Money and Capital

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Abstract

The effects of money (anticipated inflation) on capital formation is a classic issue in macroeconomics. Previous papers adopt reduced-form approaches, putting money in the utility function, or imposing cash in advance, but using otherwise frictionless models. We follow instead a literature that tries to be explicit about the frictions making money essential. This introduces new elements, including a two-sector structure with centralized and decentralized markets, stochastic trading opportunities, and bargaining. These elements matter quantitatively and numerical results differ from findings in the reduced-form literature. The analysis also reduces a gap between microfounded monetary economics and mainstream macro.

Keywords: Inflation, Investment, Holdup problems, Welfare cost of inflation

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1. Introduction

The relation between anticipated inflation and capital formation is a classic issue in macroeconomics, going back at least to Tobin (1965), Sidrauski (1967a,1967b), Stockman (1981), Cooley and Hansen (1989,1991), Gomme (1993), Ireland (1994) and many others. All these contributors adopt reduced-form approaches: they put money in the utility function, or impose cash in advance, in an attempt to capture implicitly the role of money in the exchange process, but use otherwise frictionless models. An alternative literature on money, going back to Kiyotaki and Wright (1989,1993), Aiyagari and Wallace (1991), Shi (1995), Trejos and Wright (1995), Kocherlakota (1998), Wallace (2001) and others, strives to be more explicit about the frictions that make a medium of exchange essential.[1] In doing so, these papers introduce new elements into monetary economics, including detailed descriptions of specialization, information, matching, alternative pricing mechanisms, etc. Many papers in the area show how these ingredients matter in theory. This paper shows they also matter for quantitative analysis.

We use the two-sector model in Lagos and Wright (2005), where some economic activity takes place in centralized markets and some in decentralized markets. In addition to providing microfoundations for money, the use of decentralized markets allow the introduction of ingredients like stochastic trading opportunities and bargaining, while centralized markets are useful to incorporate capital as in standard growth theory. This is a further step toward integrating theories with decentralized trade, on the one hand, and mainstream macro, on the other, which has been a challenge for some time.[2] The framework in this paper combines

[1] This literature has recently been dubbed New Monetarist Economics. See Williamson and Wright (2010a,b) and Nosal and Rocheteau (2010) for extended discussions and surveys.

[2] As Azariadis (1993) put it, "Capturing the transactions motive for holding money balances in a compact and logically appealing manner has turned out to be an enormously complicated task. Logically coherent models such as those proposed by Diamond (1984) and Kiyotaki and Wright (1989) tend to be so removed from neoclassical growth theory as to seriously hinder the job of integrating rigorous monetary theory with the rest of macroeconomics." And as Kiyotaki and Moore (2001) put it, "The matching models are without doubt ingenious and beautiful. But it is quite hard to integrate them with the rest of macroeconomic theory – not least because they jettison the basic tool of our trade, competitive markets."

components from both standard models in macro and models in monetary theory that strive for better microfoundations.

To explain how this works, relative to reduced-form models, here are the ingredients that matter most for the results.

First, stochastic trading opportunities, like those in search models, are critical for matching some observations, including observations on velocity. These observations are notoriously hard to capture in cash-in-advance models, especially when calibrating to a shorter period length (see Telyukova and Visschers, 2009 for a recent discussion). Previous models of the reduced-form variety such as those mentioned above did not incorporate stochastic trading opportunities – which is not to say they couldn't, but simply to say that they didn't – and hence miss this point.

Second, the two-sector structure highlights a channel not in the models mentioned above. When capital produced in the centralized market is used in decentralized production, since inflation is a tax on decentralized trade, monetary policy affects centralized market investment. The transmission of inflation effects from the sector where goods are traded using money to a sector where inputs for these goods are produced, is new compared to reduced-form models.

Third, as explained in more detail below, the results depend in interesting ways on what one assumes about price formation in decentralized trade. If one uses bargaining, inflation has little impact on investment, although it has a sizable impact on consumption and welfare: going from 10% inflation to the Friedman rule barely changes capital, but the welfare gain is still worth around 3% of consumption. Alternatively, with price taking, the same experiment increases long-run capital by as much as 7%, and has a welfare effect between 1% and 1.5%. There is nothing in the reduced-form literature that considers bargaining, and hence this comparison has been missed.

Fourth, fiscal policy also matters: due to tax distortions, the first best outcome cannot be obtained even at the optimal monetary policy, which increases the cost of inflation under either bargaining or price taking. Although this is certainly not the first time this has been pointed out, and some of the papers mentioned above also include taxation, the interaction between taxation and the other key ingredients (stochastic trade, the two-sector structure, and alternative pricing mechanisms) has not been studied.

The intuition for why it matters whether one assumes price taking or bargaining is the following. When agents invest in capital they not only earn income in the centralized market, they also lower their production cost in the decentralized market. But there is a *holdup problem*, well known to practitioners of bargaining theory, if somewhat neglected in macro and growth theory (but see Caballero, 1999 for some discussion). Suppose the buyer gets a big share of the surplus in bilateral trade. Then the seller does not reap much of a return on his investment above what he gets in standard models, so the demand for capital does not depend much on what happens in decentralized trade, and inflation does not affect investment much. Now suppose the buyer has low bargaining power. Then the seller does get a big share of the surplus, but the surplus is small, due to a holdup problem on money demand. So whether buyer bargaining power is high or low, inflation has a small impact on investment. This depends on calibration, of course, but the impact is quite small for a wide range of parameters. Nonetheless, due to these holdup problems, decentralized market consumption is very low, so even though inflation does not have a huge effect on decentralized trade, due to concavity of the utility function it does have a sizable impact on welfare.

With price taking these holdup problems vanish. This means investment demand depends much more on what happens in the decentralized market. Since inflation is a tax on decentralized trade, it acts as a tax on investment. Thus monetary policy can have a big impact on capital formation. However, without the holdup problems, decentralized market consumption is not nearly so low, and thus when it decreases with inflation the net effect is less painful with competitive price taking than bargaining. The cost of bargaining inefficiencies are sizable. This is true even though bargaining is used only in the decentralized market, which accounts a small share of aggregate output for the calibrated parameter values. These results, about bargaining versus price taking, in models that are otherwise similar to standard macro are novel. However, to be clear, the goal is not to determine whether bargaining or price taking better fits the data – the goal is to lay out models with each and report how it matters. This is part of ongoing research trying to better understand how the details of the micro structure matter for macro and monetary economics.

In terms of the most closely related work in the area, a previous attempt to put capital into a similar monetary model by Aruoba and Wright (2003) lead to some undesirable implications, including the following dichotomy: one can solve independently for allocations in the centralized and decentralized markets. This implies monetary policy has no impact on investment, employment or consumption in the centralized market. This is *not* the case here. Other attempts to study money and capital in models with frictions include Shi (1999), Shi and Wang (2006), and Menner (2006), who build on Shi (1997), and Molico and Zhang (2005), who build on Molico (2006). Those models have only decentralized markets. It is much easier to connect with mainstream macro in a model with some centralized trade. Thus, as a special case, in nonmonetary equilibrium the model developed in this paper reduces to the textbook growth model, while those models reduce to something quite different. It is also worth emphasizing that with either bargaining or price taking the numerical results obtained using this model differ from the reduced-form literature. Here is a short survey: Cooley and Hansen (1989, 1991) find much smaller effects, with welfare numbers substantially below 1%. Gomme (1993) gets even smaller effects in an endogenous growth version of the model. Ireland (1994) gets welfare numbers around 0.67%. Lucas (2000), without capital, gets welfare numbers below 1%; earlier efforts at this approach by Lucas (1981) and Fischer (1981) get 0.3% to 0.45%. Imrohoroglu and Prescott (1991) also get less than 1%. Quantitatively, inflation matters a lot more in the framework developed in this paper.[3]

[3] To understand why most previous models generate such low costs of inflation, it helps to remember the

This paper makes a contribution to policy-relevant quantitative economics as well as to theory, in the sense that the model brings modern monetary economics much closer to the mainstream. At the very least this should facilitate communication between different camps in macro. The rest of the paper proceeds to make the case as follows. Section 2 describes the model. Section 3 lays out the calibration strategy. Section 4 presents the quantitative results. Section 5 concludes.[4]

2. Model

In this section, the model is presented. After a description of the environment, the agents' CM and DM problems are presented. Equilibrium for the bargaining and price taking versions of the model and a brief digression on banking concludes the section.

2.1. General Assumptions

A $[0, 1]$ continuum of agents live forever in discrete time. To combine elements of standard macro and search theory, we adopt the sectoral structure in Lagos and Wright (2005), hereafter LW. Each period agents engage in two types of economic activity. Some activity takes place in a frictionless centralized market, called the CM, and some takes place in a decentralized market, called the DM, with two main frictions: a *double coincidence problem*, and *anonymity*, which combine to make a medium of exchange essential.[5] Given that a medium of exchange is essential, one issue in monetary theory is to determine endogenously which objects serve this function (e.g. Kiyotaki and Wright, 1989). In order to focus on

envelope theorem. In a typical cash-in-advance setup, for instance, at the Friedman rule the first best is obtained, and so a relatively small inflation has only a second-order effect on welfare. That is not the case in this paper since, due to holdup problems, even the Freidman rule (and even with no tax distortions) does not achieve the first best. A few papers do find larger effects, such as Dotsey and Ireland (1996), because even though inflation does not affect capital very much it affects the amount of resources used in intermediation.

[4] An appendix with alternative models and detailed information about data used is available online at http://www.boraganaruoba.com and on the journal website.

[5] For formal discussions of essentiality and anonymity see Kocherlakota (1998), Wallace (2001) or Aliprantis et al. (2007).

other questions, however, other papers avoid this by assuming there is a unique storable asset that qualifies for the role. Since one obviously cannot assume a unique storable asset in a paper called "Money and Capital," a few words about the issue are in order.

A story along the lines of the "worker-shopper pair" used to motivate cash-in-advance constraints by Lucas (1980), extended based on time-honored ideas about currency having advantages in terms of *portability* and *recognizability,* is useful: First, in terms of portability, in the DM the agents have their capital physically fixed in place at production sites. Thus, when you want to buy something from someone you must visit their location, and since you cannot bring your capital, it cannot be used in payment. This use of spatial separation is in the spirit of the "worker-shopper" idea, but one really should go beyond this, in any model, and ask why claims to capital, or claims more generally, cannot overcome this problem of spatial separation. That is to say, the "worker-shopper" idea may rule out barter, but it is logically irrelevant for ruling out credit or other more sophisticated trading arrangements, and hence cash-in-advance models are not really well motivated by this story at all. A logically coherent theory needs some additional frictions – recognizability, in this case.

