

Term Structures of Inflation Expectations and Real Interest Rates*

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Abstract

I use a statistical model to combine various surveys to produce a term structure of inflation expectations — inflation expectations at any horizon — and an associated term structure of real interest rates. Inflation expectations extracted from this model track realized inflation quite well, and in terms of forecast accuracy, they are at par with or superior to some popular alternatives. The real interest rates obtained from the model follow TIPS rates as well.

Keywords: surveys, state-space methods, inflation expectations, Nelson-Siegel model.

JEL Codes: C32, E31, E43, E58

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1 Introduction

After almost two decades of being well anchored (and low), inflation expectations in the United States have received increased interest because of the uncertainty created by the Federal Reserve’s unprecedented reaction to the financial crisis and the Great Recession between 2008 and 2015.¹ The Federal Reserve kept the federal funds rate near zero, at the zero lower bound (ZLB) during this period, temporarily ending its *conventional* monetary policy. Thus, much the aforementioned reaction to the Great Recession was in the form of *unconventional* monetary policy, in which the Federal Reserve purchased various financial assets in unprecedented quantities. Real interest rates at various horizons is a key component of monetary policy transmission as they affect households’ and firms’ decisions. All of this makes tracking the term structures of inflation expectations and real interest rates in this period, and perhaps even more importantly in the future, very important.

In this paper, I combine forecasts at various horizons from several surveys to obtain a term structure of inflation expectations for consumer price index (CPI) inflation.² Further combining this term structure of inflation expectations with the term structure of nominal interest rates, I obtain a term structure of ex-ante real interest rates. As inputs to my analysis

¹Many economists, especially in the popular press, have expressed wildly different views about the impact of the expansion of the Federal Reserve’s balance sheet on inflation. For example, in an open letter to the Federal Reserve Chairman Ben Bernanke, 23 economists warned about the dangers of this expansion (see “Open Letter to Ben Bernanke,” *Real Time Economics* (blog), *Wall Street Journal*, November 15, 2010, blogs.wsj.com/economics/2010/11/15/open-letter-to-ben-bernanke). A number of other economists argued that this expansion is not a problem (see, e.g., Paul Krugman, “The Big Inflation Scare,” *New York Times*, May 28, 2009, www.nytimes.com/2009/05/29/opinion/29krugman.html?ref=paulkrugman). This divide is also apparent within the Federal Open Market Committee. For a dovish view, see various 2010 speeches by President and CEO Charles Evans (Federal Reserve Bank of Chicago, www.chicagofed.org/webpages/publications/speeches/2010/index.cfm), which predict that inflation lower than 1.5% in three years’ time is a distinct possibility. For a hawkish view, see various 2010 speeches by President and CEO Charles Plosser (Federal Reserve Bank of Philadelphia, www.philadelphiafed.org/publications/speeches/plosser), which call for the winding down of special Fed programs to prevent an increase in inflation in the medium term.

²My analysis focuses on CPI inflation as opposed to, for example, personal consumption expenditures (PCE) price index inflation, gross domestic product (GDP) price deflator inflation, or any of the “core” versions that strip out energy and food prices. PCE inflation has been released since the mid-1990s, but it has been scarcely included in commonly followed surveys. The same goes for the core versions. Since GDP price deflator is only available quarterly, it is not a very appealing measure. Finally, most financial contracts that use inflation use some variant of CPI inflation.

I use inflation expectations from the *Survey of Professional Forecasters (SPF)* published by the Federal Reserve Bank of Philadelphia (FRBP) and *Blue Chip Economic Indicators* and *Blue Chip Financial Forecasts* published by Wolters Kluwer Law & Business. I use the structure of the Nelson and Siegel (1987) (NS) model of the yield curve, which summarizes the yield curve with three factors (level, slope, and curvature), and adapt it to the context of inflation expectations. The end result is a monthly inflation expectations curve — inflation expectations at any horizon from three to 120 months — and an associated real yield curve from 1998 to the present.

The results in this paper and the technology that produces them will be useful to policy-makers and other observers in describing how inflation expectations and real interest rates evolve both over time and across horizons. The methodology will also be useful to market participants who want to price securities with returns linked to inflation expectations of an arbitrary horizon, including forward inflation expectations, that is, inflation expectations for a period that starts in the future. It is important to emphasize the ease with which expected inflation of an arbitrary horizon can be computed. In 2016, the FRBP started producing an inflation expectations curve and a real interest rate curve using the methodology in this paper. Using the output of this production — only four numbers at any point in time are necessary — anyone can compute all the objects I mention previously using the simple formulas in the paper. In addition to providing forecasts for horizons not covered by surveys, a contribution of this paper is to convert the “fixed event” forecasts in surveys to forecasts of “fixed horizons”, which is a problem researchers face. As the lengthy derivations in Appendix A show, this problem is not trivial.³

Turning to the results, I show that the model can accurately summarize the information in surveys with reasonably small measurement errors. The inflation expectations curve has a stable long end, and the lower part of the curve fluctuates considerably more. The real

³Patton and Timmermann (2011) and Knüppel and Vladu (2016) also discuss this issue and provide a solution that uses approximations.

interest rate curve matches the yields from Treasury Inflation-Protected Securities (TIPS) closely. I find that inflation expectations from the model track actual (ex-post) realizations of inflation quite well. More specifically, the forecasts from the model are no worse than two forecasts based on statistical models that come out as the most successful ones from the comprehensive analysis of Faust and Wright (2013). Moreover, with a few minor exceptions, the forecasts from the model outperform some alternatives obtained using financial variables, and in some cases, the difference in forecast accuracy is statistically significant.

This paper is related to three strands of the literature: inflation forecasting, yield-curve modeling using the NS model, and extracting inflation expectations from models that use asset prices. The first of these strands is recently reviewed by Faust and Wright (2013). They have two findings that are most relevant for this paper. First, they show that surveys of professionals (*SPF* and *Blue Chip*) outperform many alternative statistical models. I see this as a good motivation to combine forecasts from these two sources. Second, two statistical models perform particularly well in their analysis: the unobserved-components stochastic-volatility model, which decomposes current inflation, into trend, and serially uncorrelated short-run fluctuations; and the “gap” model where the very short end of the inflation expectations curve is anchored by some survey nowcast, and the rest is filled by the forecast from a simple AR(1) regression of the “gap”, the difference between the forecast at a particular horizon and the nowcast. I show below that the resulting forecast from my model has at least the same forecast accuracy as these two models for the horizons the latter are available. Thus, one way to motivate the exercise in this paper is to see it as similar to the gap model in Faust and Wright (2013), where instead of the nowcast being anchored, a small number of fixed (and typically nonstandard) horizons are anchored by surveys every month and the rest is filled in a statistically optimal way. Combining these surveys to obtain a smooth curve that shows inflation expectations at any arbitrary horizon seems to be a useful exercise, and the NS yield curve model is a parsimonious way of obtaining such a smooth curve. The

dynamic nature of the NS model also crucially provides local smoothing in a period by using observations close to that period to inform the curve in the period.

My paper is also related to Kozicki and Tinsley (2012), who use a shifting-endpoint autoregressive (AR) model to produce a term structure of inflation expectations. Their model starts with a random-walk endpoint (inflation expectations for an infinite horizon) and inflation depends both on its own lags via a stationary AR process as well as the endpoint. Inflation expectations of any horizons can be computed using this model. They cast the model in state space and use inflation expectations data from the Livingston Survey and actual inflation data to estimate their model. My paper differs from theirs in a number of ways. First is the use of the NS model to capture the dynamics of inflation expectations. Second, I use two surveys with 20 different forecast horizons while their survey has only three horizons. Moreover, I do not use actual inflation nor do I impose that all survey expectations be consistent with a particular statistical model for inflation. Finally, I also produce a real interest rate curve.

There are two papers that use a variant of the NS model to investigate related but distinct issues. Christensen, Lopez, and Rudebusch (2010) estimate a variant of the arbitrage-free NS model, using both nominal and real (TIPS) yields. As a result of their estimation, the authors can calculate the model-implied inflation expectations and the risk premium. Gürkaynak, Sack, and Wright (2010) use nominal yield data to estimate a nominal term structure and TIPS data to estimate a real term structure, both by using a generalization of the NS structure (the so-called Nelson-Siegel-Svensson form). The authors then define inflation compensation as the difference between these two term structures. By comparing inflation compensation with survey expectations, they show that it is not a good measure of inflation expectations because it is affected by the liquidity premium and an inflation risk premium. I take the opposite route in this paper, in that I construct a term structure of inflation expectations solely from surveys and compare them with measures from financial

variables.

Three important papers set out to obtain a term-structure of inflation expectations using structural finance or macro-finance models.⁴ Chernov and Mueller (2012) use a no-arbitrage macro-finance model with two observed macro factors (output and inflation) and three latent factors. They estimate their model using nominal yields, inflation, output growth, and inflation forecasts from various surveys as well as TIPS with a sample that ends in 2008. Inflation expectations also have a factor structure, but unlike the model I use, the factors in their model are related to the yield curve and macroeconomic fundamentals, except for one factor that the authors label as the “survey factor,” the only one that affects the level of inflation expectations. D’Amico, Kim, and Wei (2014) use a similar multifactor no-arbitrage term structure model estimated with nominal and TIPS yields, inflation, and survey forecasts of interest rates. Their explicit goal is to remove the liquidity premium that existed in the TIPS market for much of its existence in order to “clean” the break-even inflation rate and identify real yields, inflation expectations, and the inflation risk premium. Their results clearly show the problem of using raw TIPS data due to the often large and time-varying liquidity premium.⁵ Haubrich, Pennacchi, and Ritchken (2012) use a model that has one factor for the short-term real interest rate, another factor for expected inflation rate, another factor that models the changing level to which inflation is expected to revert, and four volatility factors. They estimate their model using data that include nominal yields,

⁴Two other papers use a reduced-form approach. Ajello, Benzoni, and Chyruk (2012) use the nominal yields at a given point in time to forecast inflation at various horizons using a dynamic term structure model that has inflation as one of the factors. The important distinction of this paper relative to some others is that the authors separately model the changes in core, energy, and food prices because, as they show, each of these components has different dynamics. Mertens (2016) sets out to extract trend inflation (long-run inflation) from financial variables and surveys. His data consist of long-horizon surveys, realized inflation measures, and long-term nominal yields. He uses a reduced-form factor model with a level and uncertainty factor that captures stochastic volatility in the trend process. My results regarding long-run inflation expectations are similar to his.

⁵Gospodinov and Wei (2016) extend the model in D’Amico, Kim and Wei (2014) to include information from derivative markets and oil futures, which they argue improves the forecasting performance of the model. Abrahams et al. (2015) also use real and nominal bond yields for a similar purpose, though, they use observable factors to adjust TIPS yields for liquidity.

survey forecasts, and inflation swap rates.

There are three important reasons why I think the methodology of this paper, where I do not use any information from financial markets, has significant value added. First, as Ang, Bekaert, and Wei (2007) and Faust and Wright (2013) show, survey-based inflation expectations are known to be superior to those that come from models with financial variables. An important part of the reason why surveys have superior forecasts is because the forecasts that come out of the models that use financial variables inherit the inevitable volatility of the underlying financial variables.⁶

Second, all swap and TIPS variables used in these papers have maturities of two years or longer. They implicitly use cross-sectional restrictions that come from no-arbitrage considerations and combined with nominal yields with short maturities to inform the lower part of the inflation expectations curve. However, it seems unclear that the relationship between, say, the 10-year TIPS rate and the 10-year nominal yield is informative enough for the one between their one-year counterparts. In my analysis, I have inflation expectations that cover the lower end of the curve as well as the middle and the long end.

Third, the quality of the inflation expectations that come out of the structural models crucially depends on the stability of the relationship among the variables in the model. There are at least three reasons to think that there may have been structural breaks in the data: (1) TIPS and swap markets are relatively young markets, which evolved significantly since their inception; (2) elevated demand for liquidity and safety and increased borrowing by the federal government after the crisis increased the supply of government bonds, which is a point Christensen, Lopez, and Rudebusch (2010) also make; and (3) any model that uses nominal yields needs to take the ZLB seriously in the estimation of the model. As Swanson and Williams (2014) show, in addition to the federal funds rate, most of the yield curve

⁶For example, Haubrich, Pennacchi, and Ritchken (2012) use survey data that are similar to mine as well as swap and nominal yield data, and their forecast accuracy is worse than what I obtain, primarily because it is more volatile.

had been constrained at some point in the 2009–2015 period.⁷ Of the papers I cite, both Haubrich, Pennacchi, and Ritchken (2012) and D’Amico, Kim, and Wei (2014) ignore the ZLB, even though they use nominal yield data from the ZLB period. Chernov and Mueller (2012) use data that end just prior to the ZLB period. The issue of structural stability will especially be more relevant moving forward when data of various regimes will be mixed in the estimation of macro-finance models. In my analysis, because I use only inflation forecasts of forecasters, these regime changes do not affect my analysis.

The paper is organized as follows. In Section 2.1, I describe the model used in the estimation, and in Section 2.2, I provide details about the surveys used as inputs in the estimation. Section 2.3 provides a summary of the full state-space model and its estimation, and Section 2.4 explains how I construct the real interest rate curve. Section 3.1 discusses the estimation results, Section 3.2 provides some robustness analysis, and Section 3.3 compares the resulting inflation expectations curve with some alternatives. Section 4 provides some concluding remarks. An Appendix contains additional results and details of the analysis. An earlier working paper version of the paper, Aruoba (2016), contains further results regarding the evolution of the inflation and real yield curves over the period 2008-2015.

2 Model

2.1 Term Structure of Inflation Expectations

The NS yield curve model is frequently used both in academic studies and by practitioners. As restated by Diebold and Li (2006), the model links the yield of a bond with τ months to maturity, $y_t(\tau)$, to three latent factors, labeled level, slope, and curvature, according to

$$y_t(\tau) = L_t - \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) S_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) C_t + \varepsilon_t, \quad (1)$$

⁷They show that the relationship between macroeconomic surprises and yields that is strong before the crisis weakens or disappears after 2008.

where L_t , S_t , and C_t are the level, slope and curvature factors; λ is a parameter; and ε_t is a measurement error.⁸ The factors evolve according to a persistent process, inducing persistence on the yields across time. Numerous studies show that the NS model is a very good representation of the yield curve both in the cross section and dynamically.⁹ This model is very popular for at least three reasons. First, the factor loadings for all maturities are characterized by only one parameter, λ . This makes scaling up by adding more maturities relatively costless. Second, the specification is very flexible, capturing many of the possible shapes the yield curve can take: the yield curve can be upward or downward sloping, with at most one peak, whose location depends on the value of λ . Third, it imposes a degree of smoothness on the yield curve that is reasonable; wild swings in the yield curve at a point in time are not common.

Many of the properties of the yield curve, such as smoothness and persistence, are also shared by the term structure of inflation expectations. Thus, at least from a curve-fitting perspective, modeling the latter by the NS model is not too much of a stretch. Defining $\pi_t(\tau)$ as the τ -month inflation expectations from month t to month $t + \tau$, I assume that it follows the process

$$\pi_t(\tau) = L_t - \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) S_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) C_t + \varepsilon_t. \quad (2)$$

According to this specification, which I consider to be a convenient and parsimonious statistical model, L_t captures long-term inflation expectations, S_t captures the difference between long- and short-term inflation expectations, and C_t captures higher or lower medium-term expectations relative to short- and long-term expectations.

⁸The original NS model starts with the assumption that the forward rate curve is a variant of a Laguerre polynomial, which results in the function in (1) when converted to yields. As such, it has no economic foundation, unlike some of the papers cited in the introduction that contain asset-pricing models. The slope factor in Diebold and Li (2006) is defined as $-S_t$. The three factors are labeled as such because, as Diebold and Li (2006) demonstrate, $L_t = y_t(\infty)$, $S_t = y_t(\infty) - y_t(0)$ (with the definition adopted in this paper), and the loading on C_t starts at zero and decays to zero affecting the middle of the yield curve where the maximum loading is determined by the value of λ .

