_{t+1} = Z_t F(K_t, H_t) + (1-\delta) K_t, \tag{9}$$

and

$$M_t + B_t + P_t Z_t F_H(K_t, H_t) \tau_t^h H_t + P_t \tau_t^k [Z_t F_K(K_t, H_t) - \delta] K_t = P_t G_t + M_{t-1} + R_{t-1} B_{t-1}. \tag{10}$$

where $\gamma(.)$ is defined as

$$\gamma(q_t, K_{t+1}, Z_t) \equiv \frac{c_q(q_t, K_{t+1}, Z_t) g_k(q_t, K_{t+1}, Z_t) - c_k(q_t, K_{t+1}, Z_t) g_q(q_t, K_{t+1}, Z_t)}{g_q(q_t, K_{t+1} Z_t)}. \tag{11}$$

Some of these equilibrium conditions are identical to those in a standard RBC model, such as the consumption-leisure optimality condition (4) and the intertemporal Euler equation for bonds in (8). Condition (5) follows from the solution to the DM bargaining problem, and (7) is a no-arbitrage condition that links the nominal return on bonds to the implied return of holding money.

Given our special focus on capital income taxation, (6) deserves special attention. Except for the last term on the right-hand-side, (6) is a standard intertemporal Euler equation for capital investment. Because capital is used not only in the CM but also in the DM (with probability σ, if the household turns out to be a seller), optimal investment decisions take this into account. The additional term $\sigma\gamma(.)$ captures the return to capital in the DM, which reflects the fact that, all else equal, producing a given quantity of DM output is cheaper if a seller has more capital.

We also note that any monetary equilibrium must satisfy $R_t \geq 1$, which, when expressed in terms of allocations using (7), translates into what we call the zero lower bound (ZLB) constraint

$$\sigma\left(\frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1\right) \geq 0. \tag{12}$$

2.4.2 Price-Taking

Given policy processes $\{\tau_t^h, \tau_t^k, R_t\}_{t=0}^{\infty}$, the technology process $\{Z_t\}_{t=0}^{\infty}$, the government spending process $\{G_t\}_{t=0}^{\infty}$, and initial conditions (M_0, B_0, K_0), equilibrium is a collection of sequences $\{q_t, B_t, M_t, K_t, X_t, H_t, P_t\}_{t=0}^{\infty}$ satisfying (4), (8), (9), and (10), along with

$$\beta M_t\left[\frac{U'(X_{t+1})}{P_{t+1}}\right] = q_t c_q(q_t, K_{t+1}, Z_t), \tag{13}$$

$$U'(X_t) = \beta U'(X_{t+1})\left[1 + (1 - \tau_{t+1}^k)(Z_{t+1}F_K(K_{t+1}, H_{t+1}) - \delta)\right] - \sigma c_k(q_t, K_{t+1}, Z_t), \tag{14}$$

and

$$R_t = \sigma\frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma. \tag{15}$$

Finally using (15), the ZLB constraint ensuring a monetary equilibrium is

$$\sigma\left(\frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} - 1\right) \geq 0. \tag{16}$$

2.4.3 Special Case : Lagos and Wright Model

Our model nests one without capital accumulation by simply parameterizing the marginal of the DM cost function to $c_k = 0$ (or, equivalently, the marginal product of the DM technology to $f_k = 0$) and marginal product of capital in the CM to $F_K = 0$, while retaining the assumption of a constant-returns technology in both the DM and the CM.

2.5 Efficient Allocations, Holdup Problems and Intertemporal Efficiency

Before we turn to characterizing the allocations chosen by the Ramsey planner, it is instructive to characterize efficient allocations. The social welfare function can be obtained by integrating over all households with different realizations of shocks in the DM and is given by

$$\sum_{t=0}^{\infty} \beta^t \left\{ U(X_t) - AH_t + \sigma \left[u(q_t) - c(q_t, K_{t+1}, Z_t) \right] \right\}. \tag{17}$$

Proposition 1. *The first-best or efficient allocations are* $\{q_t, K_{t+1}, X_t, H_t\}$ *that satisfy*

$$u'(q_t) = c_q(q_t, K_{t+1}, Z_t), \tag{18}$$

$$U'(X_t) = \beta U'(X_{t+1}) \left[1 + (Z_{t+1} F_K(K_{t+1}, H_{t+1}) - \delta) \right] - \sigma c_k(q_t, K_{t+1}, Z_t), \tag{19}$$

$$U'(X_t) = \frac{A}{Z_t F_H(K_t, H_t)}, \tag{20}$$

and (9).

Proof. Follows from maximizing social welfare (17) subject to the resource constraint (9). □

We make several observations comparing (18)-(20) with the decentralized equilibrium conditions. First, obviously, in the presence of proportional taxes, neither price-taking nor bargaining can achieve the social optimum. Second, shutting down proportional taxes, the equilibrium under price-taking achieves the first best if $R_t = 1$, i.e. if the Friedman Rule of a zero net nominal interest rate is in place. Third, even in the absence of proportional taxes and at the Friedman Rule, the equilibrium under Nash bargaining can never achieve the social optimum. This is due to the two holdup problems present in the bargaining environment, one that afflicts money demand and one that afflicts investment in capital.

The holdup problems in our model occur because a party makes an irreversible ex-ante investment (buyers in money and sellers in capital) and the surplus of the trade is ex-post appropriated by the other party, which is the result of ex-post bargaining. The money holdup problem disappears if $\theta = 1$, when the seller does not get any part of the surplus; and the investment holdup problem would disappear if $\theta = 0$, but this would also shut down monetary equilibrium. For any intermediate value of θ, both holdup problems will be present.

The money holdup problem manifests itself in the $g_q(.)$ term in (7) instead of the $c_q(.)$ term in (18). Similarly, the investment holdup problem, which will be key in understanding a part of our results regarding capital taxation, manifests itself in the $\gamma(.)$ term in (6), instead of the $c_k(.)$ term in (19). In fact, looking at the definition of $\gamma(.)$ in (11), the difference arises from the existence of the first term, $c_q g_k / g_q$. This term captures the fact that as the seller brings in more capital to the DM, the amount he needs to produce for the buyer increases ($g_k/g_q > 0$). As a result, while an extra unit of capital helps reduce the cost of producing a given quantity for a seller, the extra capital would force the seller to produce more, reducing the benefits of the extra capital. This is precisely the impact of the investment holdup problem.

In contrast to the bargaining version, in the price-taking version of our environment, in the absence of proportional taxes and with $R_t = 1$, the equilibrium allocations coincide with those of the social optimum. This is because neither of the agents' actions directly affect the pricing in the DM, which eliminates both holdup problems.

3 Optimal Policy

This section describes the Ramsey problem and then presents the main results.

3.1 Ramsey Problem

As has been common in the public finance approach to macroeconomic policy since [32], we use the primal approach and formulate the Ramsey problem as the problem of a benevolent planner that chooses allocations subject to their decentralization as a monetary equilibrium.

Proposition 2. *The allocations in a monetary equilibrium satisfy the resource constraint (9), the ZLB constraint ((12) for the bargaining model or (16) for the price-taking model), and the present-value implementability constraint (PVIC),*

$$\sum_{t=0}^{\infty} \beta^t \left[U'(X_t)X_t - AH_t + \sigma g(q_t, K_{t+1}, Z_t) \left(\frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right) + \sigma\gamma(q_t, K_{t+1}, Z_t)K_{t+1} \right] = U'(X_0)A_0 \tag{21}$$

for the bargaining model or

$$\sum_{t=0}^{\infty} \beta^t \left[U'(X_t)X_t - AH_t + \sigma q_t c_q(q_t, K_{t+1}, Z_t) \left(\frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} - 1 \right) - \sigma c_k(q_t, K_{t+1}, Z_t)K_{t+1} \right] = U'(X_0)A_0 \tag{22}$$

for the price-taking model, where the constant A_0 depends on M_{-1}, B_{-1}, and K_0,

$$A_0 = \frac{M_{-1} + R_{-1}B_{-1}}{P_0} + \left[1 + (1 - \tau_0^k)(Z_0 F_K(K_0, H_0) - \delta) \right] K_0. \tag{23}$$

Proof. See Appendix A.1. □

Comparing these PVICs with ones from standard flexible-price models, such as the ones in [16], the third and fourth terms on the left-hand-sides of (21) and (22) are novel. The third term can be interpreted as the marginal utility of money times the amount of real money balances, where the "marginal utility" stems from being able to consume more in the DM if the household happens to be a buyer.[6] Similarly, the fourth term is nothing but the marginal DM benefit of capital times the capital holdings of the household. This extra benefit accrues when the household is a seller (which occurs with probability σ), and the benefit terms $\gamma(.)$ (bargaining) or $-c_k(.)$ (price-taking) are simply the marginal benefit of bringing capital into the DM as discussed above.

