Money, Search and Business Cycles*

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Abstract

Monetary models that specify explicit frictions to generate money demand have been developed over the last 20 years and have been used to address many questions. In this paper we investigate the short-run properties of a particular model considering a number of versions based on some modeling choices. All versions feature flexible prices. We find that in many aspects, both real and nominal, the model resembles other, more reduced-form models. Some variations of the model come closer to matching some key nominal facts than a reduced-form model. The model also generates counter-cyclical markups, in line with the data.

Keywords: micro-foundations of money, markup

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1 Introduction

Monetary models that generate a demand for holding money via explicit frictions in bilateral trade have developed significantly over the last 20 years. The seminal work of Kiyotaki and Wright (1989, 1993) lays out two fundamental frictions that lead to an intrinsically useless object being accepted in trade: lack of perfect record keeping (so that credit cannot be sustained) and some double-coincidence problem (so that the two parties want to trade in the first place). These models, often regarded as the "first generation" of money-search models, had severe restrictions such as indivisibility of goods and money.[1] The "second generation", exemplified by Trejos and Wright (1995) and Shi (1995), relaxes the restriction on goods and the "third generation", exemplified by Lagos and Wright (2005) and Shi (1997), relaxes both restrictions by utilizing two separate modeling constructs to prevent the distribution of money holdings from being non-degenerate.[2] Shi (1997) allows perfect risk-sharing for consumption across agents by assuming they are members of large households. Lagos and Wright (2005), henceforth LW, make the agents visit a second market and rebalance their money balances, in addition to the market where they do bilateral trade. Agents are assumed to have quasi-linear utility in this second market and as a result, all agents leave this market with identical money balances.

The introduction of the second market, the centralized market, opens up many possibilities to extend the model by introducing features that macroeconomists use in similar models. Aruoba and Wright (2003) and Aruoba, Waller and Wright (2008), henceforth AWW, follow this route and embed a standard real business cycle model inside the environment of LW. In particular, the model features a neoclassical firm undertaking production in the centralized market and households supplying their labor and renting their capital to this firm. Alterna-

[1] In this paper we use the term "money-search models" to refer to monetary models in the tradition of Kiyotaki and Wright (1989) even though the fact that agents in the model search for a trading partner is not a key feature in generating a demand for money.

[2] Various studies, e.g. Molico (2006), tackles the more complicated task of solving these models using numerical methods, allowing for a non-degenerate distribution of money holdings to arise in equilibrium.

tively, one can interpret the model as embedding a market where trade takes place bilaterally, the decentralized market, into the real business cycle (RBC) model of Hansen (1985). The main focus of AWW, in addition to developing the model, is the long-run relationship between money (inflation) and capital, one of the classical questions in macroeconomics. A significant portion of AWW is devoted to carefully calibrating a number of different versions of the model that vary in certain dimensions. Generally speaking, in terms of fitting the long-run facts that they use for calibration, all models are equally successful. In other words, one cannot distinguish these different versions based on their long-run predictions.

This paper has three broad objectives. First, using the benchmark model developed in AWW and considering a number of variations, we analyze the short-run properties of this model, following closely the methodology developed in Kydland and Prescott (1982) and Prescott (1986) for real models and Cooley and Hansen (1995) for nominal models. To do this, we add a technology shock and a money growth shock to the model. We report results for four versions of the model that vary along two dimensions: common capital / market-specific capital and pricing protocol in the decentralized market: generalized Nash bargaining and price taking. Second, we revisit one of the main conclusions of Cooley and Hansen (1995) and investigate whether a flexible-price model is necessarily at odds with the data in terms of some of it nominal short-run predictions.[3] Third, the model in AWW generates a markup in the decentralized market when prices in this market are determined using bargaining. We investigate whether the cyclical properties of this markup match those that can be found in the data.

Our findings suggest that except for some notable differences, the business cycle dynamics

[3]It is commonly accepted in the literature that a model that minimally departs from an RBC model, such as the cash-credit-good-model of Cooley and Hansen (1995) is not able to capture some key nominal business cycle facts such as the procyclicality and persistence of inflation or the sizable real response to monetary policy shocks. This lead to the development of many models with more departures from the RBC model. Perhaps the current state-of-the-art model is the one developed in Christiano, Eichenbaum and Evans (2005) which includes numerous real and nominal rigidities. There are also a number of papers in the literature which use flexible-price models to generate results reasonably close to those in the data. One example is the limited participation models, e.g. Cooley and Quadrini (1999), which prevent the instantaneous adjustment of all prices.

of the models developed in AWW are not different from those of a bare-bone flexible-price model. In particular, in terms of the short-run dynamics of real variables, the money-search models share all the successes and failures of their RBC counterpart. In terms of the properties of nominal variables, generally speaking, the same conclusion holds, while some versions of the model show some better results than those obtained from the model in Cooley and Hansen (1995). We find these results encouraging and we think more research is needed to find out if the versions of the model with improved nominal properties can be usefully developed. Finally, we also find that the markup generated in our model, while not as volatile, matches some of the key facts in the data.

There has been considerably limited work in exploring the business cycle properties of money-search models. Menner (2007) and Wang and Shi (2006) conduct similar exercises using the Shi (1997) model as their starting point. A key difference between their and our frameworks is that the non-monetary version of our model degenerates to the Hansen (1985) RBC model while theirs degenerate to autarky. We think having a standard RBC model as the limiting case is a useful property for comparison purposes. Moreover, Menner's (2007) analysis show some counterfactual results such as employment being counter-cyclical. Wang and Shi (2006) focuses on the volatility of velocity and develop a model that has search frictions in both labor and goods markets. While they are successful in generating a realistic process for velocity, since they abstract from capital accumulation, their model is silent regarding some key business cycle facts. Finally, Telyukova and Visschers (2009b) extend the framework in LW to have idiosyncratic uncertainty and a retail sector. The former creates a disconnect between household's choices of cash balances and their consumption which helps generate volatility in velocity of money and creates a precautionary demand for money. The latter allows them to interpret the sellers in LW as retail firms who purchase goods in the centralized market and sell them in the decentralized market. Unlike LW, there is no production in the decentralized market in their model. Both of these extensions create large departures from the original representative-agent framework of LW. Our analysis in this paper is complementary to all these papers, especially the last one. Our aim is to document the business cycle properties of the simple framework developed in LW and AWW.

The paper is organized as follows. In much of Section 2 we develop the model with two separate capital stocks for the two markets and in Section 2.7 we show how one can obtain the one-capital benchmark model in AWW. We briefly mention the reference models in Section 2.8. Section 3 discusses the data used for documenting the business cycle facts for the U.S. and the calibration strategy. In Section 4.1 we list our results grouped by each statistic of interest and in Section 4.2 we discuss some of the results that stand out. Section 5 concludes. An online appendix available at `http://www.boraganaruoba.com`. provides detailed results for the business cycle statistics obtained from the U.S. data and those obtained from each of four versions we consider as well as the reference models and some extra results.

2 Model

The model is based on the basic structure of LW. Time is infinite and there is a unit measure of ex-ante identical households. In every period, agents first enter a decentralized market (DM) where they receive idiosyncratic shocks that make them buyers or sellers, each with σ probability, or neither.[4] Agents are anonymous in this market and this, coupled with the double-coincidence problem described above, makes a medium of exchange essential.[5] In order to focus on equilibria with fiat money (as opposed to equilibria that also includes the possibility of commodity money in the form of capital) we assume that the capital of all agents are fixed in place during the DM and buyers visit the sellers location, without having

[4] This setup is mathematically equivalent to the search structure developed in the literature, e.g. Kiyotaki and Wright (1993) and Trejos and Wright (1995). The mapping works as follows. Each of these agents can produce a unique good out of a measure one of goods, placed on the unit circle and they have preferences such that they get utility only from a set of goods, represented by an arc of length σ on the unit circle. When two agens meet, with probability σ there is a single coincidence where one agent likes what the other can produce but not vice versa.

[5] AWW also allow for "monitored" meetings, where contracts can be enforced. We do not include these types of meetings.

access to their own capital.[6]

The sellers in the DM utilize a production function $q = Sf(z, e)$, where S is an aggregate technology shock that is common between the DM and CM production, z is their DM-specific capital holdings and e is the effort they exert. They have a disutility for e which we normalize to be linear. This leads to a cost of production in terms of utility for the sellers denoted by $c(q, z, S)$ with properties $c_q > 0$, $c_z < 0$ and $c_S < 0$. The buyers in the DM get utility $u(q)$ and in exchange for q, they make a payment of d units of money where (q, d) are jointly determined via the particular pricing mechanism employed. Once a round of DM trade is completed, all agents move to the centralized market (CM) where they interact with the market as price-takers, just as the agents in a typical neoclassical model would: they work for and rent their capital to a neoclassical firm, they consume, and adjust their money and capital holdings. The flow utility in the CM is given by $U(x) - Ah$ where x denotes their consumption and h denotes their labor supply.[7] All agents hold some CM-specific capital, denoted by k, that they rent to the neoclassical firm and receive a rental payment of r. They also receive wage payments of w. The government conducts monetary policy in the CM, and since we abstract from any fiscal issues, it amounts to a lump-sum transfer made to the agents where new money injected (or a lump-sum tax where some money is withdrawn), with gross money growth rate given by μ.

To solve the model, we start from the second half of a period, the CM, and work our way backwards. Since we are going to look for a recursive equilibrium, we start from the outset by dropping time subscripts for all the functions.

[6] Lagos and Rocheteau (2008) analyze a similar environment and they allow for capital to circulate. They show that as long as socially-optimal level for capital is not very large, money and capital will co-exist where the latter may or may not serve as a medium of exchange. Aruoba and Schorfheide (2010) extend the model used in the present paper to allow for capital to circulate in the DM. Their findings suggest that aggregate U.S. data does not support this version of the model.

[7] The quasi-linearity in utility is similar to that in Cooley and Hansen (1995), which has its roots in the indivisible labor models of Rogerson (1988) and Hansen (1985). It also serves an important role here, making the model tractable by eliminating wealth effects that would generate a non-degenerate distribution of money holdings.

