

# Bargaining and the Value of Money\*

S. Borağan Aruoba  
University of Maryland

Guillaume Rocheteau  
Federal Reserve Bank of Cleveland

Christopher Waller  
University of Notre Dame

February 15, 2007

## Abstract

Search models of monetary exchange have typically relied on Nash (1950) bargaining, or strategic games that yield an equivalent outcome, to determine the terms of trade. By considering alternative axiomatic bargaining solutions in a search model with divisible money, we show that the properties of the bargaining solutions do matter both qualitatively and quantitatively for questions of first-degree importance in monetary economics such as: (i) the efficiency of monetary equilibrium; (ii) the optimality of the Friedman rule and (iii) the welfare cost of inflation.

**Keywords :** Money, Bargaining, Search, Inflation.

**JEL Classification :** C70, E40.

---

\*We thank Steve Williamson and the referee for valuable comments. We also thank Ben Craig, Ed Nosal, Peter Rupert and Randall Wright for useful comments and discussions as well as participants at the FRB-Cleveland Monetary Theory Workshop August 2004 and at the 2005 annual meeting of the Society of Economic Dynamics. We thank Monica Crabtree-Reusser for editorial assistance. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Cleveland or the Federal Reserve System.

# 1 Introduction

In the last decade search-theoretic models of money have become the dominant framework for studying monetary theory. Contrary to traditional monetary models, trade is decentralized and carried out bilaterally. Since Shi (1995) and Trejos and Wright (1995), the standard approach for determining the terms of trade in bilateral matches is to impose the generalized Nash solution, or to use a strategic bargaining game that yields a similar outcome.<sup>1</sup> Recent extensions to allow for policy analysis by Shi (1997) and Lagos and Wright (2005) produce new results regarding the optimality of the Friedman rule and the welfare cost of inflation. It is unclear, however, whether these new results are robust predictions of models with search and bargaining or if they are driven by the choice of a particular bargaining solution.

We show that the bargaining solution does matter both qualitatively and quantitatively for questions of first-degree importance in monetary economics such as: (i) the efficiency of monetary equilibrium, (ii) the optimality of the Friedman rule and (iii) the welfare cost of inflation. We show that the results of Shi and Lagos and Wright are in fact very sensitive, qualitatively and quantitatively, to the use of the Nash bargaining solution and that other axiomatic bargaining solutions, such as the egalitarian (or proportional) solution proposed by Kalai (1977), yield dramatically different results.<sup>2,3</sup>

Regarding the efficiency of monetary equilibrium, a key result in LW is that the Friedman rule cannot replicate the first-best allocation and output is inefficiently low unless buyers have all the bargaining power. This inefficiency has been attributed to a holdup problem. The difficulty with this argument is that a holdup problem requires an irreversible ex ante investment cost *and* some opportunistic ex post appropriation of the surplus from trade, which typically results from ex post bargaining. In a monetary model the irreversible investment cost occurs when money is costly to hold due to a positive nominal interest rate. But at the Friedman rule the nominal interest rate is zero so money is costless to hold and there is no sunk cost to be ‘held up’. Thus the inefficiency

---

<sup>1</sup>For a related literature on alternative trading mechanisms in search-theoretic models of monetary exchange, see Coles and Wright (1998), Rupert, Schindler and Wright (2001), Curtis and Wright (2004), Rocheteau and Wright (2005) and Julien, Kennes and King (2006).

<sup>2</sup>While we only compare the Nash solution to the egalitarian solution in this paper, in our earlier working paper, Aruoba, Rocheteau and Waller (2006), we also studied the Kalai-Smorodinsky (1975) solution.

<sup>3</sup>For our purpose, the axiomatic approach to bargaining has the advantage of focusing directly on the properties of the solutions that matter for the (in)efficiency of monetary equilibrium. It complements the mechanism design approach of monetary exchange that considers as admissible all trading mechanisms that satisfy agents’ individual rationality constraints (Kocherlakota, 1998; Wallace, 2001). It should also be noted that we do not attempt to derive or justify our axiomatic solutions as outcomes of non-cooperative since this is not the focus of our analysis.

identified by LW at the Friedman rule cannot be the result of a holdup problem.<sup>4</sup> We show that, in fact, LW’s result is a consequence of the lack of *strong monotonicity* of the Nash bargaining solution. The property of strong monotonicity requires agents’ payoffs to be monotonically increasing as the bargaining set expands.<sup>5</sup> It is particularly relevant in a monetary context since the role of money is precisely to enlarge the set of incentive-feasible outcomes. To emphasize this point, we prove that the monetary equilibrium is efficient at the Friedman Rule under *any* strongly monotonic bargaining solution such as the egalitarian (or proportional) solution.

Next, we study how the bargaining solution matters for determining the optimal monetary policy. One of the main insights of Shi (1997) is that deviations from the Friedman rule can be optimal in search economies —an unusual finding for most monetary models— when the composition of the market in terms of buyers and sellers is endogenous. The potentially positive effect of inflation on participation decisions (extensive margin) can outweigh the negative effect on quantities exchanged (intensive margin) in bilateral matches. The latter distortion is of second-order magnitude provided that the Friedman rule maximizes each trade surplus. The validity of this envelope argument, however, critically depends on the monotonicity properties of the bargaining solution: for instance, it fails to hold under Nash bargaining. As a consequence, the Friedman rule is more likely to be suboptimal when terms of trade are determined in accordance with the proportional solution instead of the Nash solution.

Last but not least, we show that the choice of the bargaining solution also matters for quantitative issues such as measuring the welfare cost of inflation. Numerous papers have shown that search models generate much higher estimates of the welfare cost of inflation [see Craig and Rocheteau (2006)]. For instance, LW find that the welfare cost of 10% inflation (relative to price stability) under Nash bargaining is larger than 3% of GDP. They attribute this large welfare cost of inflation to the inability of the Friedman rule to generate the first-best allocation. We show that the welfare cost of 10% inflation in the LW model is equally large, more than 3% of GDP, for both the Nash and the egalitarian solutions, irrespective of the ability of the Friedman rule to generate the first-best

---

<sup>4</sup>While the notion of holdup problem can cover a variety of situations, a key principle is the presence of sunk investment costs that are irrelevant in the bargaining. Because the investing party is not able to appropriate the full return to his (marginal) investment, he underinvests. See, e.g., the seminal paper by Grout (1984). The sunkness of the investment usually arises because of the relationship-specific nature of the investment or because of the presence of trading frictions that prevent the investing party from finding an alternative use for his asset costlessly. Money is not a relationship-specific asset and, in LW, it can be spent in a centralized market at no cost when the monetary authority deflates at a rate equal to agents’ rate of time preference.

<sup>5</sup>The different notions of monotonicity (strong, weak and individual) for bargaining solutions and their importance for economic applications are discussed in Chun and Thomson (1988).

allocation (and over 5% of GDP under generalized Nash and proportional bargaining). However, the sharp contrast between bargaining solutions comes from the welfare gains of implementing the optimal deflation (e.g., reducing the inflation rate from zero to the Friedman rule). This welfare gain is about 0.9% of GDP under Nash bargaining compared to 0.4% under the egalitarian solution. When calibrating the buyer's bargaining power to fit the markup, this gain is up to 2.1% of GDP under the generalized Nash solution compared to 0.5% of GDP under the proportional solution.

The paper is organized as follows. Section 2 describes the environment. In Section 3 we define the bargaining problem in a match and the axiomatic bargaining solutions we consider. Section 4 contains the characterization of steady-state monetary equilibria and results regarding efficiency. The properties of the bargaining solutions are shown to matter for the optimal monetary policy in Section 5 while we examine the welfare cost of inflation in Section 6. Finally, Section 7 summarizes.

## 2 The benchmark model

The environment of the benchmark model is similar to the one in Lagos and Wright (2005) – denoted LW hereafter. Time is discrete and continues forever. There is a continuum of infinitely-lived agents with measure one. Each period is divided into two trading subperiods. In the first subperiod agents trade in a decentralized market, denoted DM, while in the second subperiod they trade in a centralized market, denoted CM. In the DM, agents are specialized in terms of the goods they produce and consume. Furthermore they are matched bilaterally: each agent meets someone who produces a good he wishes to consume with probability  $\sigma \leq 1/2$  and meets someone who likes the good he produces with the same probability  $\sigma$ . With probability  $1 - 2\sigma$  an agent has no opportunity to trade. (In Section 5, following Shi (1997) we endogenize the composition of the market in terms of buyers and sellers.) For simplicity, we rule out double-coincidence-of-wants meetings. When paired in the DM, agents bargain over the terms of trade. In the CM, all agents can produce and consume the same good. Prices in the CM are Walrasian so agents trade goods, labor and money taking prices as given. Output in the CM is produced by a linear production function in labor, which implies the (real) wage rate is equal to 1.