A stark version of the assumption that works is that agents can costlessly counterfeit claims, other than currency, say, because the monetary authority has a monopoly on the technology for producing hard-to-counterfeit notes. Given this, sellers no more accept claims to capital from anonymous buyers in the DM than they accept personal IOU's. Therefore money has a role to play in payments and exchange, even if capital is a storable factor of production. While this is not the place to go into all of the details concerning explicit information frictions and the notion of recognizability, there is ongoing research attempting to make this more rigorous (see Lester et al., 2010 and the references therein). And while by no means this is the last word on the coexistence of money and other assets, the story is at least logically coherent. It is also important to emphasize that it is by now well understood that, even if one allows capital to be used as a medium of exchange, money is still essential when the efficient stock of (portable and recognizable) capital is low, since otherwise agents overinvest (see Lagos and Rocheteau, 2008 and the references therein). The assumptions here guarantee that capital is not used as a medium of exchange for simplicity, and because it is the more relevant case during the period in question for the quantitative analysis in this paper; but there is certainly more to be done on this topic.

Moving on to the details of the specification, as in standard growth theory, in the CM there is a general good that can be used for consumption or investment, produced using labor H and capital K hired by firms in competitive markets. Profit maximization implies $r = F_K(K, H)$ and $w = F_H(K, H)$, where F is the technology, r the rental rate, and w the real wage. Constant returns implies equilibrium profits are 0. In the DM these firms do not operate, but an agent's own effort e and capital k can be used with technology $f(e, k)$ to produce a different good. Note that k appears as an input the DM, because when you go to a seller's location he has access to his capital, even though you do not have access to your capital. This is important – it is the fact that capital produced in the CM is productive in the DM that breaks the dichotomy mentioned in the Introduction, and this allows money to have interesting effects in the CM.

In the DM, each period with probability σ an agent discovers he is a *buyer*, which means he wants to consume but cannot produce, so he visits the location of someone that can produce; with probability σ he is a *seller*, which means he can produce but does not want to consume, so he waits at his location for someone to visit him; and with probability $1 - 2\sigma$ he is a *nontrader*, and he neither produces nor consumes. This taste-and-technology-shock specification is equivalent to bilateral random matching, where there is a probability σ of meeting someone that produces a good that you like, but the interpretation here fits better with the idea of spatial separation, where buyers visit sellers' locations. In some buyer-seller meetings, the former is able to pay with credit due in the next CM. As in many of the models surveyed in Williamson and Wright (2010a,b) and Nosal and Rocheteau (2010), these meetings are monitored, as opposed to anonymous. Let ℓ (for loan) be the payment made in the CM, measured in dollars without loss of generality, and assume that it is costlessly enforced. But credit is only available in meetings with probability $1-\omega$. With probability ω, the meeting is anonymous, or not monitored, and the seller requires cash.

Instantaneous utility for everyone in the CM is $U(x) - Ah$, where x is consumption and h labor. As in most applications of LW-style models, linearity in h reduces the complexity of the analysis considerably, although Rocheteau et al. (2008) show how to get the same simplification with general preferences by assuming indivisible labor and lotteries à la Rogerson (1988). Moreover, one can dispense with quasi-linear utility, or indivisible labor and lotteries, altogether as long as one is willing to use more sophisticated computational methods, as in Molico (2006) or Chiu and Molico (2010). In the DM, buyers enjoy utility $u(q)$, and sellers get disutility e, where q is consumption and e labor (normalizing the disutility of DM labor to be linear is a choice of units with no implications for the results). Assume u and U have the usual monotonicity and curvature properties. Solving $q = f(e,k)$ for $e = c(q,k)$, the function $c(.)$ denotes the utility cost of producing q given k. One can show that $c_q > 0$, $c_k < 0$, $c_{qq} > 0$ and $c_{kk} > 0$ under the usual assumptions on f, and $c_{qk} < 0$ if k is a normal input.

Government sets the money supply so that $M_{+1} = (1+\tau)M$, where $+1$ denotes next period. The policy instrument in this paper is τ. In steady state, inflation equals τ and the nominal rate is defined by the Fisher equation $1+i = (1+\tau)/\beta$, and hence either i or τ can be used as policy instruments. Government also consumes G, levies a lump-sum tax T, labor income tax t_h, capital income tax t_k, and sales tax t_x in the CM (sales taxes in the DM are omitted to ease the presentation, but it makes little difference for the results). Letting δ be the depreciation rate of capital, which is tax deductible, and p the CM price level, the government budget constraint is $G = T + t_h wH + (r-\delta)\, t_k K + t_x X + \tau M/p$ if M is interpreted as $M0$. Alternatively, if $M1$ is used as the relevant money stock, one must make an appropriate adjustment to the real revenue they earn from printing money (something that seems to have gone unnoticed in at least some of the previous literature).

2.2. Household's Problem

Let $W(m,k,\ell)$ be the value function for an agent in the CM holding m dollars and k units of capital and owing ℓ from the previous DM. Let $V(m,k)$ be the DM value function. Assuming agents discount between the CM and DM at rate $\beta \in (0,1)$, but not between the DM and CM, the problem in the CM is

$$W(m,k,\ell) = \max_{x,h,m_{+1},k_{+1}} \{U(x) - Ah + \beta V_{+1}(m_{+1},k_{+1})\} \tag{1}$$

subject to

$$(1+t_x)\,x = w\,(1-t_h)\,h + [1+(r-\delta)\,(1-t_k)]\,k - k_{+1} - T + \frac{m-m_{+1}-\ell}{p}. \tag{2}$$

One can adapt the discussion in LW to guarantee the concavity of the problem and interiority of the solution (or, in quantitative analysis, one can check it directly). Then, eliminating h using the budget, the first order conditions are

$$x \;:\; U'(x) = \frac{A\,(1+t_x)}{w\,(1-t_h)} \tag{3}$$

$$m_{+1} \;:\; \frac{A}{pw\,(1-t_h)} = \beta V_{+1,m}(m_{+1},k_{+1}) \tag{4}$$

$$k_{+1} \;:\; \frac{A}{w\,(1-t_h)} = \beta V_{+1,k}(m_{+1},k_{+1}). \tag{5}$$

Since (m,k,ℓ) does not appear in (4), for any distribution of (m,k,ℓ) across agents entering the CM, the distribution of (m_{+1},k_{+1}) exiting the CM is degenerate. Also, it is immediate that W is linear

$$W_m(m,k,\ell) = \frac{A}{pw\,(1-t_h)} \tag{6}$$

$$W_k(m,k,\ell) = \frac{A\left[1+(r-\delta)\,(1-t_k)\right]}{w\,(1-t_h)} \tag{7}$$

$$W_\ell(m,k,\ell) = \frac{-A}{pw\,(1-t_h)}. \tag{8}$$

Moving to the DM, the value of entering the DM is given by

$$V(m,k) = \sigma V^b(m,k) + \sigma V^s(m,k) + (1-2\sigma)W(m,k,0), \tag{9}$$

where $V^b(m,k)$ and $V^s(m,k)$ denote the values to being a buyer and to being a seller, which are given by

$$V^b(m,k) = \omega\left[u(q_b) + W\left(m-d_b,k,0\right)\right] + (1-\omega)\left[u(\hat{q}_b) + W\left(m,k,\ell_b\right)\right] \tag{10}$$

$$V^s(m,k) = \omega\left[-c(q_s,k) + W\left(m+d_s,k\right)\right] + (1-\omega)\left[-c(\hat{q}_s,k) + W\left(m,k,-\ell_s\right)\right]. \tag{11}$$

In these expressions q_b and d_b (q_s and d_s) denote the quantity of goods and dollars exchanged when buying (selling) for money, while $\hat{q}_b$ and ℓ_b ($\hat{q}_s$ and $-\ell_s$) denote the quantity and the value of the loan for the buyer (seller) when trading on credit. Given all this, it is now straightforward to derive

$$\begin{aligned} V_m(m,k) = {} & \frac{A}{pw\,(1-t_h)} + \sigma\omega\left[u'\frac{\partial q_b}{\partial m} - \frac{A}{pw\,(1-t_h)}\frac{\partial d_b}{\partial m}\right] \\ & + \sigma\omega\left[\frac{A}{pw\,(1-t_h)}\frac{\partial d_s}{\partial m} - c_q\frac{\partial q_s}{\partial m}\right] \end{aligned} \tag{12}$$

$$
\begin{aligned}
V_k(m,k) \;=\; & \frac{A\left[1+(r-\delta)(1-t_k)\right]}{w(1-t_h)} + \sigma\omega\left[u'\frac{\partial q_b}{\partial k} - \frac{A}{pw(1-t_h)}\frac{\partial d_b}{\partial k}\right] \\
& + \sigma\omega\left[\frac{A}{pw(1-t_h)}\frac{\partial d_s}{\partial k} - c_q\frac{\partial q_s}{\partial k} - c_k\right] \\
& + \sigma(1-\omega)\left[u'\frac{\partial \hat{q}_b}{\partial k} - \frac{A}{pw(1-t_h)}\frac{\partial \ell_b}{\partial k}\right] \\
& + \sigma(1-\omega)\left[\frac{A}{pw(1-t_h)}\frac{\partial \ell_s}{\partial k} - c_q(\hat{q}_s,k)\frac{\partial \hat{q}_s}{\partial k} - c_k(\hat{q}_s,k)\right].
\end{aligned} \tag{13}
$$

To complete the analysis, the terms of trade (q, d, $\hat{q}$ and ℓ) need to be specified, which in turn will determine the derivatives in (12) and (13).