The Ramsey problem is thus to choose sequences $\{X_t, H_t, K_{t+1}, q_t\}$ that maximize the social welfare function (17) subject to the resource constraint (9), the PVIC ((21) for the bargaining model or (22) for the price-taking model), and the ZLB constraint ((12) for the bargaining model or (16) for the price-taking model), taking as given $\{G_t, Z_t\}$ and K_0. In Appendix A.2, we list the conditions that characterize the solution to the Ramsey problem for both pricing schemes, along with the conditions that allow for construction of the policies and prices that support the Ramsey allocation.

An important consideration in the construction of any Ramsey problem is whether or not a *complete* set of policy instruments is assumed to be available to the government. If not, then additional constraints beyond those just described must be imposed on the problem to reflect the incompleteness. In Section 4.1, we show that a complete set of instruments is indeed present in our model, which validates the Ramsey problem presented here.

3.2 Optimal Deviation from the Friedman Rule

Our first main result is that the Friedman Rule is typically not optimal, which we can prove for three important versions of the model: take-it-or-leave it offers by buyers (which, recall from above, is simply generalized Nash bargaining with $\theta = 1$), price taking, and a version of the model without physical capital with price taking. We start the analysis with the latter case because it enhances comparability with the benchmark LS and CCK studies, whose monetary policy results are obtained in models without capital. Furthermore, we focus on price-taking in the version without capital because this pricing mechanism makes DM trades as conceptually close as possible to a standard CCK-type environment.

Proposition 3. ***(Optimal Deviation from the Friedman Rule in Model without Capital)*** *In the model without capital, suppose terms of trade are determined using price taking in the DM. Then the optimal policy features a strictly positive net nominal interest rate in every period $t \geq 1$.*

Proof. See Appendix B. □

[6] To see this in the bargaining version, note that from the bargaining problem and (5), $S_b(q) \equiv u(q) - g(q, K, Z)$ is the surplus of the buyer and therefore $S_b'(q) \equiv u'(q) - g_q(q, K, Z)$ is the marginal surplus of the buyer. Moreover, money has no use in the DM unless the household is a buyer, which occurs with probability σ. Thus, the marginal utility of money can be expressed as $\sigma S_b'(q)\partial q/\partial m$. From (5) we have $m = g(q, K, Z)/\beta\chi$ and $\partial q/\partial m = \beta\chi/g_q(q, Z)$. Combining these, we obtain the third term under the summation in the PVIC.

Deviations from the Friedman Rule have been obtained in other Ramsey models, as well. For example, [36] show that a positive nominal interest can tax producers' monopoly profits, and [17] shows that it can tax monopolistic labor suppliers' rents. We know from Ramsey theory that taxing rents is optimal because it is non-distorting. However, the deviations from the Friedman Rule in [36] and [17] are instances of the Ramsey planner using a positive nominal interest rate to *indirectly* tax some rent — in neither case is activity requiring money the ultimate object the Ramsey planner seeks to tax. In Section 4, we offer a rent-seizing interpretation of our result and also further connect it to the results of CCK; here, though, we develop the idea behind the result based on just the primitives of our model environment.

The Ramsey allocation, independent from its actual implementation, requires q to be smaller than the socially efficient level, which we denote by q^*. From the perspective of the results in LW, AWW, and much of the ensuing related work, which invariably find that $q = q^*$ is optimal, the finding of $q < q^*$ being *optimal* in any sense may be surprising at first. However, a Ramsey problem — which is one about financing of government activities through distortionary taxation — is inherently one about creating optimal inefficiencies. A standard result in public finance is that such inefficiencies ought to be spread across *all* final goods. Because q is a final good, we have $q < q^*$. A strictly positive net nominal interest rate ($R > 1$) achieves this outcome.

The deviation from the Friedman Rule does not arise as an imperfect substitute for some other unavailable policy instrument. We show in Section 4.1 that our model features a complete tax system; in Section 4.2, we nevertheless entertain the idea of allowing the government to use additional tax instruments.

Now returning to the full model, we prove the Friedman Rule is suboptimal for two important versions of the environment with capital, and the economic reasons are as just discussed and as further elaborated in Section 4.

Proposition 4. ***(Optimal Deviation from the Friedman Rule in Model with Capital)***

1. *When buyers make take-it-or-leave-it offers, (or generalized Nash bargaining with $\theta = 1$), the optimal policy features a strictly positive net nominal interest rate in every period $t \geq 1$.*

2. *Under price-taking, if the DM production function $f(k, e)$ is constant-returns-to-scale, the optimal policy features a strictly positive net nominal interest rate in every period $t \geq 1$.*

Proof. See Appendix B. □

3.3 Optimal Capital Taxation

Having established results regarding the monetary aspects of optimal policy, we now turn to characterizing the associated fiscal aspects of optimal policy.

Proposition 5. ***(Optimal Tax for Labor)*** *Under both bargaining and price-taking, the optimal labor income tax is positive.*

Proof. See Appendix C. □

Proposition 6. ***(Optimal Subsidy for Capital)*** *Assuming CRS production in the DM, (a) when buyers make take-it-or-leave-it offers (generalized Nash bargaining with $\theta = 1$) or (b) with price-taking, the optimal long-run policy will include a subsidy to capital income except when (1) the DM is shut down (i.e. no trades are carried out exclusively with money) or (2) capital is not used in the DM (i.e., the DM and CM are decoupled).*

Proof. See Appendix D. □

The two parts of Proposition 6 allow us to disentangle two distinct motivations for capital subsidies. One obvious motivation is the investment holdup problem. As discussed above, provided $\theta > 0$ in the bargaining environment, a investment holdup problem is present, which leads to sub-optimally low private-sector capital accumulation. A subsidy to capital income naturally alleviates this problem. We think this result is interesting because existing optimal-taxation models that descend from [14] and [25] are not suited to consider how holdup problems affect capital accumulation and hence optimal capital tax rates. And yet, as [1] and [13] argue, holdup problems are prevalent in the economy and are likely to be important for macroeconomic issues. Search-based environments featuring capital accumulation decisions made before bilateral trades naturally can give rise to holdup problems. Although optimal-capital taxation implications are not explicitly drawn by [1], it seems clear in their (non-monetary) environment that search and bargaining frictions associated with labor would also lead to optimal capital subsidies.

However, it is not just holdup problems that lead to capital subsidies in our model, as the second part of Proposition 6 makes clear. Price-taking removes investment holdup (as well as money holdup) problems, as discussed above and as AWW show. Nonetheless, the Ramsey policy features a capital subsidy in the price-taking environment because optimal monetary policy spills over into optimal fiscal policy. This idea is a second distinct motivation for capital subsidies in our model, which we discuss further in Section 4.

Regardless of the precise reasons for a non-zero capital tax, we can also connect our results to [2], who provide a unified framework with which to think about long-run capital taxation. One important distinction they make is that nonzero capital taxation may be consistent with zero intertemporal distortions. Specifically, their work highlights that the essence of the celebrated [14] and [25] result is not that zero-capital-taxation *per se* is optimal, but rather that it is optimal because it supports perfect alignment between intertemporal marginal rates of substitution and marginal rates of transformation. Whether a zero intertemporal wedge requires a zero capital tax or a nonzero capital tax then depends on the details of the economic environment. The framework of [2] does not encompass (either reduced-form or micro-founded) monetary environments.[7] We can show, for the cases covered by Proposition 6, that capital *subsidies* are in fact needed for a zero intertemporal distortion.

Proposition 7. ***(Zero Intertemporal Distortions)*** *For the cases studied in Proposition 6, the Ramsey-optimal policy features a zero intertemporal distortion in the long run.*

Proof. See Appendix E. □

4 Discussion

We now discuss several points that pertain to the model and main results.