Instantaneous utility of an agent is  $u(q^b) - \psi(q^s) + U(c) - h$ , where  $q^b$  and  $q^s$  are the quantities consumed and produced in the DM while  $c$  is consumption and  $h$  is the supply of hours in the CM. Until Section 6 where we calibrate the model, we assume  $U(c) = c$ . The utility function is well-behaved and  $q^*$  denotes the solution to  $u'(q^*) = \psi'(q^*)$ , which is the efficient DM quantity. All agents have the same discount factor  $\beta \equiv (1 + r)^{-1} \in (0, 1)$ .

The quantity of fiat money per capita at the beginning of period  $t$  is  $M_t > 0$ . We assume  $M_{t+1} = \gamma M_t$ , where  $\gamma \equiv 1 + \pi$  is constant and new money is injected by lump-sum transfers in the CM. The price of goods in terms of money in the CM is  $p_t$ . We restrict our attention to steady-state equilibria where the real value of aggregate money balances  $M/p$  is constant. This implies  $p_{t+1} = \gamma p_t$ . For notational simplicity, we omit time indices and replace  $t + 1$  by  $+1$  and so on.

Bellman's equation for an agent in the DM holding  $z = m/p$  units of real balances is

$$V(z) = \sigma \int \{u[q(z, \tilde{z})] + W[z - d(z, \tilde{z})]\} dF(\tilde{z}) \\ + \sigma \int \{-\psi[q(\tilde{z}, z)] + W[z + d(\tilde{z}, z)]\} dF(\tilde{z}) + (1 - 2\sigma)W(z), \quad (1)$$

where  $F(\tilde{z})$  is the distribution of real balances across agents, and  $W(z)$  is the value function of the agent in the CM. Equation (1) has the following interpretation. An agent meets someone who produces a good he likes with probability  $\sigma$ . He consumes  $q$  units of goods and delivers  $d$  units of real balances (expressed in terms of CM output) to his trading partner. The terms of trade  $(q, d)$  depend on his real balances  $z$  and the real balances  $\tilde{z}$  of his partner in the match. With probability  $\sigma$ , the agent meets someone who likes his good and becomes a seller. He produces  $q$  for his trading partner and receives  $d$  real balances. With probability  $1 - 2\sigma$ , no trade takes place. The CM problem of the agent is

$$W(z) = \max_{\hat{z}} \{T + z - \gamma \hat{z} + \beta V(\hat{z})\}, \quad (2)$$

where  $T$  the lump-sum transfer (expressed in general goods), and  $\hat{z}$  the real balances taken into the next day.<sup>6</sup> We have used the budget constraint according to which the net CM consumption is  $c = h + z + T - \gamma \hat{z}$  and the relative price of real balances next period in terms of current-period CM output is  $p_{+1}/p = \gamma$ . From (2), the maximizing choice of  $\hat{z}$  is independent of  $z$ ; and  $W$  is linear in  $z$  with  $W_z = 1$ . Substituting  $V(\hat{z})$  by its expression given by (1), using the linearity of  $W(z)$  and ignoring the constant terms, we can reformulate the buyer's problem as

$$\max_{\hat{z}} \left\{ -\gamma \hat{z} + \beta \left\{ \sigma \int \{u[q(\hat{z}, \tilde{z})] - d(\hat{z}, \tilde{z})\} dF(\tilde{z}) + \sigma \int \{d(\tilde{z}, \hat{z}) - \psi[q(\tilde{z}, \hat{z})]\} dF(\tilde{z}) + \hat{z} \right\} \right\}.$$

Divide the previous expression by  $\beta$  and denote  $i$  the nominal interest rate given by  $1 + i = (1 + \pi)(1 + r)$  to get:

$$\max_{\hat{z}} \left\{ -i \hat{z} + \sigma \int \{u[q(\hat{z}, \tilde{z})] - d(\hat{z}, \tilde{z})\} dF(\tilde{z}) + \sigma \int \{d(\tilde{z}, \hat{z}) - \psi[q(\tilde{z}, \hat{z})]\} dF(\tilde{z}) \right\} \quad (3)$$

---

<sup>6</sup>Note that  $\hat{m}/p = (p_{+1}/p)(\hat{m}/p_{+1}) = \gamma \hat{z}$ .

### 3 Bargaining

The prime focus of this paper is the study of alternative bargaining solutions in a model of monetary exchange. Thus, in this section we define and carefully characterize the bargaining problem for determining the terms of trade in a match between a buyer holding  $z$  units of real balances and a seller holding  $\tilde{z}$  units of real balances. We show how to apply two main bargaining solutions identified in the axiomatic literature – the Nash and egalitarian solutions – to our problem.<sup>7</sup> These two solutions differ in terms of their monotonicity properties: The Nash solution does not have a monotonicity axiom while the egalitarian solution is *strongly* monotonic. We will discuss how monotonicity of the solution is relevant in the context of our model.

#### 3.1 The bargaining problem

An agreement is a pair  $(q, d)$  where  $q$  is the amount of goods produced by the seller and  $d$  is the amount of real money transferred by the buyer to the seller. The monetary transfer is constrained by the real balances of the buyer and the seller, i.e.,  $-\tilde{z} \leq d \leq z$ . If an agreement is reached then the utility of the buyer is  $u^b = u(q) + W(z - d)$  whereas the utility of the seller is  $u^s = -\psi(q) + W(\tilde{z} + d)$ . If no agreement is reached, the utility of the buyer is  $u_0^b = W(z)$  and the utility of the seller is  $u_0^s = W(\tilde{z})$ . While  $u_0^b$  and  $u_0^s$  are taken as given within the bargaining problem, they are endogenous in equilibrium.

From the linearity of  $W(z)$ ,  $u^b = u_0^b + u(q) - d$  and  $u^s = u_0^s + d - \psi(q)$ . In order to illustrate the role of money, suppose that the buyer is restricted not to spend more than  $\tau \leq z$  real balances.<sup>8</sup> The set  $\mathcal{S}(\tau)$  of feasible utility levels associated with this problem is

$$\mathcal{S}(\tau) = \left\{ (u(q) - d + u_0^b, d - \psi(q) + u_0^s) : d \in [-\tilde{z}, \tau] \text{ and } q \geq 0 \right\}$$

The equation for the Pareto frontier of  $\mathcal{S}$  is derived from the program  $u^b = \max_{q,d} [u(q) - d] + u_0^b$  s.t.  $-\psi(q) + d \geq u^s - u_0^s$  and  $d \leq \tau$  for some  $u^s$ . It satisfies

$$u^s - u_0^s = \begin{cases} u(q^*) - \psi(q^*) - (u^b - u_0^b) & \text{if } u^s - u_0^s \leq \tau - \psi(q^*) \\ z - \psi[u^{-1}(u^b - u_0^b + \tau)] & \text{otherwise} \end{cases} \quad (4)$$

---

<sup>7</sup>In our earlier working paper, we also studied the Kalai-Smorodinsky (1975) bargaining solution in the LW framework. Despite being based on an axiom of *individual monotonicity*, both the qualitative and quantitative results are nearly identical to those obtained from the Nash solution. What matters for the result is the property of *strong monotonicity* as emphasized by Kalai (1977). Thus, for presentation purposes, we omit it here.

<sup>8</sup>For instance, the buyer could only bring a fraction of his monetary wealth into a match. For an example of a model where this is the case, see Lagos and Rocheteau (2006).

Therefore,  $d^2u^s/(du^b)^2 = 0$  if  $u^s \leq \tau - \psi(q^*)$  and  $d^2u^s/(du^b)^2 < 0$  otherwise. The Pareto-frontier is linear when  $q = q^*$  and it is strictly concave whenever  $q < q^*$ .

In Figure 1, we represent the bargaining set  $\mathcal{S}(\tau)$  for  $\tau_3 > \tau_2 > \tau_1$ . We denote  $\Delta^* = u(q^*) - \psi(q^*)$  the maximum surplus of a match. Note that the bargaining set expands as the buyer brings more money, i.e.,  $\mathcal{S}(\tau_1) \subset \mathcal{S}(\tau_2) \subset \mathcal{S}(\tau_3)$ . This transformation of the bargaining set illustrates the fact that fiat money allows traders to achieve utility and output levels that would not be attainable otherwise.

