Appendix for Money and Capital

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February 4, 2011

1 Data

Our calibration frequency is annual. We use data for the period 1959-2004, extracted in 2005.

Original series:

- Nominal consumption of non-durables (**NCND**): BEA - Table 1.1.5.
- Nominal consumption of services (**NCS**): BEA - Table 1.1.5.
- Nominal fixed investment (**NFI**): BEA - Table 1.1.5.
- Nominal government consumption expenditures and gross investment (**NG**) : BEA - Table 1.1.5.
- Nominal stock of nonresidential private fixed assets (**NNRFA**): BEA
- Nominal stock of residential private fixed assets (**NRFA**): BEA
- GDP Deflator (**DEF**): Ratio of nominal GDP (BEA - Table 1.1.5) to real GDP (BEA - Table 1.1.6)
- Nominal money stock (**NM**) : Quarterly sweep-adjusted M1, averaged over the year. (obtained from www.sweepmeasures.com, Cynamon et al. (2006))
- Monetary Base (**MB**) : Monthly St. Louis adjusted monetary base, averaged over the year (FRED - AMBSL)

- Currency Component of M1 (**CUR**) : Average of monthly measure (FRED - CURRSL)
- M2 (**M2**) : Average of monthly measure (FRED - M2SL)
- Nominal compensation of employees (**COMP**) : BEA
- Nominal national income (**NI**) : BEA
- Nominal proprietors' income (**PI**) : BEA
- Nominal interest rate (**INT**) : Corporate Bonds (Moody's) - Aaa Rated from Economic Report to the President 2005 - Table B.73.
- Labor and Capital Income Taxes : Average of the rates reported in McGrattan et al. (1997) over the period 1959-1992.
- Consumption Tax : Average of sum of excise and sales taxes over personal consumption expenditures over 1951-2003.

Derived model-consistent series:

- Real consumption of non-durables (**RCND**): NCND / DEF
- Real consumption of services (**RCS**):NCS / DEF
- Real fixed investment (**RFI**): NFI / DEF
- Real government consumption expenditures and gross investment (**G**) : NG / DEF
- Real stock of nonresidential private fixed assets (**RNRFA**): NNRFA / DEF
- Real stock of residential private fixed assets (**RRFA**): NRFA / DEF
- Real investment (**I**) : RFI
- Real GDP (**Y**) : RCND + RCS + RFI + RG
- Capital Stock (**K**) : RNRFA + RRFA
- Real money stock (**M**) : NM / DEF
- Velocity (**VEL**) : Y / M
- Inflation (**INFL**) : Percentage change in DEF
- Labor share (**LS**) : COMP / (NI - PI) which follows from LS = (COMP + LS $\times$ PI) / NI

Using $\tau^b = 0.3$ for tax rate on bond returns from NBER TAXSIM, we compute the after-tax real return as

$$\rho = \left(1 - \tau^b\right)\left[\frac{1 + INT}{1 + INFL} - 1\right] = 0.028 \tag{1}$$

and after-tax nominal return is 6.63%.

For interest rate semi-elasticity of money demand, we estimate

$$\log(M_t) = \alpha + \beta_i INT_t + \beta_y \log\left(Y_t\right) + d_r\left(L\right)\Delta INT_t + d_y\left(L\right)\Delta\log(Y_t) + \varepsilon_t \tag{2}$$

following Stock and Watson (1993) and Ball (2001) where $d_r\left(L\right)$ and $d_y\left(L\right)$ are lag polynomials with 2 leads and 2 lags, estimated using dynamic least squares (DOLS). The estimate for β_i is -0.064, with p-value 0.00.

For interest rate semi-elasticity of investment, we estimate

$$\log(I_t) = \alpha + \beta_y \log(Y_t) + \beta INT_t + d_r\left(L\right)\Delta INT_t + d_y\left(L\right)\Delta\log(Y_t) + \varepsilon_t \tag{3}$$

using DOLS as described above (with 14 lags and no leads, as selected by AIC), on quarterly U.S. data from 1959Q1-2007Q4. This yields $\beta_i = -0.0059$, with p-value 0.03.