4.1 Completeness of the Tax System

An important issue in models of optimal taxation is whether or not the assumed tax instruments constitute a *complete* tax system.[8] This subsection establishes that the tax system is complete in our model. Establishing this is important for two reasons. First, at a technical level, proving completeness reaffirms that the Ramsey problem as formulated in Section 3, in which the only constraints are the sequence of CM resource constraints and the single PVIC, is indeed the correct Ramsey problem.[9] As shown by [16] (p. 1680), [19], [5], and many others, incompleteness of the tax system requires imposing additional constraints that reflect the incompleteness. Second, it is well-known in Ramsey theory that incomplete tax systems can lead to a wide range of "non-standard" policy prescriptions in which some instruments stand in for the ability to create certain wedges that *cannot*, by assumption of the available tax instruments, be created in a decentralized economy. Proving completeness therefore establishes that none of our results is due to any policy instrument serving as imperfect proxies for other, unavailable, instruments.

To demonstrate completeness, it is useful to begin by restating the conditions describing efficient allocations in terms of marginal rates of substitution (MRS) and corresponding marginal rates of transformation (MRT). To do this, note that our definition of the cost function of DM sellers mixes

[7] In our micro-founded environment, what prevents the model from being cast in the framework of [2] canonical form is the presence of the K terms in the implementability constraint, arising from the trading arrangements in the DM. Such a feature of the Ramsey problem does not arise in standard analyses of the type [14] or [25]).

[8] For convenience, we restate the definition of [16] (pp. 1679-1680) that an *incomplete* tax system is in place if, for at least one pair of goods in the economy, the government has *no* policy instrument that drives a wedge between the marginal rate of substitution (MRS) and the corresponding marginal rate of transformation (MRT). If this is not the case, then the tax system is instead said to be *complete*.

[9] For the purpose of establishing completeness as we and [16] define it, the ZLB constraint is irrelevant because it stems from the need to implement a monetary equilibrium and has nothing to do with completeness/incompletness of the tax system.

notions of preferences with notions of technology. As described in Section 2.1, the primitives behind this reduced-form cost function are that DM production occurs as sellers operate the technology $q = Zf(k,e)$ while incurring the disutility of effort $v(e)$. It is easy to verify that the relationship between DM cost, utility, and production functions implies $c_q(q,k,Z) = v'(e)/Zf_e(k,e)$ and $c_k(q,k,Z) = -f_k(k,e)v'(e)/f_e(k,e)$.

Proposition 8. *If the DM production function is $q = Zf(k,e)$ and DM disutility of effort is $v(e)$, then the MRS and MRT for the pairs (e_t, q_t), (X_t, H_t) and (X_t, X_{t+1}) are defined by*

$$\begin{aligned} MRS_{e_t,q_t} &\equiv -\frac{u'(q_t)}{v'(e_t)} & MRT_{e_t,q_t} &\equiv -\frac{1}{Z_t f_e(K_{t+1}, e_t)} \\ MRS_{X_t,H_t} &\equiv -\frac{A}{U'(X_t)} & MRT_{X_t,H_t} &\equiv -Z_t F_H(K_t, H_t) \\ MRS_{X_t,X_{t+1}} &\equiv \frac{\beta U'(X_{t+1})}{U'(X_t)} & MRT_{X_t,X_{t+1}} &\equiv \frac{1 - \dfrac{\sigma v'(e_t) f_k(K_{t+1}, e_t) Z_t F_H(K_t, H_t)}{A f_e(K_{t+1}, e_t)}}{Z_{t+1} F_K(K_{t+1}, H_{t+1}) + 1 - \delta} \end{aligned} \tag{24}$$

Proof. See Appendix F. □

Each MRS in Proposition 8 has the standard interpretation as a ratio of marginal utilities. Similarly, each MRT has the interpretation as a ratio of the marginal products of an appropriately-defined transformation frontier.[10] It is useful to note that in the LW version of the model, there is no intertemporal margin for the economy and the only two margins are the two intratemporal margins. In other words, the social planner has no way of transferring resources from one period to the next because of the absence of any storable goods.

The intertemporal MRT (IMRT) in the third line in (24) deserves further discussion. We formally derive the IMRT in Appendix F; an intuitive description suffices here. In the standard one-sector RBC model, in which there is only one type of produced good, it is straightforward to define the IMRT using the economy-wide intertemporal resource constraint. Due to our model's two-sector structure (DM and CM), however, defining the IMRT (in terms of CM consumption goods) is not as simple. By definition, the IMRT measures how many units of X_t the economy must forego in order to gain a given amount of X_{t+1}, holding output of all other goods in the economy constant. If X_t is reduced by one unit, the economy gains one additional unit of capital K_{t+1}, which increases X_{t+1} via period-$t+1$ CM production. Following period-$t+1$ production and subsequent depreciation, the one unit reduction in X_t leads to a gain of $Z_{t+1}F_K(K_{t+1}, H_{t+1}) + 1 - \delta$. This channel is standard in an RBC model, and it is present in our environment, as well. However, a second channel that affects the IMRT is also at work in our environment. The additional unit of

[10] We have in mind a very general notion of transformation frontier as in [33] (p. 129).

K_{t+1} will also lead to increased period-t DM production. Our definition of IMRT thus adjusts for this increase in q_t.

Appendix G characterizes efficient allocations using the MRS/MRT pairs in Proposition 8. In particular, the efficient allocation equates the MRS in each line of (24) to the corresponding MRT in each line. There are thus three "zero wedge conditions" that characterize the efficient allocation, one for each of the three independent margins of adjustment in the model. Appendix G also characterizes the private-sector equilibrium using these same MRS/MRT pairs; these conditions show that there exists one unique policy instrument in each of the three independent margins that the Ramsey government can use to create the "optimal wedges." Finally, Appendix G characterizes the Ramsey solution in terms of the three MRS/MRT pairs. Based on these characterizations, we prove that the tax system in our model is complete.

Proposition 9. ***Completeness of the Tax System.*** *In a monetary equilibrium at any time $t \geq 1$, the three policy instruments R_t, τ_t^h, and τ_{t+1}^k can uniquely create, in the margins defined by (24), the wedges implied by the Ramsey allocation.*

This result implies that none of the policy prescriptions obtained above is because one (or more) of the policy instruments that is assumed available is acting as an imperfect substitute for a policy instrument that is assumed unavailable.

4.2 Optimal Deviation from the Friedman Rule

Several points are worth discussing regarding the optimality of a strictly positive nominal interest rate. To conserve on notation, in this subsection we drop the K argument from relevant functions (because, recall, the non-optimality of the Friedman Rule does not depend on the endogeneity of capital accumulation).

4.2.1 Alternative Tax Instruments

Proposition 9 shows that the Ramsey planner has exactly one tax instrument to create a wedge between the MRS and MRT of each margin in the economy. By definition of completeness, allowing the government any additional policy instruments necessarily creates (given the Ramsey allocation) indeterminacies across policy instruments, which means there is no model-based justification for preferring the use of one policy instrument over another. Nonetheless, in this section we briefly consider introducing an additional instrument, a sales tax in the DM, which of course leads to a redundancy across policy instruments.[11]

[11] The anonymous nature of DM trades makes it somewhat difficult to consider traditional fiscal policy tools, hence we think monetary policy is the natural instrument in the DM. However, the following analysis is helpful in at least clarifying ideas and results.

We introduce a DM sales tax to the price-taking version of the model without capital in the following way: after a buyer turns over to a seller $\tilde{P}_t q_t$ units of money in a DM trade ($\tilde{P}_t$ denotes the nominal price of DM goods), the seller must remit $\tau_t^d \tilde{P}_t q_t$ to the government in the next CM, which, given our timing assumptions, occurs in period $t+1$.[12] After re-solving the model with this modification, one of the key equilibrium conditions, (15), is replaced by

$$R_t = \sigma(1-\tau_t^d) Z_t u'(q_t) + 1 - \sigma, \tag{25}$$

where recall that $c_q = 1/Z_t$ in the version without capital. We then find that the PVIC and therefore the Ramsey problem and its solution (in terms of allocations) are unchanged by this modification.[13] What can now differ, of course, is the precise way in which the Ramsey allocation is decentralized. Given the Ramsey allocation, an indeterminacy arises between the nominal interest rate and the DM sales tax, as condition (25) shows. In particular, the same allocations can be supported by any non-negative nominal interest rate along with an appropriate sales tax/subsidy in the DM. One particular policy would be to set the Friedman Rule ($R = 1$) along with whatever DM sales tax rate is required. While this is certainly model-admissible, there is no justification within the context of the model for this particular decentralization. Thus, if one were to prefer this "restoration of the Friedman Rule," it must be driven by considerations outside the scope of the model.[14]

4.2.2 Comparison of Results with CCK

The conclusion that the Friedman Rule is not optimal of course differs from that of CCK. At a technical level, it can be reconciled with their result by considering basic principles of public finance. In CCK, optimality of the Friedman Rule depends on a certain class of utility functions. In particular, CCK require cash goods and credit goods to enter the utility function homothetically and separably from leisure. Similarly, in the MIU model of [16], money and consumption must enter utility homothetically and separably from leisure in order for the Friedman Rule to be optimal. These results are essentially an application of the uniform taxation result of [10], requiring cash-good

[12] Thus, we assume that it is the sellers that pass along the sales tax receipts to the government; assuming that it is buyers that remit taxes would formally lead to the same conclusion. Regarding the timing, we can suppose that the government receives the revenue in the DM but waits until the next CM to spend it. Because asset markets are not open in the DM, the government cannot invest this extra revenue in an interest-bearing asset (nor can sellers, for that matter).