⁹For an extensive survey, see Diebold and Rudebusch (2013).

Next, I turn to developing some results that will facilitate mapping various observables into measurement equations. First, I define inflation between two arbitrary dates. The Bureau of Labor Statistics, the statistical agency that measures the CPI in the United States, uses the simple growth rate formula to compute inflation. Using this formula in this context, however, leads to a nonlinear state space, which is considerably more difficult to handle. Thus, I define inflation using continuous compounding instead.¹⁰ More specifically, let P_t be the CPI price level in month t . I define

$$\pi_{t \rightarrow s} \equiv 100 \times \frac{12}{s-t} [\log(P_s) - \log(P_t)] \quad (3)$$

as the annualized inflation rate between month t and month s . In terms of the notation in (2), $\pi_t(\tau)$ is represented as $\pi_{t \rightarrow t+\tau}$ for $\tau > 0$. This notation is quite flexible. For example, $\pi_{t \rightarrow t+12}$ denotes the expected inflation between month t and month $t+12$, a conventional one-year-ahead forecast, whereas $\pi_{t+3 \rightarrow t+6}$ is the expected quarterly inflation starting from month $t+3$, which is a forward forecast. The former can be immediately written as $\pi_t(12)$, but to convert the latter to this notation, the following result is useful.

Using properties of continuous compounding, if we have $\pi_{t \rightarrow t+s}$, $\pi_{t \rightarrow t+r}$, and $\pi_{t+r \rightarrow t+s}$, where $s > r > 0$, then these are related by

$$\pi_{t+r \rightarrow t+s} = \frac{s}{s-r} \pi_{t \rightarrow t+s} - \frac{r}{s-r} \pi_{t \rightarrow t+r}. \quad (4)$$

As an intuitive example of this result, the formula yields $\pi_{t \rightarrow t+6} = 0.5 \times (\pi_{t \rightarrow t+3} + \pi_{t+3 \rightarrow t+6})$, which shows that the six-month inflation rate is simply the average of the two three-month inflation rates, one from today to three months from now and the other between three and six months from now. In general, inflation between two dates is equal to the average monthly inflation over the period from one to the other.

Finally, to map any inflation measure $\pi_{t+\tau_1 \rightarrow t+\tau_2}$ with $\tau_2 > \tau_1 \geq 0$ into the factor model

¹⁰In practice, this turns out to be a very minor issue. See footnote A-5 in the Appendix.

in (2), it's easy to show that it can be written as

$$\pi_{t+\tau_1 \rightarrow t+\tau_2} = L_t + \frac{e^{-\lambda\tau_1} - e^{-\lambda\tau_2}}{\lambda(\tau_2 - \tau_1)} (C_t - S_t) + \left(\frac{\tau_1 e^{-\lambda\tau_1} - \tau_2 e^{-\lambda\tau_2}}{\tau_2 - \tau_1} \right) C_t. \quad (5)$$

plus a measurement error. As should be clear from inspecting (5), using continuous compounding, I preserve the linearity of the state-space system, which would not be possible with a simple growth formula. Moreover, computing this for any (τ_1, τ_2) would require knowing only the factors in the current period and the estimated value of λ .

2.2 Measurement Equations

With the results of the previous section in hand, all that remains to be done is to map observed measures of inflation expectations into the framework described so far. Letting x_t^i be a generic observable, converted to annualized percentage rates, this amounts to writing

$$x_t^i = \begin{pmatrix} f_L^i & f_S^i & f_C^i \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \varepsilon_t^i, \quad (6)$$

where $\{f_L^i, f_S^i, f_C^i\}$ are the loadings on the three factors and $\varepsilon_t^i \sim N(0, \sigma_i^2)$ is an idiosyncratic error term, which accounts for deviations from the factor model.

The data comes from the *SPF* and the *Blue Chip*, where I use the median of the individual forecasts. None of the forecasts from these two surveys squarely fit the definition $\pi_{t \rightarrow t+\tau}$ or $\pi_t(\tau)$ for some τ , even though at first glance many of them appear to. A very detailed description of how each survey question can be converted to a set of factor loadings is provided in Appendix A. The Appendix also contains a discussion of why carefully mapping each survey question to the right forecast horizon is important.

2.3 State-Space System and Methodology

I set up a state-space model, following the approach in Diebold, Rudebusch, and Aruoba (2006), where (6) constitutes a generic measurement equation. To complete the state-space

representation, I assume that the three latent factors follow the independent AR(3) processes

$$\begin{aligned}
L_t &= \mu_L + \rho_{11}(L_{t-1} - \mu_L) + \rho_{12}(L_{t-2} - \mu_L) + \rho_{13}(L_{t-3} - \mu_L) + \eta_t^L \\
S_t &= \mu_S + \rho_{21}(S_{t-1} - \mu_S) + \rho_{22}(S_{t-2} - \mu_S) + \rho_{23}(S_{t-3} - \mu_S) + \eta_t^S \\
C_t &= \mu_C + \rho_{31}(C_{t-1} - \mu_C) + \rho_{32}(C_{t-2} - \mu_C) + \rho_{33}(C_{t-3} - \mu_C) + \eta_t^C,
\end{aligned} \tag{7}$$

where $\eta_t^i \sim N(0, \sigma_i^2)$ and $cov(\eta_t^i, \eta_t^j) = 0$ for $i, j = L, S,$ and C and $i \neq j$.¹¹

Once the model is cast in state space by combining (6) and (7), estimation and inference are standard. The full state space model is shown in Appendix B. The model is estimated using maximum likelihood via the prediction-error decomposition and the Kalman filter. As I explain in the Appendix A, due to the timing of surveys, there are many missing observations in the data set I use. The Kalman filter and the state-space methods associated with it are well suited to handle them.¹² A total of 75 parameters are estimated, where all but 16 of these parameters are measurement error variances. Estimates of the level, slope, and curvature factors are obtained using the Kalman smoother, though in Section 3.2 I show that results are unchanged if I use the Kalman filter or if I recursively estimate the model.

To see how the dynamic NS model works to generate the term structure of inflation expectations, consider the following example. Suppose in a particular period t , we observe $\pi_t(3), \pi_t(12), \pi_t(60)$ and $\pi_t(120)$ and in the previous period we observe $\pi_{t-1}(6), \pi_{t-1}(24), \pi_{t-1}(84)$. First, one can take the approach in Diebold and Li (2006) and treat (2) as a regression equation where λ is known and fixed to a number and the regression coefficients are $\{L_t, S_t, C_t\}$. Given observations for period t , we can obtain values for $\{L_t, S_t, C_t\}$ via OLS, and we can compute $\pi_t(\tau)$ for any τ using (2) and the assumed value for λ . The dynamic NS model imposes more structure where now there are laws of motion between L_t

¹¹In Section 3.2, I consider a VAR(1) containing all three factors as an alternative. I show that model selection criteria point to the independent AR(3) specification, and I use this as the benchmark.

¹²See, for example, Diebold, Rudebusch, and Aruoba (2006) for the details of estimating the NS model; Aruoba, Diebold, and Scotti (2009) for a specific example with a state-space model with many missing observations; and Durbin and Koopman (2012) for a textbook treatment of both.

and L_{t-1} , S_t and S_{t-1} and C_t and C_{t-1} that need to be respected by estimation. Moreover, in this approach λ is among the estimated parameters. Thus, if we want to compute π_t (72), which is a horizon for which we do not have information neither in t nor in $t - 1$, we use the four observations we have in period t , and the information in the three observations in $t - 1$, where latter comes from the dynamics of the three factors.

2.4 Term Structure of Ex-Ante Real Interest Rates

The Fisher equation links the nominal interest rate to the *ex-ante* real interest rate and expected inflation. Generalizing to a generic maturity, the linearized version can be written as

$$y_t(\tau) = r_t(\tau) + \pi_t(\tau), \quad (8)$$

where $y_t(\tau)$ denotes the nominal continuously-compounded yield for a bond that is purchased in period t and matures in period $t + \tau$, $\pi_t(\tau)$ is the expected inflation over this horizon and r_t is the ex-ante real interest rate.¹³ In generating this ex-ante real rate, I have in mind (the linearized version) of a general economics model in which the real interest rate is one of the key determinants of current economic activity. A decline in the ex-ante real interest rate stimulates private consumption demand by making consumption cheaper today as opposed to the future, and it also boosts investment by reducing the opportunity cost of funds used for investment.

Since I already obtained $\pi_t(\tau)$, all I need to do to obtain the term structure of ex-ante *real* interest rates is to use the term structure of *nominal* interest rates. To that end, I use the estimated yield curve as computed by the Board of Governors of the Federal Reserve System, following Gürkaynak, Sack, and Wright (2007). This yield curve is estimated using a generalized specification of the NS model, the so-called Nelson-Siegel-Svensson specification,

¹³It is important to note that by decomposing the nominal rate this way, I implicitly include the inflation risk premium in $r_t(\tau)$. However, this is not crucial as it is natural for the ex-ante real rate to include this risk. The debate on the size of the inflation risk premium is far from settled in the literature. See for example D'Amico, Kim, and Wei (2014), Duffee (2014) and Haubrich, Pennacchi, and Ritchken (2012).

which adds one more factor to the original NS model. The relevant information has been reported for every day since June 1961, which enables me to compute $y_t(\tau)$ for any arbitrary maturity τ . Since the frequency in this paper is monthly, I take averages over a month to calculate the monthly yields and compute the continuously compounded ex-ante real interest curve, $r_t(\tau)$, as the difference between $y_t(\tau)$ and $\pi_t(\tau)$ following (8).

3 Inflation Expectations and Real Interest Rate Curves

I estimate the state-space model presented in the previous section on a sample that covers the period from January 1998 through July 2016. I will return to the choice of the start date of the sample and also consider a slightly longer sample starting in 1992 in Section 3.2.

3.1 Estimation Results

Table 1 presents the estimated parameter values. Panel (a) shows the estimated transition equation parameters. All three factors are persistent, but roots of characteristic polynomials (not reported) show that all are comfortably covariance stationary. The long-run average of the level factor is 2.46%. It is well known (see, e.g., Hakkio, 2008) that CPI inflation and personal consumption expenditures (PCE) inflation have about a 0.4% difference on average. This means that the average long-run inflation expectation in the sample stays very close to 2% when expressed in terms of the Federal Reserve’s preferred inflation measure and its official target. The average slope of the inflation expectations curve, which is defined as the difference between long- and short-term expectations, is mildly positive at 43 basis points. The curvature factor has a mean of -18 basis points, showing that medium-term forecasts are typically lower than short- and long-term forecasts, giving the inflation expectations curve a mild U shape on average, though this estimate is not statistically significant. The variances of the transition equation innovations are small.

Panel (b) of Table 1 shows that λ is estimated as 0.12, which means that the loading

on the curvature factor is maximized at just under 15 months. The estimated measurement error variances show that as the forecast horizon of the variable increases, the measurement error variances become smaller, indicating a better fit of the model. Average measurement error standard deviation is 8 basis points, indicating a very good fit of the NS model to this expectations data.

Given these estimated parameters, I obtain estimates of the three factors using the Kalman smoother, which are presented in Figure 1.¹⁴ The level factor has a slight downward trend, at about 1 basis point per year. The slope factor is positive for much of the sample, falling below zero briefly just before the 2001 recession, in 2006, and during the Great Recession, prior to the financial crisis of 2008. During the financial crisis, the inflation expectations curve sharply steepens, with much of the movement coming from the sharp fall of the short end. This is also visible in this figure as the sharp increase in the slope near September 2008. The curvature factor has smaller and nonsystematic fluctuations.

The main output from the estimation that is of interest is the inflation expectations curve itself. In Figure 2, I show the time series of some selected inflation expectations in the full sample: those at a 6-month and at 1-, 5-, and 10-year horizons. It is apparent that as the forecast horizon increases, the forecasts become smoother; note the range of the y-axis in the figures.

The financial crisis and the Great Recession change the behavior of these forecasts, especially the shorter horizons, drastically, with a drop of 1.5% at the 6-month horizon. When viewed in a low frequency, the 10-year forecast does not seem to be affected significantly by the financial crisis: in the period from September 2008 to December 2015, it fluctuates between 2.19% and 2.47%, with an average of 2.33%, a decline of only eight basis points rel-

¹⁴In all figures, the two National Bureau of Economic Research (NBER) recessions in the sample are shown with gray shading, and September 2008 is shown with a vertical line. The latter is arguably the height of the financial crisis, and significant changes occur in both the inflation forecasts and the financial variables introduced later. Also, where relevant, I use red dashed lines to denote pointwise 95% confidence bands.

ative to the three-year period prior to the crisis. Using the model, I am also able to compute the year-3-to-year-10 and year-6-to-year-10 forward forecasts (not shown). These forecasts remain only slightly below their precrisis levels. About half of the (already small) decline in the 10-year expectations arises from the expected decline during the first two to five years. My results are in line with Mertens' (2016) results, which show that trend inflation did not change by much during the crisis.

Figure 3 shows some of the ex-ante real interest rates obtained as described in Section 2.4. The short-term rates (e.g., the six-month rate) show significant cyclicity, rising in booms and declining rapidly in recessions. The remaining panels of Figure 3 show the 5- and the 10-year ex-ante real rates, along with the corresponding TIPS yield as computed by Gurkaynak, Sack and Wright (2010), which is supposed to measure the same underlying concept.¹⁵ They disagree significantly during the financial crisis when TIPS yields are pushed to much higher levels due to the developments in the financial markets. (See footnote 21.) Except for this brief period, there seems to be a close match, which is reassuring, and it means the real interest rates I compute at any arbitrary horizon are quite useful.

Looking around the financial crisis, as of December 2008, the ex-ante real interest rate for horizons up to seven years is negative, with the two-year rate around -1.4% . Thus, the combination of the financial crisis and the Fed's conventional response leads to a downward shift of the real yield curve. The evidence in Figure 3 is clear that real interest rates of most maturities were consistently negative since 2008. One way of interpreting this finding is to conclude that these negative rates, which are over 3% below pre-crisis levels, provided a massive monetary stimulus to the economy. However, Figure 3 also reveals a long-term downward trend in real rates. This is even more striking using the results of the estimation

¹⁵In fact, since TIPS break-even rates are defined as the difference between nominal yields and the TIPS rate, and I define my ex-ante real rate as the difference between the nominal yields and my inflation expectations, the difference between TIPS yields and my real interest rate is by construction equal to the difference between the break-even rate and my inflation expectations. Note that I do not show a TIPS rate for the 6-month and 1-year maturities since Gurkaynak, Sack and Wright (2010) caution against using their model to generate TIPS rates for maturities lower than two years.

from 1992: average 10-year real rate in the 1990s is 3.4%, in the 2000s until the crisis it is 2.4% and after the crisis it is 0.4%. This observation is used as a symptom of *secular stagnation* in the literature by Summers (2016) among others. Thus, another way to interpret the results is that the neutral real interest rate, the rate at which monetary policy is neither stimulative nor contractionary, may have fallen and thus the low levels of real interest rates that I show here may be less stimulative or not stimulative at all. While I cannot provide any guidance about what the neutral real rate may be, measuring real rates at various horizons and showing how they change during and after the crisis is a useful ingredient of this debate.

3.2 Robustness of Results

In selecting my benchmark model, I made a number of choices. In this section, I briefly summarize the results under some alternative choices. First, I consider a longer sample, starting in 1992. Given that the 10-year forecast for the *SPF* starts in the last quarter of 1991, starting the estimation earlier than 1992 is not sensible. Figure A2 in the Appendix compares the results of this estimation with the benchmark results. Both the underlying long-term forecasts from surveys and the extracted long-term forecasts from my model (especially the 10-year forecast) show a clear structural break around 1998. Evidently, the forecasters did not adjust their long-term forecasts downward quickly, even though inflation settled down to much lower levels in the early 1990s. In order to avoid this structural break, I start my estimation in 1998. Despite this structural break, however, as Figure A2 clearly shows, the factors and the forecasts are virtually identical to the benchmark estimates in the sample that starts in 1998.

