Innovation, Demand for Skills, and Productivity Growth

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Abstract

Young firm activity shares have been declining in the U.S. and the decline has been particularly pronounced in the high-tech sector post-2000. Do labor market frictions play a role in declining young firm activities and the associated slower productivity growth? Using a longitudinal worker-firm matched dataset from the U.S. Census Bureau, I document that declining young firm activities are accompanied by: 1) a decline in the growth rate of the demand for skills in the high-tech sector, and 2) a flattening of the life cycle of skilled labor accumulation of high-tech firms. By developing an endogenous growth firm dynamics model that is consistent with the micro-level skilled labor accumulation over the firm life cycle, I show that rising frictions in skilled labor adjustment can explain the joint evolution of young firm employment shares and demand for skills. These frictions influence productivity growth through affecting the stock of human capital firms possess. A calibrated model shows that a rise in skilled labor adjustment costs lowers productivity growth by 75 basis points in the high-tech sector. A rise in entry costs, on the other hand, is not likely the main driver for declining young firm activities, as it implies an increase in demand for skills. Finally, productivity gain (loss) from reallocation can be offset by the general equilibrium effects of reallocation on aggregate demand for skills.

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1 Introduction

[1]The U.S. economy has been experiencing a secular decline in the pace of business formation and young firm activity shares in recent decades. In particular, the post-2000 decline has been very pronounced in the high-tech sector.[2] Should we be concerned about these trends?

Recent literature hasn't reached a consensus on what caused the decline. On the one hand, labor supply side explanations argue that declining young firm activity reflects an efficient response to broader trends such as slower population growth, which leads to lower firm entry rates (Hopenhayn et al. (2018), Karahan et al. (2019)), or skill-biased technological progress, which raises the attractiveness of becoming a worker relative to being an entrepreneur (Salgado (2019)). On the other hand, some studies argue that frictions may affect young firm activity (Davis and Haltiwanger (2014), Decker et al. (2018), Akcigit and Ates (2019)). Despite a growing literature studying these empirical trends and possible drivers behind declining dynamism, we still don't fully understand the underlying factors, or the impact of declining dynamism on long-term growth.

This chapter studies the possible connections between business dynamism and productivity growth. I focus on demand side factors that affect firms' decision to enter and grow. Using a longitudinal worker-firm matched dataset built from administrative databases from the U.S. Census Bureau, I document a novel fact about the high-tech sector: the post-2000 decline in young firm activity has been accompanied by a decline in the growth rate of *demand for skills*. Furthermore, I document that the aggregate decline in the growth of demand for skills is driven by a decline in the speed and level of firms' skilled labor's accumulation over their *life cycle*.

Motivated by these empirical facts, I develop an innovation-based firm dynamics

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[2]Literature documents the secular trends include Davis et al. (2012), Hyatt and Spletzer (2013), Decker et al. (2014a), Decker et al. (2014b), Haltiwanger et al. (2014a), Karahan et al. (2019), among others.

model that is consistent with micro-level evidence on firms' skilled labor accumulation. Using this model, I study the joint evolution of young firm employment shares and demand for skills in the high-tech sector. My quantitative exercises show that rising adjustment costs in firms' skilled labor adjustment drive down both the young firm employment share and firms' demand for skills, as observed in the data. Moreover, rising adjustment costs lead to a decline in productivity growth of 75 basis points. An increase in entry costs, on the other hand, is not likely to be the dominant driver for the decline in the young firm employment share, as it predicts a counterfactual increase in the demand for skills.

The contribution to the literature is twofold. First, empirically, this chapter is the first to examine demand for skills in the high-tech sector and to document the post-2000 slowdown in its growth. It is also the first to study firms' life cycle of skilled labor accumulation and how this has changed over time. Second, theoretically, I develop an innovation-based endogenous growth model that is consistent with firms' skilled labor accumulation over their life cycle, and use this model to study the joint evolution of the young firm employment share and demand for skills and the implication of rising adjustment costs for productivity growth.

I estimate demand for skills based on the canonical framework developed by Katz and Murphy (1992) and later analyzed in Katz et al. (1999), Autor et al. (2008), and Acemoglu and Autor (2011). The dataset I use is the Longitudinal Employer and Household Dynamics dataset (LEHD), augmented by the Longitudinal Business Database (LBD) and the American Community Survey (ACS) from the U.S. Census Bureau. This dataset covers the universe of U.S. private sector jobs and hence provides us an unbiased view of the economy with a large enough sample size even if we zoom into a narrowly defined sector. I construct composition-adjusted relative wages and relative quantities of skilled to unskilled workers. Assuming the two skill groups are imperfect substitutes, a shift in the demand curve for skills can be inferred from the relative price and quantity series. I find that in the high-tech sector, the growth in the demand for skills slowed down significantly post-2000. The timing coincides with the decline in young firm activity shares in the high-tech sector.