[13] The PVIC is the same as the no-capital version of (22), except for the fact that τ_{-1}^d appears as part of the constant term on the right-hand-side. Because optimization begins in period zero, we treat τ_{-1}^d as fixed. An implication of the equilibrium condition (25) is that the ZLB constraint is modified. However, because the ZLB is slack in all of our analytical results, it will continue to be slack upon addition of the DM sales tax.

[14] We also considered a direct tax on money balances levied in the CM. Not surprisingly, it leads to the same kind of indeterminacy of policy as the sales tax in the DM. Finally, as it is well understood from CCK, a consumption tax in the CM would create an indeterminacy between this tax and the labor income tax in the CM and will not affect the results regarding the optimality of the Friedman rule.

consumption and credit-good consumption (or money and consumption) to be taxed uniformly; a deviation from the Friedman Rule would mean that cash goods are taxed more heavily than credit goods, hence cannot be optimal.

The instantaneous social welfare function (the one that the Ramsey planner maximizes) in our model takes the form $\mathcal{U}(q, X, e, h) = \sigma[u(q) - e] + U(X) - AH$ (e denotes the effort of sellers in the DM). If we interpret q as the cash good and X as the credit good, q and X must enter $\mathcal{U}$ homothetically to satisfy the CCK requirement. Our Proposition 3 admits this case. For example, we can set $u(.) = U(.)$ and Proposition 3 of course still holds. However, realize that, given the structure of the LW model, $e = q/Z$. The *reduced-form* social utility function thus has the form $\tilde{\mathcal{U}}(q, X, h) = \sigma[u(q) - q/Z] + U(X) - AH$, and q and X will in general not enter $\tilde{\mathcal{U}}(q, X, h)$ homothetically. In other words, even though we have homothetic preferences in terms of the primitives, the reduced-form representation, which is the one relevant for the Ramsey planner, does not feature homothetic preferences. Given the lack of homotheticity of the social welfare function, there is no presumption that the CCK result carries over to our environment. Our results thus reconcile in a technical sense with those of CCK.

4.2.3 Rents Associated with DM Activity

We now offer a more conceptual reconciliation of our results with those of CCK and standard Ramsey theory. Given the fundamental need to tax activities requiring money, we think one useful way of considering the deviation from the Friedman Rule is that it stems from the presence of a rent associated with DM activity. To make ideas as clear as possible, consider the case $\theta = 1$, where the entire surplus of a DM trade is obtained by the buyer with $\theta = 1$. The instantaneous social welfare function in our model takes the form $\sigma[u(q) - e] + U(X) - AH$. Define $W(q, X) \equiv \sigma[u(q) - e] + U(X)$ where the e term in $W(.)$ can be thought of as a scarce, or fixed, factor in the social utility function. More precisely, from the perspective of a *buyer*, e is inelastic with respect to any of *his* actions because e represents the (utility) cost to the *seller*. Since maximization of the social welfare function can be interpreted as maximization of simply buyers' utility, from the Ramsey point of view, the e term can be therefore viewed as a fixed factor in preferences.

With this way of thinking about the optimal policy problem, our results and interpretation fit squarely into something pointed out by [16] (p. 1734-1735): if preferences exhibit decreasing returns in cash goods (our q) and credit goods (our X) because of a scarce factor that affects preferences of cash goods, then the Friedman Rule is not optimal. Their literal interpretation was that the fixed factor was something supplied inelastically by the representative household. In our model, the latter part of this intuition is modified to something inelastically supplied by *some* household because there is no representative household in the DM – rather, ex-post, there are three types of households.

Of course, in our model, the fixed factor is not something we arbitrarily introduce into preferences to obtain a deviation from the Friedman Rule — rather, it arises from the primitives of the environment.

4.2.4 Alternative Bargaining Schemes

Our analytical results have focused on the case in which either buyers make take-it-or-leave-it offers or on the case in which both buyers and sellers are price-takers in the DM. Analytical results for the equally interesting case of $\theta < 1$ cannot be obtained because additional incentives that affect the Ramsey planner lead to ambiguous results. Fully documenting the results for this case requires a full-blown quantitative analysis, which is beyond the scope of the present paper and is left for future work. In this section, we discuss some of the interesting details that are likely to be revealed by a full quantitative analysis.

In order to fully understand the results that are likely to emerge with generalized Nash bargaining and $\theta < 1$, it is useful to consider an alternative bargaining mechanism in the DM, proportional bargaining. Proportional bargaining is a generalized version of egalitarian bargaining which, just like Nash, is an axiomatic bargaining rule. Briefly, egalitarian bargaining retains the Pareto optimality, symmetry, and independence of irrelevant alternatives axioms of Nash bargaining, but replaces the scale invariance axiom with strong monotonicity. As a result, it does not suffer from a shortcoming of Nash bargaining, which is that a party may end up with a lower *individual* surplus if he undertakes an action that increases the size of the *total* surplus.

As first pointed out by [7], this lack of strong monotonicity of Nash bargaining significantly influences the properties of monetary equilibrium.[15] Specifically, [7] show that with Nash bargaining and $\theta < 1$, the buyer's surplus is maximized at a level of q that is smaller than the value that maximizes the joint surplus to both the buyer and the seller. This misalignment between the private surplus function and the joint surplus function leads the buyer to bring less money into the DM than the level that maximizes the joint surplus. It is important to note that this issue regarding Nash bargaining is a *different* issue than the money holdup problem that arises with $\theta < 1$, which afflicts both proportional bargaining and Nash bargaining.

Turning back to our model, in the generalized Nash bargaining version with $\theta < 1$, then, the Ramsey planner would attempt to balance four incentives: raising enough revenue to finance the

[15] Under proportional bargaining, which simply generalizes egalitarian bargaining by relaxing symmetry, it is assumed that the buyer and the seller split the surplus in the exogenous shares θ and $1-\theta$, respectively. As [7] show, the only change in the equilibrium (in the model without capital, for simplicity) is that the function $g(q, Z)$ in (3) is replaced by

$$g(q, Z) = (1-\theta)u(q) + \theta c(q, Z),$$

with all equilibrium conditions and Ramsey optimality conditions unchanged.

government's expenditures (the public finance motive), alleviating the money holdup problem, alleviating the investment holdup problem, and the lack of strong monotonicity of the generalized Nash solution. The first incentive, as was the case throughout our analysis, calls for a positive interest rate, while the second incentive, which is only present when $\theta < 1$, calls for lower nominal interest rates. As AWW show, the effect of inflation on the third incentive is of second-order importance and (if present at all) and thus is not greatly influenced by the long-run level of inflation. Finally, the fourth incentive also requires as small a nominal interest rate as possible.

As should be clear, whether or not the Friedman rule will be optimal under generalized Nash bargaining for any given $\theta < 1$ will crucially depend on the relative strengths of these four incentives. For sufficiently low values of θ, which exacerbate the second and fourth incentives, the optimal policy will feature the Friedman rule as the constrained optimal policy. On the other hand, with proportional bargaining, we are able to prove that *a strictly positive net nominal interest rate is optimal for any* $\theta > 0$. Because the only difference between the proportional bargaining and generalized Nash bargaining solutions is that the former sidesteps the non-monotonicity of the latter, we can conclude that any *qualitative* difference between the results using these two bargaining schemes is simply the result of the lack of strong monotonicity of the Nash bargaining solution. It should be noted, however, that in all these cases DM output will be distorted and capital income subsidies will still be a feature of optimal policy.

4.3 Optimal Capital Taxation

As discussed at length both immediately above and in Section 3, the deviation from the Friedman Rule in our model — a monetary tax — achieves taxation of the final good q, which is required by basic optimal-taxation principles. This result and intuition carry over to the model with capital, as Proposition 4 showed.