I use independent AR(3)s in the transition equation, which do not allow for any correlation between factors. The extracted factors show some mild correlation: 0.22 between level and slope, -0.10 between level and curvature, and -0.21 between slope and curvature. To investigate if the assumed transition equations are too restrictive, I estimate the model with

a VAR(1) in factors as the transition equation. The extracted factors remain very close to the benchmark ones with correlations of 0.98 or higher. Comparing the Schwartz or Akaike information criteria, the benchmark model is preferred.¹⁶

In some yield-curve applications, the third factor is only marginally important, and some authors prefer a two-factor model for parsimony. To investigate this in my model, I reestimate the model without the curvature factor, which eliminates five parameters. Interestingly, the log likelihood falls by 77 log-points. This difference is large enough to outweigh the parsimony it achieves, and the information criteria prefer the benchmark specification. Figure A2 shows that the differences in the resulting inflation forecasts are most evident in the short horizons, while medium- to long-horizon forecasts of the model without curvature is not distinguishable from the one from the benchmark model.

Finally, in generating the forecasts from the model, I used the Kalman smoother to extract the level, slope and curvature factors. There are two alternatives, which make the approach more real-time in nature. First, one can use the Kalman filter to extract the factors, which would use information up to the current period to compute the factors, while still using parameter estimates obtained from the full sample. Second, one can recursively estimate the model using data up to and including the current period. This way not only the filtering would use information up to the current period, but also the parameters would be obtained using the same information as well. I do both of these versions, where for the latter exercise I start estimation in January 2007. The results are essentially identical to the benchmark results. Focusing on the root-mean-square error (RMSE) of the forecasts, there is hardly a difference of 1% across the three different versions where the average absolute difference is only 0.3%. Figure A3 in the Appendix compares the factors and selected forecasts from these two versions with the benchmark version.

¹⁶The two models have the same number of parameters, and thus the difference in the log-likelihood, which is about 22 log points, means that the benchmark model fits the data better, indicating that capturing higher order autoregressive dynamics is much more important than cross-factor correlations.

3.3 Comparison of Forecasts

In this section, I compare the forecasts from the model with two sets of alternatives, with two goals in mind. First, I present results that compare the model’s forecasts with forecasts from two statistical models that are known to perform particularly well at all horizons. Since it is well known that surveys perform well, the goal here is to demonstrate that my model is able to produce accurate forecasts at *all* horizons, including those that are not explicitly covered by surveys. In other words, this would be a test of the model’s ability to successfully intrapolate forecasts for all horizons using a handful of forecasts every period, and the dynamic linkages coming from the factor model. Second, I compare the model forecasts with measures obtained from financial variables. Here, I want to explicitly demonstrate how my model forecasts compare with forecasts that use *some* information from financial markets.

Table 2 presents formal forecast comparison test results using realized inflation. The first column reports the RMSE of the model forecast, the second column reports the RMSE of the alternative measure considered, and the third column shows the number of observations available for each comparison.¹⁷ Boldface in a column indicates the rejection of the null of equal forecast accuracy in favor of the forecast in that column using the Diebold and Mariano (1995) test with the squared-error loss function.

3.3.1 Comparison with Two Successful Statistical Models

As formulated by Stock and Watson (2007), the unobserved-components stochastic-volatility (UCSV) model decomposes current inflation, π_t , into a trend, τ_t , and serially uncorrelated short-run fluctuations where the latter and the innovations to the trend exhibit stochastic

¹⁷The actual inflation measure is the appropriate difference of the natural logarithm of CPI, as extracted from Federal Reserve Economic Data (FRED) in July 2016, with the FRED code CPIAUCSL. The RMSEs for the model forecast differ across panels only due to differences in the samples used in comparisons with the alternative models.

volatility. More specifically,

$$\begin{aligned}
\pi_t &= \tau_t + \sigma \exp(h_{\epsilon,t})\epsilon_t, \\
\tau_t &= \tau_{t-1} + (\varphi\sigma) \exp(h_{\eta,t})\eta_t \\
h_{j,t} &= \nu_j h_{j,t-1} + \sqrt{1 - \nu_j^2} \sigma_j \omega_{j,t} \\
\epsilon_t, \eta_t, \omega_{j,t} &\sim N(0, 1) \text{ with } j \in \{\epsilon, \eta\}.
\end{aligned} \tag{9}$$

I estimate this model and extract a measure of trend denoted by $\hat{\tau}_t$ using Bayesian methods designed for state-space models with stochastic volatility developed by Kim, Shephard, and Chib (1998) that are also used by Schorfheide, Song, and Yaron (2014) and Aruoba and Schorfheide (2016). Parameter estimates are provided in Table A1 in the Appendix. As Stock and Watson (2007) put it, this model takes current inflation, filters out what it considers to be transitory noise, and uses the remainder, $\hat{\tau}_t$, as the forecast of inflation at *any* horizon. Faust and Wright (2013) demonstrate that this simple univariate model has superior or similar forecast accuracy relative to many other statistical models, including those that use information from other variables.

The first panel of Table 2 shows the results from comparing the forecast of the UCSV model with the model forecast. In short forecast horizons, the UCSV model provides a better forecast than the model forecast, but the RMSEs are close, and the difference in accuracy is not statistically significant. For forecasts for horizons of two years and longer, the model forecast has lower RMSEs than the UCSV model, and the difference becomes significant for horizons longer than four years.¹⁸

Faust and Wright (2013) also emphasize another forecasting model as useful. The “gap model” uses a nowcast from a survey to anchor the very short end of the inflation expectations curve, and uses an AR(1) model to forecast the “gaps”, which are the differences between

¹⁸I also compared the model forecast with a simpler no-change forecast, one that assumes that the forecast of any horizon is equal to the annual inflation at the point of the forecast. The model forecast is superior to this forecast, and this is statistically significant for all horizons.

the forecast of a given horizon and the nowcast at the short end. Their results show that this model is superior to most of the other alternatives they consider in forecasting CPI and never worse. They also show that *Blue Chip* and *SPF* forecasts by themselves are better than this forecast in terms of RMSE, though not enough to be statistically significant. In the second panel of Table 2 I compare the forecasts from this model with those from my model. This requires some small modifications since the original model in Faust and Wright (2013) is quarterly and it is designed to forecast quarterly forecasts in the future, as opposed to cumulative inflation from the present in to the future as I do. Obtaining such forecasts from my estimated model is trivial using (5) and I use the appropriately defined quarterly change in CPI as the measure of truth. To maximize the number of observations, given the quarterly frequency, I use the results from the estimation that starts in 1992. At the 1-quarter horizon, my forecast produces a RMSE that is 3% lower. At all other horizons, the RMSEs from the forecasts are within 0.5% of each other, with a forecast accuracy that is not statistically different from each other. This result establishes, indirectly as it relies on the results in Faust and Wright (2013), that my model forecast is likely to be no worse than the statistical models Faust and Wright (2013) consider.

3.3.2 Comparison with Measures Derived from Financial Variables

It is well understood that many financial variables contain information about the market participants' inflation expectations. Perhaps the two financial instruments that have the most information are inflation swaps and TIPS. An inflation swap is an agreement in which one party makes periodic payments to another party, which are linked to inflation realized in the future, in exchange for a fixed payment up front. TIPS, on the other hand, are bonds issued by the U.S. Treasury, with yields that are linked to future realized inflation rates. Both of these financial assets potentially include compensation for bearing real-interest rate risk and other risks such as liquidity risk. In the absence of these risks, the fixed payment in a swap of a certain maturity, and the difference between the yield on a TIPS at a certain

maturity and the U.S. Treasury nominal yield at the same maturity, the so-called break-even rate, would be good estimates of inflation expectations at that horizon. In fact both measures are routinely used in the popular press as direct measures of inflation expectations.

The liquidity of the TIPS market has changed significantly since its inception, which makes it very difficult to use the break-even rate as a direct estimate of inflation expectations. Similar problems also plague the inflation swaps market.¹⁹ In this section I show results from two papers I discuss in the Introduction, which provide what can be considered as cleaned versions of the TIPS data (for D’Amico, Kim, and Wei, 2014) and swap rates (for Haubrich, Pennacchi, and Ritchken, 2012).²⁰

Figure 4 shows the one-year swap rate and the results from Haubrich, Pennacchi, and Ritchken (2012), labeled “Cleveland Fed,” and the actual realization of inflation. Two things are very clear. First, the Cleveland Fed forecast and the swap rate, when it is available, are significantly more volatile than the model forecast. Second, the swap rate takes a significant dive near the financial crisis, falling to nearly -4% , while the model forecast remains slightly above 1% . It is evident that the raw swap rate suffers from the problems I list previously. Although not as extreme as the swap rate, the Cleveland Fed forecast also displays similar behavior, falling below zero in early 2009.

Figure 5 shows the 10-year swap rate, the TIPS break-even rate, and the results from D’Amico, Kim, and Wei (2014), labeled “DKW Inflation Expectation,” and actual inflation. Also shown is the Cleveland Fed forecast, with TIPS-related variables in the top panel and the swap rate-related variables in the bottom panel. The TIPS break-even rate clearly displays

¹⁹Lucca and Schaumburg (2011) provide a good summary of these problems and some others that make TIPS and swap rates noisy indicators of inflation expectations.

²⁰Both of these papers start their estimations prior to the introduction of the respective financial asset, using only nominal yields. As such, their reported inflation expectations can be considered as being related to TIPS and swaps only after 1999 for TIPS and 2004 for swaps. The forecasts of D’Amico, Kim, and Wei (2014) are graciously provided by the Federal Reserve Board. The forecasts of Haubrich, Pennacchi, and Ritchken (2012) are available from the website of the Federal Reserve Bank of Cleveland (www.clevelandfed.org). The forecasts of other studies cited in the Introduction are not publicly available; therefore, I am not able to use them in this comparison.

very different behavior before 2003 and again after the financial crisis in 2008 relative to both actual inflation and the model forecast. A similar conclusion also applies for the swap rate, which is available for a shorter sample. Both rates fall below zero during the financial crisis.²¹ Both DKW and the Cleveland Fed forecasts behave much better relative to the raw financial variables, although they are still more volatile relative to the model forecast. The same conclusions apply for the five-year forecasts (not shown).

The rest of Table 2 shows forecast comparison results for the variables discussed in this section. The raw financial variables, shown in the third and fifth panels, produce substantially worse forecasts relative to the model forecast, with improvements in the RMSEs of the latter as large as 53% for the five-year TIPS break-even rate. Looking deeper into the source of the large RMSE for this particular variable, it is twice as volatile as the model forecast and has a larger bias. The DKW and Cleveland Fed forecasts produce results that are much better, with RMSEs that are roughly half to two-thirds of those from the raw financial variables and near the values attained by the model forecast. The model forecast is significantly more accurate than the 5-year and 10-year DKW forecasts. The two-year DKW forecast has a lower RMSE than that of the model forecast, and the difference is marginally statistically significant. The model forecast comes out significantly more accurate than the 10-year Cleveland Fed forecast, with the model forecast producing better RMSEs in all other cases. This is especially interesting because the Cleveland Fed model uses survey forecasts as I do and adds nominal yields and swap rates as additional observables to a macro-finance model; this evidently reduces the forecast accuracy of the model relative to a simple model such as mine that uses only surveys.

I view the results of this section as making a strong case for the usefulness of the model forecast relative to a number of alternatives related to the financial markets. This strong

²¹As Campbell, Shiller and Viceira (2009) notes, following the failure of Lehman Brothers in September 2008, a large amount of TIPS bonds flooded the market as Lehman's holdings were being sold, followed by large institutional investors. This depressed the price, increased the TIPS yields, and with little change in the nominal yields led to a large decline in break-even rates.

case is also why I chose not to use any financial variables in the model developed in this paper. The results in this section are also a confirmation and generalization of the results of Gürkaynak, Sack, and Wright (2010), who show that inflation compensation from TIPS has been far more volatile than survey expectations from the *Blue Chip* surveys and that the two have no consistent relationship.

4 Conclusions

Starting in 2008, the Federal Reserve enacted unprecedented policies in response to the biggest decline in economic activity since the Great Depression. The impact of these policies on medium- to long-term inflation is yet to be seen. In this paper, I provide a flexible and accurate way of aggregating survey-based inflation expectations into an inflation expectations curve. I also compute a term structure of ex-ante real interest rates by combining the inflation expectations curve with a nominal yield curve. The resulting term structure of inflation expectations proves capable of providing superior forecasts relative to some of the popular alternatives. Thus, moving forward, this approach seems to be a useful tool to gauge inflation expectations at any arbitrary horizon. The ex-ante real rates I obtain also follow their counterparts from the TIPS yield curve very well.

From here, a number of further directions are possible. First, a reasonable approach may be to consider non-Gaussian errors or stochastic volatility (or both) in the model. Second, although the model in this paper explicitly excludes financial variables, there may be ways of introducing them without worsening performance. For example, similar to but distinctly different from what Christensen, Lopez, and Rudebusch (2012) do, one could model inflation expectations as I do here and add nominal yields that follow an NS structure with different factors, however, by explicitly introducing ZLB into the model. Finally, one could introduce information from various online prediction markets. I leave these directions for future work.