2.1 The Centralized Market Problem

Agents start period t with money and capital holdings given by m_t, z_t and k_t. Depending on their experiences in the DM, agents may have different holdings of money and DM-specific capital as they exit the DM. For example a buyer spends some of his money and a part of the seller's capital depreciates. To capture this, we use $\hat{m}_t$ and $\hat{z}_t$ to denote the money and DM capital holdings of an agent at the start of the CM. His value of entering the CM with $\hat{m}_t$ and $\hat{z}_t$ as well as k_t units of CM-specific capital, with aggregate states S_t and μ_t is given by

$$W(\hat{m}_t, \hat{z}_t, k_t, S_t, \mu_t) = \max_{\substack{x_t, h_t, m_{t+1}, \\ i_t^d, i_t^c, z_{t+1}, k_{t+1}}} \left\{ U(x_t) - Ah_t + \beta E_t \left[V(m_{t+1}, z_{t+1}, k_{t+1}, S_{t+1}, \mu_{t+1}) \right] \right\}$$

subject to

$$\begin{aligned} x_t &= w_t h_t + r_t k_t - i_t^c - i_t^d + T_t + \frac{\hat{m}_t - m_{t+1}}{p_t} \\ z_{t+1} &= \hat{z}_t + i_t^d \\ k_{t+1} &= (1-\delta) k_t + i_t^c \end{aligned}$$

where i_t^c and i_t^d are the investments in CM and DM capital, respectively, p_t is the price level in the CM, T_t is the real transfer from the government and $V(.)$ is the value of entering the DM. The depreciation of DM-specific capital, if used, takes place in the DM and is implicit in the $\hat{z}_t$ term.

The first order conditions of this problem are given by

$$
\begin{aligned}
x_t &: U'(x_t) = \frac{A}{w_t} \\
m_{t+1} &: \frac{A}{p_t w_t} = \beta E_t \left[V_m(m_{t+1}, z_{t+1}, k_{t+1}, S_{t+1}, \mu_{t+1}) \right] \\
z_{t+1} &: \frac{A}{w_t} = \beta E_t \left[V_z(m_{t+1}, z_{t+1}, k_{t+1}, S_{t+1}, \mu_{t+1}) \right] \\
k_{t+1} &: \frac{A}{w_t} = \beta E_t \left[V_k(m_{t+1}, z_{t+1}, k_{t+1}, S_{t+1}, \mu_{t+1}) \right]
\end{aligned}
$$

which show that as long as V_m, V_z and V_k are strictly monotonic, the investment decisions of the agent do not depend on his money and capital holdings. This is a direct result of the quasi-linear utility function which eliminates wealth effects. A degenerate distribution of money and capital holdings across agents will emerge as the agents exit the CM and this makes the problem tractable. Another implication of quasi-linearity is the linearity of $W(.)$ in its arguments

$$
\begin{aligned}
W_m(\hat{m}_t, \hat{z}_t, k_t, S_t, \mu_t) &= \frac{A}{p_t w_t} && (1) \\
W_z(\hat{m}_t, \hat{z}_t, k_t, S_t, \mu_t) &= \frac{A}{w_t} && (2) \\
W_k(\hat{m}_t, \hat{z}_t, k_t, S_t, \mu_t) &= \frac{A(1-\delta+r_t)}{w_t}
\end{aligned}
$$

To solve the model, we need the envelope conditions V_m and V_z, which depend on the details of the pricing mechanism used in the DM, and V_k.

2.2 The Decentralized Market Problem

The value of entering the DM, before the realization of the idiosyncratic shock, is given by

$$
V(m_t, z_t, k_t, S_t, \mu_t) = \sigma V^b(m_t, z_t, k_t, S_t, \mu_t) + \sigma V^s(m_t, z_t, k_t, S_t, \mu_t) + (1-2\sigma) W(m_t, z_t, k_t, S_t, \mu_t) \tag{3}
$$

where the values of being a buyer and a seller are

$$\begin{aligned} V^b(m_t, z_t, k_t, S_t, \mu_t) &= u\left(q_t^b\right) + W\left(m_t - d_t^b, z_t, k_t, S_t\right) \\ V^s(m_t, z_t, k_t, S_t, \mu_t) &= -c(q_t^s, z_t, S_t) + W\left[m_t + d_t^s, (1-\delta) z_t, k_t, S_t\right] \end{aligned}$$

and $\left(q_t^b, d_t^b\right)$ and (q_t^s, d_t^s) reflect the terms of trade in the DM, from the viewpoint of the buyer and seller, respectively. The value function for the seller shows the depreciation of DM capital. Using (1) and (2) we can simplify (3) to

$$V(m_t, z_t, k_t, S_t, \mu_t) = \sigma\left[u\left(q_t^b\right) - c(q_t^s, z_t, S_t) + \frac{d_t^s A}{p_t w_t} - \frac{d_t^b A}{p_t w_t} - \frac{\delta A z_t}{w_t}\right] + W(m_t, z_t, k_t, S_t, \mu_t)$$

This yields the envelope conditions

$$V_m(m_t, z_t, k_t, S_t, \mu_t) = \sigma\left[u'\left(q_t^b\right)\frac{\partial q_t^b}{\partial m_t} - \frac{A}{p_t w_t}\frac{\partial d_t^b}{\partial m_t}\right] + \frac{A}{p_t w_t} \tag{4}$$

$$V_z(m_t, z_t, k_t, S_t, \mu_t) = \sigma\left[-c_z(q_t^s, z_t, S_t) - c_q(q_t^s, z_t, S_t)\frac{\partial q_t^s}{\partial z_t} + \frac{A}{p_t w_t}\frac{\partial d_t^s}{\partial z_t} - \frac{\delta A}{w_t}\right] + \frac{A}{w_t} \tag{5}$$

where we imposed the results that q_t^s, d_t^s will not depend on m_t and q_t^b, d_t^s will not depend on z_t which we will show below. Since CM capital is not used in the DM we can also write

$$V_k(m_t, z_t, k_t, S_t, \mu_t) = W_k(m_t, z_t, k_t, S_t, \mu_t) = \frac{A\left(1 - \delta + r_t\right)}{w_t}$$

2.2.1 Generalized Nash Bargaining

The generalized Nash bargaining problem is

$$\begin{gathered} \max_{q,d}\left[u\left(q\right) + W\left(m_t - d, z_t, k_t, S_t\right) - W\left(m_t, z_t, k_t, S_t\right)\right]^{\theta} \\ \times\left[-c\left(q, z_t, S_t\right) + W\left(m_t + d, \left(1-\delta\right) z_t, k_t, S_t\right) - W\left(m_t, z_t, k_t, S_t\right)\right]^{1-\theta} \\ \text{s.t. } d \leq m_t^b. \end{gathered} \tag{6}$$

where the first term is the buyer's surplus, net of his threat point of walking away and going to the CM, $\theta \in (0,1)$ is his bargaining power and the second term is the seller's surplus, net of his threat point. The constraint in (6) restricts the payment to the seller with the money holdings of the buyer, reflecting the *quid pro quo* nature of trade in the DM. Using (1) and (2), we can convert this problem to a static problem

$$\max_{q,d} \left[u(q) - \frac{Ad}{p_t w_t}\right]^{\theta} \left[-c(q, z_t, S_t) - \frac{\delta A z_t}{w_t} + \frac{Ad}{p_t w_t}\right]^{1-\theta} \tag{7}$$
$$\text{s.t. } d \le m_t^b.$$

In any monetary equilibrium, the solution to this problem will feature $d = m_t^b$, and $q = q(m_t^b, z_t^s, S_t)$ that solves

$$\frac{m_t^b}{p_t} = \frac{g(q_t, z_t^s, S_t) w_t}{A} \tag{8}$$

where

$$g(q, z, S) \equiv \frac{\theta u'(q)\left[c(q, z, S) + \dfrac{\delta A z}{w}\right] + (1-\theta) u(q) c_q(q, z, S)}{\theta u'(q) + (1-\theta) c_q(q, z, S)} \tag{9}$$

The relevant derivatives in (4) and (5) are therefore $\partial d^b / \partial m^b = 1$, $\partial q^b / \partial m_t^b = A / p_t w_t g_q$ and $\partial q^s / \partial z_t^s = -g_z / g_q$ and all others are zero. This yields the envelope conditions

$$\begin{aligned} V_m(m_t, z_t, k_t, S_t, \mu_t) &= \frac{A}{p_t w_t}\left[\sigma \frac{u'(q_t)}{g_q(q_t, z_t, S_t)} + 1 - \sigma\right] \\ V_z(m_t, z_t, k_t, S_t, \mu_t) &= (1 - \sigma\delta)\frac{A}{w_t} + \sigma \Gamma(q_t, z_t, S_t) \end{aligned}$$

where[8]

$$\Gamma(q, z, S) = \frac{c_q(q, z, S)\, g_z(q, z, S) - c_z(q, z, S) g_q(q, z, S)}{g_q(q, z, S)} > 0 \tag{10}$$

An important property of the monetary models with bargaining is the holdup problem(s)

[8] Our definition of $\Gamma(.)$ is minus of the definition of $\gamma(.)$ in AWW since we want to make it explicit that having access to capital in the DM adds to the return of capital.

it features.[9] In this model there are two holdup problems: in money accumulation and in investment in DM-specific capital. These are captured by the partial derivatives $\partial q^b/\partial m_t^b$ and $\partial q^s/\partial z_t^s$ above, both of which depend on the actions of the agent as the various partial derivatives of the $g(.)$ function indicate. (10) makes the investment holdup problem very explicit: if the agent brings more capital to the DM, the second term shows that holding q constant he will incur a lower cost, but the first term shows that the q he has to produce actually increases. These holdup problems have profound normative implications as AWW and Aruoba and Chugh (2010) demonstrate.

2.2.2 Price-Taking

As an alternative to bilateral bargaining schemes, Rocheteau and Wright (2005) introduce a competitive alternative where once matched, buyers and sellers both act as price takers. While there are many ways to conceptualize this, one can image an auctioneer announcing a price, all agents computing their respective demands and supplies given this price and equilibrium price being the one that clears the market.