In the following, we will assume that money holdings are common knowledge in a match, and agents cannot commit to spend less than their monetary wealth, i.e.,  $\tau = z$ . Formally, a bargaining problem is a pair  $(\mathcal{S}, u_0)$  where  $\mathcal{S}$  is the set of feasible utility levels and  $u_0 = (u_0^b, u_0^s)$  is the disagreement outcome. A solution to the bargaining problem is a function  $\mu$  that assigns a pair of utility levels to every bargaining game.

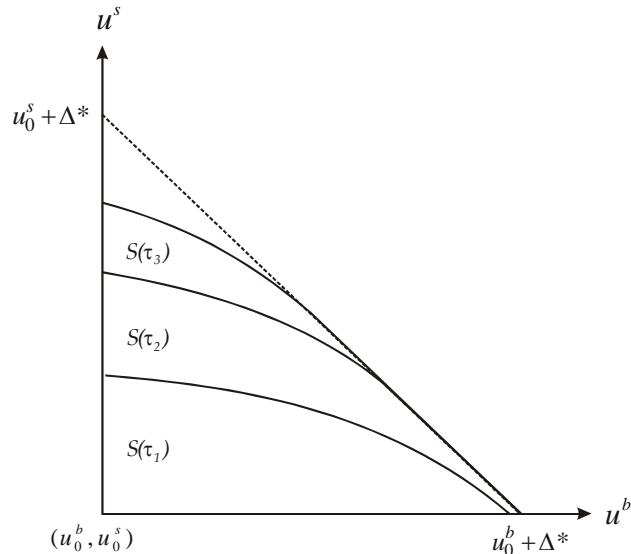


Figure 1: The bargaining set

### 3.2 The Nash solution

The Nash (1950) solution,  $\mu^N$ , is the unique solution that satisfies the axioms of Pareto optimality, scale invariance, symmetry and independence of irrelevant alternatives. It is given by

$$\mu^N(\mathcal{S}, u_0) = \arg \max_{(u^b, u^s) \in \mathcal{S}} (u^b - u_0^b)(u^s - u_0^s). \quad (5)$$

This solution is illustrated graphically in Figure 2. It is at the tangency point of the bargaining set  $\mathcal{S}$  and a Nash product curve representing the points  $u^s - u_0^s = N/(u^b - u_0^b)$  for some  $N \geq 0$ . Since  $u^b - u_0^b = u(q) - d$  and  $u^s - u_0^s = -\psi(q) + d$ ,  $(q, d)$  satisfies

$$(q, d) = \arg \max_{q, d} [u(q) - d] [-\psi(q) + d]$$

subject to  $d \leq z$ . The solution is  $q = q^*$  and  $d = z^*$  if  $z \geq z^* \equiv [u(q^*) + \psi(q^*)]/2$ , and  $d = z$  and

$$z = z(q) \equiv \frac{u'(q)\psi(q) + \psi'(q)u(q)}{u'(q) + \psi'(q)}, \quad (6)$$

otherwise. Note that the buyer's surplus from a match is given by  $u(q) - z(q) = \Theta(q)[u(q) - \psi(q)]$  where  $\Theta(q) = u'(q)/[u'(q) + \psi'(q)]$  is the share of the buyer. It is easy to show that  $u(q) - z(q)$  is non-monotonic in  $q$  and negatively sloped in the vicinity of  $q = q^*$ . This is illustrated in Figure 2 where the buyer's utility falls ( $u_2^b < u_1^b$ ) as the bargaining set expands.

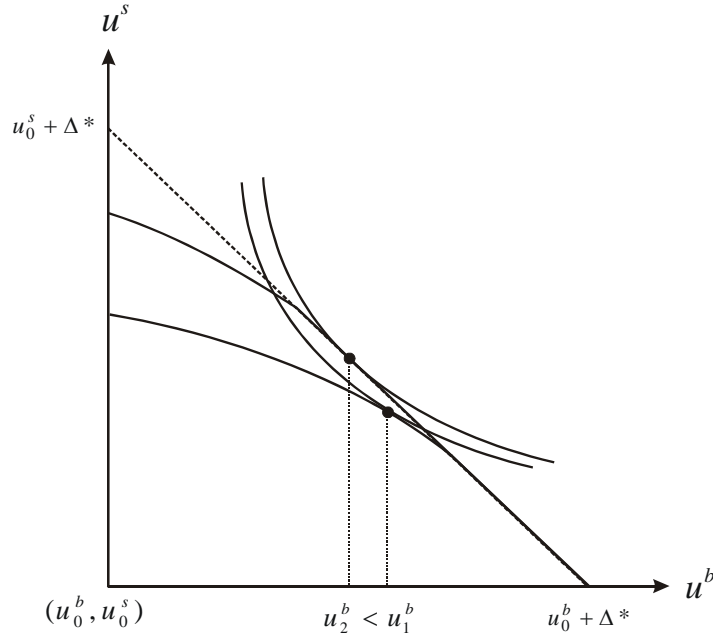


Figure 2: Nash solution

In a sense, by taking an action that expands the Pareto frontier an agent can be punished for doing so even though both parties can be made better off. This is a troubling attribute for the Nash solution that, as we will show, has serious implications for monetary search models.



Finally, if we relax the axiom of symmetry, we obtain the generalized Nash solution parameterized by  $\theta \in [0, 1]$ , the buyer's bargaining weight. In this case  $(q, d)$  maximizes  $[u(q) - d]^\theta [-\psi(q) + d]^{1-\theta}$  and the solution to the bargaining problem remains the same as before with  $z(q)$  defined as follows:

$$z(q) = \frac{\theta u'(q)\psi(q) + (1 - \theta)\psi'(q)u(q)}{\theta u'(q) + (1 - \theta)\psi'(q)}. \quad (7)$$

### 3.3 The egalitarian solution

The egalitarian solution (Luce and Raiffa, 1957; Kalai, 1977) imposes a notion of monotonicity, *strong monotonicity*, according to which no players are made worse-off if additional alternatives are made available to them.<sup>9</sup> Consider two bargaining problems  $(\mathcal{S}_1, u_0)$  and  $(\mathcal{S}_2, u_0)$  such that  $\mathcal{S}_1 \subseteq \mathcal{S}_2$ . Then, a solution  $\mu$  is strongly monotonic iff  $\mu(\mathcal{S}_1) \leq \mu(\mathcal{S}_2)$ . Kalai (1977) showed that the unique solution that satisfies Pareto optimality, symmetry and strong monotonicity is the egalitarian solution.<sup>10</sup> In our context, the egalitarian solution implies

$$u^b - u_0^b = u^s - u_0^s, \quad (8)$$

and  $(u^b, u^s)$  lies in the Pareto-frontier of  $\mathcal{S}$ .<sup>11</sup> In Figure 3, the egalitarian solution is at the intersection of the 45°-line and the Pareto frontier of  $\mathcal{S}$ .

Formally,  $(q, d)$  solves

$$(q, d) = \arg \max_{q, d} [u(q) - d] \quad (9)$$

$$\text{s.t. } u(q) - d = d - \psi(q), \quad (10)$$

and  $d \leq z$ . So, one can think of the buyer as choosing an offer so as to maximize his own surplus from trade subject to the constraint that the match surplus is shared evenly between the buyer and the seller. Substituting  $d$  by its expression as a function of  $q$  from (9), this problem can be

---

<sup>9</sup>*Strong monotonicity* does not require that all players benefit equally from the expansion of the bargaining set. Some expansions may be skewed in favor of a particular agent in which case this agent may benefit more from the presence of new opportunities.

<sup>10</sup>In contrast to the Nash solution, the egalitarian solution is not scale invariant – it is invariant only under simultaneous rescaling of the utility functions of the two players with the same rescaling factor. See the discussion in Kalai (1977) and Kalai and Samet (1985).

<sup>11</sup>Thomson and Myerson (1980) have characterized the class of bargaining solutions, named the *monotone path solutions*, that satisfy Pareto optimality and strong monotonicity.

simplified to

$$q = \arg \max_q \left[ \frac{u(q) - \psi(q)}{2} \right] \quad (11)$$

$$\frac{u(q) + \psi(q)}{2} \leq z. \quad (12)$$

From (11)-(12),  $q = q^*$  and  $d = z^*$  if  $z \geq z^* \equiv [\psi(q^*) + u(q^*)] / 2$ , and  $d = z$  with

$$z = z(q) \equiv \frac{\psi(q) + u(q)}{2}, \quad (13)$$

otherwise.