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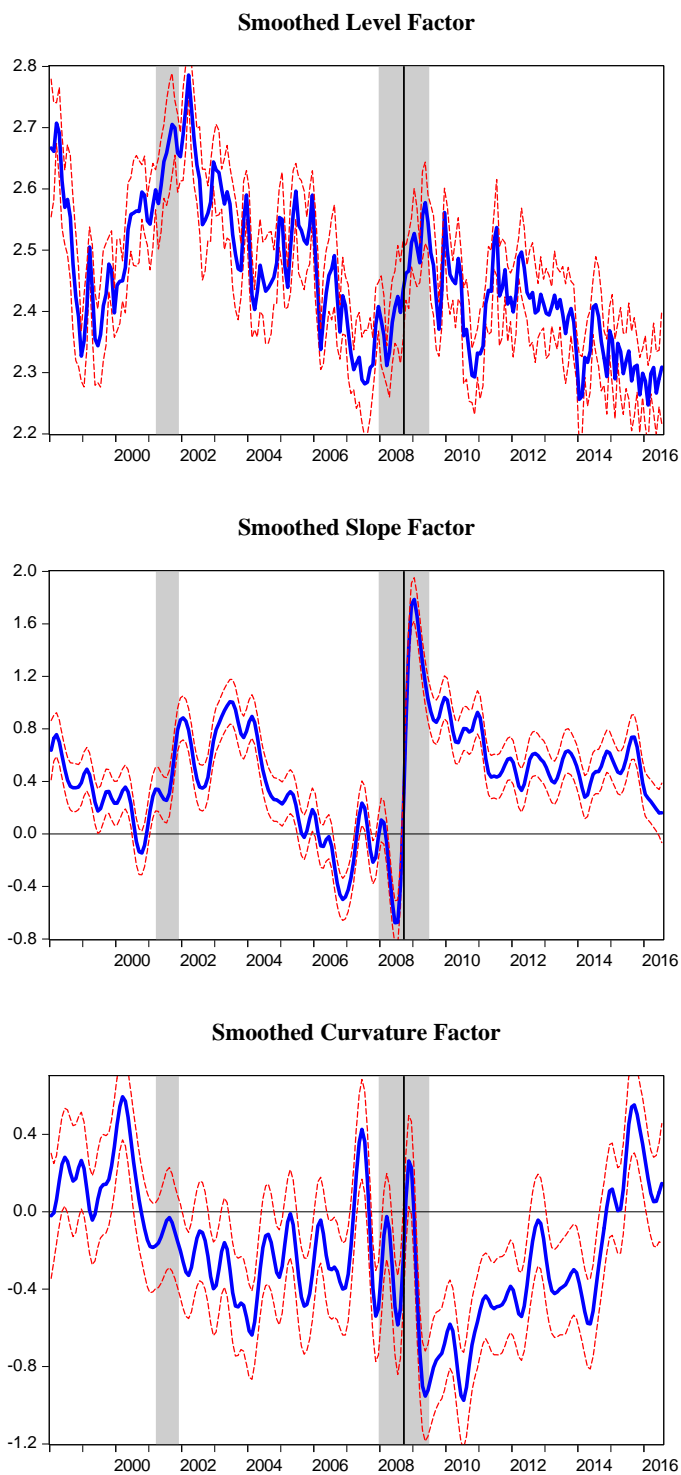
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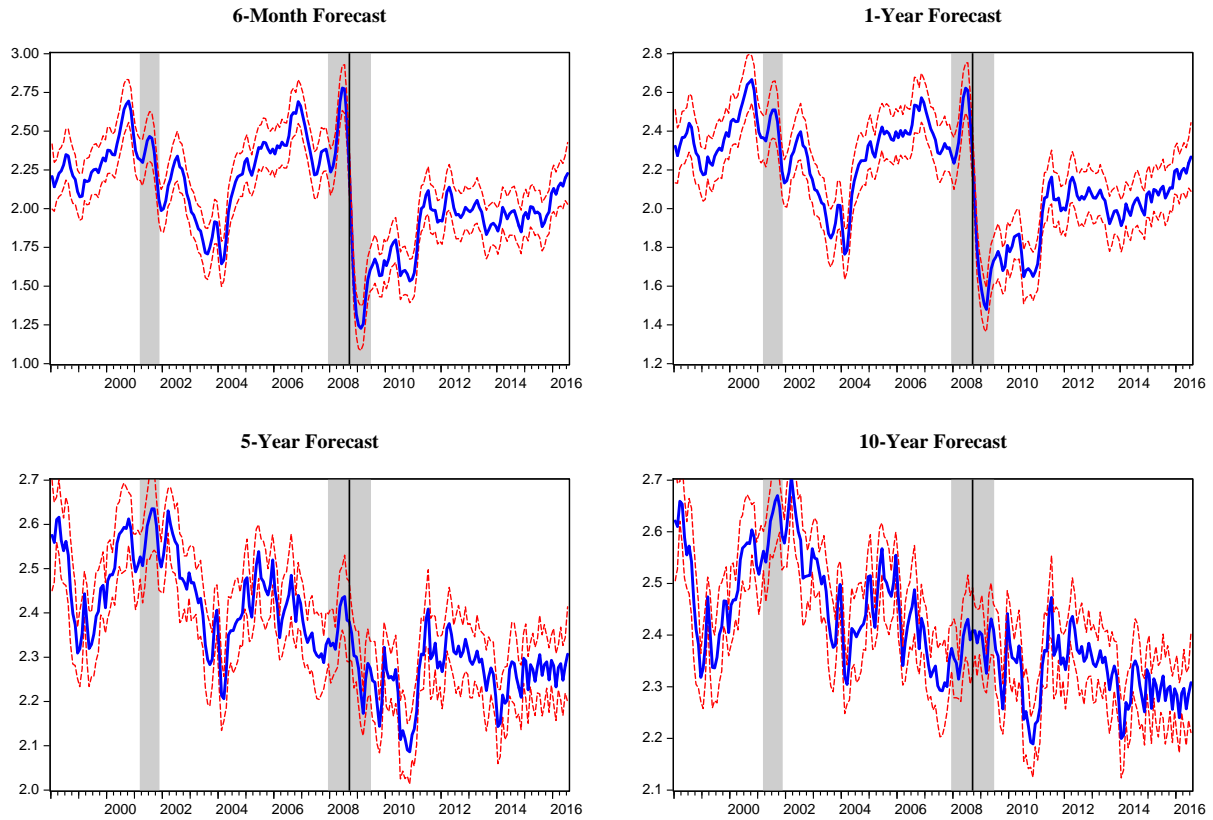
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Figure 1: Extracted Factors



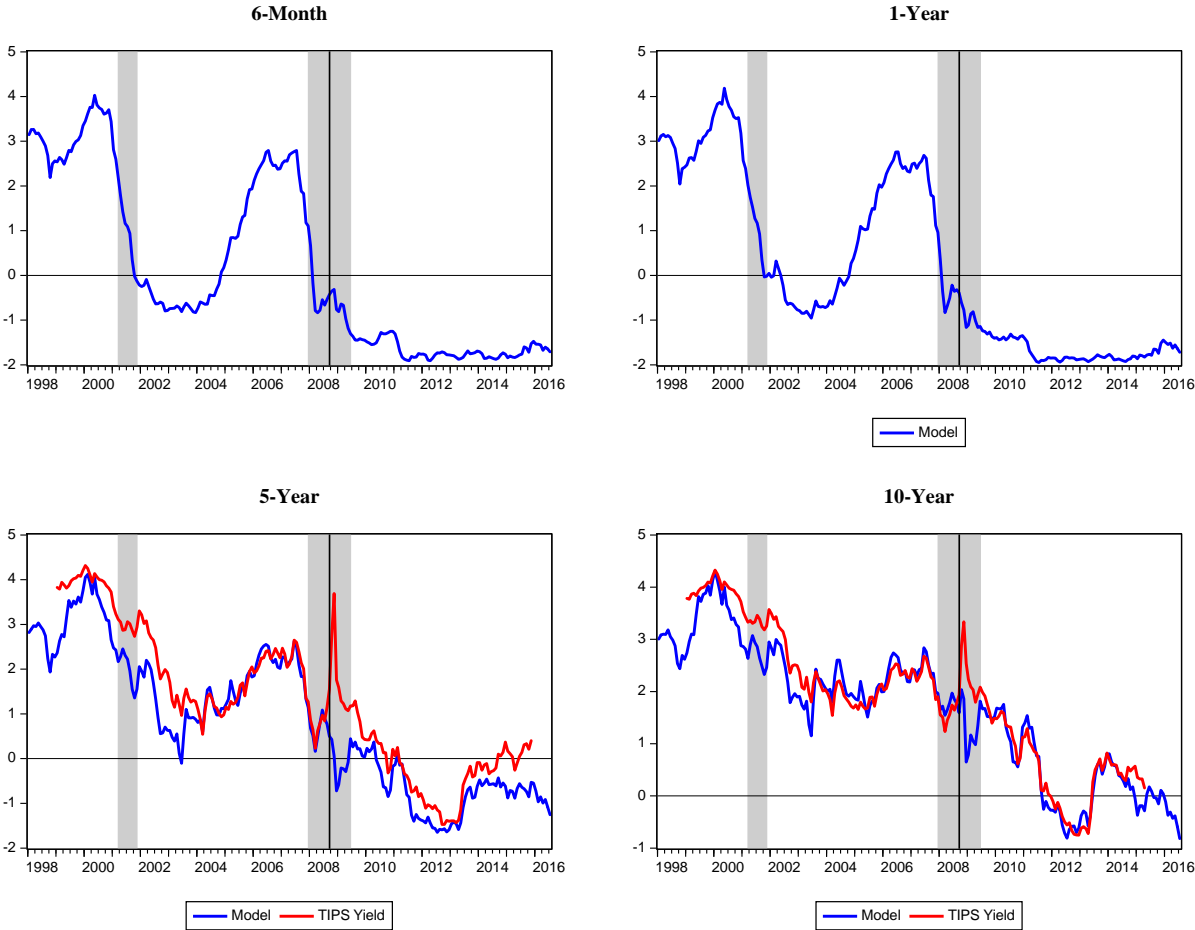
Notes: The gray bars denote NBER recessions. The vertical line denotes September 2008. The blue lines denote the smoothed factors, and the red dashed lines show their pointwise 95% confidence bands.

Figure 2: Selected Inflation Expectations



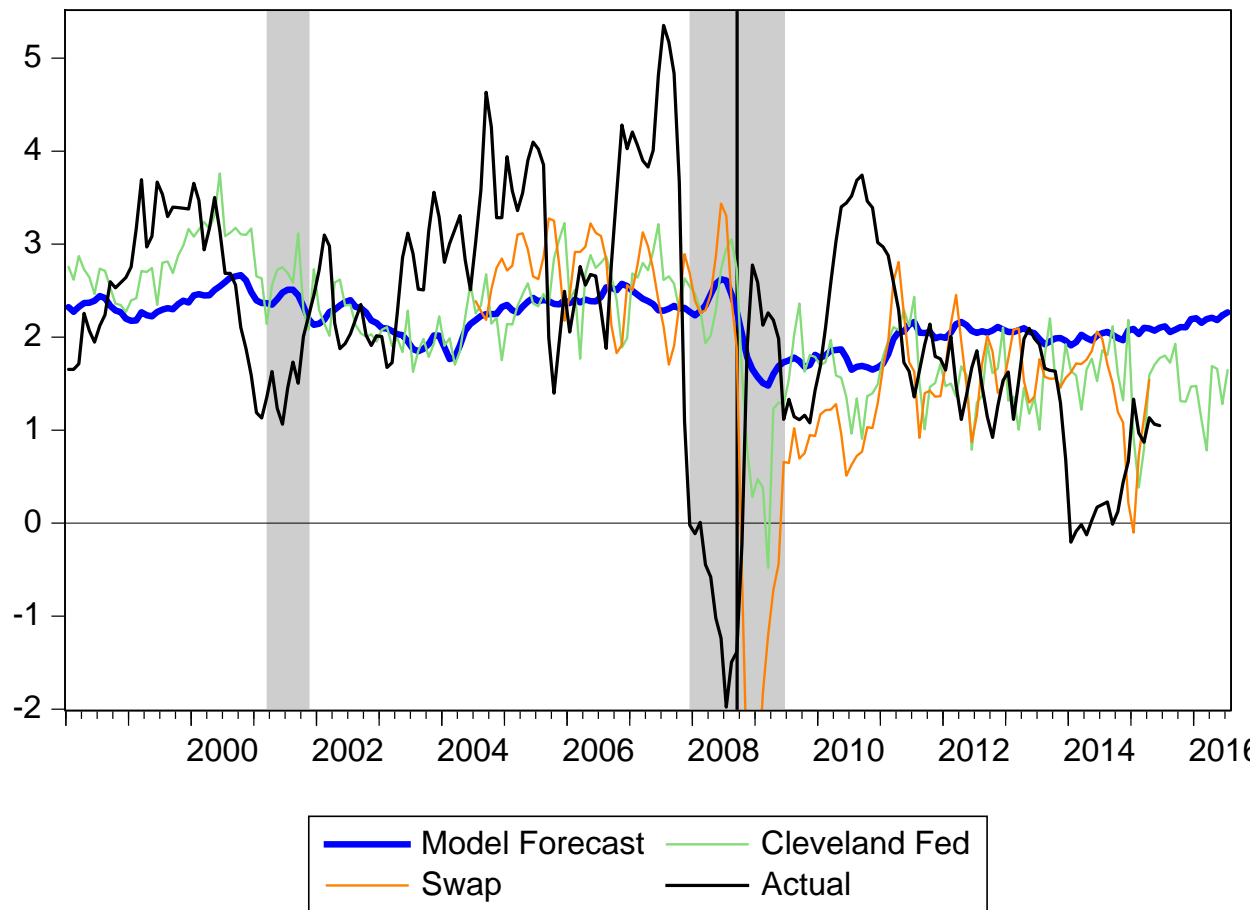
Notes: The gray bars denote NBER recessions. The vertical line denotes September 2008. The blue lines denote the smoothed factors, and the red dashed lines show their pointwise 95% confidence bands.

Figure 3: Selected Real Interest Rates



Notes: The gray bars denote NBER recessions. The vertical line denotes September 2008. The red lines in the 5-Year and 10-Year panels are the corresponding TIPS yields.

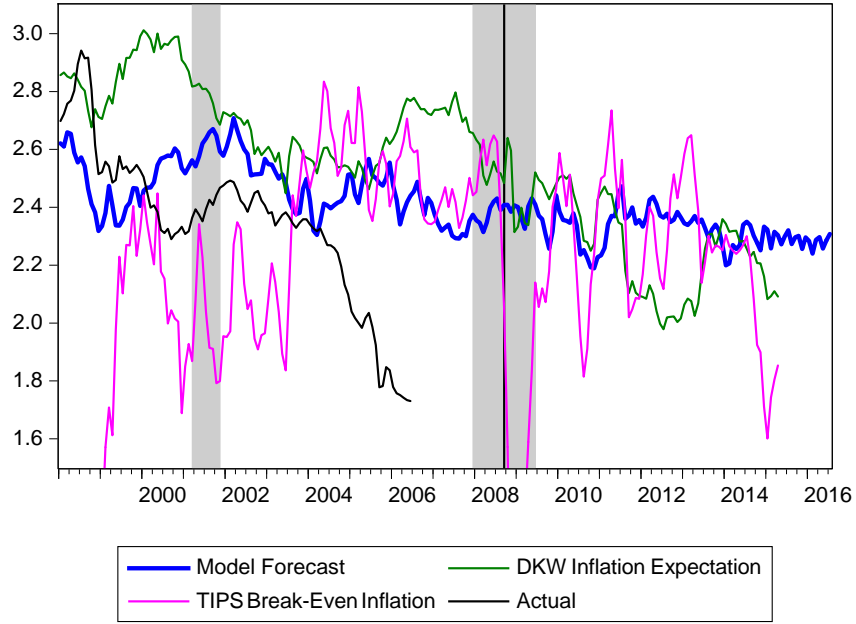
Figure 4: Comparison of One-Year Inflation Expectations with Financial Variables, and Actual



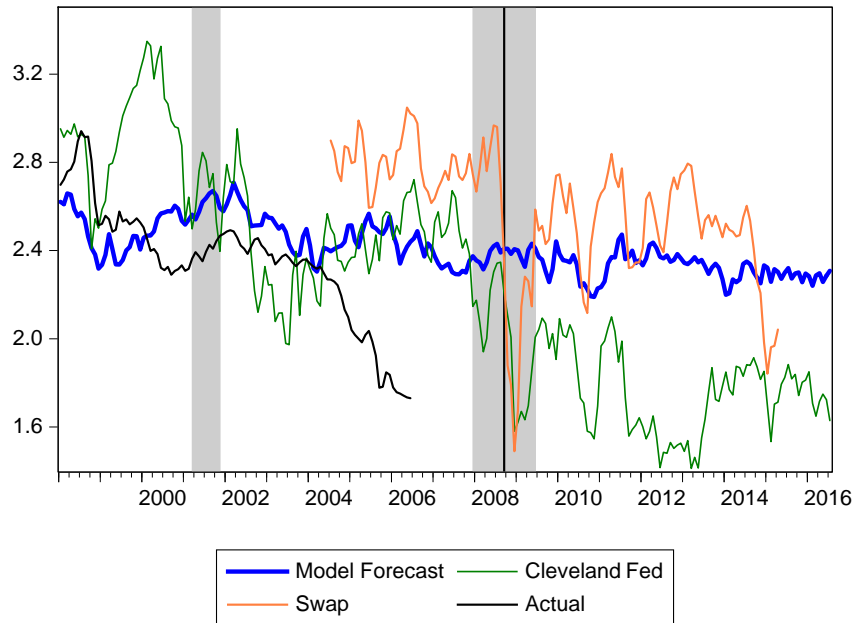
Notes: The gray bars denote NBER recessions. The vertical line denotes September 2008. The swap rate (orange line) falls to -3.83% in December 2008, but the graph is truncated at -2% .

Figure 5: Comparison of 10-Year Inflation Expectations with Financial Variables, and Actual

(a) Model Forecast, TIPS-Based Financial Variables and Actual



(b) Model Forecast, Swap-Based Financial Variables and Actual



Notes: The gray bars denote NBER recessions. The vertical line denotes September 2008. In panel (a), the TIPS break-even rate (purple line) falls to 0.52%, but the graph is truncated at 1.5%.