Before doing so, however, consider as a benchmark the planner's problem when money is *not* essential:

$$J(K) = \max_{X,H,K_{+1},q} \left\{U(X) - AH + \sigma\left[u(q) - c(q,K)\right] + \beta J_{+1}(K_{+1})\right\} \tag{14}$$

subject to

$$X = F(K,H) + (1-\delta)K - K_{+1} - G. \tag{15}$$

Eliminating X, and again assuming interiority, the first order conditions are

$$q \;:\; u'(q) = c_q(q,K) \tag{16}$$

$$H \;:\; A = U'(X)F_H(K,H) \tag{17}$$

$$K_{+1} \;:\; U'(X) = \beta J'_{+1}(K_{+1}). \tag{18}$$

The envelope condition $J'(K) = U'(X)[F_K(K,H) + 1 - \delta] - \sigma c_k(q,K)$ implies

$$U'(X) = \beta U'(X_{+1})[F_K(K_{+1},H_{+1}) + 1 - \delta] - \beta\sigma c_k(q_{+1},K_{+1}). \tag{19}$$

From the first condition in (17), $q = q^*(K)$ where $q^*(K)$ solves $u'(q) = c_q(q,K)$. Then the paths for (K_{+1}, H, X) satisfy (19), the second first order condition in (17), and the constraint in (14).

This characterizes the first best, or FB for short.[6] Note the term $-\beta\sigma c_k(q_{+1}, K_{+1}) > 0$ in (19), which reflects the fact that investment affects DM as well as CM productivity because K is used in both sectors. If K did not appear in $c(q)$ the system would dichotomize: one could first set $q = q^*$, where q^* solves $u'(q) = c'(q)$, and then solve the other conditions independently for (K_{+1}, H, X). The fact that K is used in the DM and produced in the CM breaks this dichotomy. Here the assumption is that the same K used in both sectors, but the online appendix contains a version with two distinct capital goods in the CM and DM, as well as a version where K is used only in the CM but is produced and traded in the DM. As discussed in the Section on robustness, these variations do not affect the main results much.

2.3. Bargaining

Assume the DM terms of trade are determined by generalized Nash bargaining.[7] Consider first a nonmonitored meeting where trade requires cash. If the buyer's and seller's states are (m_b, k_b) and (m_s, k_s), (q, d) solves the generalized Nash bargaining problem with the bargaining power of the buyer given by θ and threat points given by continuation values. Since the buyer's payoff from trade is $u(q) + W(m_b - d, k_b, 0)$ and his threat point is $W(m_b, k_b, 0)$, by the linearity of W, his surplus is $u(q) - Ad/pw\,(1 - t_h)$. Similarly, the seller's surplus is

[6] As is standard, one can characterize the solution by the FOC and envelope condition, or replace the FOC for K_{+1} and envelope condition by the Euler equation and transversality. One can check when there is a unique steady state to which the planner's solution converges under the usual kind of assumptions. This is less straightforward for equilibria with distortions. In the working paper we show there is a unique steady state under price taking.

[7] At the suggestion of a referee we mention the following: Often Nash bargaining is motivated by arguing that it can be considered the reduced-form for an underlying strategic bargaining game (see e.g. Osborne and Rubinstein, 1990). But those strategic foundations do not generally apply in nonstationary situations like the one in this paper (see Coles and Wright, 1998 or Ennis, 2001). This means the Nash solution is a primitive here – there is no claim here that it can be derived from a strategic bargaining game in the usual manner.

$Ad/pw\,(1-t_h) - c(q,k_s)$. Hence the bargaining solution is

$$\max_{q,d} \left[u(q) - \frac{Ad}{pw\,(1-t_h)}\right]^{\theta} \left[\frac{Ad}{pw\,(1-t_h)} - c(q,k_s)\right]^{1-\theta} \text{ s.t. } d \leq m_b. \tag{20}$$

As in LW, it is easy to show that in equilibrium $d = m_b$. Inserting this and taking the first order condition with respect to q,

$$\frac{m_b}{p} = \frac{g(q,k_s)w\,(1-t_h)}{A}, \tag{21}$$

where

$$g(q,k_s) \equiv \frac{\theta c(q,k_s)u'(q) + (1-\theta)u(q)c_q(q,k_s)}{\theta u'(q) + (1-\theta)c_q(q,k_s)}. \tag{22}$$

Writing $q = q(m_b, k_s)$, where $q(\cdot)$ is given by (21), the relevant derivatives in (12) and (13) are $\partial d/\partial m_b = 1$, $\partial q/\partial m_b = A/pw\,(1-t_h)\,g_q > 0$ and $\partial q/\partial k_s = -g_k/g_q > 0$, where

$$g_q = \frac{u'c_q[\theta u' + (1-\theta)c_q] + \theta(1-\theta)(u-c)[(u'c_{qq} - c_q u'')}{[\theta u' + (1-\theta)c_q]^2} > 0 \tag{23}$$

$$g_k = \frac{\theta u' c_k\,[\theta u' + (1-\theta)c_q] + \theta(1-\theta)(u-c)u'c_{qk}}{[\theta u' + (1-\theta)c_q]^2} < 0. \tag{24}$$

Now consider a meeting where credit is available, assuming the buyer has the same bargaining power θ. Then $(\hat{q}, \ell)$ is determined just like (q,d) above, except that there is no constraint on ℓ, the way $d \leq m_b$ needed to hold in monetary trades. Hence, the solution is given by

$$u'(\hat{q}) = c_q(\hat{q}, k_s) \tag{25}$$

$$\frac{A\ell}{pw\,(1-t_h)} = (1-\theta)\,u(\hat{q}) + \theta c(\hat{q}, k_s). \tag{26}$$

Given $k_s = K$, notice that $\hat{q}\,(K)$ is the same as the solution to the planner's problem $q^*(K)$, (bilateral credit transactions are efficient, conditional on K). It is now easy to take the relevant derivatives and insert them into (12) and (13). After imposing $(m, k) = (M, K)$, this delivers

$$V_m(M,K) = \frac{(1-\sigma\omega)A}{pw\,(1-t_h)} + \frac{\sigma\omega A u'(q)}{pw\,(1-t_h)\,g_q(q,K)} \tag{27}$$

$$V_k(M,K) = \frac{A\left[1+(r-\delta)\,(1-t_k)\right]}{w\,(1-t_h)} - \sigma\omega\gamma(q,K) - \sigma\,(1-\omega)\,(1-\theta)\,c_k(\hat{q},K) \tag{28}$$

where it is understood that $q = q(M, K)$ and $\hat{q} = \hat{q}(K)$, while

$$\gamma(q,K) \equiv c_k(q,K) + c_q \frac{\partial q}{\partial K} = c_k(q,K) - c_q\,(q,K)\,\frac{g_k(q,K)}{g_q(q,K)} < 0. \tag{29}$$

The last two terms in (28) capture the idea that if a seller has an extra unit of capital it affects marginal cost in the DM, which augments the value of investment in the CM.[8]

Substituting (27) and (28), as well as prices $p = AM/w\,(1-t_h)\,g(q,K)$, $r = F_K(K,H)$, and $w = F_H(K,H)$, into the first order condition for m_{+1} and k_{+1}, the two equilibrium conditions are

$$\frac{g(q,K)}{M} = \frac{\beta g(q_{+1},K_{+1})}{M_{+1}}\left[1-\sigma\omega+\sigma\omega\frac{u'(q_{+1})}{g_q(q_{+1},K_{+1})}\right] \tag{30}$$

$$\begin{aligned} U'(X) = {} & \beta U'(X_{+1})\left\{1+\left[F_K(K_{+1},H_{+1})-\delta\right](1-t_k)\right\} \\ & -\beta\,(1+t_x)\,\sigma\left[\omega\gamma(q_{+1},K_{+1})+(1-\omega)\,(1-\theta)\,c_k(\hat{q}_{+1},K_{+1})\right]. \end{aligned} \tag{31}$$

Two other conditions come from the first order condition for X and the resource constraint,

$$U'(X) = \frac{A\,(1+t_x)}{(1-t_h)\,F_H(K,H)} \tag{32}$$

$$X + G = F(K,H) + (1-\delta)K - K_{+1}. \tag{33}$$

An *equilibrium with bargaining* is defined as (positive, bounded) paths for (q, K_{+1}, H, X)

[8] The expression in (29) captures non-price-taking behavior in the bargaining model: the first term reflects the cost reduction due to extra capital, and the second reflects the change in cost due to the change in the terms of trade when sellers have more capital.

satisfying (30)-(33), given policy and the initial condition K_0.

We are mostly interested in *monetary* equilibrium, with $q > 0$ at every date. But consider for a moment *nonmonetary* equilibrium, with $q = 0$ at all dates. In this case, (K_{+1}, H, X) solves (31)-(33) with $\gamma = 0$, which is exactly the equilibrium for a standard neoclassical growth model, as mentioned in the Introduction. Also, notice that if capital is not used in the DM, then $c(q, K) = c(q)$ and $\gamma(q, K) = c_k(q, K) = 0$. This version dichotomizes, and since M appears in (30) but not (31)-(33), monetary policy affects q but not (K_{+1}, H, X) or $\hat{q}$. Equilibrium does not dichotomize when K enters $c(q, K)$. Notice however that if $\theta = 1$ then, although K enters $c(q, K)$, (31)-(33) can be solved for (K_{+1}, H, X), then (30) determines q since $\gamma(q, K) = 0$. So if $\theta = 1$ money still does not influence CM variables, even though anything that affects the CM (e.g. taxes) influences q. Intuitively, when $\theta = 1$ sellers do not get any of the surplus from DM trade, and so investment decisions are based solely on returns to K that accrue in the CM. Looking at (29), when $\theta = 1$, the cost reduction due to having more capital is exactly matched by the increase in cost due to higher production.

This is an extreme version of a holdup problem in the demand for capital. More generally, for any $\theta > 0$, sellers do not get the full return on capital from DM trade, and hence they underinvest. This holdup problem is not present in most standard macro models, and constitutes a distortion over and above those from taxes and monetary inefficiencies. Even under the Friedman Rule (FR) where $i = 0$ and with only lump-sum taxes, the holdup problem on capital and a related problem on money emphasized in LW remains. In some models all holdup problems can be resolved if one sets bargaining power θ correctly. This is *not* possible here: $\theta = 1$ resolves the problem in the demand for money, but this is the worst case for investment; and $\theta = 0$ resolves the problem in the demand for capital, but this this is the worst case for money. There is no θ that can eliminate the double holdup problem, which has implications for both the empirical performance of bargaining models and their welfare implications.