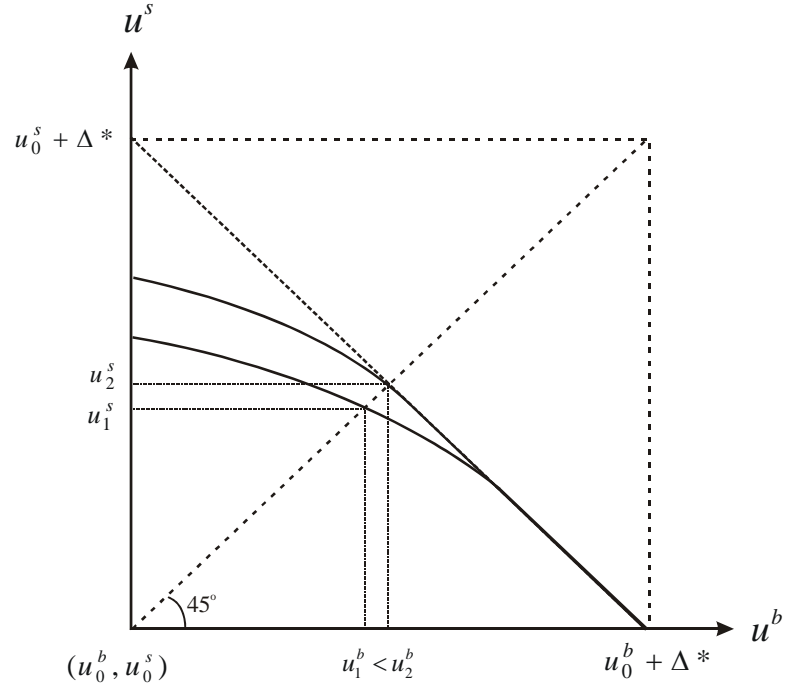


Figure 3: Egalitarian solution

Dropping the axiom of symmetry, we obtain the *proportional solution* (Kalai, 1977) that satisfies

$$(q, d) = \arg \max_{q, d} [u(q) - d] \quad (14)$$

$$\text{s.t. } [u(q) - d] = \frac{\theta}{1 - \theta} [d - \psi(q)], \quad (15)$$

and  $d \leq z$ , where  $\theta \in [0, 1]$ . Then, we get the same solution with

$$z(q) = (1 - \theta)u(q) + \theta\psi(q). \quad (16)$$

## 4 Efficiency of monetary equilibrium

In this section, we study the existence and efficiency of monetary equilibrium for a class of bargaining solutions that encompasses the ones described in Section 3. We restrict our attention to bargaining solutions with the following properties. The outcome is independent of the seller's real balances and can be represented by a continuous and strictly increasing function  $z(q)$  such that the following is true: For all  $z \leq z^* = z(q^*)$ ,  $d = z$  and  $q$  is implicitly defined by  $z = z(q)$ ; for all  $z > z^*$ ,  $q = q^*$  and  $d \geq z^*$ .<sup>12</sup> The function  $z(q)$  is given by (6) for the Nash solution and by (13) for the egalitarian solution.

Since  $q = q^*$  and  $d \geq z^*$  for all  $z \geq z^*$  a buyer's optimal choice of real balances is such that  $z \leq z^*$  for all  $i > 0$ . Using the fact that  $q(z, \tilde{z})$  is independent of  $\tilde{z}$ , the agent's problem (3) can then be reformulated as

$$\max_{\hat{z} \in [0, z^*]} \{-i\hat{z} + \sigma \{u[q(\hat{z})] - \hat{z}\}\}.$$

Finally, since there is a bijection between  $z$  and  $q$ , it is equivalent to express the agent's problem as a choice of  $q$

$$\max_{q \in [0, q^*]} \{-iz(q) + \sigma [u(q) - z(q)]\}. \quad (17)$$

The maximization problem in (17) has a simple interpretation. The agent chooses a  $q$  that maximizes his expected surplus as a buyer minus the cost of holding the real balances that are necessary to buy  $q$ . If (17) has more than one solution, we restrict our attention to symmetric equilibria where all agents choose the same real balances.

A steady-state monetary equilibrium is a  $q > 0$  solving (17). The objective function in (17) is continuous and maximized over a compact set, so a solution exists. In order to guarantee the existence of a monetary equilibrium, assume  $u(q^*) - z(q^*) > 0$ , i.e., buyers get a positive surplus if  $q = q^*$ . At  $i = 0$ , the solution to (17) is strictly positive since  $\max_{q \in [0, q^*]} \{\sigma [u(q) - z(q)]\} \geq \sigma [u(q^*) - z(q^*)] > 0$ . Since  $\max_{q \in [0, q^*]} \{-iz(q) + \sigma [u(q) - z(q)]\}$  varies continuously with  $i$ , there is an  $\bar{i} > 0$  such that a steady-state monetary equilibrium exists for all  $i < \bar{i}$ . For instance, under the proportional solution, the threshold for the nominal interest rate is  $\bar{i} = \theta\sigma/(1 - \theta)$ .

---

<sup>12</sup>To see this that the outcome of the bargaining problem does not depend on the seller's real balances, normalize the utility functions to assign zero utility to the two players in case no agreement is reached,  $u_0 = (0, 0)$ . The set  $\bar{\mathcal{S}}$  of incentive-feasible utility levels is then

$$\bar{\mathcal{S}} = \{(u(q) - d, d - \psi(q)) : \psi(q) \leq d \leq u(q), d \leq z \text{ and } q \in [0, q^*]\}.$$

Since  $(\bar{\mathcal{S}}, u_0)$  is independent of  $\tilde{z}$ , a bargaining solution that maps  $(\bar{\mathcal{S}}, u_0)$  into an element of  $\bar{\mathcal{S}}$  is independent of the seller's real balances.

We now investigate some implications of the choice of the bargaining solution for the efficiency of monetary equilibrium.

**Proposition 1** (i) *There exists an equilibrium with  $q = q^*$  at  $i = 0$  iff  $q^*$  is a maximizer of  $u(q) - z(q)$  over  $[0, q^*]$ . (ii) If  $u(q) - z(q)$  is (strictly) increasing over  $[0, q^*]$  then  $q = q^*$  is an (the) equilibrium at  $i = 0$ . (iii) Assuming  $z(q)$  is differentiable, if  $u'(q^*) < z'(q^*)$  then  $q = q^*$  is not an equilibrium for any  $i \geq 0$ .*

**Proof.** Direct from (17). ■

According to part (i) of Proposition 1, the equilibrium is efficient if the buyer's surplus  $u(q) - z(q)$  is maximum at  $q = q^*$ , the value of  $q$  that maximizes the surplus of a match. According to part (ii) of Proposition 1, this condition is satisfied if the bargaining solution is such that the buyer's payoff is increasing in  $q$  over  $[0, q^*]$ . A sufficient condition for this requirement to hold is that the bargaining solution is strongly monotonic since a higher  $z$  generates an expansion of the bargaining set and therefore a higher payoff for both players. In contrast, as indicated by part (iii) of Proposition 1, if the solution is non-monotonic, and if the buyer's surplus decreases when  $q$  gets close to  $q^*$ , then the Friedman rule fails to achieve the efficient allocation.

The next Corollary uses Proposition 1 to establish the ability, or inability, of the Friedman rule to generate the first-best allocation under the two standard bargaining solutions described in Section 3.

**Corollary 1** *For all  $i \geq 0$ , equilibria under the Nash solution are inefficient, i.e.  $q < q^*$ . Equilibria under the egalitarian solution are efficient iff  $i = 0$ .*

**Proof.** Consider first the Nash solution. From Proposition 1, it is sufficient to show that  $u'(q^*) < z'(q^*)$ . Recall that  $u(q) - z(q) = \Theta(q) [u(q) - \psi(q)]$  where  $\Theta(q) = u'(q) / [u'(q) + \psi'(q)]$ . Hence,  $z'(q^*) = u'(q^*) - \Theta'(q^*) [u(q^*) - \psi(q^*)] > u'(q^*)$  since  $\Theta'(q^*) < 0$ . Consider next the egalitarian solution. The first-order condition for  $q$  gives

$$\frac{i}{\sigma} = \frac{u'(q) - \psi'(q)}{\psi'(q) + u'(q)}. \quad (18)$$

From (18),  $q = q^*$  iff  $i = 0$ . ■

The results in Corollary 1 can be easily extended to asymmetric bargaining solutions.

**Corollary 2** For all  $i \geq 0$  and all  $\theta < 1$ , equilibria under the generalized Nash solutions are inefficient. For all  $\theta \in (0, 1]$ , equilibria under (asymmetric) proportional solutions are efficient iff  $i = 0$ .