The unique marriage between worker and firm characteristics in LEHD allows me to study not only aggregate demand for skills, but also underlying patterns of firms' skilled labor accumulation. I document how the ratio of the stock of skilled labor to that of unskilled labor changes as firms age in the high-tech sector. I show that firms accumulate skill rapidly when they are young, but the pace of accumulation slows as firms age. I also find that the shape of this life cycle pattern has changed over time. Compared to firms that entered the economy before 2000, firms that were born after 2000 accumulate skilled labor more slowly and tend to have a lower stock of skilled labor when they mature. We refer to this below as a flattening of the life cycle pattern.

The empirical findings motivate me to develop a firm dynamics model with skilled labor accumulation to study the joint evolution of young firm employment shares and demand for skills. This model builds on the endogenous technological change literature and in particular the framework developed by Klette and Kortum (2004), as it delivers a general equilibrium model of technological change while capturing firm entry and exit, and hence is well suited for the analysis of firm dynamics and productivity growth. My model is closely related to the model of Acemoglu et al. (2018), which considers differences between skilled and unskilled labor. I extend those models by introducing adjustment costs to firms when changing their stock of skilled labor. This feature is key to generating a life cycle pattern in the model as we observe in the micro data.

In my model, firms hire skilled labor to perform R&D. A successful innovation provides a technological advantage that enables a firm to take over a competitor's product line. Adjusting the current stock of skilled labor is costly due to the presence of adjustment costs, and this implies that a firm's stock of skilled labor increases as it ages. After a successful innovation, a firm's markup depends on its stock of skilled labor. This assumption captures the fact that it takes time for a young firm to establish a customer base and gain market share. I prove that under general conditions there exists a solution to the model in which a firm's stock of skilled labor per product line increases over firm age and converges to a unique long-run level.

I calibrate the model to be consistent with key features of the high-tech sector for the 1990-2000 period and study how factors that depress young firms' employment share could also affect demand for skills and productivity growth in the high-tech sector. I find that rising frictions in skilled labor adjustment, which raise the marginal cost of hiring skilled labor for all firms, lead to a decline in aggregate demand for skills. Such frictions hurt young firms disproportionately, as young firms have a higher incentive to adjust the stock of their skilled labor. Young firm employment shares also decline as a result. Long-term growth is hampered in this cases, as a lower stock of human capital (skill) leads to less innovation and hence lower productivity growth. An increase in skilled labor adjustment costs that is sufficient to generate the decline in the young firm employment share observed in the data post-2000 leads to a reduction in the long-term productivity growth of 75 basis points.

Rising costs of entry, on the other hand, reduce the entry rate and young firm employment share, but do not necessarily lead to lower productivity growth. This is because incumbents' probability of survival increases when they face less threat from entrants. An increase in the expected firm value due to higher survival rates incentivizes incumbents to hire more skilled labor, and aggregate demand for skills increases. In equilibrium, incumbents have a higher stock of skilled labor, and that leads to an increase in productivity growth. The quantitative impact of reallocation from young to old firms on productivity growth is therefore ambiguous in this case, depending on the relative strength of the two competing forces in equilibrium: the loss from the relative innovation capacity of young vs. old firms, and the gain from an increase in the expected future value of old firms.

In sum, the quantitative study suggests that rising entry costs are unlikely to be driving the decline in young firms' employment shares, as they should be associated with a rising demand for skills. Rising frictions in hiring skilled labor reconcile both patterns, and are concerning as they imply lower long-term growth.

This chapter is connected to several strands of literature. First, it contributes to the literature that studies declining business dynamism, its drivers and implications. Many papers have documented declining entrepreneurship and young firm activities along with declining labor market fluidity in the United States (Davis et al. (2012), Hyatt and Spletzer (2013), Decker et al. (2014a), Decker et al. (2014b), Davis and Haltiwanger (2014), Decker et al. (2018), Molloy et al. (2016), etc.). This chapter complements these empirical studies by documenting a companion feature - declining growth in demand for skills - in the high-tech sector.

Existing studies that develop theoretical frameworks to understand the causes of declining business dynamism cannot also explain this new fact about declining growth in demand for skills. The labor supply story of slower population growth (Hopenhayn et al. (2018) and Karahan et al. (2019)) cannot explain why the share of skilled labor in firms has declined relative to unskilled labor.[3] The skill-based technological progress argument from Salgado (2019) is also unable to reconcile this empirical fact. Salgado (2019) focuses on the entire economy, but if his mechanism - entrepreneurship declines in response to a rising skill premium - holds broadly, we should expect to observe an increase (or at least a slower decline) in the share of young firms in the high-tech sector post-2000, as data suggest that the growth rate of the skill premium fell during that period.