2.4. Price Taking

While the holdup problems cannot simultaneously be solved by bargaining, some other solution concepts work much better. For example, it is by now well known that *competitive search equilibrium*, based on directed search and price posting, rather than random matching and bargaining, resolves multiple holdup problems (see e.g. Acemoglu and Shimer, 1999 or Mortensen and Wright, 2002). And *competitive equilibrium* with Walrasian price taking also does the job here, even though this is not true in general (for example Rocheteau and Wright, 2005 show that competitive search equilibrium can do better than competitive equilibrium in environments with search externalities, but there are no such externalities in the model). Since it is easier to present, relative to price posting with directed search, in this section we consider price taking.[9]

For simplicity assume that there are two distinct trading locations in the DM, one for anonymous traders where cash is needed, and one where credit is available. Agents do not get to choose, but are randomly assigned to one location. The DM value function then has the same form as (9), but now, in the location with anonymous meetings

$$V^s(m,k) = \max_q \{-c(q,k) + W(m+\tilde{p}q,k,0)\} \tag{34}$$

$$V^b(m,k) = \max_q \{u(q) + W(m-\tilde{p}q,k,0)\} \text{ s.t. } \tilde{p}q \le m, \tag{35}$$

[9] At the suggestion of a referee we emphasize the following: One can think of the two models – the one with bargaining and the one with price taking – as representing two alternative environments, one with bilateral meetings and one with multilateral meetings. In the former case it makes sense to let agents bargain, while in the latter it makes sense to assume they take as given the price that clears the market as is standard in competitive equilibrium. By analogy, one could think about the labor market search models of Mortensen and Pissarides (1994), with bilateral meetings and bargaining, and Lucas and Prescott (1974), with multilateral meetings and price taking. However, it is important to emphasize that the only impact of assuming multilateral matching here is to motivate (Walrasian) price taking, and it has no implications for the set of allocations that are feasible, since all agents with the same trading status (buyer versus seller) in DM are identical. This would not be the case in some search models of exchange (e.g. Kiyotaki and Wright, 1989), where the switch from multilateral to bilateral meetings would make a big difference for feasible allocations.. Alternatively, one can simply interpret the analysis of the price-taking model in this paper as an analytic short cut to deriving the allocation that obtains in compeitive search equilibrium, which does not require multilateral meetings.

where $\tilde{p}$ is the price (which generally differs from the CM price p), and in the location with monitored meetings

$$\hat{V}^s(m,k) = \max_{\hat{q}} \{-c(\hat{q},k) + W(m,k,-\hat{p}\hat{q})\} \tag{36}$$
$$\hat{V}^b(m,k) = \max_{\hat{q}} \{u(\hat{q}) + W(m,k,\hat{p}\hat{q})\}. \tag{37}$$

The first order condition for the sellers in the two DM locations are

$$c_q(q,k) = \tilde{p}W_m = \tilde{p}A/pw(1-t_h) \tag{38}$$
$$c_q(\hat{q},k) = -\hat{p}W_\ell = \hat{p}A/pw(1-t_h). \tag{39}$$

Market clearing implies buyers and sellers choose the same q, and the same $\hat{q}$ As in the previous model, in the anonymous market buyers spend all their money so $q = M/\tilde{p}$. Inserting $\tilde{p} = M/q$, the analog to (21) from the bargaining model is given by

$$\frac{M}{p} = \frac{qc_q(q,k)w(1-t_h)}{A}. \tag{40}$$

Similarly, when credit is available, $\hat{q} = \hat{q}(K)$, as in the bargaining model, but now $\ell = pw(1-t_h)u'(\hat{q})\hat{q}/A$. Then the analogs to (27) and (28) are

$$V_m(M,K) = \frac{(1-\sigma\omega)A}{pw(1-t_h)} + \frac{\sigma\omega u'(q)}{\tilde{p}} \tag{41}$$
$$V_k(M,K) = \frac{A + A(r-\delta)(1-t_k)}{w(1-t_h)} - \sigma\omega c_k(q,K) - \sigma(1-\omega)c_k(\hat{q},K). \tag{42}$$

Inserting these into (4) yields the analogs to (30) and (31)

$$\frac{c_q(q,K)q}{M} = \frac{\beta c_q(q_{+1},K_{+1})q_{+1}}{M_{+1}}\left[1 - \sigma\omega + \sigma\omega\frac{u'(q_{+1})}{c_q(q_{+1},K_{+1})}\right] \tag{43}$$
$$U'(X) = \beta U'(X_{+1})\{1 + [F_K(K_{+1},H_{+1}) - \delta](1-t_k)\} \tag{44}$$
$$-\beta(1+t_x)\sigma[\omega c_k(q_{+1},K_{+1}) + (1-\omega)c_k(\hat{q}_{+1},K_{+1})].$$

The other equilibrium conditions (32)-(33) are the same as above.

An *equilibrium with price taking* is given by (positive, bounded) paths for (q, K_{+1}, H, X) satisfying (43)-(44) and (32)-(33), given policy and K_0. The difference between bargaining and price taking is the difference between (30)-(31) and (43)-(44). The equilibrium condition for q here looks like the one from the bargaining model when $\theta = 1$, and the condition for K looks like the one from the bargaining model when $\theta = 0$, indicating that price taking avoids both holdup problems.[10]

2.5. A Digression on the Relevant Concept of Money

At first blush, it might seem the relevant notion of money here is $M0$, but that is not the only interpretation. Although it has not yet been done it a fully satisfactory way, one can imagine introducing banks into the model following the approach in Berentsen et al. (2007) (see also He et al., 2008 and Chiu and Meh, 2010). Assume that after production and exchange stops in the CM, at which point agents have decided their m_{+1}, it is revealed which ones want to consume and which ones are able to produce while banks are still open but before the DM convenes. As the sellers have no use for money, they deposit it in banks, who then lend it to buyers, at interest. One can think of banks either lending out the same physical currency, or perhaps keeping that in the vault and issuing bank-backed securities usable for payments (assuming these are not easily counterfeitable). However, in neither case does one get anything like the "money multiplier" from undergraduate monetary economics that would allow one to take seriously the relationship between $M0$ and $M1$.

Some of the models discussed in Williamson and Wright (2010a,b) or Nosal and Ro-

[10] To show this formally, set $t_k = t_h = t_x = 0$. Then under price taking the equilibrium conditions for (K_{+1}, H, X) are the same as those for the planner problem. Hence the equilibrium coincides with the FB iff $u'(q) = c_q(q, K)$. From (43), this means $c_q(q, K)q/M = \beta c_q(q_{+1}, K_{+1})q_{+1}/M_{+1}$. Using (40) this reduces to $1/pw = \beta/p_{+1}w_{+1}$. Since $w = A/U'(X)$, it further reduces to $p/p_{+1} = U'(x)/\beta U'(X_{+1})$. Since in any equilibrium the slope of the indifference curve $U'(x)/\beta U'(X_{+1})$ equals the slope of the budget line $1 + \rho$, with ρ equal to the real interest rate, the relation in question finally reduces to $p_{+1}/p = 1/(1 + \rho)$. Using the Fisher equation, this holds and hence $q = q^*(K)$ solves (43) iff the nominal rate is set as to zero, i.e. $i = 0$. This implies that under price taking with lump-sum taxes, setting $i = 0$ yields efficiency.

cheteau (2010) are better in this regard, but still provide nothing like a definitive banking setup that can be inserted seamlessly into the environment here. The model in He et al. (2005) actually does generate an explicit "money multiplier" very much like the one in undergraduate economics, but only for second-generation search models of monetary exchange. Second-generation models assume for technical reasons that assets, including currency, are indivisible, making them ill suited for quantitative analyses like the one in this paper. It is not hard to see why a model with indivisible assets might generate a role for inside (bank) money – simply put, there may not be enough outside money to go around – and to see why this can lead to a "money multiplier." This is more difficult to capture formally when money is divisible. Evidently, much more work remains to be done in order to address issues related to financial intermediation in these kinds of models.

Having said that, in the quantitative work, we do not necessarily want to take M to be currency per se. Results for several measures of money, including $M0$, $M1$, and so on, are presented below and the reader can pick and choose as desired. But $M1$ – actually, $M1S$, the so-called sweep-adjusted version of the series – is perhaps most relevant, for several reasons. First, $M1$ is the measure used by most of the previous studies cited above, and so its use here facilitates comparisons. Second, when the fraction of DM trades where credit is available is calibrated using micro data, monetary trade is interpreted to include all transactions that use cash, check and debit card, but not credit card, purchases. This is based on two criteria: (a) checks and debit cards can be thought of (simplistically?) as convenient ways to access deposits, which like cash have the property that they are very liquid and pay 0 or close to 0 interest; and (b) the relevant feature of credit cards, like credit in general, is that they allow you to consume now and work later, while with either cash or demand deposits you have to raise the funds before you spend them (as discussed by Dong, 2008).

There is a tension here, and indeed there is a tension whenever one tries to implement monetary theory empirically, irrespective of the extent to which the theory has any claim to microfoundations. To quote Lucas (2000, p.270):

> Another set of questions about the time series estimates concerns the fact that $M1$ – the measure of money that I have used – is a sum of currency holdings that do not pay interest and demand deposits that (in some circumstances) do. Moreover, other interest bearing assets beside these may serve as means of payment. One response to these observations is to formulate a model of the banking system in which currency, reserves, and deposits play distinct roles. Such a model seems essential if one wants to consider policies like reserve requirements, interest on deposits, and other measures that affect different components of the money stock differently.
>
> A second response to the arbitrariness of $M1$ ... is to replace $M1$ with an aggregate in which different monetary assets are given different weights. The basic idea, as proposed in Barnett (1978,1980), and Poterba and Rotemberg (1987), is that if a treasury bill yielding 6 percent is assumed to yield no monetary services, then a bank deposit yielding 3 percent can be thought of as yielding half the monetary services of a zero-interest currency holding of equal dollar value. Implementing this idea avoids the awkward necessity of classifying financial assets as either entirely money or not monetary at all, and lets the data do most of the work in deciding how monetary aggregates should be revised over time as interest rates change and new instruments are introduced.

Having understood this, Lucas still uses $M1$, and suggests that it may not be too bad an approximation for the issues at hand and the period under consideration (although perhaps not for all issues or all periods). The use of the sweep-adjusted data $M1S$ in this paper is somewhat parallel to the second approach he mentions but this does not solve all of the issues in terms of measurement.[11] And we very much agree with the first idea, to model

[11] By way of example, Lucas in the quotation takes it for granted that "a treasuly bill yielding 6 percent is

banking and payments more seriously, although obviously this is far from a trivial exercise in terms of theory.

3. Quantitative Analysis

We now turn to describing the quantitative methodology. Since the model is essentially a two-sector model, a careful accounting of aggregate variables is necessary which is presented in the next section, followed by a description of the calibration strategy.