Table 1: Estimation Results

(a) Transition Equation

Level		Slope		Curvature	
ρ_{11}	0.99	ρ_{21}	2.11	ρ_{31}	2.19
ρ_{12}	-0.28	ρ_{22}	-1.65	ρ_{32}	-1.87
ρ_{13}	0.20	ρ_{23}	0.51	ρ_{33}	0.65
μ_1	2.46	μ_2	0.43	μ_3	-0.18
σ_L^2	0.003	σ_S^2	0.004	σ_C^2	0.003

(b) Measurement Equation

$\lambda = 0.12$					
SPF Quarterly		Blue Chip Short-Run		Blue Chip Long-Run	
σ_1^2	0.034	σ_{21}^2	0.008	σ_{36}^2	0.002
σ_2^2	0.011	σ_{22}^2	0.016	σ_{37}^2	0.003
σ_3^2	0.007	σ_{23}^2	0.007	σ_{38}^2	0.003
σ_4^2	0.006	σ_{24}^2	0.004	σ_{39}^2	0.001
SPF Annual		σ_{25}^2	0.007	σ_{40}^2	0.001
σ_5^2	0.009	σ_{26}^2	0.020	σ_{41}^2	0.011
σ_6^2	0.005	σ_{27}^2	0.017	σ_{42}^2	0.005
σ_7^2	0.008	σ_{28}^2	0.009	σ_{43}^2	0.003
σ_8^2	0.006	σ_{29}^2	0.005	σ_{44}^2	0.003
σ_9^2	0.008	σ_{30}^2	0.007	σ_{45}^2	0.004
σ_{10}^2	0.006	σ_{31}^2	0.089	σ_{46}^2	0.006
σ_{11}^2	0.005	σ_{32}^2	0.011	σ_{47}^2	0.004
σ_{12}^2	0.005	σ_{33}^2	0.009	σ_{48}^2	0.003
SPF 5-Year		σ_{34}^2	0.005	σ_{49}^2	0.003
σ_{13}^2	0.012	σ_{35}^2	0.008	σ_{50}^2	0.002
σ_{14}^2	0.009			σ_{51}^2	0.011
σ_{15}^2	0.012			σ_{52}^2	0.010
σ_{16}^2	0.011			σ_{53}^2	0.001
SPF 10-Year				σ_{54}^2	0.001
σ_{17}^2	0.009			σ_{55}^2	0.004
σ_{18}^2	0.009			σ_{56}^2	0.002
σ_{19}^2	0.015			σ_{57}^2	0.002
σ_{20}^2	0.009			σ_{58}^2	0.002
				σ_{59}^2	0.001

Notes: Boldface indicates significance at the 5% level.

Table 2: Forecast Comparison Results

Forecast	RMSE - Model	RMSE - Alternative	T
UC-SV 1-year	1.29	1.21	210
UC-SV 2-year	0.89	0.93	198
UC-SV 5-year	0.43	0.79	162
UC-SV 10-year	0.27	0.69	102
FW Gap Model 1-Quarter	2.62	2.70	80
FW Gap Model 2-Quarter	2.62	2.63	80
FW Gap Model 4-Quarter	2.61	2.61	80
FW Gap Model 8-Quarter	2.63	2.63	80
SWAP 1-year	1.51	1.89	130
SWAP 2-year	1.01	1.30	119
SWAP 5-year	0.51	0.77	83
Cleveland 1-year	1.29	1.33	210
Cleveland 2-year	0.89	0.90	198
Cleveland 5-year	0.45	0.54	162
Cleveland 10-year	0.31	0.45	102
TIPS BE 2-year	1.02	1.43	126
TIPS BE 5-year	0.46	0.97	150
TIPS BE 10-year	0.31	0.50	90
DKW 2-year	0.89	0.82	198
DKW 5-year	0.45	0.52	162
DKW 10-year	0.31	0.45	102

Notes: In all cases except for the FW Gap Model, forecasts are monthly, they cover the largest sample starting from January 1998 and they are cumulative. For the FW Gap Model the forecast is made quarterly, it covers 1992Q1 through 2011Q4 and it is for a particular quarter in the future. Boldface for RMSEs indicates rejection of the null of equal accuracy at the 5% level using the Diebold-Mariano (1995) test with squared errors in favor of the model that has the boldface RMSE.

Appendix

A Measurement Equations

All the data I use in the estimation come from surveys. In virtually all cases, the question asked of the forecasters does not correspond exactly to a simple τ -month-ahead forecast, in the form of $\pi_t(\tau)$ for some $\tau > 0$, so I do some transformations as I explain in detail below. I convert all raw data to annualized percentage points to conform with the previous notation. In other words, I show how the “fixed event” forecasts in surveys can be converted to forecasts of “fixed horizons”.

Unless otherwise noted, all data start in 1998. In all cases, the forecasters are asked to forecast the seasonally adjusted CPI inflation rate. Both the *SPF* and *Blue Chip* forecasts are released around the middle of the month, with the forecasts due a few days prior to the release.^{A-1} I will thus consider both of the forecasts released in month t as forecasts made in month t .

As will be clear below, in some cases what is asked of the forecasters is a mixture of realized (past) inflation and a forecast of future inflation. Most of the realized inflation will be in the form of $\pi_{s \rightarrow r}$, where $s < r \leq t - 2$ so that in period t the forecasts are able to observe the official data release before making their forecasts.^{A-2} I use Archival Federal Reserve Economic Data (ALFRED) at the Federal Reserve Bank of St. Louis to obtain the exact inflation rate the forecasters would have observed in real time.^{A-3} Furthermore, there will be instances in which I need $\pi_{t-2 \rightarrow t-1}$, $\pi_{t-1 \rightarrow t}$, or $\pi_{t-2 \rightarrow t}$, none of which are observed by the time forecasters make their forecasts in period t . Since it is difficult to know explicitly what the forecasters knew when they sent their forecasts in month t about these inflation

^{A-1}See Figure 1 in Stark (2010) that shows the timing of *SPF* forecasts. Similar information is confirmed for the *Blue Chip* forecasts.

^{A-2}For example, $\pi_{t-3 \rightarrow t-2}$ would involve P_{t-3} and P_{t-2} , and the latter is released (and perhaps the former is revised) in the second half of the month $t - 1$. Remember that both the *SPF* and *Blue Chip* forecasts are made in the first half of month t , before P_{t-1} is released.

^{A-3}The data are available at <http://alfred.stlouisfed.org/series?seid=CPIAUCSL>.

rates that are realized (but not yet released by statistical agencies), I assume that these expectations are equal to the longer horizon being forecast. Once I show the equations below, what I mean by this will be clear.

A.1 *Survey of Professional Forecasters*

The *SPF* is a quarterly survey that has been conducted by the FRBP since 1990. The forecasters are asked to make forecasts for a number of key macroeconomic indicators several quarters into the future, and in the case of CPI inflation, they are also asked to make 5-year and 10-year forecasts. I use the median of these forecasts.

A.1.1 *SPF Quarterly Forecasts*

The *SPF* reports six quarterly forecasts ranging from “minus 1 quarter” to “plus 4 quarters” from the current quarter. The forecasts labeled “3,” “4,” “5,” and “6” are forecasts for one, two, three, and four quarters after the current quarter, respectively.^{A-4} More specifically, the forecasters are asked to forecast the annualized percentage change in the quarterly average of the CPI price level. Using my notation, the “4” forecast made in period t is

$$\text{SPF-4}_t = 100 \left[\left(\frac{\frac{P_{t+5} + P_{t+6} + P_{t+7}}{3}}{\frac{P_{t+2} + P_{t+3} + P_{t+4}}{3}} \right)^4 - 1 \right],$$

where the numerator is the average CPI price level in the second quarter following the current one and the denominator is the average CPI price level for the next quarter. Using

^{A-4}The “1” and “2” forecasts contain at least some realized inflation rates, and I do not use them since I want to focus as much as possible on pure forecasts.

continuous compounding and geometric averaging, this forecast can be written as^{A-5}

$$\begin{aligned}
\text{SPF-4}_t &\approx 400 \left\{ \log \left[(P_{t+5}P_{t+6}P_{t+7})^{1/3} \right] - \log \left[(P_{t+2}P_{t+3}P_{t+4})^{1/3} \right] \right\} \\
&= \frac{400}{3} \{ \log [(P_{t+5}P_{t+6}P_{t+7})] - \log [(P_{t+2}P_{t+3}P_{t+4})] \} \\
&= \frac{400}{3} [\log (P_{t+5}) - \log (P_{t+2}) + \log (P_{t+6}) - \log (P_{t+3}) + \log (P_{t+7}) - \log (P_{t+4})] \\
&= \frac{\pi_{t+2 \rightarrow t+5} + \pi_{t+3 \rightarrow t+6} + \pi_{t+4 \rightarrow t+7}}{3},
\end{aligned}$$

which is the arithmetic average of three quarterly inflation rates.

The SPF-3 forecast is special (as will be the other *SPF* forecasts I turn to next) in that a part of the object being forecast refers to the past and not to the future. Using similar derivations as above, the SPF-3 forecast in period t can be written as

$$\begin{aligned}
\text{SPF-3}_t &= \frac{\pi_{t-1 \rightarrow t+2} + \pi_{t \rightarrow t+3} + \pi_{t+1 \rightarrow t+4}}{3} \\
&= \frac{\left(\frac{\pi_{t-1 \rightarrow t} + 2\pi_{t \rightarrow t+2}}{3} \right) + \pi_{t \rightarrow t+3} + \pi_{t+1 \rightarrow t+4}}{3} \\
&= \frac{1}{9} (\pi_{t-1 \rightarrow t} + 2\pi_{t \rightarrow t+2} + 3\pi_{t \rightarrow t+3} + 3\pi_{t+1 \rightarrow t+4}) \\
&= \frac{1}{8} (2\pi_{t \rightarrow t+2} + 3\pi_{t \rightarrow t+3} + 3\pi_{t+1 \rightarrow t+4}),
\end{aligned}$$

where in the last line I replace $\pi_{t-1 \rightarrow t}$ with SPF-3_t . This is the assumption I will maintain whenever formulas call for $\pi_{t-2 \rightarrow t-1}$, $\pi_{t-1 \rightarrow t}$, or $\pi_{t-2 \rightarrow t}$ – I will assume each of them are equal to the main object being forecast.^{A-6}

Using similar derivations for the “5” and “6” forecasts, and using definitions $x_t^1 \equiv \text{SPF-3}_t$, $x_t^2 \equiv \text{SPF-4}_t$, $x_t^3 \equiv \text{SPF-5}_t$, and $x_t^4 \equiv \text{SPF-6}_t$, the measurement equations for the quarterly *SPF*

^{A-5}The correlation of actual inflation computed using the exact formula and the approximation I use is 0.9993.

^{A-6}This creates a small inconsistency across different forecasts when the same object, say $\pi_{t-2 \rightarrow t-1}$, is set equal to different forecasts with different values. Given that these terms receive small weights and the absence of a clearly better alternative, I choose this route.

forecasts are

$$\begin{aligned}
x_t^1 &= \frac{1}{8} (2\pi_{t \rightarrow t+2} + 3\pi_{t \rightarrow t+3} + 3\pi_{t+1 \rightarrow t+4}) + \varepsilon_t^1 \\
x_t^2 &= \frac{1}{3} (\pi_{t+2 \rightarrow t+5} + \pi_{t+3 \rightarrow t+6} + \pi_{t+4 \rightarrow t+7}) + \varepsilon_t^2 \\
x_t^3 &= \frac{1}{3} (\pi_{t+5 \rightarrow t+8} + \pi_{t+6 \rightarrow t+9} + \pi_{t+7 \rightarrow t+10}) + \varepsilon_t^3 \\
x_t^4 &= \frac{1}{3} (\pi_{t+8 \rightarrow t+11} + \pi_{t+9 \rightarrow t+12} + \pi_{t+10 \rightarrow t+13}) + \varepsilon_t^4.
\end{aligned}$$

Once stated as combinations of $\pi_{t+\tau_1 \rightarrow t+\tau_2}$, it is straightforward, though somewhat tedious, to write the full measurement equations for these forecasts using (5).^{A-7}

A.1.2 *SPF* Annual Forecasts

The *SPF* provides three annual forecasts, one for the survey calendar year and one each for the next two calendar years. I use the latter two, the “B” forecast and the “C” forecast, since they are (mostly) pure forecasts into the future. The “C” forecast is available starting in 2005Q3. More specifically, in every quarter of the survey year, for the “B” forecast the forecasters are asked to forecast the change in average price level of the last quarter of the year after the survey year relative to the last quarter of the survey year. Similarly for the “C” forecast, they need to forecast the change in the average price level of the last quarter of the year that is two years after the survey year, relative to the last quarter of the year that is one year after the survey year. As such, as we progress further into the current year, the distance between the period being forecast and the point of forecast gets shorter. This requires me to define forecasts made in particular quarters as separate variables.^{A-8}

^{A-7}For example, the second measurement equation will be

$$\begin{aligned}
x_t^2 &= L_t + \left[\frac{e^{-2\lambda} - e^{-5\lambda} + e^{-3\lambda} - e^{-6\lambda} + e^{-4\lambda} - e^{-7\lambda}}{9\lambda} \right] (C_t - S_t) \\
&\quad + \left(\frac{2e^{-2\lambda} - 5e^{-5\lambda} + 3e^{-3\lambda} - 6e^{-6\lambda} + 4e^{-4\lambda} - 7e^{-7\lambda}}{9} \right) C_t + \varepsilon_t^2.
\end{aligned}$$

^{A-8}To be clear, I split each variable into four variables, each of which is observed only once a year.

The “B” forecast released in February (denoted by t) is thus

$$\text{SPF-B-Q1}_t = 100 \left[\left(\frac{P_{t+20} + P_{t+21} + P_{t+22}}{P_{t+8} + P_{t+9} + P_{t+10}} \right) - 1 \right].$$

Using the same derivations as in *SPF4*, this simplifies to

$$\text{SPF-B-Q1}_t \approx \frac{\pi_{t+8 \rightarrow t+20} + \pi_{t+9 \rightarrow t+21} + \pi_{t+10 \rightarrow t+22}}{3}.$$

Doing the same derivations for the other quarters 1 through 3, and using definitions $x_t^5 \equiv \text{SPF-B-Q1}_t$, $x_t^6 \equiv \text{SPF-B-Q2}_t$, and $x_t^7 \equiv \text{SPF-B-Q3}_t$, I get the measurement equations

$$\begin{aligned} x_t^5 &= \frac{1}{3} (\pi_{t+8 \rightarrow t+20} + \pi_{t+9 \rightarrow t+21} + \pi_{t+10 \rightarrow t+22}) + \varepsilon_t^5 \\ x_t^6 &= \frac{1}{3} (\pi_{t+5 \rightarrow t+17} + \pi_{t+6 \rightarrow t+18} + \pi_{t+7 \rightarrow t+19}) + \varepsilon_t^6 \\ x_t^7 &= \frac{1}{3} (\pi_{t+2 \rightarrow t+14} + \pi_{t+3 \rightarrow t+15} + \pi_{t+4 \rightarrow t+16}) + \varepsilon_t^7. \end{aligned}$$

For the last quarter, I need to take into account that a small part of the object being forecast is realized by the time the forecast is made. In particular, the Q4 forecast is

$$\text{SPF-B-Q4}_t = \frac{1}{35} (11\pi_{t \rightarrow t+11} + 12\pi_{t \rightarrow t+12} + 12\pi_{t+1 \rightarrow t+13}),$$

where, again, I replaced $\pi_{t-1 \rightarrow t}$ with SPF-B-Q4_t . Defining $x_t^8 \equiv \text{SPF-B-Q4}_t$, I get the measurement equation

$$x_t^8 = \frac{1}{35} (11\pi_{t \rightarrow t+11} + 12\pi_{t \rightarrow t+12} + 12\pi_{t+1 \rightarrow t+13}) + \varepsilon_t^8.$$

For the “C” forecast, in the first quarter of a year, the expression being forecast is

$$\text{SPF-C-Q1}_t = 100 \left[\left(\frac{P_{t+32} + P_{t+33} + P_{t+34}}{P_{t+20} + P_{t+21} + P_{t+22}} \right) - 1 \right],$$

and defining $x_t^9 \equiv \text{SPF-C-Q1}_t$, $x_t^{10} \equiv \text{SPF-C-Q2}_t$, $x_t^{11} \equiv \text{SPF-C-Q3}_t$, $x_t^{12} \equiv \text{SPF-C-Q4}_t$, the

measurement equations are

$$\begin{aligned}
 x_t^9 &= \frac{1}{3} (\pi_{t+20 \rightarrow t+32} + \pi_{t+21 \rightarrow t+33} + \pi_{t+22 \rightarrow t+34}) + \varepsilon_t^9 \\
 x_t^{10} &= \frac{1}{3} (\pi_{t+17 \rightarrow t+29} + \pi_{t+18 \rightarrow t+30} + \pi_{t+19 \rightarrow t+31}) + \varepsilon_t^{10} \\
 x_t^{11} &= \frac{1}{3} (\pi_{t+14 \rightarrow t+26} + \pi_{t+15 \rightarrow t+27} + \pi_{t+16 \rightarrow t+28}) + \varepsilon_t^{11} \\
 x_t^{12} &= \frac{1}{3} (\pi_{t+11 \rightarrow t+23} + \pi_{t+12 \rightarrow t+24} + \pi_{t+13 \rightarrow t+25}) + \varepsilon_t^{12}.
 \end{aligned}$$

Given these expressions, I directly apply (5) to get the measurement equations.