This chapter instead highlights the role of frictions. Davis and Haltiwanger (2014) argue that rising regulations such as occupational licensing or employment protection decrease labor market fluidity and may hurt startups and young firms more. Decker et al. (2018) document weaker marginal responsiveness of businesses to productivity shocks and rising within-industry dispersion of TFP and output per worker in the post-2000 period, consistent with an increase in adjustment frictions. Akcigit and Ates (2019) highlight the decline in knowledge diffusion from frontier firms to laggard firms as the dominant driver underlying declining business dynamism. My work complements this line of literature by highlighting that frictions in adjusting skilled labor could lead to the decline in both young firm activities and demand for skills as observed in the data.

Second, this chapter connects to the literature that studies firm dynamics and productivity. The seminal work of Hopenhayn and Rogerson (1993) shows that an increase in frictions on labor adjustment reduces productivity as it reduces allocative efficiency of factor inputs. Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) also study the impact of allocative efficiency on aggregate productivity. The influential work by Hsieh and Klenow (2009) quantifies the role of allocative efficiency for aggregate productivity using firm-level data. They show that distortions that drive wedges between the marginal products of labor and capital across firms will lower aggregate TFP. While they emphasize the role of *allocation* of factor inputs, I focus on the role of the *stock* of

[3]The argument from Hopenhayn et al. (2018) that the non-production to production worker ratio declined as a result of the aging firm distribution could potentially shed light on the decline in the share of skilled labor, if we assume that non-production workers are mostly skilled labor.

factor inputs. Moreover, they focus on the *level* of productivity, while I instead study the *growth* of productivity through an endogenous growth model.

Third, this chapter connects to the endogenous growth literature. The inspiration to look at the impact of the stock of human capital on growth comes from the seminal work of Romer (1990), in which the stock of human capital determines the rate of growth. Romer (1990) builds on a Solow-type model with technological change where human capital affects growth through the nonrivalry property of ideas. I instead model human capital as a direct input to R&D in a firm dynamics model. This model is closely related to the endogenous growth firm dynamics literature (Grossman and Helpman (1991), Aghion and Howitt (1992), Klette and Kortum (2004)) and in particular builds upon Acemoglu et al. (2018). The difference between my model and this line of literature is twofold. First, my model focuses on the age dimension of firms, and in particular is the first to match micro-level life cycle patterns of firms' human capital accumulation. Second, Acemoglu et al. (2018) explore the positive impact on growth from the reallocation of skill from old to young firms, which are assumed to be more innovative. My model, on the other hand, suggests that such a gain from reallocation is not always guaranteed, since in equilibrium reallocation of skilled labor from old to young firms increases the destruction rate, which lowers the expected value of firms. With lower expected future value, firms hire less skilled labor, which leads to lower growth. In other words, I consider how structural changes affect the stock of human capital, how this affects growth, and how this channel offsets the composition effects from reallocation.

Finally, this chapter connects to the literature that studies the evolution of aggregate demand for skills. Autor and Price (2013), Beaudry et al. (2016) and Valletta (2018) document a reversal in the demand for skill and cognitive tasks in the post-2000 period for the U.S. economy. While these studies look at the overall U.S. economy and the labor market impacts of the decline in demand for skills, I focus on the high-tech sector and study the impact of declining demand for skills on the slowdown in productivity growth. I also study a potential cause of this decline in skill demand: a rise in skilled labor adjustment costs.

The rest of the chapter is organized as follows. Section 2 documents the aggregate demand for skills in the high-tech sector and the underlying firms' life cycle of skilled labor accumulation. Section 3 presents the model. Section 4 presents the quantitative results and the last section concludes.

2 Empirical Facts

In this section, I document changes over time in the demand for skills and life cycle patterns of skilled labor accumulation for high-tech firms in the United States.

2.1 Data

The main dataset used for the analysis is the Longitudinal Employer-Household Dynamic (LEHD) dataset from the U.S. Census Bureau. LEHD is a matched employer-employee dataset that covers 95% of U.S. private sector jobs.[4] I use the 2014 snapshot of the data, which covers information from 1990 to 2014. LEHD tracks individual earnings at a quarterly frequency and provides information on worker demographics (e.g. age, gender, education). The unique combination of worker and firm characteristics in LEHD allows me to analyze firm-level behavior regarding their skilled labor accumulation and demand for skills.