3.1. Preliminaries

The price levels in the CM and DM are p and $\tilde{p} = M/q$, respectively, where p satisfies

$$p = \frac{AM}{(1 - t_h)\, g\,(q, K)\, F_H(K, H)} \tag{45}$$

in the bargaining version of the model by (21), and

$$p = \frac{AM}{(1 - t_h)\, qc_q\,(q, K)\, F_H(K, H)} \tag{46}$$

in the price-taking version by (40). Nominal output is $pF\,(K, H)$ in the CM, and $\sigma\omega M + \sigma(1-\omega)\ell$ in the DM. Using p as the unit of account, real output in the CM is $Y_C = F\,(K, H)$ and in the DM is $Y_D = \sigma\omega M/p + \sigma(1-\omega)\sigma\ell/p$. Total real output is $Y = Y_C + Y_D$.

Define the share of output produced in the DM by $s_D = Y_D/Y$, the share of output where money is essential by $s_M = Y_M/Y$ where $Y_M = \sigma\omega M/p$, and the share where credit is used by $s_\ell = Y_\ell/Y$ where $Y_\ell = \sigma(1-\omega)\ell/p$. These shares are not calibrated, but they are indirectly computed from other variables. To see how, note that velocity is $v = pY/M = \sigma\omega Y/Y_M$.

assumed to yield no monetary services," while recent work by Krishnamurthy and Vissing-Jorgensen (2009) puts these self-same T Bills into households' utility functions in order to capture in a reduced-form way their "convenience yields." If this it is meant to stand in for anything at all, presumably it stands for an assets' liquidity or monetary services.

Hence, $s_M = Y_M/Y = \sigma\omega/v$. The maximum σ can be is 1/2, and the maximum ω can be is 1, so given $M1$ velocity is around 5, s_M is bounded above by 10%. In fact, given the calibrated parameters it is actually even smaller. There are two points to emphasize. First, to think about the size of the different sectors, one does not have to take a stand on which goods are traded in each. Second, the results presented below do not depend on having an excessive amount of monetary trade – at least 90% of economic activity looks just like what one sees in nonmonetary models.

The markup μ, is an aggregate of markups (price over marginal cost) in the two markets. The markup in the CM market is 0, since it is competitive. The markup in the DM under price taking is also 0. With bargaining, the markup in the DM is derived as follows. First consider monetary trades. Marginal cost in terms of utility is $c_q(q,K)$. Since a dollar is worth $A/p(1-t_h)w$ utils, marginal cost in dollars is $c_q(q,K)\,p(1-t_h)\,w/A$. Since the price is M/q, the markup in monetary trade is given by

$$1+\mu_M = \frac{M/q}{c_q(q,K)\,p(1-t_h)\,w/A} = \frac{g(q,K)}{qc_q(q,K)}, \tag{47}$$

after eliminating M using (45). Similarly, the markup in credit trade in the DM is

$$1+\mu_\ell = \frac{\ell/\hat{q}}{c_q(\hat{q},K)\,p(1-t_h)\,w/A} = \frac{(1-\theta)\,u(\hat{q})+\theta c(\hat{q},k_s)}{\hat{q}c_q(\hat{q},K)}. \tag{48}$$

The average markup in the DM is then $\mu_D = \omega\mu_M + (1-\omega)\,\mu_\ell$, while the average markup for the whole economy is $\mu = s_D\mu_D$.

3.2. Calibration

Consider the following functional forms for preferences and technology: in the CM $U(x) = B\left[x^{1-\varepsilon}-1\right]/(1-\varepsilon)$ and $F(K,H) = K^\alpha H^{1-\alpha}$; in the DM $u(q) = C\left[(q+b)^{1-\eta}-b^{1-\eta}\right]/(1-\eta)$ and $c(q,k) = q^\psi k^{1-\psi}$.The cost function $c(\cdot)$ comes from the technology $q = e^{1/\psi}k^{1-1/\psi}$; if

$\psi = 1$ then the model dichotomizes. The parameter b in $u(q)$ is introduced merely so that $u(0) = 0$, which is useful for technical reasons, and is set to $b = 0.0001$. This means relative risk aversion is not constant, but if $b \approx 0$, it is approximately constant at $\eta q/(q + b) \approx \eta$. Risk aversion parameters are set to $\varepsilon = \eta = 1$ as a benchmark, to facilitate comparison with the literature, and also because one can show these choices are consistent with a balanced growth path in an extended version of the model with long run technical change (see Waller, 2010). In any case, the results are robust to these choices, as discussed below. C is normalized at $C = 1$, with no loss in generality.

In terms of calibrating the remaining parameters, we begin with a heuristic description, and then provide details. It is useful to point out that the approach here is a natural extension of standard methods. To pick a typical application, Christiano and Eichenbaum (1992) study the one-sector growth model, parameterized by $U = \log(x) + A(1 - h)$ and $Y = K^\alpha h^{1-\alpha}$ for their indivisible-labor version; for their divisible-labor version replace $A(1-h)$ by $A \log(1-h)$. One calibrates the parameters as follows: Set the discount factor $\beta = 1/(1 + \rho)$ where ρ is some observed average interest rate. Then set depreciation $\delta = I/K$ to match the investment-capital ratio. Then set α to match *either* labor's share of income LS or the capital-output ratio K/Y, since these yield the same result given there are no taxes (see below). Finally, set A to match observed average hours worked h.

This method can be adapted to many scenarios. For example, Greenwood et al. (1995) calibrate a two-sector model, with home production, as follows. Consider $U = \log(x) + A(1 - h_m - h_n)$, $Y_m = K_m^{\alpha_m} h_m^{1-\alpha_m}$ and $Y_n = K_n^{\alpha_n} h_n^{1-\alpha_n}$, where $x = [Dx_m^\kappa + (1 - D)x_n^\kappa]^{1/\kappa}$, and x_m, h_m and k_m are consumption, hours and capital in the market while x_n, h_n and k_n are consumption, hours and capital in the nonmarket or home sector. The two-sector version of the standard method is this: again set $\beta = 1/(1 + \rho)$; set δ_m and δ_n to match I_m/K_m and I_n/K_n; set α_m and α_n to match K_m/Y_m and K_n/Y_m; and set A and D to match h_m and h_n. This leaves κ, which is hard to pin down based on steady state observations, and is therefore typically set based on direct estimates of the relevant elasticities.

Since the model in this paper is also a two-sector model, a variant of the home-production method is appropriate. Thus, first set β, δ and A as above. Then set α and ψ to match *both* K/Y and LS. In the standard one-sector model, without taxes, it does not matter if one calibrates α to LS or K/Y, but with taxes calibrating α to LS yields a value for K/Y that is too low (Greenwood et al., 1995; Gomme and Rupert, 2007). The idea here is to set α to match LS, then try to use ψ to match K/Y, since DM production provides an extra kick to the return on K. Given this, the utility parameter B and probabilities σ and ω are set to match some money demand observations, as discussed below, which is the analog of picking κ in home production framework, and is similar to what is done in any calibrated monetary model. This completes the heuristic description.

An online appendix describes in more detail the data used to obtain the calibration targets, and a summary is provided here. The benchmark model is annual, but as discussed below, the results are basically the same for quarterly and monthly calibrations (which is a big advantage over the typical cash-in-advance model, as mentioned in the introduction). The benchmark calibration period is 1959-2004 and some alternatives are considered below. Model-consistent measures from the data are used, where available. For example, the definition of GDP excludes consumption expenditures on durables and net exports since these are not explicitly modeled. Also the measure of money is the sweep-adjusted measure for $M1$, which have some distinct advantages, as discussed in Cynamon et al. (2006). Table 1 lists the calibration targets and parameters.

Some parameters can be directly pinned down: $\beta = 1/(1+\rho)$ with $\rho = 0.028$; $t_h = 0.251$ and $t_k = 0.533$; $t_x = 0.069$; $G/Y = 0.241$; $\delta = I/K = 0.070$; $\alpha = 0.293$ to get $LS = 0.707$.[12]

[12] ρ is the annual after-tax real interest rate based on an average pre-tax nominal rate on Aaa-rated corporate bonds of 7.8%, an inflation rate from the GDP deflator of 3.7%, and a tax on real bond returns of 30% from the NBER TAXSIM model. As is standard, a bond market is not included in the definition of equilibrium, but bonds can be priced in the usual way. t_h and t_k are the average effective marginal tax rates in McGrattan et al. (1997) (Gomme and Rupert, 2007 report similar numbers); t_x is the average of excise plus sales tax revenue divided by consumption, LS is obtained using the method in Prescott (1986).

In order to pin down the fraction ω of DM trades where credit is not available, two sources are used. First, Klee (2008) finds that shoppers use credit cards in 12% of total transactions in the supermarket scanner data. The remaining transactions use cash, checks and debit cards which, recall from the digression on banking, fit with the notion of money in the model. The DM does not literally correspond to supermarket shopping, but since this is the best available data, it is nevertheless informative. Second, using earlier consumer survey data, Cooley and Hansen (1991) come up with a similar measure of around 16%. Thus ω is calibrated using $\omega = 0.15$, which seems to be a good compromise, but it turns out that over a reasonable range ω does not matter much.

[Table 1 About Here]

This determines all the parameters in panel (a) of Table 1. The remaining ones in panel (b) are: A and B from utility, the cost parameter ψ, the probability of being a buyer σ, and, in the bargaining model, θ. These parameters are determined simultaneously to match the following targets. First, the standard measure of work as a fraction of discretionary time $H = 1/3$. Second, average velocity $v = 5.381$. Third, $K/Y = 2.337$. Fourth, a money demand semi-elasticity of $\xi = -0.064$.[13] Fifth, in models with bargaining a DM markup of 0.3 is targeted.[14] These parameters are calibrated simultaneously to minimize the squared percentage distance between the targets in the data and model with equal weights on each target. All of the targets, except for the money demand semi-elasticity, can be directly obtained using straightforward formulas. The semi-elasticity is computed using the change

[13] As explained in the Appendix, the money demand relationship is estimated using the cointegration methods in Stock and Watson (1993). The resulting semi-elasticity estimate of -0.064 is perfectly in line with other estimates: Stock and Watson (1993) get semi-elasticities between -0.05 and -0.10; Ball (2001) gets -0.05; and Lucas (2000) argues that -0.07 fits the data best.