**Proof.** The proof is similar to one of Corollary 1 and is therefore omitted. ■

Interestingly, the Friedman rule achieves the first best for any proportional solution. In contrast, under the generalized Nash solution, output is always too low, except when  $\theta = 1$  (in which case both solutions coincide).

As noticed in LW, the quantity traded under Nash bargaining is inefficiently low even at the Friedman Rule ( $i = 0$ ) when  $\theta < 1$ . LW attribute this inefficiency to a *holdup problem* on money balances.<sup>13</sup> A holdup problem requires several elements: an irreversible sunk cost of investment, bargaining over the proceeds of the investment and an inability to contract ex ante with one's trading partner over the proceeds. In search models of money the last condition arises naturally since agents do not know who their trading partners are. The irreversible sunk cost is measured by  $i$ , the nominal interest rate. Finally, when  $\theta < 1$ , buyers do not receive the full return from bringing money into a match. Thus, when an agent brings an additional unit of money into a match, he bears the full sunk cost of acquiring the money but has to share the proceeds of that investment (the additional surplus in a match) with the seller. Thus, a holdup problem on money holdings always exists when  $\theta < 1$  and  $i > 0$ . However, the second necessary condition is violated at the Friedman rule since  $i = 0$ . In this case, there is no sunk cost of holding money – any cost of acquiring money is completely reversible since agents can sell a unit of money in the next period for more goods than they gave up to acquire it and this exactly compensates them for discounting. Consequently, a holdup problem cannot occur when  $i = 0$  even though  $\theta < 1$ .

What then is the reason for the inefficiency identified by LW at the Friedman rule? The insight comes from Proposition 1 and (17). At  $i = 0$  there are no costs to holding money so the buyer chooses his real balances to maximize his own surplus from a trade,  $u(q) - z(q)$ . It then follows that the reason for the inefficiently low  $q$  comes from the fact that the Nash solution is non-monotonic and the buyer's surplus  $u(q) - z(q)$  reaches a maximum at some  $\tilde{q} < q^*$  (See Figure 4). Intuitively, the buyer's surplus is the product of the buyer's share  $\Theta(q)$ , which is decreasing in  $q$ , and the surplus of the match  $u(q) - \psi(q)$ , which is increasing in  $q$ . For  $q$  close to  $q^*$ , the effect of a change in  $q$  on the buyer's share dominates the effect on the match surplus. In Figure 2, an increase of the buyer's real balances shifts the bargaining set upward which in principle could allow the buyer to

---

<sup>13</sup>See the explanation in LW on page 574.

reach a higher surplus. However, the tangency with the Nash product curve requires the buyer's surplus to fall when the Pareto-frontier is sufficiently close to the  $(u_0^b + \Delta^*, u_0^s + \Delta^*)$  line.

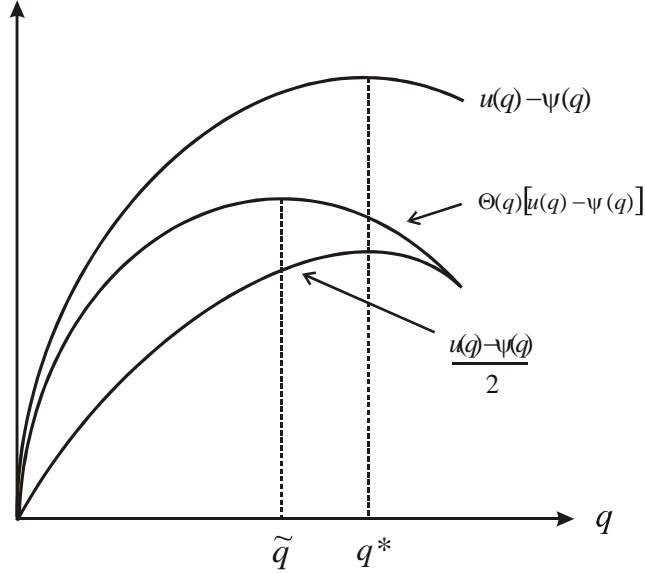


Figure 4: Buyer's surplus under Nash and egalitarian solutions

Under the egalitarian solution, the Friedman rule achieves the first-best allocation. Since the buyer's surplus increases with his money holdings, and strictly increases if  $z < z^*$ , the buyer will invest up to  $z^*$  when  $i = 0$ . To put it differently, the buyer's surplus  $u(q) - z(q)$  is half of the total surplus of the match,  $u(q) - \psi(q)$ . It is therefore maximized when the match surplus is maximized (See Figure 4). In Figure 3, since the outcome of the bargaining is located on the 45°-line, any expansion of the bargaining set raises the buyer's surplus.

## 5 Optimal monetary policy

In the model studied so far the optimal monetary policy corresponds to the Friedman rule irrespective of the choice of the bargaining solution. Shi (1997), however, established that a deviation from the Friedman rule may be desirable when the composition of the market between buyers and sellers is endogenous. This result is striking since there are few monetary models where the Friedman rule is not optimal. In this section we show that, when participation is endogenous, the optimality of the Friedman depends critically on the monotonicity properties of the bargaining solution.

Following Shi (1997) we now let each agent choose to be either a buyer or a seller in the DM. Let  $n \in (0, 1)$  be the fraction of sellers in the DM. Matching is random in the DM and the matching probabilities of buyers and sellers are  $\sigma^b = n$  and  $\sigma^s = 1 - n$ , respectively. With no loss of generality, suppose that the choice of being a buyer or a seller in the period  $t + 1$  DM is made at the beginning of the period  $t$  CM. Let  $W^b$  ( $W^s$ ) denote the value function of an agent in the CM who chooses to be a buyer (seller) in the next DM with  $V^b$  ( $V^s$ ) denoting the value function for a buyer (seller) in the DM. The value functions in the CM satisfy Bellman equations analogous to (2),

$$W^j(z) = \max_{\hat{z}} \{T + z - \gamma\hat{z} + \beta V^j(\hat{z})\}, \quad (19)$$

where  $j \in \{b, s\}$ . As before, the value functions are linear in wealth. Using the result according to which buyers spend all their money holdings in the DM, the value of being of a buyer in the DM satisfies

$$V^b(z) = n \{u[q(z)] - z\} + \max [W^b(z), W^s(z)]. \quad (20)$$

Substituting (20) into (19), the value of a buyer with  $z$  units of real balances in the CM satisfies

$$W^b(z) = T + z + \max_{q \in [0, q^*]} \beta \{-iz(q) + n[u(q) - z(q)]\} + \beta \max [W^b(0), W^s(0)]. \quad (21)$$

From (21) the buyer chooses the quantity to trade in the next DM taking as given the measure  $n$  of sellers, and therefore his matching probability. By a similar reasoning, the value of being a seller with  $z$  units of real balances satisfies

$$W^s(z) = T + z + \beta(1 - n)[z(q) - \psi(q)] + \beta \max [W^b(0), W^s(0)]. \quad (22)$$

From (22) sellers do not carry money balances into the DM and they take the quantity traded  $q$  (or, equivalently, buyers' real balances) as given.

Since both  $W^b(z)$  and  $W^s(z)$  are linear in  $z$ , the choice of being a buyer or a seller is independent of  $z$ . In equilibrium, agents must be indifferent between being a seller or a buyer. Consequently,  $W^b(z) = W^s(z)$  and, from (21) and (22),  $n$  satisfies

$$(1 - n)[z(q) - \psi(q)] = n[u(q) - z(q)] - iz(q). \quad (23)$$

The left hand-side of (23) is the seller's expected surplus in the decentralized market whereas the right hand-side is the buyer's expected surplus minus the cost of holding real balances. Solving for  $n$  we get

$$n = \frac{(1 + i)z(q) - \psi(q)}{u(q) - \psi(q)}. \quad (24)$$

So, for given  $q$  an increase in  $i$  raises the fraction of sellers and reduces the measures of buyers. Intuitively, higher inflation raises the cost of holding real balances and reduces the incentives to be a DM buyer. From (21),  $q$  solves

$$\max_{q \in [0, q^*]} \{-iz(q) + n[u(q) - z(q)]\}. \quad (25)$$

A steady-state monetary equilibrium is a pair  $(q, n)$  such that  $q$  is a solution to (25) and  $n$  satisfies (24). Equilibrium is unique at the Friedman rule:  $q$  solves  $u'(q) = z'(q)$ , and given  $q$  the measure of sellers is uniquely determined by (24). The effects of a change in  $i$  in the neighborhood of  $i = 0$  are given by

$$\left. \frac{dq}{di} \right|_{i=0} = \frac{z'(q)}{n[u''(q) - z''(q)]}, \quad (26)$$

$$\left. \frac{dn}{di} \right|_{i=0} = [u(q) - \psi(q)]^{-1} \left\{ \left( \frac{1-n}{n} \right) \frac{z'(q)[u'(q) - \psi'(q)]}{u''(q) - z''(q)} + z(q) \right\}, \quad (27)$$

where  $n$  and  $q$  are evaluated at  $i = 0$ .<sup>14</sup> Inflation has a direct effect on the equilibrium allocation by raising the cost of holding real balances and therefore by reducing  $q$ . The effect of inflation on  $n$  is ambiguous in general. For strongly monotonic bargaining solutions,  $n$  increases with inflation since  $\frac{dn}{di} = \frac{z(q^*)}{u(q^*) - \psi(q^*)} > 0$ .