However, in the environment with capital, there is an additional consequence of this monetary tax. Intuitively, by taxing DM goods, the monetary tax is also a tax on capital used for DM production. To correct this distortion, capital income must be subsidized. This connection is the second of the two distinct motivations for a capital subsidy to which we referred in the discussion following Proposition 6. Although of course everything is endogenous here, the capital-income subsidy can be thought of as being "caused by" the presence of cash-intensive activity in a way not present in the models of [37] or [18].

This point can be made clear by the following thought exercise. Suppose the Ramsey planner is forced to implement a zero nominal interest rate — that is, impose the ZLB (16) as an equality constraint on the Ramsey problem. We can prove that for the empirically-relevant case of log utility (in both CM consumption and DM consumption), which is a necessary condition for balanced growth, the optimal policy displays $\tau^k = 0$. That is, if the Ramsey planner cannot create a distortion in the DM using $R_t > 1$, capital accumulation is not distorted and thus the need for a capital subsidy disappears.

To sum up, we think of the deviation from a zero capital tax as being "caused by," or the flip side of, the deviation from a zero nominal interest rate. Our results thus illustrate that capital-tax policy can fundamentally be driven by monetary issues, rather than simply co-existing with them; moreover, capital-tax policy can depend on the primitive reasons for money demand.

5 Conclusion

We view our work and results as a first step in taking more seriously the new class of micro-founded models of money as a laboratory for studying policy questions. Our central findings are that the Friedman Rule is typically not the optimal policy and that the long-run optimal capital-income tax is not zero. Our analysis of optimal monetary policy connects broadly with those being conducted by others in micro-founded models of money demand — for example, [11]. In light of recent results regarding asset taxation in the new dynamic public finance literature — for example, [21] and [3] — and the attempt of [2] at reconciling them with standard Ramsey results, it may be interesting to know how or whether the capital-taxation implications of a micro-founded model of monetary exchange square with this growing body of knowledge.

Besides this, there are of course a number of ways one might want to modify our framework. Monopoly power in goods and labor markets are thought by many to be important realistic features. It would be straightforward to introduce monopoly power in the CM. The results of [36] and [17] suggest that inflation in such an environment would be partly a direct tax on the money rent we identify and partly an indirect tax on producers' and labor suppliers' rents. It may be interesting to know quantitatively how these direct and indirect uses of the inflation tax interact.

Once one has monopoly power in the CM, one could go further in adding elements monetary policy makers often think are important, such as nominal rigidities in prices and wages. For example, [8] show that when one replaces the typical "cashless" assumption of a Calvo-type model with micro-founded frictions for the use of cash, welfare implications are altered significantly. Investigating both long-run and short-run optimal policy — be it monetary alone or monetary and fiscal jointly — in the presence of both temporary nominal rigidities and deep-rooted frictions underlying monetary trade also seems likely to yield new insights.

Pushing our first step in different directions, another interesting issue to study may be the nature of and solution to the time-inconsistency problem of the Ramsey policy in this sort of environment. It is not clear how the time-consistency results of, say, [4] or [34], would extend to our environment. Neither is it clear how the emerging results in the aforementioned new dynamic public finance literature, which places at center stage distributional concerns, might extend to a version of our environment in which money holdings were allowed to differ across households.

This paper is also part of a larger effort underway in the literature studying the policy implications of deep-rooted, non-Walrasian frictions in goods markets, money markets, and labor markets. A central focus of this larger project has been to think about what sorts of departures from typical Walrasian frameworks impinge importantly on conventional policy prescriptions derived from standard models. Much progress has recently been made using micro-founded models of labor market transactions — for example, [40], [39], [30], [6], and [20], to name just a few. We think much progress is in the offing using micro-founded models of money as well.

Appendix

A The Ramsey Problem

A.1 Proof of Proposition 2

That allocations from a monetary equilibrium should satisfy the CM resource constraint (9), the ZLB constraint ((12) for the bargaining model or (16) for the price-taking model) is obvious. Here we derive the PVIC for the bargaining version of the model. The expression for the price-taking version follows the same steps.

Before we derive the PVIC, we need a compact expression for real money balances; combining (5), (7) and (8) we can express real money balances as

$$\frac{M_t}{P_t} = \frac{g(q_t, K_{t+1}, Z_t)}{U'(x_t)} \left[\sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma \right]. \tag{26}$$

In the price-taking environment we can express real money balances as

$$\frac{M_t}{P_t} = \frac{q_t c_q(q_t, K_{t+1}, Z_t)}{U'(X_t)} \left[\sigma \frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma \right],$$

which follows from (8), (13) and (15).

We begin by summing the budget constrains of the three types of agents (buyer, sellers and nonparticipants in the previous DM) to get

$$P_t X_t + B_t + M_t + P_t K_{t+1} = P_t w_t (1 - \tau_t^h) H_t + M_{t-1} + R_{t-1} B_{t-1} + \left[1 + (1 - \tau_t^k)(F_K(K_t, H_t) - \delta) \right] K_t.$$

Multiplying by $\beta^t U'(X_t)/P_t$ and summing from $t = 0, 1, ..., \infty$, we get

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t U'(X_t) X_t + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{B_t}{P_t} + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{M_t}{P_t} + \sum_{t=0}^{\infty} \beta^t U'(X_t) K_{t+1} = \\
&\quad \sum_{t=0}^{\infty} \beta^t U'(X_t)(1-\tau_t^h) w_t H_t + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{M_{t-1}}{P_t} + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{R_{t-1} B_{t-1}}{P_t} \\
&\quad + \sum_{t=0}^{\infty} \beta^t U'(X_t) \Big[1 + (1-\tau_t^k)(F_K(K_t, H_t) - \delta)\Big] K_t.
\end{aligned} \tag{27}$$

Substitute into the second term on the left-hand-side of (27) using expression (8) to get

$$\sum_{t=0}^{\infty} \beta^{t+1} U'(x_{t+1}) \frac{R_t B_t}{P_{t+1}}.$$

This term cancels with the the last summation on the right-hand-side of (27) to leave only the initial bond position,

$$U'(x_0) \frac{R_{-1} B_{-1}}{P_0}.$$

Next, substitute into the third term on the left-hand-side of (27) using (7) and (8) to get

$$\sum_{t=0}^{\infty} \beta^{t+1} U'(x_{t+1}) \frac{M_t}{P_{t+1}} \left[\sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma\right];$$

expanding this summation, we have

$$\sum_{t=0}^{\infty} \beta^{t+1} U'(x_{t+1}) \frac{M_t}{P_{t+1}} + \sigma \sum_{t=0}^{\infty} \beta^{t+1} U'(x_{t+1}) \frac{M_t}{P_{t+1}} \left[\frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1\right].$$

Canceling the first summation in this last expression with the second summation on the right-hand-side of (27) to leave only the initial money holdings,

$$U'(x_0) \left(\frac{M_{-1}}{P_0}\right),$$

and writing $\frac{M_t}{P_{t+1}} = \frac{M_t}{P_t} \frac{P_t}{P_{t+1}}$, we can express the second summation just above as

$$\sigma \sum_{t=0}^{\infty} \beta^{t+1} U'(x_{t+1}) \frac{M_t}{P_t} \frac{P_t}{P_{t+1}} \left[\frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1\right].$$

Use (26) to substitute for M_t/P_t,

$$\sigma \sum_{t=0}^{\infty} \beta^{t+1} \frac{U'(x_{t+1})}{P_{t+1}} \frac{P_t}{U'(X_t)} g(q_t, K_{t+1}, Z_t) \left[\sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma \right] \left[\frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right].$$

Using (7) and (8), we can make the substitution $\beta \left[\frac{U'(x_{t+1})}{P_{t+1}} \right] = \frac{U'(X_t)}{P_t} \left[\sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma \right]^{-1}$ which yields

$$\sigma \sum_{t=0}^{\infty} \beta^t g(q_t, K_{t+1}, Z_t) \left[\frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right].$$

Next, using (4), we can substitute into the first term on the right-hand-side of (27) to get

$$\sum_{t=0}^{\infty} \beta^t A H_t;$$

and using (6), we can express the fourth term on the left-hand-side of (27) as

$$\sum_{t=0}^{\infty} \beta^{t+1} U'(X_{t+1}) \left[1 + (1 - \tau_{t+1}^k)(F_K(K_{t+1}, H_{t+1}) - \delta) \right] K_{t+1} + \sum_{t=0}^{\infty} \beta^t \sigma \gamma(q_t, K_{t+1}) K_{t+1}.$$

Canceling the first summation with the last term on the right-hand-side of (27) yields

$$U'(X_0) \left[1 + (1 - \tau_0^k)(F_K(K_0, H_0) - \delta) \right] K_0.$$

Defining A_0 as

$$A_0 \equiv \frac{M_{-1} + R_{-1} B_{-1}}{P_0} + \left[1 + (1 - \tau_0^k)(F_K(K_0, H_0) - \delta) \right] K_0$$

and collecting all remaining terms, we arrive at

$$\sum_{t=0}^{\infty} \beta^t \left[U'(X_t) X_t - A H_t + \sigma g(q_t, K_{t+1}, Z_t) \left(\frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right) + \sigma \gamma(q_t, K_{t+1}, Z_t) K_{t+1} \right] = U'(X_0) A_0,$$

which is expression (21) in the text.