[14] Since this is somewhat novel, our markup target uses the evidence discussed by Faig and Jerez (2005) from the Annual Retail Trade Survey of retail establishments. At the low end, Warehouse Clubs and Superstores come in around 17%, Automotive Dealers 18%, and Gas Stations 21%. At the high end Specialty Foods come in at 42%, Clothing and Footware 43%, and Furniture 44%. The value used, $\mu_D = 0.3$, is in the middle of the data, but the robustness discussion shows that this does not matter much.

in money demand when the interest rate changes from $i + 0.5$ to $i - 0.5$ where i is the benchmark value. This concludes the baseline calibration strategy. In Section 4.3, two variations on this baseline are discussed and while the details differ the results are very similar.

3.3. Decision Rules

All nominal variables are scaled by M, so that $\hat{m} = m/M$, $\hat{p} = p/M$ etc. Then the individual state becomes $(\hat{m}, k, K)$, where in equilibrium $\hat{m} = 1$ and $k = K$. Although the above presentation was more general, the recursive equilibrium is given by time-invariant decision rules $[q(K), K_{+1}(K), H(K), X(K)]$ and value functions $[W(K), V(K)]$ solving the relevant equations. These equations are solved numerically using a nonlinear global approximation, which is important for accurate welfare computations.

4. Results

This section presents the quantitative results obtained using the methodology outlined above.

4.1. Calibration Results

[Table 2 About Here]

In Table 2, one column lists the relevant moments in the data, while the others list moments from three specifications of the model. Model 1 uses bargaining in the DM with bargaining power $\theta = 1$, giving up on the DM markup μ_D as a target; it is presented mainly as a benchmark since as proved earlier, when $\theta = 1$ money cannot affect the CM variables at all. Model 2 uses bargaining with θ calibrated along with the other parameters. Model 3 uses price taking in the DM, so there is no θ, calibrating the rest of parameters to match the targets other than the markup. The targets are matched with two exceptions. First, the DM markup μ_D can be matched only under bargaining and calibrate θ, rather than fixing it at 1 or assuming price taking, for obvious reasons. Second, K/Y can be matched only in the price-taking model, for reasons that we now explain.

Intuitively, the calibration sets the CM technology parameter α to match LS and then tries to hit K/Y using the technology parameter ψ (although this way of looking at things is instructive, it is meant only to be suggestive, since in fact all parameters are calibrated simultaneously). When $\psi = 1$, K is not used in the DM, and K/Y is too low, as in the standard model once taxes are introduced. As ψ increases above 1, the return on K from its use in the DM increases and hence so does K/Y. But, in practice, with bargaining, this effect is tiny because the holdup problem eats up most of the DM return on K. Of course, this depends on bargaining power, but even if θ is picked to maximize K/Y, this still is not enough. Intuitively, if θ is big then buyers have all the bargaining power, which makes q big, other things being equal, but gives little return from DM trade to sellers; and if θ is small then sellers have all the bargaining power, which gives them a big share of the return, but only on a very small q. There is no way around it with bargaining. With price taking, the holdup problems vanish and ψ can be picked to match K/Y exactly.

All models deliver a DM share s_D of only around 3%. At first this may seem to imply that the DM is very small and hence it cannot have a significant impact on welfare. As the results below show, however, this is not true for reasons explained below. Also, because s_D is relatively small the aggregate markup for Model 2 is only around 1%.[15] This is lower than the numbers some macroeconomists use, but remember that the CM has no markups.[16] In any case, the robustness discussion shows that the results do not hinge much on μ_D. For

[15] For Model 1, i.e. when $\theta = 1$ the markup is actually negative in Table 2, because take-it-or-leave-it offers by buyers means price equals AC which is below MC. Hence, just to get $\mu_D > 0$, θ needs to be significantly below 1.

[16] Aruoba and Schorfheide (2011) introduce markups in the CM by incorporating monopolistic competition, calibrating to around 15% in each sector.

example an aggregate markup target of 10% yield similar results.

Finally, Table 2 also reports the semi-elasticity of investment (or capital) with respect to the interest rate (or inflation), denoted ζ. This can be used as a testable implication of the model since it is not directly targeted. It is especially useful as such since the main focus in this paper is on the effect of inflation on capital accumulation. As discussed in the online appendix, the empirical counterpart of this is -0.006, which means that a one percentage point increase in inflation reduces investment by around 0.6%. Model 1 delivers exactly $\zeta = 0$ since as a matter of theory this specification implies inflation has no impact on capital accumulation. Model 2 delivers $\zeta = -0.0001$, due to the holdup problem explained above. Model 3 delivers $\zeta = -0.004$ which is fairly close to the empirical counterpart. Section 4.3 also shows results where ζ is added to the list of calibration targets. To conclude, the model, especially the price-taking version is very much in line with the data in terms of its implications for the effect of inflation on capital accumulation.

4.2. Policy Experiments

In the experiments considered in this section, starting in a steady state, make a once-and-for-all change in the growth rate of money τ and track the behavior of the economy over time. Since inflation in steady state equals τ, with a slight abuse of language, the experiments are presented as a change in inflation, but note that inflation actually does not jump to the new steady state level in the short run (i.e. inflation may not equal τ during the transition).

4.2.1. 10% Inflation to the Friedman Rule

[Table 3 About Here]

Table 3 contains results for a common experiment in the literature where τ changes from $\tau^1 = 0.1$ to the FR, which is $\tau^2 = -0.027$ for the baseline calibration. For now, any change in government revenue is made up using the lump-sum tax T, but other other fiscal options are considered below. Table 3 presents ratios of equilibrium values of several variables at the two inflation rates.

The first thing to note is that q^1/q^2 is considerably less than 1. Looking across models, q is considerably lower in Model 2 than the other specifications, reflecting the impact of the money holdup problem. Intuitively, inflation is a tax on DM activity, and these results show that this tax is quantitatively very important for q. In Model 1 this is the only effect, since $\theta = 1$ implies monetary policy has no impact on the CM. In Model 2, monetary policy does affect the CM, in principle, but the impact is tiny as one should expect from the discussion in Section 4.1. Models 1 and 2 predict that going to the FR increases aggregate output Y by 2%, essentially all due to the change in q. In Model 3 the effects are very different. Now K changes and by a lot, around 7%. This makes CM consumption X change by about 2%, and the net impact on Y is 4%.

Now consider welfare. As is standard, welfare is measured by the required percentage increase in the agents' consumption under the high τ regime that makes the agents indifferent between the two τ regimes. The table shows the answer comparing across steady states – jumping instantly from τ^1 and K^1 to τ^2 and K^2 – as well as the cost of the transition from K^1 to K^2 and the net gain to changing τ starting at K^1. This net gain is the true benefit of the policy change, although the steady state comparison is also interesting (it shows how much an agent facing τ^1 and K^1 would pay to trade places with someone facing τ^2 and K^2). In Model 1 there is no transition since τ does not affect K, and in Model 2 it is unimportant, since τ does not affect K much, but in Models 3 the transition is significant. The table also reports the net gain to reducing τ to 0, instead of all the way to FR, to check how much of the gain comes from eliminating inflation and how much comes from deflation (most comes from the former).

[Figure 1 About Here]

In Model 1, with $\theta = 1$, going from 10% inflation to the FR is worth 1.4% of consumption. This is larger than the findings from reduced-form models discussed in the Introduction, but not a lot larger. In Model 2, with $\theta \approx 0.9$, this same policy is worth just under 3% of consumption. Intuitively, at $\theta \approx 0.9$ the money holdup problem makes q very low, so any additional reduction is very costly. In Model 3 the steady state gain is close to the one in Model 2. Inflation has a sizable impact on K and X in the former, but since much of the gain accrues in the long run, and agents work more and consume less during the transition, reducing the net gain to 1.8%. Figure 1 shows the transitions for Models 2 and 3. In Model 3, e.g., in the short run H increases over 2% and X falls slightly before settling down to the new steady state, while DM output jumps on impact over 50% and quickly settles down. The difference between the two panels of Figure 1 is the size of the adjustment in CM variables: with bargaining, K changes only about 0.2% in in the long run, while with price-taking K changes over 6%.

The size of the DM is roughly 3%, yet as Table 3 indicates the welfare results are sizable. One may think that since DM activity is a small part of the economy, changes in inflation cannot have a large impact on welfare, but this logic is flawed. As hinted above, the matching parameter σ is key to determining the size of DM, and it is almost directly pinned down by the semi-elasticity of money demand.[17] As Bailey (1956) and many others since emphasized, the slope of the money demand curve is key for calculating the welfare cost of inflation, and this semi-elasticity is intimately related to that slope. As a result, since the model displays a realistic semi-elasticity, when inflation changes from 10% to the FR, real-money balances fall by about 45%, leading to a similar size change in the quantity of goods produced in the

[17] In a log-linearized environment, Aruoba and Schorfheide (2011) shows that the interest semi-elasticity of money demand is approximately equal to $(1+i)/(i+\sigma)$.

DM. Even with the relatively small σ (and the size of the DM) this yields a welfare cost of $2-3\%$.

4.2.2. Friedman Rule to the First Best

[Table 4 About Here]

Table 4 reports the welfare gain of going from the FR to the FB under three different assumptions: the benchmark calibration; a version with no distorting taxes (i.e., $t_h = t_k = t_x = 0$) and parameters recalibrated; and a version with no distorting taxes but the original parameter values. The differences in the first panel are big, mainly due to taxation (McGrattan et al. 1997 find similar results in standard nonmonetary models). Once taxes are shut down, Model 3 shows a gain of 0 because the FR implements the FB. In Model 1, with capital holdup but no money holdup, the steady state gain is around 2%, although much is lost in transition. In Model 2, with both holdup problems, the steady state gain is around 2% and about half remains after incorporating the transition. These calculations provide measures of the impact of holdup problems: based on the steady state comparisons from the third panel, say, 1.85% of consumption is the cost of capital holdup and an additional 0.30% is the cost of money holdup. Although there is no single 'correct' way to decompose these effects, this suggests holdup can be quantitatively important, even though bargaining occurs only in the DM and the DM is small.