In this version, the DM value function has the same form as (3), but now after being matched, buyers and sellers solve the problems

$$\begin{aligned} V^s(m_t, z_t, k_t, S_t, \mu_t) &= \max_q \left\{-c(q, z_t, S_t) + W\left(m_t + \tilde{p}_t q, (1-\delta)\, z_t, k_t, S_t, \mu_t\right)\right\} \quad (11) \\ V^b(m_t, z_t, k_t, S_t, \mu_t) &= \max_q \left\{u(q) + W(m_t - \tilde{p}_t q, z_t, k_t, S_t)\right\} \text{ s.t. } \tilde{p}_t q \leq m_t \end{aligned}$$

taking $\tilde{p}_t$, the DM price level, as given. Market clearing implies buyers and sellers choose the same q. As with bargaining, buyers spend all their money, so $q_t = m_t^b/\tilde{p}_t$. The FOC from (11) is $c_q(q_t, z_t^s, S_t) = \tilde{p}_t W_m = \tilde{p}_t A/p_t w_t$. Inserting $\tilde{p}_t = m_t^b/q_t$, we get the analog to (8) from

[9] A holdup problem occurs when a party makes a costly and irreversible investment whose benefits he shares with another party. This would cause an under-investment.

the bargaining model

$$\frac{m_t^b}{p_t} = \frac{q_t c_q(q_t, z_t^s, S_t) w_t}{A}. \tag{12}$$

The relevant derivatives in (4) and (5) are $\partial d_t^b / \partial m_t^b = \tilde{p}_t (\partial q_t^b / \partial m_t^b)$, $\partial q_t^b / \partial m_t^b = 1/\tilde{p}_t$, and $\partial d_t^s / \partial z_t^s = \tilde{p}_t (\partial q_t^s / \partial z_t^s)$. $\partial q_t^s / \partial z_t^s$ is non-zero but unnecessary for our calculations and all other partial derivatives are zero. In contrast to the versions with bargaining, $\partial q_t^b / \partial m_t^b$ and $\partial q_t^s / \partial z_t^s$ no longer depend on the actions of the agents and the holdup problems are eliminated.

The envelope conditions are given by

$$\begin{aligned} V_m(m_t, z_t, k_t, S_t, \mu_t) &= \frac{A}{p_t w_t} \left[\frac{\sigma u'\left(q_t^b\right)}{c_q(q_t, z_t, S_t)} + 1 - \sigma \right] \\ V_z(m_t, z_t, k_t, S_t, \mu_t) &= (1 - \sigma\delta) \frac{A}{w_t} - \sigma c_z(q_t^s, z_t, S_t) \end{aligned}$$

2.3 Firms in the Centralized Market

We have standard neoclassical firms, without loss of generality only one firm, that has access to the constant-returns-to-scale technology $Y = SF(K, H)$, taking as given the price of its good p, and its factor prices w and r, yielding

$$\begin{aligned} w_t &= S_t F_H(K_t, H_t) \\ r_t &= S_t F_K(K_t, H_t) \end{aligned}$$

2.4 Shocks and Government

Government's monetary policy consists of controlling the money supply via transfers/taxes in the CM to all agents. Money supply follows

$$M_{t+1} = \mu_t M_t$$

where M_{t+1} denotes aggregate money balances at the end of period t. The gross money growth rate is stochastic following the process

$$\log\left(\frac{\mu_t}{\mu_*}\right) = \rho_\mu \log\left(\frac{\mu_{t-1}}{\mu_*}\right) + \varepsilon_t^\mu \text{ and } \varepsilon_t^\mu \sim N\left(0, \sigma_\mu^2\right) \tag{13}$$

where μ_* is the mean of gross money growth. We assume μ_t is realized at the beginning of period t and the transfer to implement it is done in the CM at the end of period t. This transfer is given by

$$T_t = \frac{(\mu_t - 1) M_t}{p_t}$$

The common technology shock is also stochastic and it follows the process

$$\log(S_t) = \rho_S \log(S_{t-1}) + \varepsilon_t^S \text{ and } \varepsilon_t^S \sim N\left(0, \sigma_S^2\right) \tag{14}$$

2.5 Equilibrium

Combining the results so far, imposing $k_t = K_t$, $z_t = Z_t$, $m_t = M_t$, $x_t = X_t$ and defining $\mathcal{M}_{t+1} \equiv M_{t+1}/p_t$ as the end-of-period real money balances for period t and $\pi_{t+1} \equiv p_{t+1}/p_t$ as the CM inflation rate in period $t+1$, the equilibrium conditions that define

$(X_t, q_t, H_t, K_{t+1}, Z_{t+1}, \mathcal{M}_{t+1}, \pi_t, R_t)$ in the generalized Nash bargaining version are given by

$$U'(X_t) = \frac{A}{S_t F_H(K_t, H_t)} \tag{15}$$

$$U'(X_t) = \beta E_t \left\{ \frac{U'(X_{t+1})}{\pi_{t+1}} \left[\sigma \frac{u'(q_{t+1})}{g_q(q_{t+1}, Z_{t+1}, S_{t+1})} + 1 - \sigma \right] \right\} \tag{16}$$

$$U'(X_t) = \beta E_t \left\{ U'(X_{t+1}) \left[S F_K(K_{t+1}, H_{t+1}) + 1 - \delta \right] \right\} \tag{17}$$

$$U'(X_t) = \beta E_t \left[(1 - \sigma\delta) U'(X_{t+1}) + \sigma \Gamma \left(q_{t+1}, Z_{t+1}, S_{t+1} \right) \right] \tag{18}$$

$$X_t = S_t F(K_t, H_t) + (1 - \delta) K_t - K_{t+1} + (1 - \sigma\delta) Z_t - Z_{t+1} \tag{19}$$

$$\mathcal{M}_t = \frac{g(q_t, Z_t, S_t) S_t F_H(K_t, H_t) \pi_t}{A} \tag{20}$$

$$\mathcal{M}_{t+1} = \frac{\mu_t}{\pi_t} \mathcal{M}_t$$

$$U'(X_t) = \beta R_t E_t \left[\frac{U'(X_{t+1})}{\pi_{t+1}} \right] \tag{21}$$

where R_t is the gross nominal interest rate of a one-period bond and (21) shows how it is priced.[10]

In the appendix we also show results for the version where terms of trade in the DM are determined via proportional bargaining, following Aruoba, Rocheteau and Waller (2007). In this version, the equilibrium conditions listed above remain the same and the only change is the definition of $g(.)$ in (9) is replaced by

$$g(q, z, S) \equiv \theta \left[c(q, z, S) + \frac{\delta A z}{w} \right] + (1 - \theta) u(q) \tag{22}$$

As Aruoba, Rocheteau and Waller (2007) and Aruoba and Chugh (2010) show, the normative implications of this version of the model is very different but it turns out the positive implications we investigate in this paper are similar to those from the version with generalized Nash and thus we do not report these results in the paper.

[10]While we do not explicitly model bonds, one can introduce inside or outside bonds that are issued and redeemed in the CM, to obtain this condition.

For the price-taking version (16), (18) and (20) need to be replaced by

$$\begin{aligned} U'(X_t) &= \beta E_t \left\{ \frac{U'(X_{t+1})}{\pi_{t+1}} \left[\sigma \frac{u'(q_{t+1})}{c_q(q_{t+1}, Z_{t+1}, S_{t+1})} + 1 - \sigma \right] \right\} \\ U'(X_t) &= \beta E_t \left[(1 - \sigma\delta) U'(X_{t+1}) - \sigma c_z (q_{t+1}, Z_{t+1}, S_{t+1}) \right] \\ \mathcal{M}_t &= \frac{q_t c_q(q_t, Z_t, S_t) S_t F_H(K_t, H_t) \pi_t}{A} \end{aligned} \tag{23}$$

2.6 National Income Accounting

Since we have a two-sector model, we need to define aggregate measures. Nominal output in the DM is given by σM_t and in the CM it is $p_t Y_t$. Using these, the share of the DM in total nominal output can be defined as

$$s_t \equiv \frac{\sigma \mathcal{M}_t}{\sigma \mathcal{M}_t + \pi_t Y_t}$$

We define the aggregate price level[11] as

$$\mathcal{P}_t \equiv s_t \frac{M_t}{q_t} + (1 - s_t) p_t$$

and aggregate inflation as

$$\Pi_t \equiv \frac{\mathcal{P}_t}{\mathcal{P}_{t-1}} = \frac{s_t \frac{\mathcal{M}_t}{q_t} + (1 - s_t) \pi_t}{s_{t-1} \frac{\mathcal{M}_{t-1}}{\pi_{t-1} q_{t-1}} + (1 - s_{t-1})}$$

We can also define aggregate real output, real consumption, real investment, capital-to-

[11]This would roughly correspond to constructing a price index, based on the shares of each sector.

output ratio and the real wage as

$$\begin{aligned}
\mathcal{Y}_t &\equiv \frac{\sigma M_t + p_t Y_t}{\mathcal{P}_t} = \frac{\sigma \mathcal{M}_t + \pi_t Y_t}{s_t \frac{\mathcal{M}_t}{q_t} + (1-s_t)\,\pi_t} \\
\mathcal{C}_t &\equiv \frac{\sigma M_t + p_t X_t}{\mathcal{P}_t} = \frac{\sigma \mathcal{M}_t + \pi_t X_t}{s_t \frac{\mathcal{M}_t}{q_t} + (1-s_t)\,\pi_t} \\
\mathcal{I}_t &= \left[K_{t+1} - (1-\delta)\,K_t + Z_{t+1} - (1-\sigma\delta)\,Z_t\right] \frac{p_t}{\mathcal{P}_t} \qquad (24) \\
KY_t &\equiv \frac{(K_t + Z_t)\,\frac{p_t}{\mathcal{P}_t}}{\mathcal{Y}_t} \qquad (25) \\
\mathcal{W}_t &= w_t \frac{p_t}{\mathcal{P}_t}
\end{aligned}$$

all of which are deflated by the aggregate price level.