Welfare is measured by the sum of all trade surpluses in a period, i.e.,  $\mathcal{W} = n(1-n)[u(q) - \psi(q)]$ . It is maximized for  $q = q^*$  and  $n = 0.5$ . (Recall that the number of trades is maximized when the composition of the market is symmetric.) Totally differentiating the social welfare function and using (26) and (27), we obtain

$$\left. \frac{d\mathcal{W}}{di} \right|_{i=0} = \frac{u'(q)[u'(q) - \psi'(q)](1-n)^2}{[u''(q) - z''(q)]n} + (1-2n)z(q). \quad (28)$$

The intuition provided by Shi (1997) for a positive welfare effect of inflation is based on an envelope argument. If the Friedman rule achieves  $q^*$  then the first term on the right-hand side of (28) is zero and a deviation from the Friedman Rule is optimal provided that  $n < 1/2$  or, from (24), if  $\frac{u(q^*) - z(q^*)}{u(q^*) - \psi(q^*)} > 1/2$ .<sup>15</sup> For instance, under proportional bargaining, a deviation from the Friedman rule is optimal whenever  $\theta > 0.5$ . In this case, the policy maker would be willing to trade off

<sup>14</sup>The first-order condition of the problem (25) is necessary and sufficient under proportional bargaining —the problem is strictly concave— and under Nash when some conditions on primitives enumerated in LW are imposed.

<sup>15</sup>Since  $q = q^*$  at  $i = 0$ , the welfare effect of a deviation from the Friedman rule is given by  $\frac{d\mathcal{W}}{di} = z(q^*)(1-2n^*)$ . From (24),  $n^* = \frac{z(q^*) - \psi(q^*)}{u(q^*) - \psi(q^*)}$ . Therefore,  $\frac{d\mathcal{W}}{di} > 0$  iff  $\frac{z(q^*) - \psi(q^*)}{u(q^*) - \psi(q^*)} < 1/2$  or, equivalently,  $\frac{u(q^*) - z(q^*)}{u(q^*) - \psi(q^*)} > 1/2$ .



efficiency on the intensive margin to improve the extensive margin by raising inflation so as to increase the number of sellers and reduce the number of buyers.<sup>16</sup> However, under generalized Nash bargaining the first term on the right-hand side of (28) is not equal to 0 at the Friedman rule ( $q < q^*$ ) due to the lack of strong monotonicity. Thus, it is much more difficult to show that a deviation from the Friedman rule is optimal.

We demonstrate this point more forcefully by comparing the bargaining solutions numerically. The results are reported in Figure 5. For sake of comparison, we follow Shi (1997) and adopt the following functional forms:  $u(q) = q$  and  $\psi(q) = q^\eta/\eta$  where  $\eta > 1$  and  $q^* = 1$ . In the left charts a green (shaded) area indicates the region in the space of parameter values where a deviation from the Friedman rule is optimal. In these figures we use  $\eta$  and  $\theta$  which are the two important structural parameters.<sup>17</sup> The charts in the second column use a “topographic map” to indicate the value for  $q$  at the Friedman rule under the three bargaining solutions. The contours join points on the surface that generate the same value for  $q$  in  $\{0.4, 0.5, \dots, 1\}$ . The region for  $q = 1$  is white and regions with lower values for  $q$  are darker. So the inefficiency of the intensive margin at the Friedman rule gets more severe as the region gets darker. Similarly, the charts in the third column plot the topographic map for  $n$ . The regions are darker for values of  $n$  close to 0 and lighter for values of  $n$  close to 1. So efficiency on the extensive margin occurs in regions which are neither too dark nor too light.

---

<sup>16</sup>Notice that  $\frac{u(q^*)-z(q^*)}{u(q^*)-\psi(q^*)} = 1/2$  corresponds to the Hosios (1990) condition for efficiency. The argument according to which a deviation from the Friedman rule could be optimal when the Hosios (1990) condition is violated has been spelled out by Berentsen, Rocheteau and Shi (2006) for a particular bargaining protocol. Also, despite some similarities, our results differ from those in Shi (1997) in that an increase in the money growth rate raises the number of buyers in Shi’s model. Therefore, a deviation from the Friedman rule in Shi’s model is welfare improving when the number of buyers is too low.

<sup>17</sup>In principle,  $\theta$  is a different parameter under the generalized Nash and the proportional bargaining solutions. However, in both cases  $\theta$  represents the buyer’s share of the match surplus when  $q = q^*$ .

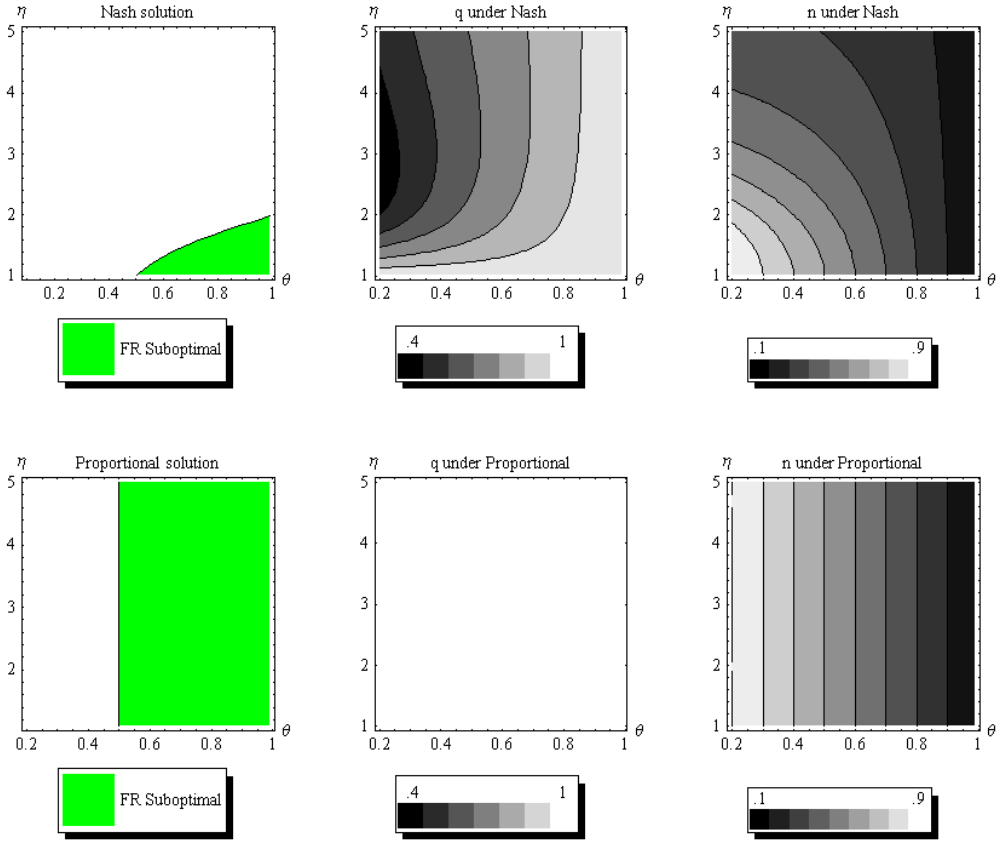


Figure 5: (Sub)Optimality of the Friedman rule

The charts in the first column of Figure 5 reveal that a deviation from the Friedman rule is more likely to be optimal under the proportional bargaining solution than under the Nash solution: the set of parameter values under which  $dW/di > 0$  with proportional bargaining encompasses the set of parameter values under which  $dW/di > 0$  with Nash bargaining. Put differently, for  $\eta$  sufficiently large, there is no  $\theta \in [0, 1]$  such that  $dW/di > 0$  at  $i = 0$  under Nash or proportional bargaining solutions. In contrast, for any  $\eta > 1$ , the Friedman rule is suboptimal under the proportional solution whenever  $\theta > 0.5$ .