A.2 Ramsey First-Order Conditions

In the Ramsey problem described in Section 3, we associate multipliers $\beta^t \rho_t$ with the time-t resource constraint, ξ with the PVIC, and $\beta^t \iota_t$ with the time-t ZLB constraint.

A.2.1 Bargaining

The Kuhn-Tucker conditions for the problem above are the first-order conditions

$$\begin{aligned} \boldsymbol{q_t} &: \sigma \left[u'(q_t) - c_q(q_t, K_{t+1}, Z_t)\right] + \sigma\xi \left[g_q(q_t, K_{t+1}, Z_t) \left(\frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1\right)\right] \\ &+\sigma\xi \left[g(q_t, K_{t+1}, Z_t) \left(\frac{u''(q_t)g_q(q_t, K_{t+1}, Z_t) - u'(q_t)g_{qq}(q_t, K_{t+1}, Z_t)}{[g_q(q_t, K_{t+1}, Z_t)]^2}\right) + \gamma_q(q_t, K_{t+1}, Z_t)K_{t+1}\right] \\ &+\sigma\iota_t \left[\frac{u''(q_t)g_q(q_t, K_{t+1}, Z_t) - u'(q_t)g_{qq}(q_t, K_{t+1}, Z_t)}{[g_q(q_t, K_{t+1}, Z_t)]^2}\right] = 0, \end{aligned} \tag{28}$$

$$\boldsymbol{X_t} : U'(X_t) - \rho_t + \xi \left[U''(X_t)X_t + U'(X_t)\right] = 0, \tag{29}$$

$$\tag{30}$$

$$\begin{aligned} \boldsymbol{K_{t+1}} &: -\sigma c_k(q_t, K_{t+1}, Z_t) - \rho_t + \beta \left\{\rho_{t+1} \left[Z_{t+1}F_K(K_{t+1}, H_{t+1}) + 1 - \delta\right]\right\} \\ &+\sigma\xi \left[g_k(q_t, K_{t+1}, Z_t) \left(\frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1\right) - \frac{g(q_t, K_{t+1}, Z_t)u'(q_t)g_{qk}(q_t, K_{t+1}, Z_t)}{[g_q(q_t, K_{t+1}, Z_t)]^2}\right] \\ &+\sigma\xi\gamma_k(q_t, K_{t+1}, Z_t)K_{t+1} + \sigma\xi\gamma(q_t, K_{t+1}, Z_t) - \sigma\iota_t \frac{u'(q_t)g_{qk}(q_t, K_{t+1}, Z_t)}{[g_q(q_t, K_{t+1}, Z_t)]^2} = 0, \end{aligned} \tag{31}$$

$$\boldsymbol{H_t} : -A + \rho_t Z_t F_H(K_t, H_t) - \xi A = 0, \tag{32}$$

along with (9), (21), and the complementary slackness condition

$$\iota_t\sigma \left[\frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1\right] = 0, \text{ and } \iota_t \geq 0. \tag{33}$$

We can represent the right-hand side of the PVIC in terms of allocations as

$$U'(X_0)A_0 = U'(X_0) \left[\frac{g(q_{-1}, K_0, Z_{-1})}{\beta U'(X_0)} + \frac{B_{-1}/P_{-1}}{\beta}\right] + \frac{U'(X_0)}{\beta}K_0 \left[1 - \sigma\frac{\gamma(q_{-1}, K_0, Z_{-1})}{U'(X_0)}\right],$$

in which the initial real bond position B_{-1}/P_{-1} is a parameter.

With these Ramsey FOCs in hand, we proceed as follows. Imposing steady state on (28)-(32), and taking the timeless perspective, i.e. setting time-zero allocations equal to their steady state value, we solve for the steady state values of allocations and the multiplier ξ. Next, given ξ and $\{Z_t, G_t\}$, (28)-(33) characterize $\{q_t, X_t, K_t, H_t, \iota_t\}$. We back out policies $\{\tau_t^h, R_t\}$ from (4) and (7).

A.2.2 Price-Taking

The Kuhn-Tucker conditions for the problem above are the first-order conditions

$$\boldsymbol{q_t} : \sigma\left[u'(q_t) - c_q(q_t, K_{t+1}, Z_t)\right] + \sigma\xi\left[u'(q_t) + q_t u''(q_t) - c_q(q_t, K_{t+1}, Z_t) - q_t c_{qq}(q_t, K_{t+1}, Z_t)\right]$$
$$-\sigma\xi c_{qk}(q_t, K_{t+1}, Z_t)K_{t+1} + \sigma\iota_t\left[\frac{u''(q_t)c_q(q_t, K_{t+1}, Z_t) - u'(q_t)c_{qq}(q_t, K_{t+1}, Z_t)}{[c_q(q_t, K_{t+1}, Z_t)]^2}\right] = 0, \tag{34}$$

$$\boldsymbol{X_t} : U'(X_t) - \rho_t + \xi\left[U''(X_t)X_t + U'(X_t)\right] = 0, \tag{35}$$

$$\boldsymbol{K_{t+1}} : -\sigma c_k(q_t, K_{t+1}, Z_t) - \rho_t + \beta\left\{\rho_{t+1}\left[Z_{t+1}F_K(K_{t+1}, H_{t+1}) + 1 - \delta\right]\right\}$$
$$-\sigma\xi\left[q_t c_{qk}(q_t, K_{t+1}, Z_t) + c_{kk}(q_t, K_{t+1}, Z_t)K_{t+1} + c_k(q_t, K_{t+1}, Z_t)\right] - \sigma\iota_t\frac{u'(q_t)c_{qk}(q_t, K_{t+1}, Z_t)}{[c_q(q_t, K_{t+1}, Z_t)]^2} = 0 \tag{36}$$

$$\boldsymbol{H_t} : -A + \rho_t Z_t F_H(K_t, H_t) - \xi A = 0, \tag{37}$$

along with (9), 22), and the complementary slackness condition

$$\iota_t\sigma\left[\frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} - 1\right] = 0, \text{ and } \iota_t \geq 0. \tag{38}$$

We can represent the right-hand side of the PVIC in terms of allocations as

$$U'(X_0)A_0 = U'(X_0)\left[\frac{q_{-1}c_q(q_{-1}, K_0, Z{-}1)}{\beta U'(X_0)} + \frac{B_{-1}/P_{-1}}{\beta}\right] + \frac{U'(X_0)}{\beta}K_0\left[1 + \sigma\frac{c_k(q_{-1}, K_0, Z_{-1})}{U'(X_0)}\right],$$

in which, once again, the initial real bond position B_{-1}/P_{-1} is a parameter.

With these FOCs in hand, we proceed as follows. Imposing steady state on (34)-(37), and taking the timeless perspective, i.e. setting time-zero allocations equal to their steady state value, we solve for the steady state values of allocations and the multiplier ξ. Next, given ξ and $\{Z_t, G_t\}$, (34)-(38) characterize $\{q_t, X_t, K_t, H_t, \iota_t\}$. We back out policies $\{\tau_t^h, R_t\}$ from (4) and (15).