Another key object we compute is the gross aggregate markup which is the weighted sum of the gross markup in the DM and the gross markup in the CM (one) and is given by

$$\Lambda_t \equiv s_t \frac{g\left(q_t, Z_t, S_t\right)}{q_t c_q\left(q_t, Z_t, S_t\right)} + (1-s_t) \qquad (26)$$

Finally, we define the velocity of money as

$$VEL_t \equiv \frac{\mathcal{Y}_t}{\mathcal{M}_{t+1}}$$

where according to our timing conventions it is calculated using the end-of-period money balances.

2.7 The Model with Single Capital Stock

In order to recover the equilibrium conditions of AWW, who assume the same capital stock is used both in the DM and the CM, we need to replace all Z_t terms with K_t, delete (18) for bargaining or (23) for price-taking and replace (17) with

$$U'(X_t) = \beta E_t \left\{ (1-\sigma\delta) U'(X_{t+1}) \left[S F_K(K_{t+1}, H_{t+1}) + 1 - \delta \right] + \sigma \Gamma \left(q_{t+1}, K_{t+1}, S_{t+1} \right) \right\}$$

for bargaining and with

$$U'(X_t) = \beta E_t \left\{ (1-\sigma\delta) U'(X_{t+1}) \left[S F_K(K_{t+1}, H_{t+1}) + 1 - \delta \right] - \sigma c_k \left(q_{t+1}, K_{t+1}, S_{t+1} \right) \right\}$$

for price-taking.[12] Finally, we replace (19) with

$$X_t = S_t F(K_t, H_t) + (1 - \delta - \sigma\delta) K_t - K_{t+1}$$

(24) with

$$\mathcal{I}_t = \left[K_{t+1} - (1 - \delta - \sigma\delta) K_t \right] \frac{p_t}{\mathcal{P}_t}$$

and (25) with

$$KY_t \equiv \frac{K_t \dfrac{p_t}{\mathcal{P}_t}}{\mathcal{Y}_t}$$

2.8 Reference Models

We use two reference models to compare our results to. First, to provide a *real* benchmark, we use the real business cycle (RBC) model of Hansen (1985) and we will simply refer to it as the RBC model. Second, we use the cash-credit-good model of Cooley and Hansen (1995) as a *monetary* benchmark and refer to it as CH. An important reason that underlie our choices is the fact that both monetary models, ours and CH, simplify to Hansen's (1985) RBC model in their nonmonetary equilibria.[13] Since both of these models are very standard

[12] The only material difference between AWW and the specification here is the fact that capital depreciates in the DM when it is used.

[13] This can be seen by setting $\sigma = 0$ in our model which will shut down the DM, and by removing the cash-in-advance constraint in the CH model.

and well-studied, we do not repeat their details here and refer the readers to the respective papers.

3 Data and Calibration

3.1 Data

We use quarterly data for the U.S. covering the period 1959:Q1-2007:Q4 obtained from the FRED database maintained at the Federal Reserve Bank of St. Louis in April 2010, unless otherwise stated. We do not include data after 2007 because they are still subject to revisions and, more importantly, to exclude the data during the financial crisis of 2008-2009, which temporarily changed the nature of a number of the variables we use.[14] All of our data are seasonally adjusted and we convert series observed at higher frequencies to quarterly by averaging over the quarter. To the extent possible, we use measures that are consistent with our model. For example our measure of consumption includes nondurables and services and consumption of durables is included in investment. Below we list the details of each of the series that we use along with the FRED Series IDs.

- **Consumption:** Sum of real services consumption (PCESVC96) and real nondurable good consumption (PCNDGC96).
- **Investment**: Sum of real private fixed investment (FPIC96) and real durable good consumption (PCDGCC96).
- **Real Output:** Sum of the consumption and investment measures defined above.

[14] Our exercise here, in an effort to stay as close to CH and similar papers, is to analyze the business-cycle properties of our model, when the driving forces are "standard" shocks. Clearly the events in 2008-2009 deserve a careful study and will certainly require the introduction of some financial frictions or shocks. Since this will move the focus of the paper significantly, we abstract from using data from the crisis.

- **Nominal Output**: Sum of the nominal counterparts of the consumption and investment measures defined above (PCES, PCEND, FPI and PCEDG).

- **Hours**: Average weekly hours, total private industries, from the establishment survey (AWHNONAG).

- **Labor Productivity:** Ratio of real output to hours as defined above.

- **Price Level:** Ratio of nominal output to real output as defined above.

- **Wages:** Hourly compensation for nonfarm sector (COMPNFB) deflated by the price level as defined above.

- **Markup:** From Rotemberg and Woodford (1999) for 1973:1-1993:1, not seasonally adjusted.[15]

- **Nominal Money Balances:** Sweep-adjusted M1, from Cynamon et al. (2006).[16]

- **Real Money Balances:** Nominal money balances deflated by the price level as defined above.

- **Velocity:** The ratio of nominal output and nominal money balances as defined above.

- **Money Growth Rate:** Quarterly change in nominal money balances as defined above.

- **Inflation:** Quarterly change in the price level as defined above.

- **Nominal Interest Rate:** The secondary market rate for the 3-month treasury bill. (TB3MS).

[15] The markup series do not contain any discernible seasonality.

[16] Since the mid-1990s, banks in the U.S. have systematically moved funds from their customers' demand deposit accounts to money market accounts in an effort to reduce their statutory reserve requirements. This operation, labeled "sweep", systematically and significantly distorts the official M1 statistics and Cynamon et al. (2006) produce sweep-adjusted measures and publish them on the web at `www.sweepmeasures.com`.

The only possibly nonstandard data series is the one we use for the aggregate markup. Rotemberg and Woodford (1999) provide four different model-based measures of aggregate markup for a subset of our sample. Since we do not have a strong stand on the appropriate measure of markup, we compute the first principal component of these series to use as our markup series. These measures must nevertheless be used with caution since they are obtained using a particular economic model. We turn to this issue in Section 4.2.

For the purpose of computing business-cycle statistics, we first take natural logarithms of all series except for those that are expressed in percentages (like interest rates and inflation) and we annualize all series expressed in percentages. Finally, we apply the Hodrick and Prescott (1997) (HP) filter to all series to focus on their business-cycle-frequency fluctuations.

3.2 Calibration

Given that we have different versions of our model and two reference models, we want to carefully calibrate each model so that they have identical long-run implications. We choose the following functional forms for our model.

$$\text{CM:} \qquad U(x) = B\log(x) \text{ and } F(K,H) = K^{\alpha}H^{1-\alpha}$$

$$\text{DM:} \qquad u(q) = \log(q+b) - \log(b) \text{ and } c(q,k,S) = S^{-\frac{1}{1-\alpha}}q^{\frac{1}{1-\alpha}}k^{-\frac{\alpha}{1-\alpha}}$$

where B and α are parameters to be calibrated. In choosing the utility functions, we follow Waller (2010), who shows that in the benchmark AWW model a necessary condition for balanced growth is that both utility functions are natural logarithms. The production function in the CM is a standard Cobb-Douglas function. In the DM, we assume that the sellers have access to the same production technology as in the CM given by $q = Sk^{\alpha}e^{1-\alpha}$, where e represents the effort of the seller. This leads to the particular cost function above.

We fix $b = 0.0001$ which means $u(q)$ is very close to a standard CRRA utility function.[17]

We calibrate the parameters of our models except for those related to the shocks by matching certain long-run calibration targets obtained from post-war U.S. data, using the steady states of our models. These calibration targets are:[18]

- Labor share of output : 70%
- Annual investment-capital ratio : 0.07
- Interest semi-elasticity of money demand : −0.064
- Annual capital-output ratio : 2.3
- Average hours : 0.33
- Annual Velocity : 4.59
- DM markup: 30%

By fixing $\alpha = 0.3$, we match the labor share of output for both the DM and the CM. In our models with one capital stock, investment-capital ratio at the steady state is given by $(1+\sigma)\delta$ and once we fix the value for σ, as we explain below, we use δ to match this ratio. We fix $\sigma = 0.026$ which matches the long-run semi-elasticity of money demand.[19]

[17] We need a utility function in the DM that satisfies $u(0) = 0$ since the threat point of a buyer involves walking away from the match and consuming nothing in the DM. While this is a technical assumption, in the relevant parts of the domain, i.e. when $q > 0$, this function behaves just like a standard CRRA function.

[18] Labor share and average hours are standard targets commonly used in the literature. The investment-capital ratio and capital-output ratio are taken from AWW who compute these ratios using a very similar dataset. Velocity is computed from our dataset as defined above. Finally, aggregate markup measures used in the literature vary from 10% to 15% and more disaggregated studies find values as high as 45%. We follow AWW in using a target of 30% for the DM markup. In our experience the exact level of markup targeted simply changes θ and keeps other parameters and long-run targets intact.

[19] The target and the value of σ come from the quarterly calibration of AWW. Aruoba and Schorfheide (2011) estimate σ to be around 0.3 using quarterly time-series data and argue that this estimate captures the short-run elasticity of money demand which is much lower than its long-run counterpart. In an earlier version of this paper we repeated the analysis for this higher value of σ and the results were similar.

We match the remaining four targets jointly by calibrating parameters A, B, β and θ, except for the price-taking version where markup is not defined, in which case we drop θ from the list of parameters to be calibrated. In models with two capital stocks, the investment-capital ratio cannot be pinned down independent from the equilibrium outcomes and we add it to the list of calibration targets that are jointly determined.

The parameters that define the properties of the two stochastic processes are calibrated as follows. We fix $\rho_Z = 0.95$. We set $\sigma_Z = 0.0056$ so that the volatility of output in the RBC model matches that obtained from the data. We set $\mu_* = 1.0154$ which corresponds to an annual money growth rate of 6.15%.[20] Finally, estimating (13) using HP-filtered money growth rate we get $\rho_\mu = 0.4813$ and $\sigma_\mu = 0.0066$. The RBC and CH models are also calibrated using identical targets, to the extent possible. The CH model cannot match the target level of velocity we use even with very high weight in the utility function for cash goods (α in their notation) and we use the value they provide for this parameter. Calibration results are reported in Table 1.