The intuition for this result is clear from the charts in the second column. Under proportional bargaining,  $q = q^* = 1$  at the Friedman rule for all parameter values. Therefore, from an envelope-type argument it is optimal to raise  $\pi$  to reduce the number of buyers whenever  $n < 1/2$ , which

occurs when  $\theta > 1/2$  (see charts in the third column). However, under the Nash solution,  $q < q^*$  and the negative welfare effect of a reduction in  $q$  is first-order. Furthermore, the first-order welfare loss from reducing  $q$  gets even larger as  $\theta$  decreases and  $\eta$  increases (the regions in the topographic map which are darker). From all this, we conclude that a deviation from the Friedman rule is more likely to be optimal under proportional bargaining than generalized Nash bargaining.

## 6 Welfare cost of inflation

Lagos and Wright (2005, p. 481) argue that the result according to which the Friedman rule does not achieve the first-best for  $\theta < 1$  has "*sizable implications for the cost of inflation in a calibrated version of the model*". We showed in Section 4 that the Friedman rule does achieve the first-best output under strongly monotonic bargaining solutions. Thus, the welfare cost of inflation should be significantly lower under the egalitarian solution than it is under the Nash solution. We check this conjecture by quantifying the welfare cost of inflation under different bargaining solutions.

We follow closely the methodology of LW and Lucas (2000). We set a period to a year,  $\sigma = 0.5$  and  $\beta^{-1} = 1.03$ . For calibration purposes, the utility in the CM is now  $U(c) - h$  with  $U(c) = B \ln c$ . Furthermore,  $u(q) = q^{1-\eta}/(1-\eta)$  and  $\psi(q) = q$ . We choose the parameters  $(\eta, B)$  to fit money demand in the model to the data.<sup>18</sup> Money demand is defined as  $L \equiv M/PY$ . In the model, nominal CM output is  $pB$ , and nominal output in the DM is  $\sigma M$ . Hence,  $PY = pB + \sigma M$  and  $Y = B + \sigma M/p$ . In equilibrium,  $M/P = z(q)$ , and so

$$L = \frac{M/P}{Y} = \frac{z(q)}{B + \sigma z(q)},$$

where  $q$ , and hence  $L$ , is a function of  $i$ . Following Lucas (2000),  $i$  is taken to be the commercial paper rate and let  $M$  be  $M1$ . The sample period is 1900-2000. We recalibrate the model for each bargaining solution. While we do not show it here, both bargaining solutions can fit the money demand data equally well.

Our measure of the cost of inflation is the fraction  $1 - \Delta_0(\pi)$  of total consumption that agents would be willing to give up to have zero inflation instead of  $\pi$ . Let  $q_\pi$  be the output traded in bilateral matches in the DM when the inflation rate is  $\pi$ . For a given  $\pi$ ,  $\Delta_0$  solves

$$U(B) - B + \sigma [u(q_\pi) - q_\pi] = U(B\Delta_0) - B + \sigma [u(q_0\Delta_0) - q_0].$$

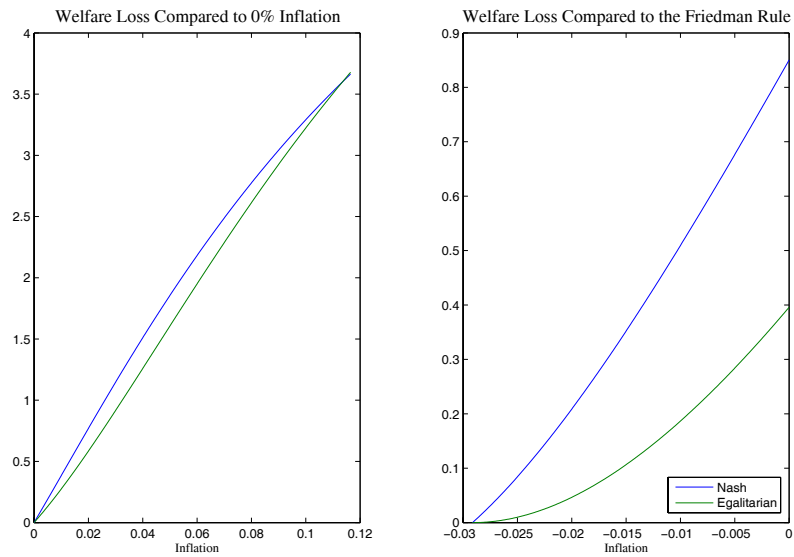
---

<sup>18</sup>Alternatively, one could fix  $\eta$  to some arbitrary value and choose  $(\sigma, B)$  to fit money demand. This alternative calibration method works well for the Nash solution but not for proportional bargaining. Intuitively, the values of  $\sigma$  that generate a good fit are too low to support a monetary equilibrium.

We also consider  $\Delta_F(\pi)$ , which is how much they would give up to have the Friedman rule rather than  $\pi$ . In Table 1, we report  $1 - \Delta_0(0.1)$ ,  $1 - \Delta_F(0.1)$  and  $1 - \Delta_F(0)$  as well as the equilibrium value for  $q$  for  $\pi = 0.1$ ,  $\pi = 0$  and at the Friedman rule ( $q_F$ ) while in Figure 6, we plot  $1 - \Delta_0$  and  $1 - \Delta_F$  for a range of interest rates.

	Nash	Egalitarian	Gen. Nash	Proportional
	$\eta = 0.26$ $B = 1.77$	$\eta = 0.26$ $B = 2.19$	$\eta = 0.36$ $B = 1.65$ $\theta = 0.32$	$\eta = 0.37$ $B = 2.65$ $\theta = 0.34$
$q_{0.1}$	0.14	0.13	0.09	0.07
$q_0$	0.50	0.64	0.34	0.61
$q_F$	0.81	1	0.60	1
$1 - \Delta_0(0.1)$	3.31%	3.24%	5.19 %	5.25 %
$1 - \Delta_F(0.1)$	3.95%	3.45%	6.92 %	5.56 %
$1 - \Delta_F(0)$	0.86%	0.39%	2.10 %	0.55%

**Table 1:** Cost of inflation



**Figure 6:** Welfare cost of inflation

As suggested by Table 1 and Figure 6, under the Nash solution output is inefficiently low at the Friedman rule ( $q_F = 0.81 < 1$ ) and the welfare cost of increasing  $\pi$  from 0 to 10% is about 3.3% of GDP.

For the egalitarian solution the welfare cost of increasing  $\pi$  from 0 to 10% is also large – 3.2% of GDP – and is very similar to the estimates found under Nash bargaining. This suggests that, for any bargaining solution, the welfare costs of inflation are much larger in a search model of money than arise in standard models. Thus, in contrast to LW’s conjecture above, it appears that the large welfare costs of inflation are due to bargaining per se and not the fact that the Nash fails to generate the first-best allocation at the Friedman rule.

So what is it about bargaining that leads to such similar and large welfare costs? Here is where the LW logic about the holdup problem is correct. Whenever  $\theta < 1$  and  $i > 0$  (as it is when inflation goes from zero to 10%) *any* bargaining solution generates a holdup problem on money holdings, therefore it is not surprising that the welfare cost of going from 0% inflation to 10% inflation is very similar across both models. Understanding the holdup problem now permits us to see why the welfare costs are high compared to those of Lucas (2000) for example. The expected utility gain from holding an additional unit of money is equal to the probability of a single-coincidence match,  $\sigma$ , times the increase in the buyers’ surplus. Under proportional bargaining, this gain is  $\sigma\theta[u'(q) - \psi'(q)]\frac{dq}{dz}$  and it is equal to the opportunity cost of holding money,  $i$ . The expected utility gain for society is  $\sigma[u'(q) - \psi'(q)]\frac{dq}{dz}$  which is equal to  $i/\theta$ . Thus, the social marginal return of real balances is  $\frac{1}{\theta}$  times the private marginal return. Put it differently, the individual money demand does not accurately capture the social value of holding money since it ignores the seller’s surplus. For  $\theta = 1/2$  the social welfare cost of inflation is approximately *twice* the private cost for money holders which is often approximated by the area underneath the money demand curve (e.g., Lucas, 2000).<sup>19</sup> So the LW explanation that the welfare costs of inflation are large under bargaining due to a holdup problem is, in fact, correct. What is incorrect about their argument is the attribution of these large welfare costs to the fact that  $q < q^*$  at the Friedman rule.