For reference, we define

$$C_{1t} = \left[q_t c_{qk}(q_t, K_{t+1}, Z_t) + c_{kk}(q_t, K_{t+1}, Z_t)K_{t+1} + c_k(q_t, K_{t+1}, Z_t)\right]$$

$$C_{2t} = \frac{u'(q_t)c_{qk}(q_t, K_{t+1}, Z_t)}{[c_q(q_t, K_{t+1}, Z_t)]^2}$$

B Proofs of Propositions 3 and 4

B.1 Proposition 4

For the case of bargaining with $\theta = 1$ (part 1 of the proposition), consider the first-order condition of the Ramsey problem with respect to q_t, which is expression (28) in Appendix A.2. With $\theta = 1$, we have $g(q, K, Z) = c(q, K, Z)$ and $\gamma(q, K, Z) = 0$. Dropping all arguments of functions, dropping time subscripts because everything is period-t, and assuming the ZLB never binds (i.e., the multiplier $\iota_t = 0$ in expression (28)), this first-order condition simplifies to

$$u' - c_q = -\left(\frac{\xi}{1+\xi}\right)\frac{c}{c_q}\left[u'' - \frac{u' c_{qq}}{c_q}\right]. \tag{39}$$

Because the multiplier on the PVIC $\xi > 0$ under the Ramsey allocation, u is strictly concave, $c > 0$, $c_q > 0$, and $c_{qq} > 0$, the right-hand-side of (39) is strictly positive. This in turn implies $u' > c_q$, and

$$\sigma \frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma > 1.$$

But this implies, by the equilibrium condition (12) with $\theta = 1$ imposed, that $R_t > 1$. We have thus proven that the Friedman Rule is never optimal and that the ZLB never binds.

For the case of price taking (part 2 of the proposition), start from the first-order condition of the Ramsey problem with respect to q_t, which is expression (34) in Appendix A.2. Once again dropping all arguments of functions, dropping time subscripts because everything is period-t, (except, recall, that K chosen in period t is K_{t+1} by our timing and notational convention), and assuming the ZLB never binds (i.e., the multiplier $\iota_t = 0$ in expression (34)), this first-order condition simplifies to

$$u' - c_q = -\left(\frac{\xi}{1+\xi}\right)\left[qu'' - qc_{qq} - c_{qk}K\right].$$

If the DM production function is constant-returns-to-scale, then $qc_{qq} + c_{qk}K = 0$, which follows by Euler's Theorem. In this case, we get

$$u' - c_q = -\left(\frac{\xi}{1+\xi}\right)qu'' > 0, \tag{40}$$

which together with (16) implies that $R_t > 1$. The Friedman Rule is thus never optimal and the ZLB constraint never binds. Comparing (40) with (52) it is clear that supporting the Ramsey allocation requires creating a wedge in the (e_t, q_t) margin; condition (49) shows this is achieved by setting $R_t > 1$.

To show the application of Euler's Theorem in the above argument, we have

$$c_q q + c_{qk} k = \frac{1}{Z^\psi}\left[\psi(\psi-1)q^{\psi-2}K^{1-\psi}q + \psi(1-\psi)q^{\psi-1}K^{-\psi}K\right] = 0.$$

B.2 Proposition 3

The proof of Proposition 3 is a simple special case of the proof of Proposition 4, so we can rely on the immediately preceding derivation. All that is required is the assertion that physical capital K is exogenous. In the preceding derivations, this amounts to simply omitting K as an argument to the function $c(.)$; the analysis and conclusion nonetheless holds as just described.

C Proof of Proposition 5

Combining (51) and (54) we get

$$1 - \tau_t^h = 1 + \left(\frac{\xi}{1+\xi}\right)\frac{U''(X_t)X_t}{U'(X_t)},$$

which shows that $\tau_t^h > 0$ as $\xi > 0$, $U''(X) < 0$ and $U'(X) > 0$, all of which hold under our assumptions.

D Proof of Proposition 6

(a) (Bargaining with $\theta = 1$) Imposing steady state on the Ramsey planner's first-order condition for capital, (31), dropping arguments of functions, and imposing both $\iota = 0$ (because we showed the ZLB does not bind) and $\theta = 1$ (which implies $\gamma(.) = 0$ and $g(.) = c(.)$), we have

$$-\sigma c_k - \rho + \beta\left[\rho\left(F_K + 1 - \delta\right)\right] + \sigma\xi\left[c_k\left(\frac{u'}{c_q} - 1\right) - \frac{cu'c_{qk}}{[c_q]^2}\right] = 0.$$

The last term in square brackets on the left-hand-side can be rearranged to

$$-c_k + \frac{u'}{c_q}\left[\frac{c_k c_q - c c_{qk}}{c_q}\right] = -c_k;$$

the equality follows because $c_k c_q - c c_{qk} = 0$ due to our CRS assumption (further details are provided in Appendix B). Collecting terms, we have

$$\beta\left[1 + (F_K - \delta)\right] - 1 = \frac{\sigma(1+\xi)c_k}{\rho}.$$

Also, note that the multiplier on the resource constraint, ρ can be solved as

$$\rho = \frac{A(1+\xi)}{F_H} \tag{41}$$

from (32), which is the Ramsey planner's first-order condition for CM labor. This leads to

$$\beta\left[1 + (F_K - \delta)\right] - 1 = \frac{\sigma c_k F_H}{A} \tag{42}$$

Similarly imposing steady state and $\theta = 1$ on the monetary equilibrium condition (6) yields

$$\beta\left[1 + (1-\tau^k)(F_K - \delta)\right] - 1 = 0. \tag{43}$$

Combining (42) and (43), we get the Ramsey-optimal capital tax rate,

$$\tau^k = \frac{\sigma F_H c_k}{A\beta\left(F_K - \delta\right)}.$$

Standard assumptions imply $F_H > 0$ and $(F_K - \delta) > 0$, and of course $\beta > 0$ and $A > 0$ by assumption. The only way, therefore, τ^k can equal zero is if $\sigma = 0$ (which means DM trades never occur) or $c_k = 0$ (which means capital is not used for DM production). So long as both the DM exists ($\sigma > 0$) and capital is used for DM production, we must have $\tau^k < 0$ because $c_k < 0$.

(b) (Price-Taking) Following similar algebra, the Ramsey planner's first-order condition for capital in the price-taking case can be simplified to exactly the expression in (42), using the CRS property of the DM production function (in particular, applying Euler's theorem to the marginals of the DM cost function, which yields $c_{qk}q + c_{kk}k = 0$). The steady state multiplier ρ on the resource constraint is also again given by (41).

For price-taking, the steady-state version of the monetary equilibrium condition (14) is given by

$$\beta\left[1 + (1-\tau^k)(F_K - \delta)\right] - 1 = \frac{\sigma c_k}{U'}; \tag{44}$$

solving (42), and (44) for the Ramsey-optimal capital tax rate,

$$\tau^k = \frac{\sigma c_k\left[\dfrac{F_H}{A} - \dfrac{1}{U'}\right]}{\beta(F_K - \delta)}. \tag{45}$$

Note that we showed in Proposition 5 that $\tau^h > 0$ and using (4) this implies $F_H/A > 1/U'$. As in case (a), standard assumptions on production and this result guarantee that the only way τ_k can equal zero is if $\sigma = 0$ or $c_k = 0$. Otherwise, we must have $\tau_k < 0$.

To demonstrate the reliance on Euler's Theorem in the above arguments, note that we get

$$c_q c_k - c c_{qk} = \frac{1}{Z^{2\psi}} \left[\psi q^{\psi-1} K^{1-\psi} (1-\psi) q^{\psi} K^{-\psi} - q^{\psi} K^{1-\psi} \psi (1-\psi) q^{\psi-1} K^{-\psi} \right] = 0;$$

E Proof of Proposition 7

In Proposition 8, we derived the IMRT and IMRS for our economy. Imposing steady state, the IMRS is simply β. To prove the result, it will be sufficient to show that in a deterministic steady-state, the IMRT at the Ramsey-optimal allocation equals β.

We established above that (42) is the Ramsey planner's first-order condition for capital at the steady state for both of the cases considered in this Proposition. Rearranging (42), we have

$$\beta = \frac{1 + \dfrac{\sigma c_k F_H}{A}}{1 + F_K - \delta},$$

in which the right-hand-side is precisely the IMRT we derived in Proposition 8 (recall that $c_k = -v'(e) f_k / f_e$). For the price-taking case, another way to prove this result would be to substitute the optimal tax rate we derived in (45) into (50) to get IMRS equals IMRT.