4 Results

4.1 Business Cycle Statistics

Our objective is to compare the business cycle implications of our model to those computed from the U.S. data and also to the two benchmark models. Table A1 in the appendix shows the business cycle statistics computed from the particular data series and the sample we use in this paper. Our results are by and large consistent with those reported in Cooley and Hansen (1995) and Stock and Watson (1999), who compute similar statistics for different samples. The vast business cycles literature, including the two papers cited, contains many

[20] Both of our monetary model have the property that the long-run inflation rate is equal to the long-run money growth rate. In the data the former is 3.56%, which is substantially lower than the latter. We choose to use properties of money supply process in the data to calibrate the parmeters for μ in our model.

descriptions of the stylized facts and as such we will not have a detailed exposition here. Instead, in the next sections we focus on one statistic or group of statistics at a time and report our results. In Section 4.2 we discuss our findings and put them in context.

Our methodology for computing model-based statistics is standard. We solve each of our models using a second-order log-linear approximation around their steady states.[21] We then simulate the model 1000 times for 300 periods each and eliminate the first 100 observations from each simulation. Finally we compute the relevant statistics in each simulation, after applying the same transformations we applied to the data. We report the average of the statistics across the simulations, along with the standard error across the simulations. We use the same set of shocks in simulating different models to enhance comparability. In the tables and the discussion below, we use GN to denote the version with generalized Nash and PT to denote the version with price taking.

4.1.1 Volatility

Table 2 provides a summary of our results regarding the volatility of variables. For output we report the level of volatility, while for all other variables we report its volatility relative to the model-implied volatility of output. As we explained above, the volatility of the technology shock innovation is calibrated so that output volatility in the RBC model matches the data. Focusing on the real variables first, and comparing the results for the RBC model with the CH model and the 6 versions of our model, we see that results are similar. All monetary models generate roughly the right level of volatility for output. Consumption is less volatile and investment is more volatile than in the data for all of our models, which is a well-known feature of the RBC model without adjustment costs. The volatility of hours is higher than what is in the data. We should note, however, due to the indivisible labor interpretation

[21] Results from AWW and Aruoba and Chugh (2010), who solve related models using nonlinear global methods, show that the decision rules from our model will be closely approximated by a second-order approximation given business cycle magnitude shocks. The results do not change if we use a linear approximation instead of a log-linear approximation.

of our quasi-linear preferences in the CM, it is also reasonable to compare our results to employment or total hours in the data, both of which are much more volatile than average hours. Finally, volatilities of wages and labor productivity (which have identical properties in the RBC and CH models) are generally lower than the data. Turning to nominal variables, our models generate too little volatility in markup, velocity, nominal interest rate and real money balances and too much volatility in inflation.

4.1.2 Persistence

Table 3 reports the first-order autocorrelation coefficients of the variables. Generally speaking, our models do not appear to create sufficient persistence. A part of this finding, especially related to the real variables, is well-known and typically corrected by adding some features to the model that create more persistence such as habit persistence. In this regard the monetary models perform very similar to the RBC model. As for monetary variables, perhaps the most important failure of a typical flexible-price monetary model is the lack of any persistence in inflation. In the data inflation is mildly persistent with an autocorrelation of 0.22 and the CH model produces an autocorrelation of 0.06 and our models produce very small negative autocorrelations except for GN with two capital stocks which has an average autocorrelation of 0.02. However, for all models the distribution of this statistic across simulations is very wide – ranging from -0.24 to 0.34. Similarly all of our models, including CH generate somewhat less persistence for velocity.

4.1.3 Contemporaneous Correlations

Table 4 reports the contemporaneous correlations of variables with output. The RBC model generates a very tight relationship between all real variables and output, some of which (consumption and investment) is in line with data, but some of which, especially those related to the labor market (hours, wages and labor productivity) are stronger than in the data. The CH model and our models display similar results. On the nominal side, the most important result is the significantly higher correlation of velocity and output delivered by all the monetary models, relative to the data. The one-capital model with GN produces a somewhat lower correlation but that is still twice as large as the one in the data.

Panel (a) of Table 5 reports the contemporaneous correlations of variables with money growth. The data suggests a weakly negative to zero correlation between money growth and real quantities (output, consumption, investment hours and labor productivity) and mildly positive correlations between money growth and wages. The first observation is roughly matched by all monetary models exceot for the GN version with two capital stocks, which displays too much correlation. Moreover, the same model displays positive a correlation between money growth and output. One counterfactual implication of the CH model is the strong negative correlation it generates between money growth and consumption due to the cash-in-advance constraint. This correlation is much milder in most of our models.

As for the second observation, it is one many models struggle with. For example, the CH model shows exactly zero correlation for these variables. Interestingly, the two-capital model with GN, which shows a positive correlation between output and money growth, generate a reasonable positive correlation between wages and money growth.

Panel (b) of Table 5 reports the correlations between velocity and the nominal interest rate generated by our models. The CH model generates a lower correlation than the one in the data, while our models are broadly consistent with the data.

4.1.4 Cyclicality of Variables

While the contemporaneous correlation with output is an important statistic, in order to determine the cyclicality of a variable it may not be the most informative one, given possible phase shifts. For example, in the data the nominal interest rate has a mild positive correlation with output contemporaneously but the fifth lag of the interest rate has a strong negative correlation with output. This would classify the interest rate as a countercyclical variable that leads the cycle. Given the relatively simple structure of our models (no rigidities that would generate significant phase shifts), in Table 6 we report the largest correlation coefficient for each variable and compare the U.S. data. For all cases except for three that are noted, this highest correlation is contemporaneous.

The key lead/lag relationships in the data are for markup (lag by 4 quarters), nominal interest rate (lead by 5 quarters), inflation (lag by four quarters) and money growth (lag by 2 quarters). With one exception we point out below, none of our models are able to capture these relationships. Focusing on the cyclicality of variables, on the real side, all our models are able to capture the qualitative result that all variables except the wage rate are procyclical, though the strength of the correlations varies. On the nominal side, while most models are successful in matching the cyclical properties of velocity, they are having a hard time matching the procyclicality of inflation. As it is well known, this is a key failure of flexible-price models. One exception to this finding comes from the two-capital model with GN, which displays a mild procyclical behavior, very much in line with the data. Moreover, it shows a phase shift by one period, versus the four-quarter shift in the data. This model fails, however, in matching the mild counter-cyclicality of the money growth rate.

A key implication of our model, about which the CH model is silent, is the countercyclicality of markup. The GN version generates a countercyclical markup, as does the one with proportional bargaining.

4.1.5 Impulse Responses

We compute impulse-response functions to the money growth and technology shocks and Figures 1 and 2 report those obtained from the two reference models and the two versions of our model with generalized Nash. It is well-known that a flexible model such as ours will have a hard time matching impulse responses obtained through vector autoregressions. Moreover, in this model we use a simple exogenous money growth rule instead of, for example, an interest-rate feedback rule, which would generate what could be better related to monetary policy shocks in the data. As such, we compare our model-based impulse responses to those from the reference models.

Responses to the technology shock, which are shown in Figure 1, display the usual characteristics: an increase on impact and a monotone decline in output, hours and investment and a somewhat hump-shaped response of consumption. In the version with two capital stocks, when a positive technology shock hits the economy, we see that the CM capital goes up and DM capital goes down on impact (not reported). This is due to the fact that the return to CM capital goes up by more and some DM capital is initially converted to CM capital. After a few periods, both capital stocks smoothly increase, eventually settling back at the steady state. This decline in the DM capital in period 1 creates a decline in DM output in period 2, which creates some non-monotonicities in the responses of output and consumption for these models. Another result of the decline in DM capital is the increase in inflation in period 2. This is behind the procyclicality of inflation for this model and we discuss this further below. Inflation quickly returns to its steady state in all models.

Responses to the money growth shock, which are shown in Figure 2, are almost invariably very short-lived. In all three models inflation and the nominal interest rate increase on impact and following the persistence of the shock, return to the steady state. As a result, money becomes a worse investment relative to capital and real money balances fall (not shown) and investment increases. In CH and the model with one capital stock, this results in a decline in output and consumption since the initial buildup of capital comes at the cost of loss of consumption initially. This is behind the mild negative correlation of output and money growth in these models. In contrast, in the model with two capital stocks, in addition to these, the agents also have the option of converting some DM capital into CM capital. This overcomes the initial drop in consumption and output, increasing both on impact.

4.2 Discussion of Results

A number of results stand out from our analysis which we discuss in turn.

Looking at the results for all models, one can generally conclude that the short-run dynamics of the money-search model we consider are similar to those from a more reduced-form model like CH. There are some exceptions to this conclusion, especially the model with two capital stocks and generalized Nash, which we discuss in detail below. In particular, the dynamics of real variables resemble closely those from the RBC model. On the nominal side, the CH model is known to have a number of shortcomings. First, an important shortcoming of CH and similar monetary models is their inability to match properties of velocity of money. Hodrick et al. (1991) is an early paper which points out that a simple cash-in-advance model is by construction unable to generate time-variation in consumption velocity and that even after the introduction of a cash versus credit good choice, some problems remain. These problems are also replicated in our implementation of the CH model: velocity is about 54% as volatile as in the data, it is not as persistent, it is too highly correlated with output, and it is mildly positively correlated with money growth, instead of the mild negative correlation in the data. Our models share the same qualitative problems. Wang and Shi (2006) and Telyukova and Visschers (2009a,b) are recent papers that show various ways to address the standard models' inability to match the dynamics of velocity.[22] Aruoba and Schorfheide (2011) use modern Bayesian estimation techniques to estimate a variant of the model used in this paper and find that the two main shocks that drive fluctuations in (inverse) velocity are government spending shocks (or more generally aggregate demand shocks) and money demand shocks. This result suggests an internal propagation mechanism that would generate fluctuations in velocity is currently missing in our model, as it does

[22] Wang and Shi (2006) develop a search model of money without capital and show that with high risk aversion in utility and search frictions in both goods and labor market, they can generate the right level of velocity volatility. Telyukova and Visschers (2009a) develop a model that uses a cash-in-advance constraint where idiosyncratic shocks cause the cash-in-advance constraint to be slack for some consumers. This breaks the close link between consumption and real money balances allowing the model to address some of the issues discussed above. Telyukova and Visschers (2009b) is a similar model that is based on LW. They key feature of the latter papers is the idiosyncratic uncertainty which we abstract from.

in many monetary models. Since money demand (and therefore velocity) is an important outcome of monetary models more research is necessary to understand this often neglected variable.