4.2.3. Using Proportional Taxes to Make up Lost Revenue

[Table 5 About Here]

One can also consider lowering τ and making up the revenue with proportional taxes. The first panel of Table 5 reports results when lump-sum taxes are used to make up revenue, reproducing Table 3. The second and third panels use labor and consumption taxes, respectively.[18] In the data, the monetary base is on average 32.9% of $M1$ and as such government seigniorage revenue is a third of τ times the change in M. Going to the FR and making up the revenue with labor taxes requires raising t_h from 25.1% to between 26.2% and 26.9% Now there are two effects of this policy change. On the one hand there is the channel from before: the reduction in inflation increases q and K (and therefore all other CM variables), which leads to welfare gain, that is partially offset by the extra work and reduced consumption to accumulate the extra capital. On the other hand, the increase in labor income tax reduces K and all related variables. The effect of the first channel on welfare can be read from the first panel, which is reported in Table 3. The second channel reduces welfare in the long-run and thus all steady state numbers in the second and third panel are lower than those in the first panel.

Turning to the transition, in Models 1 and 2 the net effect of the change in policy on K is a decrease since the change in inflation barely increases K due to the hold up problems while the increase in taxes reduce K significantly. In Model 3, the effect of inflation on K is large enough so the net effect is still an increase. Thus on the transition path there is a welfare gain for Models 1 and 2 and a loss of Model 3. On net, the overall impact of lower inflation, however it is financed, is positive in all models. The last rows of each panel report results for the extreme assumption that the government is able to collect seigniorage revenue from all of $M1$ (as in Cooley and Hansen, 1991). This makes the lost revenue of the government much larger than before, forcing the labor income tax to increase to around 31% or the consumption tax to increase to around 13%. As a result, the welfare loss due to these tax increases are sufficiently large to offset the gains from lower inflation in for all models in the case of when labor income tax is used. This result makes it clear that how the lost seigniorage revenue is financed makes a difference for welfare results.[19]

[18] The case where the lost revenue is made up with capital taxes cannot be solved, since increasing t_k lowers K by so much that sufficient revenue is not forthcoming.

[19] Note that the analysis here is not about optimal monetary policy when fiscal policy is *also* set to maximize

4.3. Robustness

[Table 6 About Here]

We redid all the calculations for many alternative specifications; in the interest of space, Table 6 reports the results in terms of one statistic: the net welfare gain of going from 10% inflation to the FR. Detailed results for each specification are available upon request. The first row is the benchmark model. The first robustness check involves shutting down the distorting taxes, both for the case where other parameters are kept at benchmark values, and when they are recalibrated. Most of the results are similar to the benchmark calibration, although the cost of inflation is somewhat lower, especially under price taking. This is because the FR achieves the FB under price taking without distortionary taxes, and hence the cost of moderate inflation is low, by the envelope theorem. It is no surprise that some results depend on what one assumes about taxation, and since taxes are a fact of life, the benchmark calibration should be trusted.[20]

Next are the preference parameters b, ε and η. Clearly, the results are not overly sensitive, although lowering η generally does increase the cost of inflation somewhat. One can also vary β, δ etc. over reasonable ranges without affecting things too much (not reported). Similarly changing the target for the DM markup does not change the results much. When the aggregate markup μ is targeted instead of the DM markup, however, welfare cost increases substantially since matching this markup requires a very different θ, and this decrease in θ increases the money holdup problem.

The table also shows that the results are not very sensitive to using the so-called Great Moderation period (1985-2004) for the calibration, and not at all sensitive to assuming a

utility, since the existing tax rates are taken as given from the data; see Aruoba and Chugh (2010).

[20]Throughout the table, Model 3 provides the clearest picture of how a certain change affects the results since, as in the benchmark calibration, all calibration targets can be matched exactly. In Models 1 and 2, due to the trade off between competing targets and because K/Y cannot be matched exactly, results are more sensitive.

different length for a period (quarterly, monthly and annual models deliver very similar predictions). This is easy to understand: to go from an annual to a quarterly or monthly model, inflation, velocity, interest rates, K/Y and I/K are simply adjusted by the relevant factor. The calibrated σ declines, because a shorter period reduces the probability of consuming in any given DM, but the welfare conclusions do not change. This is important because changing frequency typically *does* change the results in some models, including standard cash-in-advance models, where agents spend all their money every period.

Perhaps surprisingly, the results are robust to changed in the payment parameter ω within a wide range. Even when only 25% of DM trades require cash, the welfare costs are similar. To understand this, first note it is certainly true that a reduction in ω reduces the cost of inflation when other parameters are fixed. But when parameters are recalibrated as ω changes, in order to match the calibration targets, σ increases and B falls. On net, this renders DM activity just about as important for welfare as before. Obviously, $\omega = 0$ means money is not valued and hence inflation is irrelevant, but if $\omega = 0$ then the calibration targets cannot be matched. For values of ω in a reasonable range, as long as the same targets are matched, the net effects are very similar.

What does matter is the empirical measure of money: we repeat the calibration using currency component of M1 and M2. These alternatives imply different values for average velocity and interest elasticity, and given the calibration method, this changes the cost of inflation. Intuitively, consider the traditional method of computing the cost of inflation by the area under the money demand curve. With a broader definition of M (i.e. lower velocity), the curve shifts up and increases the estimated cost. At the same time, remember that the slope of this curve is linked to the interest elasticity of money. When currency is used for the calibration, the elasticity is slightly lower and the velocity is much larger relative to $M1$, both of which reduces the welfare cost of inflation. However, for M2, even though velocity is lower so is the interest elasticity and the end result is a decrease in the welfare cost. In any case, these results indicate that the measure of money does matter, as it should, and as it will in any monetary theory.

Two alternative calibration strategies are also considered. In Strategy 1, the investment semi-elasticity, $\zeta = -0.0059$, is added to the list of calibration targets in Models 2 and 3.[21] As expected, Model 2 has a very hard time improving on the benchmark calibration due to the holdup problems and ζ is only -0.0002. Model 3, on the other hand, is able to match this target exactly with very little sacrifice from other targets – essentially, the only substantive change is a slight increase in K/Y. Since inflation creates a bit more reduction in investment, it is slightly more costly – 1.87 versus 1.75 in the benchmark calibration. In Strategy 2 reverts to the benchmark calibration but it no longer uses α to target the labor share of income but calibrates it. In turn, ψ is now restricted to be $1/(1-\alpha)$. This is a natural restriction – it follows from the assumption that the production in the DM and the CM uses the same technology. With this strategy, α will adjust to match K/Y exactly in all models (not just Model 3) and it does so with only a slightly larger α than the benchmark calibration. The calibrated values of α using this strategy are 0.310, 0.311 and 0.297 for the three models. There are also small changes in the remaining parameters, but the bottom line is the welfare results here are virtually identical to the benchmark results.

Finally, robustness with respect to some larger modeling choices is also considered. One can study a version of the model with two capital stocks, K_C and K_D (see the online appendix). This version of the model can be calibrated exactly as the benchmark version, paying attention to the definition of capital since now there are two types. The last row of Table 6 shows the welfare results for this version of the model. By and large the quantitative results of the benchmark model extend to this version of the model.

[21] As argued above in Model 1, $\zeta = 0$ by definition, independent of parameter values.

5. Conclusion

This paper shows that one can integrate elements from models with explicit trading frictions into capital theory in a way that generates interesting effects of money on investment. One can also use standard methods to calibrate the model, even though it contains some parameters like σ or θ that are not in standard models. This strategy performs fairly well, doing a good job matching most targets, although with price taking the markup cannot be matched, and with bargaining K/Y cannot be matched very well. Backing out the size of the two sectors from observables, the DM accounts for around 3% of total output.

There are number of policy implications, some similar to the previous literature but some not (recall footnote 3). Inflation is a tax on DM consumption q, and its impact is big. Qualitatively, given K is useful for producing q, inflation reduces investment; quantitatively, this effect is tiny under bargaining but big (around 7%) under price taking. In terms of welfare, under price taking, reducing inflation from 10% to the FR is worth 2.5% across steady states, and 1.7% taking into account the transition; it is worth around 3% under bargaining. With either price taking or bargaining, much of the gain is achieved by reducing inflation to 0 rather than going all the way to the FR. Not surprisingly, the costs of fiscal distortions are big. The holdup problems for both money and investment are is important. Most of these results are robust, but the empirical measure of M does matter. Finally, a key element of the framework is the explicit two-sector structure, although it does not matter much if the same or different capital stocks are used in the two sectors, or whether capital is traded in one sector or the other.

Perhaps the most surprising result is that the impact of inflation on output and investment can be so large. The model predicts that going from the FR to 10% inflation decreases output by up to 4%, and decreases investment by up to 7%, depending on the specification. How plausible are these findings? As discussed above, the model, especially the price-taking version roughly matches the U.S. data, both as an independent testable implication as well as a calibration target. Further work, perhaps using cross-country variation, is certainly warranted.[22] Related to the response of output, it is also true that the theory predicts an upward-sloping long-run Phillips curve, or a positive relation between inflation and unemployment (at least a negative correlation between inflation and employment, since there is no notion of unemployment per se in the model). This is as it should be: whatever one believes about the short-run Phillips curve, it is documented in Berentsen et al. (2011) and Haug and King (2009) that after filtering out business cycle frequencies, the US data displays a clear positive correlation between inflation and unemployment, and a negative correlation between inflation and employment. Again, while more work is necessary on this, there is nothing in this data obviously inconsistent with the model in this paper.

Our overall conclusion is that it is quantitatively relevant for capital formation to incorporate elements from the microfoundations literature, including bargaining, alternating centralized and decentralized markets, and stochastic trading opportunities. In terms of future work, it may be interesting to consider more general preferences, perhaps still quasi-linear, but not nonseparable between x and q. This allows one to parameterize more flexibly substitutability between CM and DM goods, and breaks the dichotomy even if K is not used in the DM. In terms of other ideas, one really should try to take financial intermediation more seriously, or study optimal fiscal and monetary policy, or examine business cycle properties of the model. All of this is left to other research.

[22] As simple cross-country stylized facts, inflation has a significantly negative correlation with output and investment/gdp ratio has a weakly negative correlation with output, which are qualitatively in line with the model.

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Table 1 - Benchmark Calibration

(a) 'Simple' Parameters

Parameters	b	$\varepsilon = \eta$	β	t_h	t_k	t_x	G/Y	δ	α	ω
Targets	0.0001	1	0.973	0.251	0.533	0.069	0.241	0.070	0.293	0.85

(b) Remaining Parameters

Parameters	A	B	ψ	σ	θ
Targets	H	v	K/Y	ξ	μ_D
Target Values	0.33	5.381	2.337	-0.064	0.3

Notes: Panel (a) shows the values for the parameters and the calibration targets used for the parameters than can be directly calibrated. Panel (b) shows the parameters that are jointly calibrated using the calibration targets shown.