F Proof of Proposition 8

The expression for IMRS, $\beta U'(X_{t+1})/U'(X_t)$, simply follows from the social welfare function. To see where the expression for IMRT comes from, consider a decrease in X_t by one unit. This will increase K_{t+1} by one unit. This marginal increase in capital will have two effects. First, as occurs in a standard RBC model with capital, output in period $t+1$ will increase by $Z_{t+1} F_K(K_{t+1}, H_{t+1}) + 1 - \delta$ units, which in period $t+1$ can be converted one-for-one into X_{t+1}. Because of the existence of the DM, though, a second effect arises: q_t will rise by $\sigma Z_t f_k(K_{t+1}, e_t)$ units. In order to properly define IMRT in our environment, then, this increase in q_t needs to be taken into account.

To properly account for this second effect, consider the following thought experiment. In order to hold production of q_t fixed following an initial reduction of X_t by one unit, DM effort must be reduced by $\sigma f_k(K_{t+1}, e_t)/f_e(K_{t+1}, e_t)$. This reduction in DM effort will increase utility $\sigma v'(e_t) f_k(K_{t+1}, e_t)/f_e(K_{t+1}, e_t)$. This increase in utility is equivalent to a decrease in CM labor by $\sigma v'(e_t) f_k(K_{t+1}, e_t)/f_e(K_{t+1}, e_t) A$, under the maintained assumption of linear disutility of CM labor. Next, this reduction in CM labor would lead to a reduction of period-t CM output by the amount $\sigma Z_t F_H(K_t, H_t) v'(e_t) f_k(K_{t+1}, e_t)/f_e(K_{t+1}, e_t) A$, which, because there is of course a unit rate of transformation between CM output and CM consumption, means a decrease in X_t by the same amount.

Thus, we have demonstrated that a *reduction* in X_t in the amount $1-\sigma Z_t F_H(K_t, H_t)v'(e_t)f_k(K_{t+1}, e_t)/f_e(K_{t+1}, e_t)A$ leads to an *increase* in X_{t+1} by $Z_{t+1}F_K(K_{t+1}, H_{t+1})+1-\delta$ units. Clearly, if $\sigma = 0$ as in the standard RBC model, we have that $MRT_{X_t,X_{t+1}} = Z_{t+1}F_K(K_{t+1}, H_{t+1}) + 1 - \delta$, which has the usual interpretation that a one unit decrease in X_t leads to increased period-$t+1$ CM consumption by the amount $Z_{t+1}F_K(K_{t+1}, H_{t+1}) + 1 - \delta$. With $\sigma > 0$, in order to achieve the same increase in period-$t+1$ CM consumption, the required decrease in X_t is *less* than one unit. This is due to the fact that DM output also increases when one unit of X_t is foregone.

In the text we claimed that c_k corresponds to $-v'(e)f_k/f_e$. To see this, remember that c_k refers to the marginal change in utility for having more capital, holding output constant. Using the production function $q = Zf(k, e)$, we can write $dq = Zf_k dk + Zf_e de$. If we consider a change in k with no change in q, this corresponds to a change in e in the amount $-f_k dk/f_e$. The change in utility due to this change in k will therefore be $-v'(e)f_k/f_e$. A similar argument shows that c_q corresponds to $v'(e)/Zf_e$.

G Completeness of the Tax System

The following series of propositions establishes that the model features a complete set of policy instruments. Begin with a characterization of the efficient allocation described in Section 2.5 in terms of the marginal rates of substitution (MRS) and marginal rates of transformation (MRT) defined in Proposition 8.

Corollary 1. *The solution to the social planner's problem is characterized by the CM resource constraint (9) along with*

$$\frac{MRS_{e_t,q_t}}{MRT_{e_t,q_t}} \equiv \frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} = 1, \tag{46}$$

$$\frac{MRS_{X_t,X_{t+1}}}{MRT_{X_t,X_{t+1}}} \equiv \frac{\beta U'(X_{t+1})\left[Z_{t+1}F_K(K_{t+1}, H_{t+1}) + 1 - \delta\right]}{U'(X_t)\left[1 + \dfrac{\sigma c_k(q_t, K_{t+1}, Z_t)Z_t F_H(K_t, H_t)}{A}\right]} = 1, \tag{47}$$

and

$$\frac{MRS_{X_t,H_t}}{MRT_{X_t,H_t}} \equiv \frac{A}{U'(X_t)\left[Z_t F_H(K_t, H_t)\right]} = 1. \tag{48}$$

Proof. Follows from the definition of the cost function $c(.)$, Proposition 8, and (18)-(20) $\square$

Corollary 1 shows that the efficient allocations in our model can be described in terms of "zero-wedge conditions" between MRSs and MRTs. This way of understanding efficiency is of course standard, but given the novelty of our model, it is important to show how to precisely express efficiency in terms of the zero-wedge expressions (46), (47), and (48). This is especially important because, as [16] (p. 1674) emphasize, optimal tax theory is really about the determination of optimal wedges between MRSs and MRTs. In what follows, we take expressions (46), (47), and (48) as the conditions that define zero wedges.

Completeness of the tax system requires that each of the three margins defined by the MRS/MRT pairs in Corollary 1 is affected by (at least) one policy instrument. To establish completeness, we first express explicitly in terms of MRS/MRT pairs the monetary equilibrium conditions that are the analogs of the efficiency conditions (46)-(48). For simplicity and brevity, we do this for the price-taking version of the model, but the ensuing arguments and logic hold for bargaining as well.

Using (4), (14), (15), and the definitions of MRSs and MRTs presented in Proposition 8, we have that in the decentralized economy

$$\frac{MRS_{e_t,q_t}}{MRT_{e_t,q_t}} = 1 + \frac{R_t - 1}{\sigma}, \tag{49}$$

$$\begin{aligned} \frac{MRS_{X_t,X_{t+1}}}{MRT_{X_t,X_{t+1}}} &= \left[1 + \frac{\sigma c_k\left(q_t, K_{t+1}, Z_t\right) Z_t F_H(K_t, H_t)}{A}\right]^{-1} \times \\ & \left\{\left[1 + \frac{\sigma c_k\left(q_t, K_{t+1}, Z_t\right)}{U'(X_t)}\right] + \beta\left[\frac{\tau^k_{t+1} U'(X_{t+1})\left[Z_{t+1}F_K(K_{t+1}, H_{t+1}) - \delta\right]}{U'(X_t)}\right]\right\} \end{aligned} \tag{50}$$

and

$$\frac{MRS_{X_t,H_t}}{MRT_{X_t,H_t}} = 1 - \tau^h_t. \tag{51}$$

Next, we express in the same way the first-order conditions of the Ramsey planner (which are derived in Appendix A.2); doing so gives

$$\begin{aligned} \frac{MRS_{e_t,q_t}}{MRT_{e_t,q_t}} &= 1 - \frac{\xi}{1+\xi}\left[\frac{q_t u''(q_t) - q_t c_{qq}(q_t, K_{t+1}, Z_t) - c_{qk}(q_t, K_{t+1}, Z_t)K_{t+1}}{c_q(q_t, K_{t+1}, Z_t)}\right] \\ &- \frac{\iota_t}{1+\xi}\left[\frac{u''(q_t)c_q(q_t, K_{t+1}, Z_t) - u'(q_t)c_{qq}(q_t, K_{t+1}, Z_t)}{[c_q(q_t, K_{t+1}, Z_t)]^3}\right], \end{aligned} \tag{52}$$

$$\begin{aligned} \frac{MRS_{X_t,X_{t+1}}}{MRT_{X_t,X_{t+1}}} &= \left[1 + \frac{\sigma c_k(q_t, K_{t+1}, Z_t) Z_t F_H(K_t, H_t)}{A}\right]^{-1} \times \\ & \left\{1 + \frac{\sigma c_k\left(q_t, K_{t+1}, Z_t\right) + \xi U''\left(X_t\right) X_t + \sigma\xi C_{1t} + \sigma\iota_t C_{2t} - \beta\xi U''\left(X_{t+1}\right) X_{t+1}\left[Z_{t+1}F_K\left(t+1\right) + 1 - \delta\right]}{(1+\xi)U'(X_t)}\right\} \end{aligned} \tag{53}$$

and

$$\frac{MRS_{X_t,H_t}}{MRT_{X_t,H_t}} = 1 + \left(\frac{\xi}{1+\xi}\right)\frac{U''(X_t)X_t}{U'(X_t)}. \tag{54}$$

In (53), C_{1t} and C_{2t} are expressions defined in Appendix A.2, and ξ and ι_t are the Lagrange multipliers of the Ramsey problem associated, respectively, with the PVIC and the sequence of ZLB constraints.

With these results, we can prove completeness of the tax system, as stated in Proposition 9.

G.1 Proof of Proposition 9

Compare (49)-(51) with (52)-(54).

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