A.1.3 *SPF* 5-Year and 10-Year Forecasts

Although much of the *SPF* contains short- to medium-term forecasts, the forecasters are asked to provide five-year and 10-year forecasts for inflation as well. In particular, they are asked to forecast five and 10 years into the future, starting from the last quarter of the previous year. In other words, as with the other forecasts, these forecasts compare the average price level over the last quarter of the previous year with the average price level over the last quarter of four or nine years following the current year. Similar to the annual forecasts, I divide the 10-year and the five-year forecasts into four separate variables, taking into account the different forecast horizons at each quarter. The 10-year forecast has been a part of the *SPF* since 1991Q4, and the five-year forecast was added in 2005Q3.

In all cases below, what is reported by the forecasters includes some inflation that is already realized. Denoting February with period t , the five-year forecast made in the first

quarter is

$$\begin{aligned}
\text{SPF-5YR-Q1}_t &= 100 \left[\left(\frac{P_{t+56} + P_{t+57} + P_{t+58}}{P_{t-4} + P_{t-3} + P_{t-2}} \right)^{\frac{1}{5}} - 1 \right] \\
&\approx \frac{\pi_{t-4 \rightarrow t+56} + \pi_{t-3 \rightarrow t+57} + \pi_{t-2 \rightarrow t+58}}{3} \\
&= \frac{1}{3} \left\{ \left[\frac{4}{60} \pi_{t-4 \rightarrow t} + \frac{56}{60} \pi_{t \rightarrow t+56} \right] + \left[\frac{3}{60} \pi_{t-3 \rightarrow t} + \frac{57}{60} \pi_{t \rightarrow t+57} \right] \right. \\
&\quad \left. + \left[\frac{2}{60} \pi_{t-2 \rightarrow t} + \frac{58}{60} \pi_{t \rightarrow t+58} \right] \right\} \\
&= \frac{1}{180} (\pi_{t-4 \rightarrow t-3} + \pi_{t-3 \rightarrow t-2} + 2\pi_{t-2 \rightarrow t}) + \frac{1}{180} (\pi_{t-3 \rightarrow t-2} + 2\pi_{t-2 \rightarrow t}) \\
&\quad + \frac{1}{180} 2\pi_{t-2 \rightarrow t} + \frac{1}{180} (56\pi_{t \rightarrow t+56} + 57\pi_{t \rightarrow t+57} + 58\pi_{t \rightarrow t+58}) \\
&= \frac{1}{180} (\pi_{t-4 \rightarrow t-3} + 2\pi_{t-3 \rightarrow t-2} + 6\pi_{t-2 \rightarrow t}) + \frac{1}{180} (56\pi_{t \rightarrow t+56} + 57\pi_{t \rightarrow t+57} + 58\pi_{t \rightarrow t+58}) \\
&= \frac{1}{174} (\pi_{t-4 \rightarrow t-3} + 2\pi_{t-3 \rightarrow t-2}) + \frac{1}{174} (56\pi_{t \rightarrow t+56} + 57\pi_{t \rightarrow t+57} + 58\pi_{t \rightarrow t+58}),
\end{aligned}$$

where I use the properties of continuous compounding to simplify the expressions, and the last line uses $\pi_{t-2 \rightarrow t} = \text{SPF-5YR-Q1}_t$. Note that this forecast contains two realized inflation terms and three terms that are forecasts from period t onward. It is important to emphasize that this correction causes sizable differences between what the SPF reports and what enters the estimation. Figure A1 demonstrates this. It shows the raw SPF 10-year data as downloaded from FRBP, SPF-5YR-Q1_t , the adjusted version – for example for Q1 this is $\text{SPF-5YR-Q1}_t - \frac{1}{174} (\pi_{t-4 \rightarrow t-3} + 2\pi_{t-3 \rightarrow t-2})$ – and the filtered counterpart of the adjusted version from the model – this will be $\frac{1}{174} (56\pi_{t \rightarrow t+56} + 57\pi_{t \rightarrow t+57} + 58\pi_{t \rightarrow t+58})$. This adjustment is particularly visible as we progress in the year and more of the raw data contains inflation that is already realized. For example looking at the Q4 panel, there are a large number of 2.5% entries in the raw data but these get adjust down to as low as 2.1%.

Turning to the second-quarter forecast, where period t now denotes May, I have

$$\begin{aligned}\text{SPF-5YR-Q2}_t &= 100 \left[\left(\frac{P_{t+53} + P_{t+54} + P_{t+55}}{P_{t-7} + P_{t-6} + P_{t-5}} \right)^{\frac{1}{5}} - 1 \right] \\ &\approx \frac{1}{174} (\pi_{t-7 \rightarrow t-6} + 2\pi_{t-6 \rightarrow t-5} + 9\pi_{t-5 \rightarrow t-2}) \\ &\quad + \frac{1}{174} (53\pi_{t \rightarrow t+53} + 54\pi_{t \rightarrow t+54} + 55\pi_{t \rightarrow t+55}).\end{aligned}$$

The third-quarter forecast, where period t now denotes August, is given by

$$\begin{aligned}\text{SPF-5YR-Q3}_t &= 100 \left[\left(\frac{P_{t+50} + P_{t+51} + P_{t+52}}{P_{t-10} + P_{t-9} + P_{t-8}} \right)^{\frac{1}{5}} - 1 \right] \\ &\approx \frac{1}{174} (\pi_{t-10 \rightarrow t-9} + 2\pi_{t-9 \rightarrow t-8} + 18\pi_{t-8 \rightarrow t-2}) \\ &\quad + \frac{1}{174} (50\pi_{t \rightarrow t+50} + 51\pi_{t \rightarrow t+51} + 52\pi_{t \rightarrow t+52}).\end{aligned}$$

Finally, the fourth-quarter forecast, with t denoting November, is given by

$$\begin{aligned}\text{SPF-5YR-Q4}_t &= 100 \left[\left(\frac{P_{t+47} + P_{t+48} + P_{t+49}}{P_{t-13} + P_{t-12} + P_{t-11}} \right)^{\frac{1}{5}} - 1 \right] \\ &\approx \frac{1}{174} (\pi_{t-13 \rightarrow t-12} + 2\pi_{t-12 \rightarrow t-11} + 27\pi_{t-11 \rightarrow t-2}) \\ &\quad + \frac{1}{174} (47\pi_{t \rightarrow t+47} + 48\pi_{t \rightarrow t+48} + 49\pi_{t \rightarrow t+49}).\end{aligned}$$

Using the definitions

$$\begin{aligned}x_t^{13} &\equiv \text{SPF-5YR-Q1}_t - \frac{1}{174} (\pi_{t-4 \rightarrow t-3} + 2\pi_{t-3 \rightarrow t-2}) \\ x_t^{14} &\equiv \text{SPF-5YR-Q2}_t - \frac{1}{174} (\pi_{t-7 \rightarrow t-6} + 2\pi_{t-6 \rightarrow t-5} + 9\pi_{t-5 \rightarrow t-2}) \\ x_t^{15} &\equiv \text{SPF-5YR-Q3}_t - \frac{1}{174} (\pi_{t-10 \rightarrow t-9} + 2\pi_{t-9 \rightarrow t-8} + 18\pi_{t-8 \rightarrow t-2}) \\ x_t^{16} &\equiv \text{SPF-5YR-Q4}_t - \frac{1}{174} (\pi_{t-13 \rightarrow t-12} + 2\pi_{t-12 \rightarrow t-11} + 27\pi_{t-11 \rightarrow t-2}).\end{aligned}$$

The measurement equations for the 5-year forecasts are

$$\begin{aligned}
x_t^{13} &= \frac{1}{174} (56\pi_{t \rightarrow t+56} + 57\pi_{t \rightarrow t+57} + 58\pi_{t \rightarrow t+58}) + \varepsilon_t^{13} \\
x_t^{14} &= \frac{1}{174} (53\pi_{t \rightarrow t+53} + 54\pi_{t \rightarrow t+54} + 55\pi_{t \rightarrow t+55}) + \varepsilon_t^{14} \\
x_t^{15} &= \frac{1}{174} (50\pi_{t \rightarrow t+50} + 51\pi_{t \rightarrow t+51} + 52\pi_{t \rightarrow t+52}) + \varepsilon_t^{15} \\
x_t^{16} &= \frac{1}{174} (47\pi_{t \rightarrow t+47} + 48\pi_{t \rightarrow t+48} + 49\pi_{t \rightarrow t+49}) + \varepsilon_t^{16}.
\end{aligned}$$

Applying the same idea to 10-year forecasts, I define

$$\begin{aligned}
x_t^{17} &\equiv \text{SPF-10YR-Q1}_t - \frac{1}{354} (\pi_{t-4 \rightarrow t-3} + 2\pi_{t-3 \rightarrow t-2}) \\
x_t^{18} &\equiv \text{SPF-10YR-Q2}_t - \frac{1}{354} (\pi_{t-7 \rightarrow t-6} + 2\pi_{t-6 \rightarrow t-5} + 9\pi_{t-5 \rightarrow t-2}) \\
x_t^{19} &\equiv \text{SPF-10YR-Q3}_t - \frac{1}{354} (\pi_{t-10 \rightarrow t-9} + 2\pi_{t-9 \rightarrow t-8} + 18\pi_{t-8 \rightarrow t-2}) \\
x_t^{20} &\equiv \text{SPF-10YR-Q4}_t - \frac{1}{354} (\pi_{t-13 \rightarrow t-12} + 2\pi_{t-12 \rightarrow t-11} + 27\pi_{t-11 \rightarrow t-2}),
\end{aligned}$$

and the remaining measurement equations are

$$\begin{aligned}
x_t^{17} &= \frac{1}{354} (116\pi_{t \rightarrow t+116} + 117\pi_{t \rightarrow t+117} + 118\pi_{t \rightarrow t+118}) + \varepsilon_t^{17} \\
x_t^{18} &= \frac{1}{354} (113\pi_{t \rightarrow t+113} + 114\pi_{t \rightarrow t+114} + 115\pi_{t \rightarrow t+115}) + \varepsilon_t^{18} \\
x_t^{19} &= \frac{1}{354} (110\pi_{t \rightarrow t+110} + 111\pi_{t \rightarrow t+111} + 112\pi_{t \rightarrow t+112}) + \varepsilon_t^{19} \\
x_t^{20} &= \frac{1}{354} (107\pi_{t \rightarrow t+107} + 108\pi_{t \rightarrow t+108} + 109\pi_{t \rightarrow t+109}) + \varepsilon_t^{20}.
\end{aligned}$$

The full measurement equations follow from applying (5) to the right-hand sides of these equations.

A.2 *Blue Chip* Quarterly Forecasts

I use the quarterly forecasts published in the *Blue Chip Economic Indicators*. Every month, the forecasters are asked to make between five and nine short-term forecasts. The table below summarizes the availability of forecasts for the year 2016 as an example in which “X”

denotes a forecast, “-” reflects the absence of a forecast, and “*” next to an “X” shows a forecast that I use in this paper. I use the median of the individual forecasts.

	2015Q4	2016Q1	2016Q2	2016Q3	2016Q4	2017Q1	2017Q2	2017Q3	2017Q4
January	X	X	X*	X*	X*	X*	X*	X	X
February	-	X	X*	X*	X*	X*	X*	X	X
March	-	X	X*	X*	X*	X*	X*	X	X
April	-	X	X	X*	X*	X*	X*	X*	X
May	-	-	X	X*	X*	X*	X*	X*	X
June	-	-	X	X*	X*	X*	X*	X*	X
July	-	-	X	X	X*	X*	X*	X*	X*
August	-	-	-	X	X*	X*	X*	X*	X*
September	-	-	-	X	X*	X*	X*	X*	X*
October	-	-	-	X	X	X*	X*	X*	X*
November	-	-	-	-	X	X*	X*	X*	X*
December	-	-	-	-	X	X*	X*	X*	X*

The table shows that in most months I use five forecasts, each of which reflects the change in the average level of CPI over a quarter, relative to the previous quarter. The first quarter forecast I use is the one that follows the month the forecast is made – for example, in months that are in the first quarter, I use forecasts that are about the second, third, and fourth quarters of the current year and the first and second quarters of the following year. In the fourth quarter, I use only four forecasts. These choices follow from the timing and the unbalanced structure of the survey.

The first four of the forecasts I use follow the “3,” “4,” “5,” and “6” forecasts of the *SPF*. Since the *SPF* forecasts were done in the second month of a quarter, I treated them identically. However, *Blue Chip* forecasts are made monthly and thus in different months of a quarter. This means I need to create three different versions of each *Blue Chip* forecast, depending on which month of the quarter it is made.

Starting with those made in the second month of a quarter, the expressions exactly mimic those from the *SPF* – for example the next-quarter forecast in February, May, August or

November will be

$$\text{BC-1Q-M2}_t = \frac{1}{8} (2\pi_{t \rightarrow t+2} + 3\pi_{t \rightarrow t+3} + 3\pi_{t+1 \rightarrow t+4}),$$

where the notation is BC-XQ-MY means the forecast made in the Y^{th} month of a quarter covering the quarter that is X quarters after the current one. And the BC-5Q-M2 forecast, which was not available in the *SPF*, is given by

$$\text{BC-5Q-M2}_t = \frac{1}{3} (\pi_{t+11 \rightarrow t+14} + \pi_{t+12 \rightarrow t+15} + \pi_{t+13 \rightarrow t+16}).$$

A forecast made in the first month of a quarter for the next quarter is

$$\begin{aligned} \text{BC-1Q-M1}_t &= 100 \left[\left(\frac{P_{t+3} + P_{t+4} + P_{t+5}}{P_t + P_{t+1} + P_{t+2}} \right)^4 - 1 \right] \\ &\approx \frac{\pi_{t \rightarrow t+3} + \pi_{t+1 \rightarrow t+4} + \pi_{t+2 \rightarrow t+5}}{3}, \end{aligned}$$

and others are obtained as

$$\begin{aligned} \text{BC-2Q-M1}_t &= \frac{\pi_{t+3 \rightarrow t+6} + \pi_{t+4 \rightarrow t+7} + \pi_{t+5 \rightarrow t+8}}{3} \\ \text{BC-3Q-M1}_t &= \frac{\pi_{t+6 \rightarrow t+9} + \pi_{t+7 \rightarrow t+10} + \pi_{t+8 \rightarrow t+11}}{3} \\ \text{BC-4Q-M1}_t &= \frac{\pi_{t+9 \rightarrow t+12} + \pi_{t+10 \rightarrow t+13} + \pi_{t+11 \rightarrow t+14}}{3} \\ \text{BC-5Q-M1}_t &= \frac{\pi_{t+12 \rightarrow t+15} + \pi_{t+13 \rightarrow t+16} + \pi_{t+14 \rightarrow t+17}}{3}. \end{aligned}$$

Finally, a forecast made in the last month of a quarter for the next quarter is

$$\begin{aligned} \text{BC-1Q-M3}_t &= 100 \left[\left(\frac{P_{t+1} + P_{t+2} + P_{t+3}}{P_{t-2} + P_{t-1} + P_t} \right)^4 - 1 \right] \\ &\approx \frac{1}{6} (\pi_{t \rightarrow t+1} + 2\pi_{t \rightarrow t+2} + 3\pi_{t \rightarrow t+3}), \end{aligned}$$

where in the last line I used $2\pi_{t-2 \rightarrow t} + \pi_{t-1 \rightarrow t} = 3\text{BC-1Q-M3}_t$.