However, differences across bargaining solutions become more apparent if one considers an increase from the Friedman rule to 10% inflation. Under Nash bargaining 10% inflation relative to the Friedman rule costs almost 4% of GDP while under the egalitarian solution the cost is 3.5%. As suggested by the right panel of Figure 6, most of this difference occurs for very low interest rates. The welfare gain of reducing inflation from 0 to the Friedman rule is worth 0.9% of GDP

---

<sup>19</sup>For an elaboration of this idea, see Craig and Rocheteau (2006).

under Nash bargaining, while under the egalitarian solution it is 0.4% of GDP. So understanding the formation of terms of trade is crucial for the assessment of the welfare cost of inflation at low (negative) inflation rates.

The exercise above is done assuming  $\theta = 1/2$  for both bargaining solutions. In general, we would like to calibrate this parameter rather than fixing it arbitrarily. Since  $1 - \theta$  is an indirect measure of the seller's pricing power, we follow LW and calibrate  $\theta$  to match the aggregate markup (the ratio of price over marginal cost) in the data. The ratio of price over marginal cost in the DM is  $M/pq$ , while in the CM it is 1. The aggregate markup  $\mu$  averages the markups in the two sectors using the shares of output in each sector. As in LW we target  $\mu = 1.1$  when the inflation rate is 4%. The results are reported in the last two columns of Table 1. Again, both bargaining solutions predict very large but similar welfare costs of 10% inflation relative to price stability – around 5% of GDP. Consequently, as before, the ability of the Friedman rule to achieve the first-best allocation does not seem to matter for the welfare cost of moderate inflation. On the other hand, the welfare gain of reducing inflation from 0 to the Friedman rule is about four times larger under the generalized Nash solution than under the proportional bargaining solution.

In our earlier working paper, we pursued our analysis by considering a fully calibrated version of the model that introduces capital and taxation along the lines of Aruoba, Waller and Wright (2006). Aruoba, Waller and Wright find that capital is fairly inelastic to the inflation rate under Nash bargaining whereas it is much more responsive under competitive pricing: they attribute this result to an holdup problem under Nash bargaining. Using the egalitarian solution we show on the contrary that inflation does have a significant and negative effect on capital formation despite the presence of an holdup inefficiency inherent to bargaining. Lowering the inflation rate from 10 percent to the Friedman rule raises the aggregate capital stock by about 33 percent – compared to a mere 3 percent under Nash bargaining. As a result of the sensitivity of capital to inflation, the welfare cost of inflation is greater under egalitarian bargaining than it is under Nash bargaining.

## 7 Summary

Bargaining is an integral part of models of decentralized trading. Yet very little work has been done to understand how various bargaining solutions affect the qualitative and quantitative predictions of these models. In this paper we examined two standard bargaining solutions to do just that in the context of a monetary search model. Our analysis provided qualitative and quantitative insights as to which properties of the bargaining solution matter for the efficiency of equilibrium, the optimal

policy and the welfare cost of inflation.

We first showed that the ability of the Friedman rule to generate the first-best allocation is linked to the monotonicity properties of the bargaining solution. Under the Nash solution – a non-monotonic solution – the quantities traded are inefficiently low even when the nominal interest rate is driven to zero. In contrast, under the strongly monotonic egalitarian solution the Friedman rule achieves the first best.

With endogenous participation, the optimal monetary policy crucially depends on the assumed bargaining solution. The key finding is that the Friedman rule is more likely to be suboptimal under proportional bargaining solutions than under generalized Nash bargaining. Once again, this finding is driven by the differences in monotonicity properties of the two bargaining solutions.

We then examined how these efficiency results affect the welfare cost of inflation. Based on the Lucas (2000) methodology, we showed that the welfare cost of 10 percent inflation (relative to price stability) is similar across both bargaining solutions due to the holdup problem on money (away from the Friedman rule). However, the welfare gain from reducing inflation from zero to the Friedman rule is larger than 2% of GDP under generalized Nash bargaining but about half a percent of GDP under proportional bargaining. This latter difference is due to the monotonicity properties of the bargaining solution as opposed to the holdup problem.

As a final thought, one is left wondering if strong monotonicity is an appealing, or even realistic, attribute of a bargaining solution for monetary models. According to Kalai (1977, p. 1623), monotonicity is "*a bargaining principle since a player who is asked to lose utility because of the new options may have a very convincing case in threatening to break cooperation.*"<sup>20</sup> Even though we also view monotonicity as a natural requirement for a bargaining solution, we do not want to dismiss the Nash solution which has strong strategic foundations. However, it is sensible to check the robustness of the the results obtained under Nash bargaining in search-theoretic models of money to alternative bargaining solutions such as the one proposed in this paper.

---

<sup>20</sup>The appeal of the monotonicity property is discussed in Section 4 of Kalai (1977) and in the introduction section of Kalai and Samet (1985).

## References

- Aruoba, S. Boragan, Guillaume Rocheteau and Christopher Waller (2006). “*Bargaining and the Value of Money*,” Policy Discussion Paper of the Federal Reserve Bank of Cleveland.
- Aruoba, S. Boragan, Waller, Christopher and Wright, Randall (2006). “*Money and capital*,” mimeo.
- Berentsen, Aleksander, Rocheteau, Guillaume, and Shi, Shouyong (2006). “*Friedman meets Hosios: Efficiency in search models of money*,” Economic Journal (Forthcoming).
- Chun, Youngsub and Thomson, William (1988). “*Monotonicities properties of bargaining solutions when applied to economics*,” Mathematical Social Sciences 15, 11-27.
- Coles, Melvyn and Wright, Randall (1998). “*A dynamic equilibrium model of search and bargaining*,” Journal of Economic Theory 78, 32-54.
- Craig, Ben and Rocheteau, Guillaume (2006). “*Inflation and welfare: A search approach*,” Policy Discussion Paper of the Federal Reserve Bank of Cleveland.
- Curtis, Elisabeth and Wright, Randall (2004). “*Price setting, price dispersion, and the value of money: or, the law of two prices*,” Journal of Monetary Economics 51, 1599-1621.
- Grout, Paul (1984). “*Investment and wages in the absence of a binding contract*,” Econometrica 52, 449-460.
- Hosios, Arthur (1990). “*On the efficiency of matching and related models of search and unemployment*,” Review of Economic Studies 57, 279-298.
- Julien, Benoit, Kennes, John and Ian King (2006). “*Monetary Exchange with Multilateral Matching*,” Journal of Economic Theory (Forthcoming).
- Kalai, Ehud (1977). “*Proportional solutions to bargaining situations: Interpersonal utility comparisons*,” Econometrica 45, 1623-1630.
- Kalai, Ehud and Samet, Dov (1985). “*Monotonic solutions to general cooperative games*,” Econometrica 53, 307-327.
- Kalai, Ehud and Smorodinsky, Meir (1975). “*Other solutions to Nash’s bargaining problem*,” Econometrica 43, 513-518.
- Kocherlakota, Narayana (1998). “*Money is memory*,” Journal of Economic Theory 81, 232-251.



- Lagos, Ricardo and Rocheteau, Guillaume (2006). “*Money and capital as competing media of exchange,*” *Journal of Economic Theory* (Forthcoming).
- Lagos, Ricardo and Wright, Randall (2005). “*A unified framework for monetary theory and policy analysis,*” *Journal of Political Economy* 113, 463-484.
- Lucas, Robert (2000). “*Inflation and welfare,*” *Econometrica* 68, 247-274.
- Nash, John (1950). “*The bargaining problem,*” *Econometrica* 18, 155-162.
- Rocheteau, Guillaume, and Wright, Randall (2004). “*Inflation and Welfare in Models with Trading Frictions,*” in *Monetary Policy in Low Inflation Economies*, David Altig and Ed Nosal, eds., Cambridge University Press (Forthcoming).
- Rocheteau, Guillaume and Wright, Randall (2005). “*Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium,*” *Econometrica* 73, 175-202.
- Rupert, Peter, Schindler, Martin and Wright, Randall (2001). “*Generalized search-theoretic models of monetary exchange,*” *Journal of Monetary Economics* 48, 605-622.
- Shi, Shouyong (1995). “*Money and prices: A model of search and bargaining,*” *Journal of Economic Theory* 67, 467-496.
- Shi, Shouyong (1997). “*A divisible search model of fiat money,*” *Econometrica* 65, 75-102.
- Thomson, William, and Myerson, Roger (1980). “*Monotonicity and independence axioms,*” *International Journal of Game Theory*, 9, 37-49.
- Trejos, Alberto and Wright, Randall (1995). “*Search, bargaining, money, and prices,*” *Journal of Political Economy* 103, 118-141.
- Wallace, Neil (2001). “*Whither monetary economics?*” *International Economic review* 42, 847-869.