2 Alternative Models

2.1 Two Capital Goods

Suppose that k_C is used in production in the CM and k_D in the DM, but both are produced in the CM. They depreciate at rates δ_C and δ_D. For illustration, there is no tax on k_D, and we present only the bargaining version (price taking is similar). Also, to reduce notation, we set $\omega = 1$. The CM problem is

$$\begin{aligned} W(m, k_C, k_D) &= \max_{x,h,m_{+1},k_{C+1},k_{D+1}} \left\{U(x) - Ah + \beta V(m_{+1}, k_{C+1}, k_{D+1})\right\} \\ \text{s.t. } \left(1 + t_x\right)x &= w\left(1 - t_h\right)h + \left[1 + \left(r - \delta_C\right)\left(1 - t_k\right)\right]k_C - k_{C+1} - T + \frac{m - m_{+1}}{p} \\ &\quad + \left(1 - \delta_D\right)k_D - k_{D+1}. \end{aligned}$$

Eliminating h using the budget equation, we have the FOC

$$\begin{aligned}
x &: U'(x) = \frac{A(1+t_x)}{w(1-t_h)} \\
m_{+1} &: \frac{A(1+t_x)}{pw(1-t_h)} = \beta V_m(m_{+1}, k_{C+1}, k_{D+1}) \\
k_{+1} &: \frac{A}{w(1-t_h)} = \beta V_k(m_{+1}, k_{C+1}, k_{D+1}) \\
z_{+1} &: \frac{A}{w(1-t_h)} = \beta V_z(m_{+1}, k_{C+1}, k_{D+1}).
\end{aligned}$$

The envelope conditions for W_m, W_k and W_z are derived in the obvious way, and the usual logic implies the distribution of (m, k_C, k_D) is degenerate leaving the CM. The DM is as before, except we replace $c(q,k)$ with $c(q,k_D)$ and $g(q,k)$ with $g(q,k_D)$. The value function in the DM and the envelope conditions for V_m, V_k and V_z are derived in the obvious way. This leads to

$$\frac{g(q,K_D)}{M} = \frac{\beta g(q_{+1},K_{D+1})}{M_{+1}}\left[1-\sigma+\sigma\frac{u'(q_{+1})}{g_q(q_{+1},K_{D+1})}\right] \tag{4}$$

$$U'(X) = \beta U'(X_{+1})\left\{1+\left[F_K(K_{C+1},H_{+1})-\delta_C\right](1-t_k)\right\} \tag{5}$$

$$U'(X) = \beta U'(X_{+1})\left[1-\delta_D-\frac{(1+t_x)\,\sigma\gamma(q_{+1},K_{D+1})}{U'(x_{+1})}\right] \tag{6}$$

$$U'(X) = \frac{A(1+t_x)}{F_H(K_C,H)(1-t_h)} \tag{7}$$

$$X+G = F(K_C,H)+(1-\delta_C)K_C-K_{C+1}+(1-\delta_D)K_D-K_{D+1} \tag{8}$$

where $\gamma(\cdot)$ is defined in (29) in the paper. An *equilibrium* is given by (positive, bounded) paths for $(q, K_{C+1}, K_{D+1}, H, X)$ satisfying (4)-(8).

2.2 Capital Acquired in the DM

Here new k is acquired in the DM. Agents do not consume DM output q, but use it as an input that is transformed one-for-one into k, an input to CM production. Each period a fraction σ of agents in the DM can produce q, and a fraction σ can transform it into k. Although agents cannot acquire new capital in the CM, they are allowed to trade used capital. Let k be the amount of capital held by an agent entering the CM and k'_{+1} the amount of capital taken out, into the next DM. The CM problem is

$$\begin{aligned}
W(m,k) &= \max_{x,h,m_{+1},k'_{+1}} U(x) - Ah + \beta V_{+1}(m_{+1}, k'_{+1}) \\
\text{s.t. } (1+t_x)\,x &= w(1-t_h)\,h + [r-(r-\delta)\,t_k]k + (1-\delta)\,\phi k - \phi k'_{+1} - T + \frac{m-m_{+1}}{p}
\end{aligned}$$