Second, our models generate inflation which has too little persistence and except for the model with two capital stocks and generalized Nash, inflation is countercyclical. Third, our models, with the same exception, generate a very small real response to changes in monetary policy. Despite these failures, we think that our results are encouraging since they show that the key properties of the RBC model, such as its ability to match the dynamics of real variables, are preserved in our models.

Unlike the CH model, our model cannot generate nominal interest rates that are as volatile as in the data, though other properties of the nominal interest rates are quite similar. In both models, the equation that determines the dynamics of the nominal interest rate is (21) where the key difference is in the CH model the marginal utility refers to the one from cash-good consumption, while in our models it is the marginal utility of CM consumption. According to the impulse-responses in Figure 1, the nominal interest rate does not respond to technology shocks. As such, to understand the dynamics of the interest rate, we need to analyze how the economy responds to a money growth shock. In our models due to quasi-linearity, $U'(X_t)$ is essentially constant in response to a money supply shock, since through (15) it is linked to the wage rate which does not respond to money growth shocks. Thus the response of the interest rate on impact equals the response of inflation one period after the shock, which is very small as Figure 2 shows. In contrast, in the CH model, the cash-in-advance constraint ensures that $c_{1t}p_t = M_t$ for all t, where c_{1t} is the cash-good consumption. Then (21) simplifies to $1 = \beta R_t E_t\left(1/\mu_{t+1}\right)$ and this shows that the interest rate response on impact equals the expected response of money growth in the period after the shock, which, due to the persistence of the shock, is fairly large. This differential response explains a number of differences across the two models: interest rate volatility, response of investment (since nominal interest rate goes up by more in the CH model, real money demand falls more, which results in a higher demand for capital) and the response of consumption to a money growth shock.

Let us now turn to the successes of our model. The DM features bilateral trade among households and when the terms of trade is determined via bargaining, as long as the sellers has a sufficiently large bargaining power, they are able to extract some of the surplus, leading to a positive markup.[23] Unlike models that employ the Dixit-Stiglitz construct of monopolistic competition, the level of this markup is not fixed and it varies due to the internal forces of the model. Our results show that the markup generated by our models, while not nearly as volatile as the measures in Rotemberg and Woodford (1999)[24], is similarly persistent, positively correlated with money growth, and most important of all, it is countercyclical – all in line with the data.[25] Referring to the definition of the aggregate markup in (26), we see that changes in it can be decomposed into two: changes in the DM markup and changes in the share of the DM. To make this clear, we can rewrite the aggregate markup as

$$\Lambda_t \equiv 1 + s_t \left(\frac{g\left(q_t, Z_t, S_t\right)}{q_t c_q \left(q_t, Z_t, S_t\right)} - 1 \right) \tag{27}$$

where the term in parentheses is the net markup in the DM. When the economy experiences a positive technology shock, one that affects both markets, we see that on impact the DM markup barely moves and the share of the DM goes down. This creates the countercyclical response of the markup in our model since, naturally, the same shock increases output. To understand the response of the share of the DM, remember that investment occurs in the

[23] Roughly speaking, the total surplus of the match between a buyer and seller is the difference between the opportunity cost of holding money for the buyer and the cost of production for the seller. When $\theta = 1$, the price is equal to the average cost, which is typically less than the marginal cost. With θ sufficiently below unity, the seller is able to charge a price that is greater than the marginal cost.

[24] We should acknowledge that the countercylicality of markups, while very commonly thought to be a feature of the data, is not without controversy. Recent work by Nekarda and Ramey (2009) show that markups in various sectors are either acyclical or procyclical. This stands in contrast to similar empirical evidence by Chevalier et al. (2003) for the U.S. or Martins et al. (1997) for 14 OECD countries, among many others.

[25] Edmond and Veldkamp (2009) also explain countercyclical markups in a flexible-price model where there is heterogeneity in households' earnings.

CM. When the economy experiences a positive technology shock, this increases the incentives for capital accumulation (for both CM- and DM-related reasons), creating a relative increase in CM activity.[26]

Among all the models we considered, the model with two capital stocks and generalized Nash deserves some special attention. As evidenced by the responses we report in Figures 1 and 2, this model delivers three key results: output (and consumption) respond positively to a positive monetary policy shock, inflation is mildly procyclical and the response of inflation to a monetary policy shock does not die out right after the impact. It should be noted, however, that none of the models we considered display the hump-shaped delayed responses to monetary policy shocks as characterized by, for example, Christiano, Eichenbaum and Evans (1999).

While the first result, when expressed as the contemporaneous correlation between money growth and output, is counterfactual, as Christiano Eichenbaum and Evans (1999) and many others have argued using impulse responses from structural VARs, is a key fact of the data. In the versions with two capital stocks and generalized Nash bargaining, when the economy experiences a money growth shock, the value of money (arising from the DM) falls on impact, reducing demand and, therefore, output in the DM. This creates an incentive for households to move their capital from the DM into the CM. This increase in CM capital boosts CM production. While the response of total output, which is the sum of DM and CM outputs is ambiguous in principle, quantitatively we find that the increase in CM dominates. The mechanism that makes households move capital between the DM and the CM is available in all pricing mechanisms we consider, but its effect is the strongest with generalized Nash bargaining. With bargaining, as a reaction to the same money growth shock, DM demand falls more compared to price taking due to the money holdup problem under the former scheme. Between the two bargaining schemes, the additional inefficiency due to the non-

[26] Some experimentation with the calibration reveals that there is a negative relationship between θ, the bargaining power of the buyer in the generalized Nash version and the volatility of markup. However, lowering θ also increases the average markup.

monotonicity of generalized Nash bargaining as pointed out by Aruoba, Rocheteau and Waller (2007), generates a larger fall, creating a larger capital transfer between markets.

In most monetary models an increase in the growth rate of money lowers the real return on money and reduces the amount of real money balances held by households. As a result, the activity that is related to money, cash-good consumption in the CH model or DM consumption in our models, falls, in turn causing reductions in output and therefore capital. In addition to this, there is also the Tobin (1965) effect where the agents substitute between real money balances and capital when the former becomes less attractive, increasing capital and therefore output supply. Typically in flexible-price models the inflation-tax effect dominates the Tobin effect. But in the model with two capital stocks and generalized Nash bargaining, due to the possibility of converting DM capital into CM capital, the Tobin effect is much stronger and it dominates.

The second result, procyclicality of inflation, which occurs with one period lag according to Table 6, is also the result of the transfer of capital between the DM and CM. When the economy experiences a positive technology shock, inflation falls on impact since with money balances fixed, the decrease in the cost for the seller reduces prices. When the households get to the CM, where they can move capital between markets, they reduce their DM capital which increases the costs for producers in the following period. As a result, one period after impact, inflation goes up.

The third result, mild persistence of inflation, is also due to the transfer of capital between markets. Focusing on the money growth shock, inflation goes up on impact since the DM good is purchased using money. In the CM that follows the initial DM, households move some of their DM capital in to the CM, since the DM is now less attractive due to higher prices. In models with only one capital, in the second period inflation goes back to a level very close to the steady state as households are able to adjust their money balances. In models with two capital stocks, however, the transfer of capital from the DM to the CM creates a further increase in inflation, making the response of inflation more long-lived relative to other models.

5 Conclusion and Future Work

Taken as a whole, our results show that the model developed in AWW share the same successes and failures as the CH model, in dimensions that they overlap. In particular, the real variables behave similar to the underlying RBC model and in models with one capital stock, the nominal variables display three important problems that many flexible-price models share: inflation is not persistent, it is countercyclical and monetary policy does not create a significant response in real variables. The models with two capital stocks, on the other hand, produce some interesting results, making progress in three key nominal predictions that flexible-price models fail in. The model also delivers an endogenous markup through the bilateral bargaining in the DM, that is counter-cyclical, though not as volatile as in the data.

We see four broad conclusions and directions for further research from this paper. First, more research seems to be necessary on the models with two capital stocks to investigate the channel through which monetary policy is effective and to compare them more rigorously with the data. Second, more progress needs to be made on explaining money demand. Current macro models either completely ignore issues related to money demand or do not do a good job in explaining the dynamics of money demand, or velocity. We think that monetary models that generate a role for money by explicitly modeling money demand may be successful in making progress. Third, we show that the models developed in AWW could be useful a starting point for extensions that improve the performance of the model in matching some key nominal facts. This may include using the enhancements that have been discussed in the literature: introducing adjustment costs in capital accumulation to reduce the volatility of investment, habit persistence to increase the persistence of real variables and, perhaps most importantly, introducing nominal rigidities to make inflation persistent and procyclical and monetary policy non-neutral. Of course many of these enhancements are "reduced-form" ones that may be seen as contradictory to the serious micro-foundations developed in the money-search literature. We nevertheless see this as an interesting intermediate step. Finally, a very interesting and fruitful direction would be to extend the search-based models to create micro-founded reasons for nominal rigidities by modeling the trading frictions between agents more carefully.

The first two directions above are subject to ongoing research. Aruoba and Schorfheide (2011) take up the third direction by replacing the neoclassical firm in the CM in the one-capital version of the AWW model with monopolistically competitive firms subject to pricing frictions. Head et al. (2010) make progress in the last direction but more work in the context of dynamic stochastic general equilibrium models is certainly necessary.