Other forecasts follow from

$$\begin{aligned}
\text{BC-2Q-M3}_t &= \frac{\pi_{t+1 \rightarrow t+4} + \pi_{t+2 \rightarrow t+5} + \pi_{t+3 \rightarrow t+6}}{3} \\
\text{BC-3Q-M3}_t &= \frac{\pi_{t+4 \rightarrow t+7} + \pi_{t+5 \rightarrow t+8} + \pi_{t+6 \rightarrow t+9}}{3} \\
\text{BC-4Q-M3}_t &= \frac{\pi_{t+7 \rightarrow t+10} + \pi_{t+8 \rightarrow t+11} + \pi_{t+9 \rightarrow t+12}}{3} \\
\text{BC-5Q-M3}_t &= \frac{\pi_{t+10 \rightarrow t+13} + \pi_{t+11 \rightarrow t+14} + \pi_{t+12 \rightarrow t+15}}{3}.
\end{aligned}$$

Collecting all these, I define

$$\begin{aligned}
x_t^{21} &\equiv \text{BC-1Q-M1}_t, \quad x_t^{22} \equiv \text{BC-2Q-M1}_t, \quad x_t^{23} \equiv \text{BC-3Q-M1}_t, \quad x_t^{24} \equiv \text{BC-4Q-M1}_t, \\
x_t^{25} &\equiv \text{BC-5Q-M1}_t, \quad x_t^{26} \equiv \text{BC-1Q-M2}_t, \quad x_t^{27} \equiv \text{BC-2Q-M2}_t, \quad x_t^{28} \equiv \text{BC-3Q-M2}_t, \\
x_t^{29} &\equiv \text{BC-4Q-M2}_t, \quad x_t^{30} \equiv \text{BC-5Q-M2}_t, \quad x_t^{31} \equiv \text{BC-1Q-M3}_t, \quad x_t^{32} \equiv \text{BC-2Q-M3}_t, \\
x_t^{33} &\equiv \text{BC-3Q-M3}_t, \quad x_t^{34} \equiv \text{BC-4Q-M3}_t, \quad x_t^{35} \equiv \text{BC-5Q-M3}_t.
\end{aligned}$$

The measurement equations are

$$\begin{aligned}
x_t^{21} &= \frac{1}{3} (\pi_{t \rightarrow t+3} + \pi_{t+1 \rightarrow t+4} + \pi_{t+2 \rightarrow t+5}) + \varepsilon_t^{21} \\
x_t^{22} &= \frac{1}{3} (\pi_{t+3 \rightarrow t+6} + \pi_{t+4 \rightarrow t+7} + \pi_{t+5 \rightarrow t+8}) + \varepsilon_t^{22} \\
x_t^{23} &= \frac{1}{3} (\pi_{t+6 \rightarrow t+9} + \pi_{t+7 \rightarrow t+10} + \pi_{t+8 \rightarrow t+11}) + \varepsilon_t^{23} \\
x_t^{24} &= \frac{1}{3} (\pi_{t+9 \rightarrow t+12} + \pi_{t+10 \rightarrow t+13} + \pi_{t+11 \rightarrow t+14}) + \varepsilon_t^{24} \\
x_t^{25} &= \frac{1}{3} (\pi_{t+12 \rightarrow t+15} + \pi_{t+13 \rightarrow t+16} + \pi_{t+14 \rightarrow t+17}) + \varepsilon_t^{25} \\
\\
x_t^{26} &= \frac{1}{8} (2\pi_{t \rightarrow t+2} + 3\pi_{t \rightarrow t+3} + 3\pi_{t+1 \rightarrow t+4}) + \varepsilon_t^{26} \\
x_t^{27} &= \frac{1}{3} (\pi_{t+2 \rightarrow t+5} + \pi_{t+3 \rightarrow t+6} + \pi_{t+4 \rightarrow t+7}) + \varepsilon_t^{27} \\
x_t^{28} &= \frac{1}{3} (\pi_{t+5 \rightarrow t+8} + \pi_{t+6 \rightarrow t+9} + \pi_{t+7 \rightarrow t+10}) + \varepsilon_t^{28} \\
x_t^{29} &= \frac{1}{3} (\pi_{t+8 \rightarrow t+11} + \pi_{t+9 \rightarrow t+12} + \pi_{t+10 \rightarrow t+13}) + \varepsilon_t^{29} \\
x_t^{30} &= \frac{1}{3} (\pi_{t+11 \rightarrow t+14} + \pi_{t+12 \rightarrow t+15} + \pi_{t+13 \rightarrow t+16}) + \varepsilon_t^{30}
\end{aligned}$$

$$\begin{aligned}
x_t^{31} &= \frac{1}{6} (\pi_{t \rightarrow t+1} + 2\pi_{t \rightarrow t+2} + 3\pi_{t \rightarrow t+3}) + \varepsilon_t^{31} \\
x_t^{32} &= \frac{1}{3} (\pi_{t+1 \rightarrow t+4} + \pi_{t+2 \rightarrow t+5} + \pi_{t+3 \rightarrow t+6}) + \varepsilon_t^{32} \\
x_t^{33} &= \frac{1}{3} (\pi_{t+4 \rightarrow t+7} + \pi_{t+5 \rightarrow t+8} + \pi_{t+6 \rightarrow t+9}) + \varepsilon_t^{33} \\
x_t^{34} &= \frac{1}{3} (\pi_{t+7 \rightarrow t+10} + \pi_{t+8 \rightarrow t+11} + \pi_{t+9 \rightarrow t+12}) + \varepsilon_t^{34} \\
x_t^{35} &= \frac{1}{3} (\pi_{t+10 \rightarrow t+13} + \pi_{t+11 \rightarrow t+14} + \pi_{t+12 \rightarrow t+15}) + \varepsilon_t^{35}.
\end{aligned}$$

A.3 *Blue Chip* Long-Range Forecasts

In the March and October issues of the *Blue Chip Economic Indicators* and the June and December issues of the *Blue Chip Financial Forecasts*, the forecasters are asked about their long-term forecasts. They are asked to make six forecasts of long-range inflation: five annual forecasts, each covering one calendar year, and one five-year forecast covering the five years following the five years in the last forecast. The annual forecasts are labeled as “year-over-year” forecasts, which means they are the percentage change in the average price level across years. More specifically, since October 2008, both of these publications ask the forecasters to forecast five years following the next year – for 2008 this would be years 2010, 2011, 2012, 2013, and 2014 – as well as the five-year forward forecast of 2015-2019. Prior to October 2008, in most years the format remained the same, but in some years the horizon shifted earlier by one year. To keep variables consistent throughout the sample, I use the format since 2008, and in years in which there is a shift, I use missing observations where appropriate.

In March of a year, the first object being forecast is defined as

$$\begin{aligned}
\text{BCLR-2Y-M}_t &= 100 \left[\left(\frac{\sum_{s=22}^{33} P_{t+s}}{21} \right) - 1 \right] \\
&\approx \frac{1}{12} \sum_{s=10}^{21} \pi_{t+s \rightarrow t+s+12},
\end{aligned}$$

which I label as a two-year forecast simply because the forecast window is in the calendar year following the next. I will continue using the same notation for the rest of the three months in which these publications are released to keep things simple even though the forecasting window moves and it approaches the period in which the forecast is made. The “M” at the end of the variable name reflects the “March” forecast, and I use “J,” “O,” and “D” to represent June, October, and December, respectively, below. The first annual forecast made in June, October, and December refer to the year that is 19, 15, and 13 months following the month the forecast is made, respectively.

Defining

$$\begin{aligned}
x_t^{36} &\equiv \text{BCLR-2Y-M}_t, & x_t^{37} &\equiv \text{BCLR-3Y-M}_t, & x_t^{38} &\equiv \text{BCLR-4Y-M}_t, & x_t^{39} &\equiv \text{BCLR-5Y-M}_t \\
x_t^{40} &\equiv \text{BCLR-6Y-M}_t, & x_t^{41} &\equiv \text{BCLR-2Y-J}_t, & x_t^{42} &\equiv \text{BCLR-3Y-J}_t, & x_t^{43} &\equiv \text{BCLR-4Y-J}_t \\
x_t^{44} &\equiv \text{BCLR-5Y-J}_t, & x_t^{45} &\equiv \text{BCLR-6Y-J}_t, & x_t^{46} &\equiv \text{BCLR-2Y-O}_t, & x_t^{47} &\equiv \text{BCLR-3Y-O}_t \\
x_t^{48} &\equiv \text{BCLR-4Y-O}_t, & x_t^{49} &\equiv \text{BCLR-5Y-O}_t, & x_t^{50} &\equiv \text{BCLR-6Y-O}_t, & x_t^{51} &\equiv \text{BCLR-2Y-D}_t \\
x_t^{52} &\equiv \text{BCLR-3Y-D}_t, & x_t^{53} &\equiv \text{BCLR-4Y-D}_t, & x_t^{54} &\equiv \text{BCLR-5Y-D}_t, & x_t^{55} &\equiv \text{BCLR-6Y-D}_t,
\end{aligned}$$

the measurement equations for March are

$$\begin{aligned}
x_t^{36} &= \frac{1}{12} \sum_{s=10}^{21} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{36} \\
x_t^{37} &= \frac{1}{12} \sum_{s=22}^{33} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{37} \\
x_t^{38} &= \frac{1}{12} \sum_{s=34}^{45} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{38} \\
x_t^{39} &= \frac{1}{12} \sum_{s=46}^{57} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{39} \\
x_t^{40} &= \frac{1}{12} \sum_{s=58}^{69} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{40}.
\end{aligned}$$

Then the measurement equations for June are

$$\begin{aligned}
 x_t^{41} &= \frac{1}{12} \sum_{s=7}^{18} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{41} \\
 x_t^{42} &= \frac{1}{12} \sum_{s=19}^{30} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{42} \\
 x_t^{43} &= \frac{1}{12} \sum_{s=31}^{42} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{43} \\
 x_t^{44} &= \frac{1}{12} \sum_{s=43}^{54} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{44} \\
 x_t^{45} &= \frac{1}{12} \sum_{s=55}^{66} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{45}.
 \end{aligned}$$

And for October the measurement equations are

$$\begin{aligned}
 x_t^{46} &= \frac{1}{12} \sum_{s=3}^{14} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{46} \\
 x_t^{47} &= \frac{1}{12} \sum_{s=15}^{26} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{47} \\
 x_t^{48} &= \frac{1}{12} \sum_{s=27}^{38} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{48} \\
 x_t^{49} &= \frac{1}{12} \sum_{s=39}^{50} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{49} \\
 x_t^{50} &= \frac{1}{12} \sum_{s=51}^{62} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{50}.
 \end{aligned}$$

And finally December forecasts use

$$\begin{aligned}
x_t^{51} &= \frac{1}{12} \sum_{s=1}^{12} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{51} \\
x_t^{52} &= \frac{1}{12} \sum_{s=13}^{24} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{52} \\
x_t^{53} &= \frac{1}{12} \sum_{s=25}^{36} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{53} \\
x_t^{54} &= \frac{1}{12} \sum_{s=37}^{48} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{54} \\
x_t^{55} &= \frac{1}{12} \sum_{s=49}^{60} \pi_{t+s \rightarrow t+s+12} + \varepsilon_t^{55}.
\end{aligned}$$

These publications contain two more forecasts. Once again using the October 2008 issue as an example, there are forecasts for “2010-2014” and “2015-2019.” The former is an arithmetic average of the five annual forecasts I use and thus is not independently useful. In order to use the latter, I take its simple average with the former and label this the forecast for the 10-year period of 2010-2019. This forecast is defined as the average of the 10 annual price changes, each of which is in the format I use above – annual change in the average price level between two years. The March forecast can be written as

$$\begin{aligned}
\text{BCLR-10Y-M}_t &= \frac{1}{10} \left\{ \frac{1}{12} \left[\sum_{s=10}^{21} \pi_{t+s \rightarrow t+s+12} + \sum_{s=22}^{33} \pi_{t+s \rightarrow t+s+12} + \sum_{s=34}^{45} \pi_{t+s \rightarrow t+s+12} \right. \right. \\
&\quad + \sum_{s=46}^{57} \pi_{t+s \rightarrow t+s+12} + \sum_{s=58}^{69} \pi_{t+s \rightarrow t+s+12} + \sum_{s=70}^{81} \pi_{t+s \rightarrow t+s+12} \sum_{s=82}^{93} \pi_{t+s \rightarrow t+s+12} \\
&\quad \left. \left. + \sum_{s=94}^{105} \pi_{t+s \rightarrow t+s+12} + \sum_{s=106}^{117} \pi_{t+s \rightarrow t+s+12} + \sum_{s=118}^{129} \pi_{t+s \rightarrow t+s+12} \right] \right\} \\
&= \frac{1}{12} \sum_{s=10}^{21} \pi_{t+s \rightarrow t+s+120},
\end{aligned}$$

where the last equality follows from the properties of continuous compounding.^{A-9} Denoting $x_t^{56} \equiv \text{BCLR-10Y-M}_t$, $x_t^{57} \equiv \text{BCLR-10Y-J}_t$, $x_t^{58} \equiv \text{BCLR-10Y-O}_t$, and $x_t^{59} \equiv \text{BCLR-10Y-D}_t$, the measurement equations are

$$\begin{aligned} x_t^{56} &= \frac{1}{12} \sum_{s=10}^{21} \pi_{t+s \rightarrow t+s+120} + \varepsilon_t^{56} \\ x_t^{57} &= \frac{1}{12} \sum_{s=7}^{18} \pi_{t+s \rightarrow t+s+120} + \varepsilon_t^{57} \\ x_t^{58} &= \frac{1}{12} \sum_{s=3}^{14} \pi_{t+s \rightarrow t+s+120} + \varepsilon_t^{58} \\ x_t^{59} &= \frac{1}{12} \sum_{s=1}^{12} \pi_{t+s \rightarrow t+s+120} + \varepsilon_t^{59}. \end{aligned}$$

The measurement equations are directly obtained from (5) with measurement errors ε_t^i .