Table 2 - Calibration Results

	Data	Model 1	Model 2	Model 3
		$\theta = 1$	calibrate θ	price taking
Calibrated Parameters				
σ		0.10	0.10	0.10
B		1.09	1.23	1.87
ψ		1.73	1.52	1.65
A		2.75	3.12	4.80
θ		–	0.92	–
Calibration Targets				
μ_D	30.00	-42.08 (*)	30.00	0.00 (*)
K/Y	2.34	2.19	2.20	2.34
H	0.33	0.33	0.33	0.33
v	5.38	5.39	5.39	5.38
ξ	-0.06	-0.06	-0.06	-0.06
Miscellaneous				
s_D		3.09	2.65	2.47
s_M		1.56	1.64	1.63
μ		-1.30	0.80	0.00
$q/\hat{q}$		0.72	0.44	0.71
ζ	-0.0059	0	-0.0001	-0.004
Squared Error		0.0031	0.0034	0.0000

Note: Model 1 refers to the version with buyer-take-all bargaining ($\theta = 1$), Model 2 refers to the version with generalized Nash bargaining and Model 3 refers to the version with price taking. Squared error is the sum of the squared differences between the calibration targets and the model-implied values. The calibration targets marked with (*) are not targeted in the corresponding model and is not included in the computation of the squared error.

Table 3 - Comparing 10% Inflation and the Friedman Rule

	Model 1	Model 2	Model 3
Allocation			
q^1/q^2	0.58	0.54	0.56
$\hat{q}^1/\hat{q}^2$	1.00	*1.00*	0.98
K^1/K^2	1.00	*1.00*	0.93
H^1/H^2	1.00	*1.00*	*1.00*
X^1/X^2	1.00	*1.00*	0.98
Y_C^1/Y_C^2	1.00	*1.00*	0.98
Y^1/Y^2	0.98	0.98	0.96
Welfare Gains			
Steady State	1.36	2.78	2.55
Transition	0.00	-0.02	-0.80
Net	1.36	2.76	1.75
Net from $\tau = 0.1$ to 0	1.20	2.03	1.31

Note: This table reports the differences in allocations and welfare of the models with 10% steady state inflation versus the Friedman rule. Superscript of 1 refers to the model with 10% inflation and 2 refers to the model under the Friedman rule. Model 1 refers to the version with buyer-take-all bargaining ($\theta = 1$), Model 2 refers to the version with generalized Nash bargaining and Model 3 refers to the version with price taking. Italics denote numbers that are close to but not exactly equal to unity. The welfare results report the welfare gain of changing inflation from 10% to the Friedman rule, as percentage of consumption.

Table 4 - Welfare - Friedman Rule versus First Best

	Model 1	Model 2	Model 3
Benchmark Calibration			
Steady State	23.06	22.67	17.46
Transition	−10.87	−10.30	−8.33
Net	12.18	12.37	9.13
No Taxes (Recalibrated)			
Steady State	1.52	1.78	0.00
Transition	−1.39	−0.99	0.00
Net	0.13	0.79	0.00
No Taxes (Not Recalibrated)			
Steady State	1.85	2.16	0.00
Transition	−1.65	−1.20	0.00
Net	0.20	0.96	0.00

Note: This table reports the welfare gain of going from the equilibrium under the Friedman rule to the first best under different assumptions about calibration and taxes, as a percentage of consumption.Model 1 refers to the version with buyer-take-all bargaining ($\theta = 1$), Model 2 refers to the version with generalized Nash bargaining and Model 3 refers to the version with price taking.

Table 5 - 10% **Inflation versus the Friedman Rule - Alternative Fiscal Policies**

	Model 1	Model 2	Model 3
Making up Revenue by T			
Steady State Gain	1.36	2.78	2.55
Transition	–	−0.02	−0.80
Net Gain	1.39	2.75	1.75
Net Gain (Full Seignorage)	1.36	2.75	1.75
Making up Revenue by t_h (Old $t_h = 0.251$)			
New t_h	0.269	0.269	0.262
Steady State Gain	0.33	1.76	1.93
Transition	0.19	0.17	−0.67
Net Gain	0.52	1.92	1.26
Net Gain (Full Seignorage)	−1.49	−0.10	−0.58
Making up Revenue by t_x (Old $t_x = 0.069$)			
New t_x	0.088	0.088	0.081
Steady State Gain	0.62	2.04	2.12
Transition	0.14	0.11	−0.72
Net Gain	0.75	2.15	1.40
Net Gain (Full Seignorage)	−0.55	0.84	0.20

Note: This table reports the results of the policy experiment of reducing inflation from 10% to the Friedman rule under various fiscal arrangements (all assuming the seignorage revenue is one third of the change in money supply): making up the loss revenue by increasing the lumpsum tax, increasing the labor income tax and increasing the consumption tax. The rows labelled Net Gain (Full Seignorage) shows the results where all of the change in money supply can be considered as seignorage revenue of the government. Model 1 refers to the version with buyer-take-all bargaining ($\theta = 1$), Model 2 refers to the version with generalized Nash bargaining and Model 3 refers to the version with price taking.

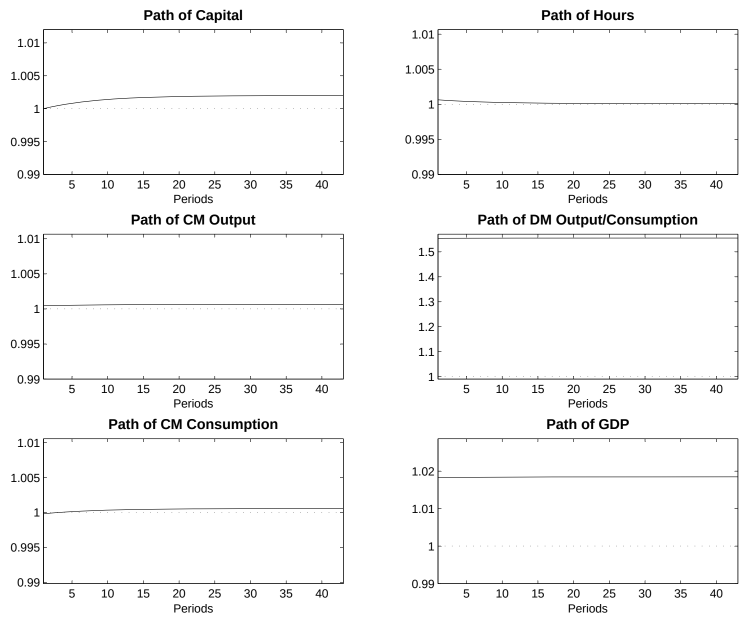
Table 6 - Robustness

	Model 1	Model 2	Model 3
Benchmark	1.36	2.76	1.75
Only Lump-sum Tax			
Recalibrated	1.57	3.01	1.62
Not	1.36	2.75	0.87
CM (ε) **and DM** (η) **Risk Aversion (Benchmark** $\varepsilon = \eta = 1$)			
$\varepsilon = 0.5$, $\eta = 0.5$	1.26	3.84	2.82
$\varepsilon = 2$, $\eta = 2$	1.57	2.31	1.14
$\varepsilon = 1$, $\eta = 0.5$	1.27	3.84	2.19
$\varepsilon = 1$, $\eta = 2$	1.36	2.85	1.35
Utility Parameter b **(Benchmark** $b = 0.0001$)			
$b = 0.00001$	1.36	2.91	1.75
$b = 0.001$	1.36	2.61	1.75
$b = 0.1$	1.37	2.66	1.86
Markup Target (Benchmark $\mu_D = 30\%$)			
$\mu_D = 10\%$	–	2.94	–
$\mu_D = 100\%$	–	2.87	–
$\mu = 10\%$	–	3.75	–
Measures of Money (Benchmark M1)			
Currency	0.26	0.74	0.54
$M2$	0.91	1.23	1.26
Frequency (Benchmark Annual)			
Quarterly	1.31	2.28	1.61
Monthly	1.28	1.93	1.59
Period (Benchmark 1959-2004)			
1985-2004	1.79	2.88	1.71
Payment Parameter ω **(Benchmark** $\omega = 0.85$)			
$\omega = 1$	1.36	2.74	1.81
$\omega = 0.25$	1.29	2.08	1.59
Alternative Calibration Strategies			
$\#1:$ Add ζ	–	3.19	1.87
$\#2: \psi = 1/(1-\alpha)$	1.39	2.74	1.68
Alternative Model			
Two-Capital	–	4.81	1.33

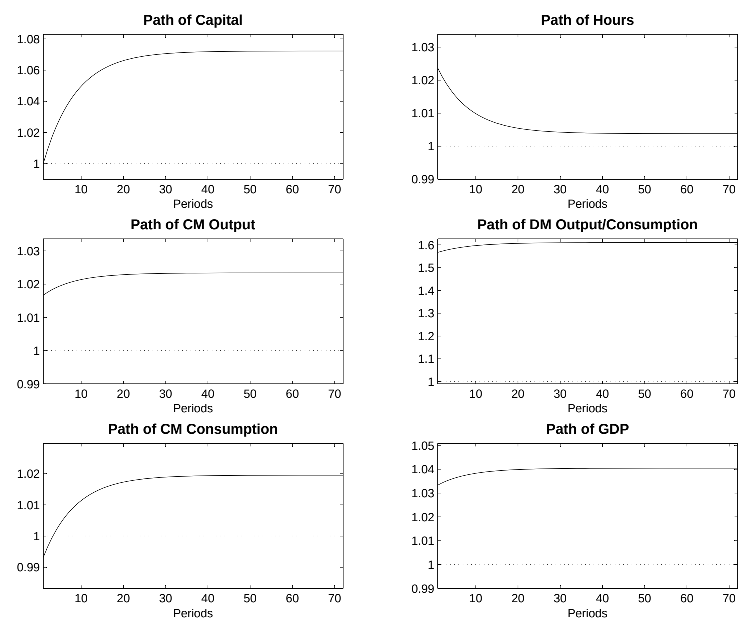
Note: This table reports the net welfare gain of going from 10% inflation to the Friedman rule under various changes in the calibration strategy. Model 1 refers to the version with buyer-take-all bargaining ($\theta = 1$), Model 2 refers to the version with generalized Nash bargaining and Model 3 refers to the version with price taking.

Figure 1 -10% to FR: Transitions

(a) Model 2



(b) Model 3



Note: Each panel of this figure shows the path of key variables during the transition after policy changes from 10% inflation to the Friedman rule.