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Table 1 - Calibration Results

Parameter	Target	RBC	CH	Single Capital Stock GN	Single Capital Stock PT	Two Capital Stocks GN	Two Capital Stocks PT
α	Labor share = 70%	0.3	θ = 0.3	0.3	0.3	0.3	0.3
δ	I/K = 0.07	0.0175	0.0175	0.0171	0.0171	0.0175	0.0183
A	H = 0.33	2.5283	γ = 2.4649	0.941	1.107	0.764	1.107
B	Velocity = 4.59	-	α = 0.84	0.371	0.436	0.301	0.436
β	K/Y = 2.3	0.985	0.985	0.986	0.985	0.986	0.985
σ	Int. El. of Money Demand	-	-	0.026	0.026	0.026	0.026
θ	DM Markup = 30%	-	-	0.958	-	0.914	-
ρ_S	Fixed	0.95	0.95	0.95	0.95	0.95	0.95
σ_S	Output volatility in RBC	0.0056	0.0056	0.0056	0.0056	0.0056	0.0056
ρ_μ	Estimated process	-	0.4813	0.4813	0.4813	0.4813	0.4813
σ_μ	Estimated process	-	0.0066	0.0066	0.0066	0.0066	0.0066
μ^*	Estimated process	-	0.0154	0.0154	0.0154	0.0154	0.0154
Memo							
Share of DM	-	-	-	2.23%	2.23%	2.23%	2.23%
Agg. Markup	-	-	-	0.67%	-	0.67%	-
DM Markup	-	-	-	30.00%	-	30.00%	-

Notes: I/K, velocity and K/Y are annualized targets. RBC column corresponds to the AWW model with σ = 0. The parameters in the CH column correspond to the notation in their paper and α = 0.84 is fixed since the model cannot match the target velocity level. In the other models α is set individually to match labor share. In models with a single capital stock, δ is set individually to match I/K and A, B, β and θ (except for price-taking) are jointly calibrated to match the corresponding calibration targets. In models with two capital stocks, δ, A, B, β and θ (except for price-taking) are jointly calibrated to match the corresponding calibration targets. σ_S is calibrated to match the volatility of output in the RBC model and is fixed in other models.

Table 2 - Standard Deviations of Key Variables

Variable	U.S. Data	RBC	CH	Single Capital Stock GN	Single Capital Stock PT	Two Capital Stocks GN	Two Capital Stocks PT
Output	1.51	1.52	1.50	1.50	1.49	1.64	1.57
		0.15	*0.15*	*0.14*	*0.14*	*0.15*	*0.16*
Consumption	0.52	0.30	0.34	0.30	0.30	0.36	0.30
		0.01	*0.02*	*0.01*	*0.01*	*0.01*	*0.01*
Investment	2.94	4.94	5.00	4.90	4.92	4.57	4.90
		0.06	*0.13*	*0.10*	*0.09*	*0.10*	*0.10*
Hours	0.27	0.75	0.76	0.76	0.77	0.70	0.76
		0.01	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*
Labor Productivity	0.86	0.30	0.30	0.29	0.29	0.35	0.29
		0.01	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*
Wages	0.58	0.30	0.30	0.30	0.30	0.37	0.30
		0.01	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*
Markup	1.04	-	-	0.01	-	0.01	-
				0.00		*0.00*	
Velocity	1.45	-	0.78	1.09	0.83	0.76	0.82
			0.02	*0.09*	*0.04*	*0.02*	*0.04*
Nominal Interest Rate	0.80	-	0.87	0.11	0.09	0.08	0.08
			0.10	*0.01*	*0.01*	*0.01*	*0.01*
Inflation	0.82	-	2.57	3.17	3.21	2.54	2.98
			0.28	*0.33*	*0.33*	*0.26*	*0.33*
Money Growth Rate	1.99	-	1.81	1.81	1.82	1.66	1.74
			0.21	*0.20*	*0.20*	*0.18*	*0.20*
Real Money Balances	1.48	-	0.36	0.48	0.48	0.39	0.45
			0.02	*0.04*	*0.04*	*0.03*	*0.04*

Notes: The table reports the level of the standard deviation for output and for all other variables the standard deviation relative to that of output. For model-based statistics, the number reported is the average across 1000 simulations and the number in italics is the standard deviation across the simulations.

Table 3 - Autocorrelations of Key Variables

Variable	U.S. Data	RBC	CH	Single Capital Stock GN	Single Capital Stock PT	Two Capital Stocks GN	Two Capital Stocks PT
Output	0.91	0.69	0.69	0.69	0.70	0.64	0.72
		0.05	*0.05*	*0.05*	*0.05*	*0.06*	*0.05*
Consumption	0.88	0.80	0.68	0.79	0.79	0.58	0.76
		0.04	*0.06*	*0.04*	*0.04*	*0.07*	*0.05*
Investment	0.90	0.68	0.67	0.68	0.68	0.67	0.72
		0.10	*0.06*	*0.05*	*0.05*	*0.06*	*0.05*
Hours	0.76	0.68	0.68	0.68	0.68	0.68	0.72
		0.05	*0.05*	*0.05*	*0.05*	*0.05*	*0.05*
Labor Productivity	0.90	0.80	0.80	0.80	0.81	0.58	0.76
		0.04	*0.04*	*0.04*	*0.04*	*0.07*	*0.05*
Wages	0.78	0.80	0.80	0.79	0.80	0.57	0.75
		0.04	*0.04*	*0.04*	*0.04*	*0.07*	*0.05*
Markup	0.61	-	-	0.68	-	0.53	-
				0.05		*0.06*	
Velocity	0.91	-	0.66	0.50	0.63	0.62	0.66
			0.06	*0.07*	*0.06*	*0.06*	*0.05*
Nominal Interest Rate	0.81	-	0.36	0.35	0.35	0.35	0.35
			0.07	*0.07*	*0.07*	*0.07*	*0.07*
Inflation	0.22	-	0.06	-0.05	-0.05	0.02	-0.04
			0.07	*0.07*	*0.07*	*0.07*	*0.07*
Money Growth Rate	0.48	-	0.36	0.35	0.35	0.35	0.35
			0.07	*0.07*	*0.07*	*0.07*	*0.07*
Real Money Balances	0.93	-	0.65	0.54	0.52	0.58	0.54
			0.06	*0.07*	*0.06*	*0.07*	*0.07*

Notes: For model-based statistics, the number reported is the average across 1000 simulations and the number in italics is the standard deviation across the simulations.

Table 4 - Contemporaneous Correlations of Key Variables with Output

				Single Capital Stock		Two Capital Stocks	
Variable	**U.S. Data**	**RBC**	**CH**	**GN**	**PT**	**GN**	**PT**
Consumption	0.93	0.88	0.77	0.89	0.88	0.92	0.90
		0.02	*0.03*	*0.02*	*0.02*	*0.01*	*0.02*
Investment	0.97	0.99	0.97	0.99	0.99	0.98	0.99
		0.05	*0.01*	*0.00*	*0.00*	*0.00*	*0.00*
Hours	0.69	0.98	0.98	0.98	0.98	0.98	0.98
		0.00	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*
Labor Productivity	0.97	0.88	0.88	0.87	0.86	0.91	0.88
		0.02	*0.02*	*0.02*	*0.02*	*0.01*	*0.02*
Wages	-0.18	0.88	0.88	0.89	0.88	0.91	0.89
		0.02	*0.02*	*0.02*	*0.02*	*0.01*	*0.02*
Markup	-0.36	-	-	-0.94	-	-0.70	-
				0.01		*0.05*	
Velocity	0.31	-	0.95	0.65	0.88	0.94	0.90
			0.01	*0.06*	*0.02*	*0.01*	*0.02*
Nominal Interest Rate	0.25	-	-0.03	-0.14	-0.01	0.13	0.01
			0.09	*0.09*	*0.10*	*0.10*	*0.10*
Inflation	0.27	-	-0.19	-0.16	-0.14	-0.09	-0.13
			0.07	*0.07*	*0.07*	*0.07*	*0.07*
Money Growth Rate	-0.07	-	-0.03	-0.01	-0.01	0.13	0.02
			0.09	*0.09*	*0.10*	*0.10*	*0.10*
Real Money Balances	0.37	-	0.73	0.59	0.56	0.74	0.59
			0.04	*0.06*	*0.07*	*0.04*	*0.07*

Notes: For model-based statistics, the number reported is the average across 1000 simulations and the number in italics is the standard deviation across the simulations.

Table 5 - Correlations of Key Variables with Money Growth Rate and Correlation of Velocity with the Interest Rate

Variable	U.S. Data	CH	Single Capital Stock GN	Single Capital Stock PT	Two Capital Stocks GN	Two Capital Stocks PT
(a) Correlation with Money Growth Rate						
Output	-0.07	-0.03	-0.01	-0.01	0.13	0.02
		0.09	*0.09*	*0.10*	*0.10*	*0.10*
Consumption	-0.03	-0.50	-0.05	-0.06	0.36	0.03
		0.08	*0.09*	*0.10*	*0.08*	*0.09*
Investment	-0.09	0.14	0.00	0.01	0.03	0.01
		0.09	*0.09*	*0.10*	*0.10*	*0.10*
Hours	-0.21	-0.05	0.00	0.01	0.00	0.00
		0.09	*0.09*	*0.10*	*0.10*	*0.10*
Labor Productivity	-0.02	0.04	-0.04	-0.05	0.38	0.05
		0.08	*0.09*	*0.10*	*0.08*	*0.09*
Wages	0.36	0.04	0.02	0.02	0.41	0.10
		0.09	*0.09*	*0.10*	*0.08*	*0.09*
Markup	0.16	-	-0.08	-	-0.78	-
		-	*0.09*	-	*0.04*	-
Velocity	-0.29	0.23	0.73	0.43	0.43	0.42
		0.09	*0.05*	*0.08*	*0.08*	*0.09*
Nominal Interest Rate	-0.46	1.00	0.99	1.00	1.00	1.00
		0.00	*0.00*	*0.00*	*0.00*	*0.00*
Inflation	-0.04	0.91	0.87	0.87	0.85	0.87
		0.01	*0.01*	*0.01*	*0.02*	*0.01*
Real Money Balances	0.23	-0.57	-0.75	-0.78	-0.50	-0.74
		0.07	*0.05*	*0.05*	*0.07*	*0.05*
(b) Correlation of Velocity with the Interest Rate						
	0.54	0.23	0.64	0.43	0.43	0.42
		0.09	*0.06*	*0.08*	*0.08*	*0.09*