B State Space Model

The preceding section show the measurement equations of the observable variables I use in my analysis – all in all, 59 variables. Combining all the measurement equations and the transition equation for the three factors, I obtain a state-space system

$$\mathbf{x}_t = \mathbf{Z}\boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t \tag{A-1}$$

$$(\boldsymbol{\alpha}_t - \boldsymbol{\mu}) = \mathbf{T}(\boldsymbol{\alpha}_{t-1} - \boldsymbol{\mu}) + \boldsymbol{\eta}_t \tag{A-2}$$

^{A-9}To see this, consider the simplified example

$$\begin{aligned} \frac{1}{2} \left(\frac{1}{2} \sum_{s=13}^{14} \pi_{t+s \rightarrow t+s+2} + \frac{1}{2} \sum_{s=15}^{16} \pi_{t+s \rightarrow t+s+2} \right) &= \frac{1}{4} (\pi_{t+13 \rightarrow t+15} + \pi_{t+14 \rightarrow t+16} + \pi_{t+15 \rightarrow t+17} + \pi_{t+16 \rightarrow t+18}) \\ &= \frac{1}{4} (\pi_{t+13 \rightarrow t+15} + \pi_{t+15 \rightarrow t+17} + \pi_{t+14 \rightarrow t+16} + \pi_{t+16 \rightarrow t+18}) \\ &= \frac{1}{2} (\pi_{t+13 \rightarrow t+17} + \pi_{t+14 \rightarrow t+18}) \\ &= \frac{1}{2} \sum_{s=13}^{14} \pi_{t+s \rightarrow t+s+4}. \end{aligned}$$

with

$$\boldsymbol{\varepsilon}_t \sim N(0, \mathbf{H}) \text{ and } \boldsymbol{\eta}_t \sim N(0, \mathbf{Q}), \quad (\text{A-3})$$

where the notation follows the standard notation in Durbin and Koopman (2012). The vector \mathbf{x}_t is a 59×1 vector containing all observed variables in period t , and $\boldsymbol{\alpha}_t$ is a 9×1 vector that collects the three inflation expectation factors and their two lags in period t :

$$\boldsymbol{\alpha}_t = [L_t \ S_t \ C_t \ L_{t-1} \ S_{t-1} \ C_{t-1} \ L_{t-2} \ S_{t-2} \ C_{t-2}]'. \quad (\text{A-4})$$

and the constant $\boldsymbol{\mu}$ is given by

$$\boldsymbol{\mu} = [\mu_L \ \mu_S \ \mu_C \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]'. \quad (\text{A-5})$$

The vector $\boldsymbol{\varepsilon}_t$ contains the measurement errors, and thus \mathbf{H} is a diagonal matrix with

$$\mathbf{H} = \text{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_{35}^2). \quad (\text{A-6})$$

The measurement matrix \mathbf{Z} collects the factor loadings described in the previous section and is given by

$$\mathbf{Z} = \begin{bmatrix} f_L^1 & f_S^1 & f_C^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ f_L^2 & f_S^2 & f_C^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_L^{59} & f_S^{59} & f_C^{59} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (\text{A-7})$$

The transition matrix \mathbf{T} takes the form

$$\mathbf{T} = \begin{bmatrix} \rho_{11} & 0 & 0 & \rho_{12} & 0 & 0 & \rho_{13} & 0 & 0 \\ 0 & \rho_{21} & 0 & 0 & \rho_{22} & 0 & 0 & \rho_{23} & 0 \\ 0 & 0 & \rho_{31} & 0 & 0 & \rho_{32} & 0 & 0 & \rho_{33} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad (\text{A-8})$$

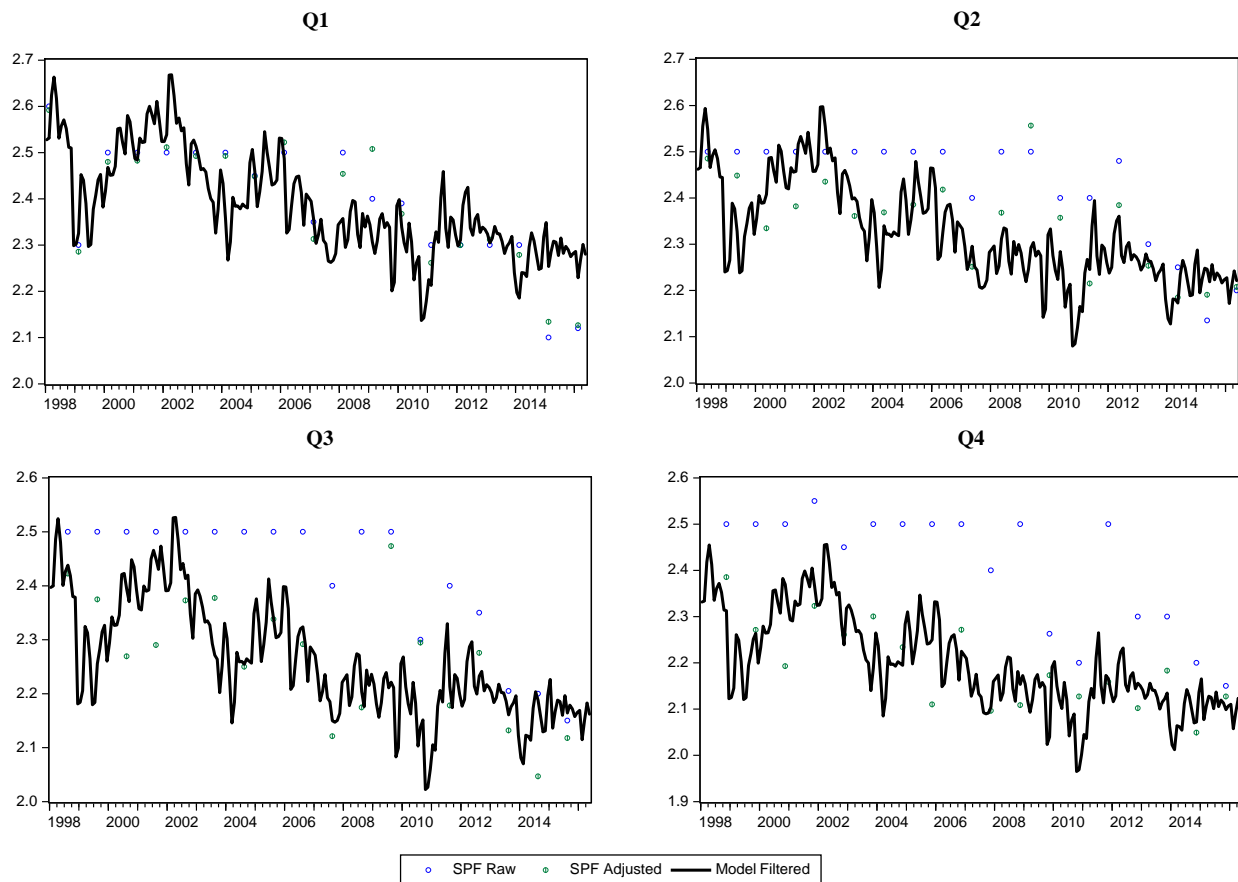
Finally, \mathbf{Q} is a diagonal matrix with

$$\mathbf{Q} = \text{diag} (\eta_t^L, \eta_t^S, \eta_t^C, 0, 0, 0, 0, 0, 0) . \quad (\text{A-9})$$

References

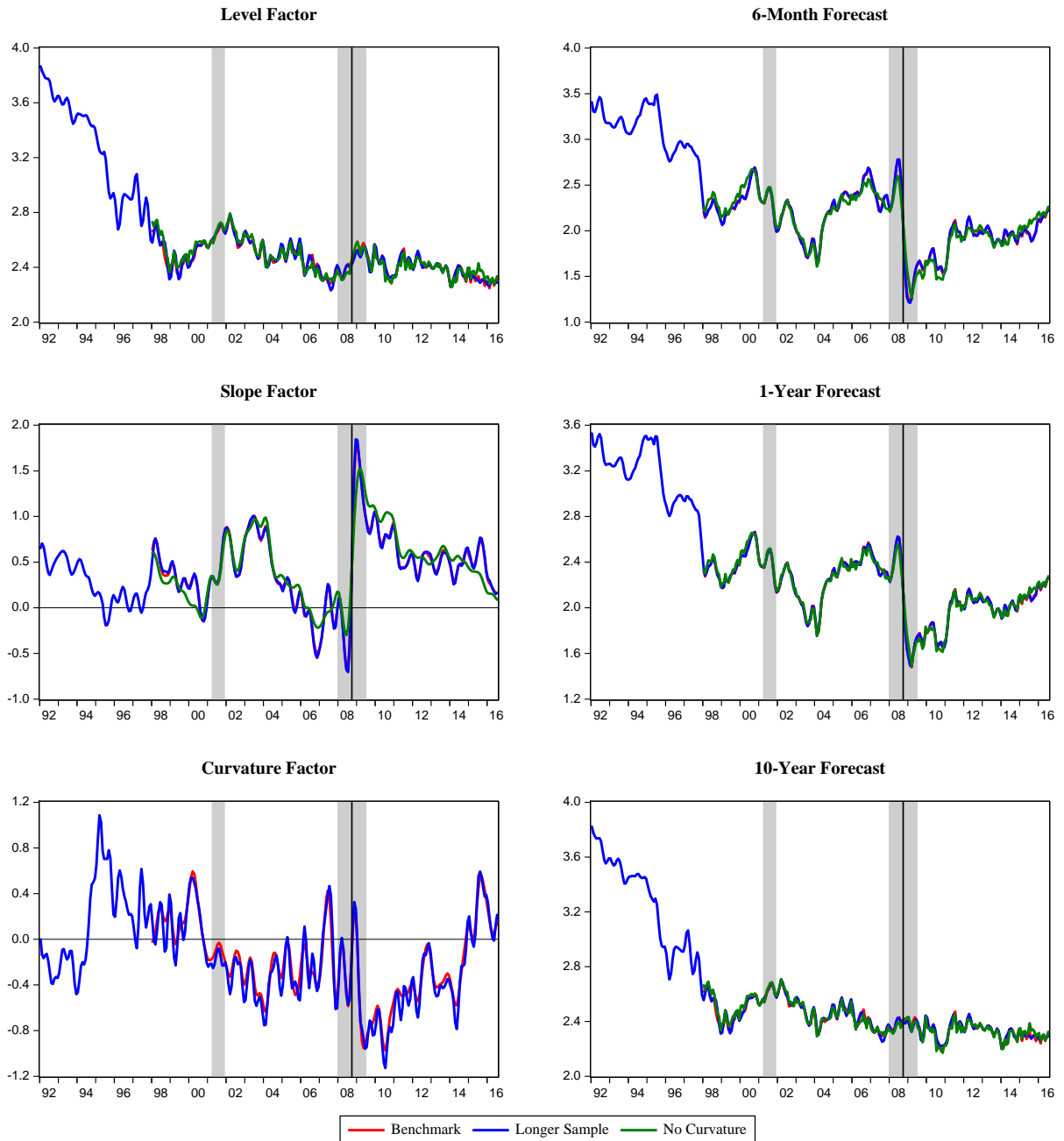
- [1] Stark, T. (2010), “Realistic Evaluation of Real-Time Forecasts in the Survey of Professional Forecasters,” Federal Reserve Bank of Philadelphia *Research Rap* Special Report.

Figure A1: Adjustment of the Raw SPF 10-Year Data



Notes: This figure shows how the adjustments explained in Appendix A.1 change the raw data for the SPF 10-year forecast.

Figure A2: Comparison of Factors and Forecasts in Alternative Specifications



Notes: The gray bars denote NBER recessions. The vertical line denotes September 2008.

Figure A3: Comparison of Factors and Forecasts in Alternative Methods for Extracting Factors

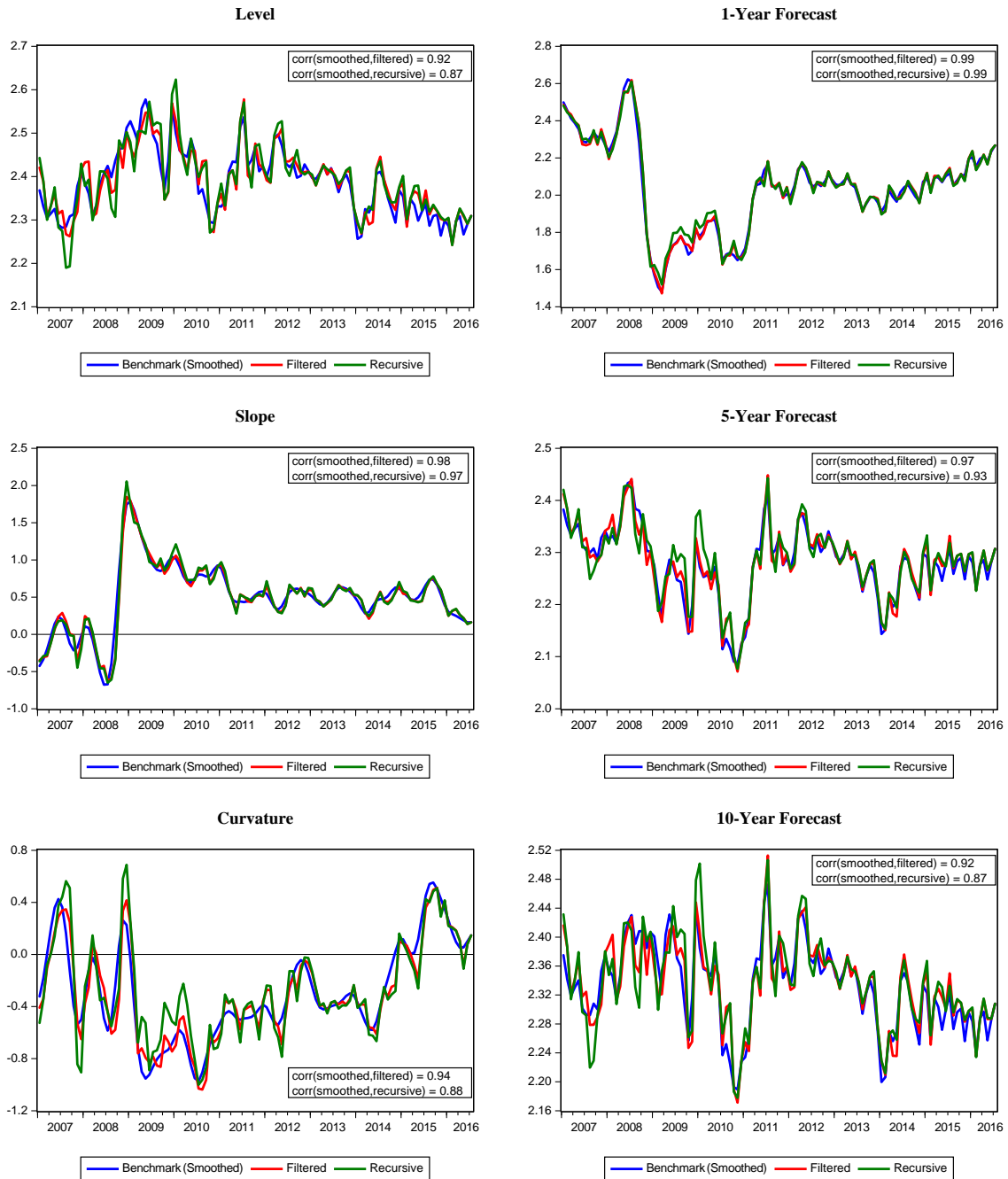


Table A1: Parameter Estimates for the UCSV Model

	Prior	5%	Median	95%
φ	U[0,1]	0.08	0.14	0.20
σ	IG(3,5)	1.25	1.39	1.65
ρ_η	N(0.9,5)	-0.40	0.33	0.80
ρ_ϵ	N(0.9,5)	0.85	0.93	0.97
σ_{ν_η}	IG(3,0.1)	0.14	0.24	0.44
σ_{ν_ϵ}	IG(3,0.1)	0.51	0.65	0.90

Notes: The first column shows the marginal prior distribution for each parameter where $U[a, b]$ means the uniform distribution between a and b , $N(a, b)$ means the normal distribution with mean a and variance b , and IG means the inverse gamma distribution $IG(a, b)$, which is parameterized as $p_{IG}(\sigma | a, b) \propto \sigma^{-a-1} \exp(b/\sigma)$. The priors for ρ_η and ρ_ϵ are truncated to ensure stationarity. The remaining columns show the given percentiles of the posterior distribution. Estimation uses data from January 1984 to December 2015.