where ϕ is the goods price of used capital in terms of x. The FOC are:

$$\begin{aligned} x &: U'(x) = \frac{A(1+t_x)}{w(1-t_h)} \\ m_{+1} &: \frac{A}{pw(1-t_h)} = \beta V_{+1,m}(m_{+1}, k'_{+1}) \\ k'_{+1} &: \frac{A\phi}{w(1-t_h)} = \beta V_{+1,k}(m_{+1}, k'_{+1}) \end{aligned} \tag{9}$$

The envelope conditions are obtained as usual. Buyers in the DM spend all their money, and bring $k = k' + q$ to the CM. The bargaining solution implies q solves $m_b/p = g(q, r, w, \phi)$ where

$$g(q,r,w,\phi) \equiv \frac{(1-t_h)\,w\,[\theta c(q) + (1-\theta)c'(q)\,q]\,[r-(r-\delta)t_k + (1-\delta)\phi]}{\theta A[r-(r-\delta)t_k + (1-\delta)\phi] + (1-\theta)(1-t_h)\,wc'(q)}.$$

In the DM, we have

$$V(m,k') = W(m,k') + \sigma\left\{\frac{A[r-(r-\delta)t_k+(1-\delta)\phi]q(m)}{w(1-t_h)} - \frac{Am}{pw(1-t_h)}\right\} + \sigma E\left\{\frac{A\tilde{m}}{pw(1-t_h)} - c\left[q\left(\tilde{m}\right)\right]\right\},$$

where the expectation is with respect to the money holdings $\tilde{m}$ of agents and we assume you visit one at random. Then

$$\begin{aligned} V_m(m,k') &= \frac{(1-\sigma)A}{pw(1-t_h)} + \frac{\sigma\,[r-(r-\delta)t_k + (1-\delta)\phi]}{pw(1-t_h)\,g_q(q,r,w,\phi)} \\ V_k(m,k') &= \frac{A\,[r-(r-\delta)t_k + (1-\delta)\phi]}{(1-t_h)\,w}. \end{aligned}$$

Since V_m is independent of k', the FOC for m_{+1} in (9) implies m_{+1} is independent of k'_{+1} and hence degenerate. Now the analog to (30) in the paper is

$$\frac{\hat{g}(q,K,H,\phi)}{F_H(K,H)\,M} = \frac{\beta\hat{g}\left(q_{+1},K_{+1},H_{+1},\phi_{+1}\right)}{F_H(K_{+1},H_{+1})\,M_{+1}}\left[1-\sigma+\sigma\Xi(q_{+1},K_{+1},H_{+1},\phi_{+1})\right] \tag{10}$$

where

$$\begin{aligned} \hat{g}(q,K,H,\phi) &\equiv g\,[q, F_K(K,H), F_H(K,H), \phi] \\ \Xi(q,K,H,\phi) &\equiv \frac{F_K(K,H)(1-t_k) + \delta t_k + (1-\delta)\phi}{\hat{g}(q,K,H,\phi)}. \end{aligned}$$

The FOC for k'_{+1} is

$$\frac{\phi}{F_H(K,H)} = \frac{\beta\left[F_K(K_{+1},H_{+1})(1-t_k) + \delta t_k + (1-\delta)\phi_{+1}\right]}{F_H(K_{+1},H_{+1})}, \tag{11}$$

which is an arbitrage condition that implies the demand for k'_{+1} is indeterminate. Hence we can set $k'_{+1} = (1-\delta)K$ for all agents, so (m_{+1}, k'_{+1}) is degenerate. The other conditions are

$$K_{+1} = (1-\delta)K + \sigma q_{+1} \tag{12}$$

$$U'(X) = \frac{A(1+t_x)}{(1-t_h)\, F_H\,(K,H)} \tag{13}$$

$$X + G = F(K,H) \tag{14}$$

An *equilibrium* is given by paths for (q, ϕ, K_{+1}, H, X) satisfying (10)-(14).