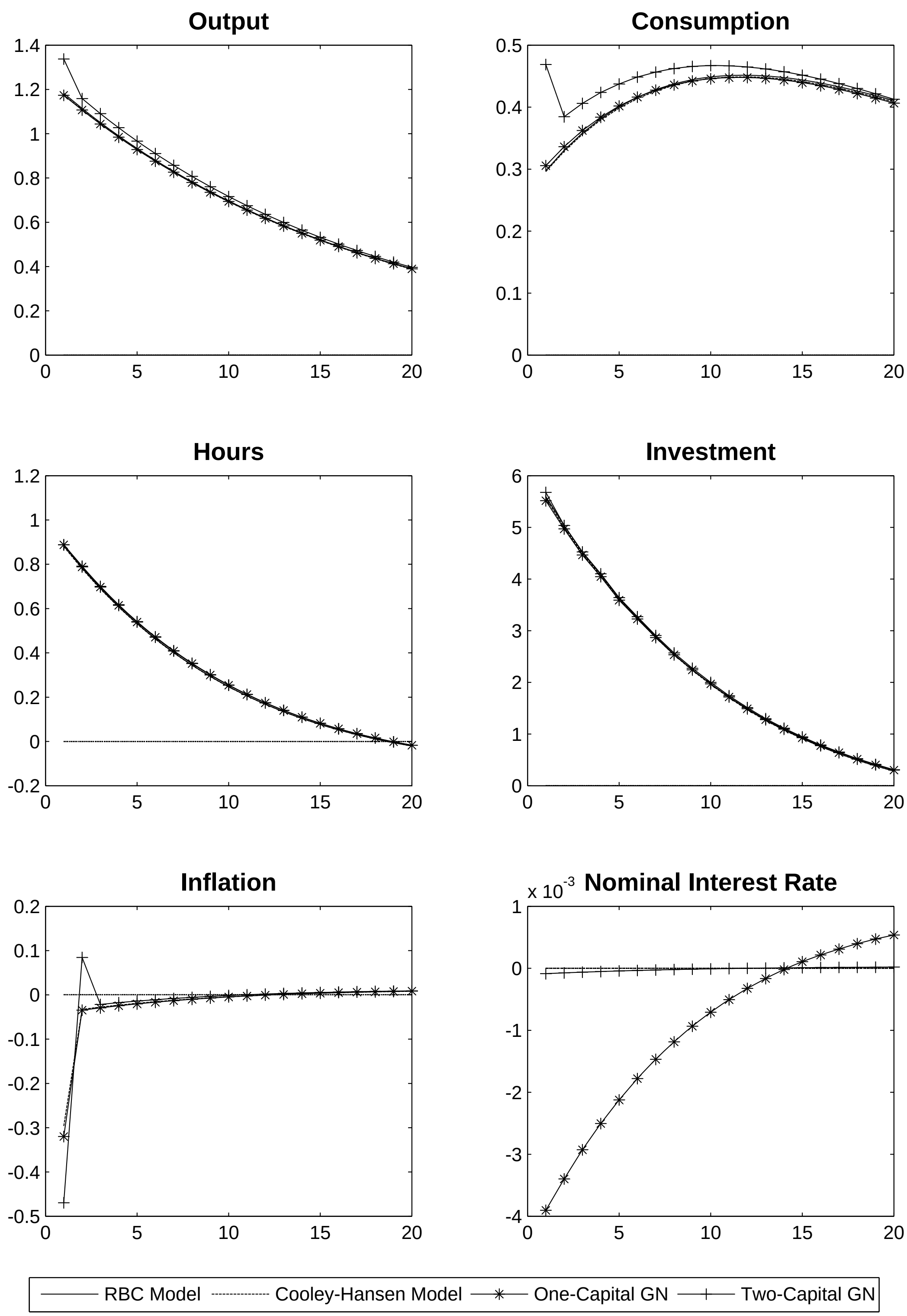
Notes: For model-based statistics, the number reported is the average across 1000 simulations and the number in italics is the standard deviation across the simulations.

Table 6 - Cyclicality of Key Variables

Variable	U.S. Data		RBC	CH	Single Capital Stock GN	Single Capital Stock PT	Two Capital Stocks GN	Two Capital Stocks PT
Consumption	0.93	(0)	0.88	0.77	0.89	0.88	0.92	0.90
Investment	0.97	(0)	0.99	0.97	0.99	0.99	0.98	0.99
Employment	0.69	(0)	0.98	0.98	0.98	0.98	0.98	0.98
Labor Productivity	0.97	(0)	0.88	0.88	0.87	0.86	0.91	0.88
Wages	-0.22	(+2)	0.88	0.88	0.89	0.88	0.91	0.89
Markup	-0.60	(+4)	-	-	-0.94	-	-0.70	-
Velocity	0.35	(+1)	-	0.95	0.65	0.88	0.94	0.90
Nominal Interest Rate	-0.59	(-5)	-	-0.03	-0.14	-0.01	0.13	0.02 (-1)
Inflation	0.28	(+4)	-	-0.19	-0.16	-0.14	0.17 (+1)	-0.13
Money Growth Rate	-0.22	(+2)	-	-0.03	-0.01	-0.01	0.13	0.02 (-1)
Real Money Balances	0.45	(-2)	-	0.73	0.59	0.56	0.74	0.59

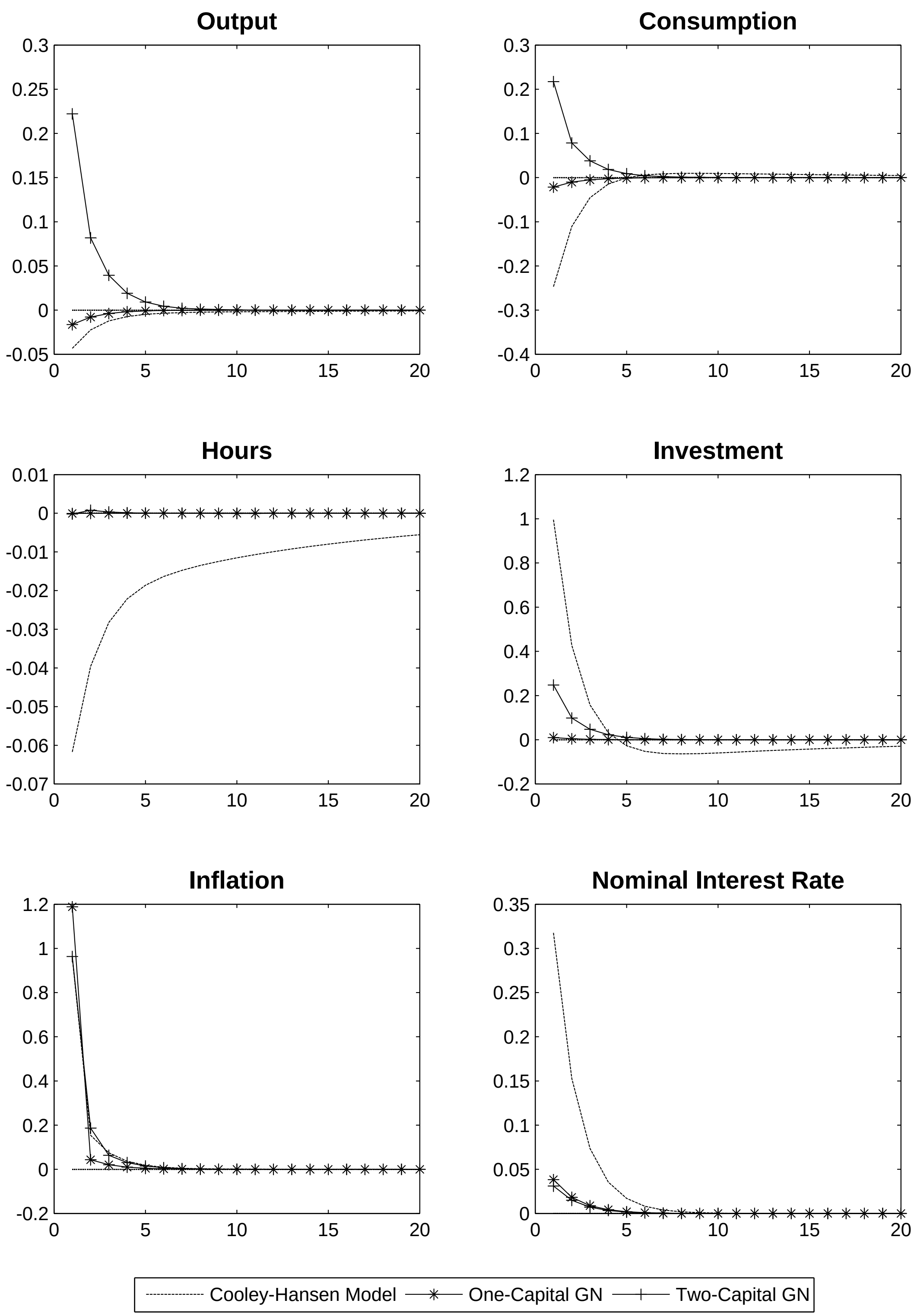
Notes: In the U.S. Data column, the first number is the highest absolute correlation coefficient of the variable with output and the number in the parantheses is the lead (negative) or lag (positive) it appears in. For all the models the number reported is the highest absolute correlation coefficient, all of which occur contemporaneously except as shown.

Figure 1: Responses to a Technology Shock



Note: Each panel reports the response of the respective variable in percentage points.

Figure 2: Responses to a Money Growth Shock



Note: Each panel reports the response of the respective variable in percentage points.