I augment LEHD by the Longitudinal Business Database (LBD). LBD is a census of business establishments and firms in the U.S. with paid employees comprised of survey and administrative records. The LBD tracks business activity information on an annual basis. Data include industry, location, employment, annual payroll, birth, death and ownership changes (if any) at the establishment level.[5] In my analysis, an accurate measure of firm age is important. The fact that LBD provides information on establishments whose identifiers are longitudinally stable as opposed to firm identifiers which can change over time due to ownership, single/multi-unit status or other changes, allows me to have a more robust measure of firm age than the direct usage of firm identifiers from the LEHD, and this is crucial for my analysis. The highest level of business unit ID in the LEHD is the federal EIN. To merge LBD to LEHD, I integrate the federal EIN from the Business

[4] Detailed discussion of LEHD data can be found in Abowd et al. (2009)
[5] Jarmin and Miranda (2002) provides a detailed description of the data.

Register with the LBD and use the crosswalk developed by Haltiwanger et al. (2014c).

Finally, I augment the merged LEHD-LBD data by the American Community Survey (ACS). The education variable in LEHD is heavily imputed, with about 92% of individuals having imputed education (Vilhuber et al. (2018)). To get a sample with a better measure of education, I integrate the demographic information from ACS to the LEHD. After the merge, I keep only the sample with non-imputed education variable which is around 25% of the full sample. As the non-imputed sample is from the Decennial Census and ACS which are random samples of the households in the United States, the 25% non-imputed sample I take is representative of the underlying overall sample.

The focus of this paper is on the high-tech sector. I use the methodology developed by Heckler (2005) and define the high-tech sector as a group of industries with very high shares of workers in the STEM occupations of science, technology, engineering, and math. This sector includes 14 four-digit NAICS industries and covers ICT and biotechnology industries.[6]. A list of high-tech industries is provided in Appendix A.

The final dataset consists of over 600000 firms in the high-tech sector from 1990 to 2014.

2.2 Demand for Skills in the High-Tech Sector

I measure demand for skills following the canonical model methodology developed by Katz and Murphy (1992) and further analyzed by Katz et al. (1999), Autor et al. (2008) and Acemoglu and Autor (2011). The canonical model provides a parsimonious framework for thinking about the skill premium. The key assumption underlying the canonical model is that skilled and unskilled labor are separate inputs for production and they are imperfect substitutes. Any force mimicking skilled-biased technological progress can lead to a shift in the demand curve for skills, and the shift can be derived based on how the price and quantity of skilled labor change relative to unskilled labor. In such models, skilled workers are defined as those with college and above education and unskilled workers as those with high school equivalent education. I follow the same definition as the literature.

[6]Similar definitions have been adopted in Haltiwanger et al. (2014b), Decker et al. (2018)

To calculate demand for skills from the canonical framework, the key is to compute the underlying composition-adjusted relative wages and relative quantity series between skilled and unskilled workers. The demand for skills can be computed as a linear combination of the relative wage and quantity series given the elasticity of substitution between high and low-skilled labor.[7] In LEHD, I define wage for a particular worker in year t as the average full quarter earnings for that worker in year t and labor supply as the total number of quarters the worker worked in that year. I classify workers into 64 demographic cells by gender (x2), education (x4) and experience (x8), and compute the average wage and total labor supply for each cell in each year. The average wage for each cell is a weighted average of quarterly wages where the weights are annual quarters worked. I use fixed weights for each cell to compute aggregate wages, where the fixed weight for each cell is its average share of labor supply over all years (1990 to 2014). Using fixed weights to aggregate wage across groups has the benefit of keeping the composition of the labor force fixed so that the results are not driven by changes in composition.

I aggregate the sample into 4 cells by education level in every year (the 4 education categories are college and above, some college, high school and below high school). The relative wage is the wage ratio of college and above workers to high school workers. The relative supply is the labor supply ratio of college equivalent to high school equivalent workers in efficiency units (where efficiency units of labor supply is defined as the total hours multiplied by the average wage for that cell over the entire sample). The college equivalent labor supply is defined as the sum of labor supply from college and above educated workers plus half of the supply from some college educated workers. The high school equivalent supply is the sum of labor supply from high school educated workers, supply from below high school workers and half of the supply from some college educated workers.

Figure 1 plots the evolution of demand for skills in the high-tech sector between 1990 and 2014. The demand for skills was on a steep upward trend from 1990 until 2000, after which the growth in skill demand slowed down significantly. This finding on the

[7] In particular, I follow Autor et al. (2008) and calculate demand for skills base on $\ln(w_t^H/w_t^L) = (1/\sigma)[D_t - \ln(N_t^H/N_t^L)]$. w^H and w^L are wages for high and low skilled labor respectively and N^H and N^L are corresponding quantities (supply). D_t indexes relative demand shifts favoring high skilled labor. σ represents the elasticity of substitution.

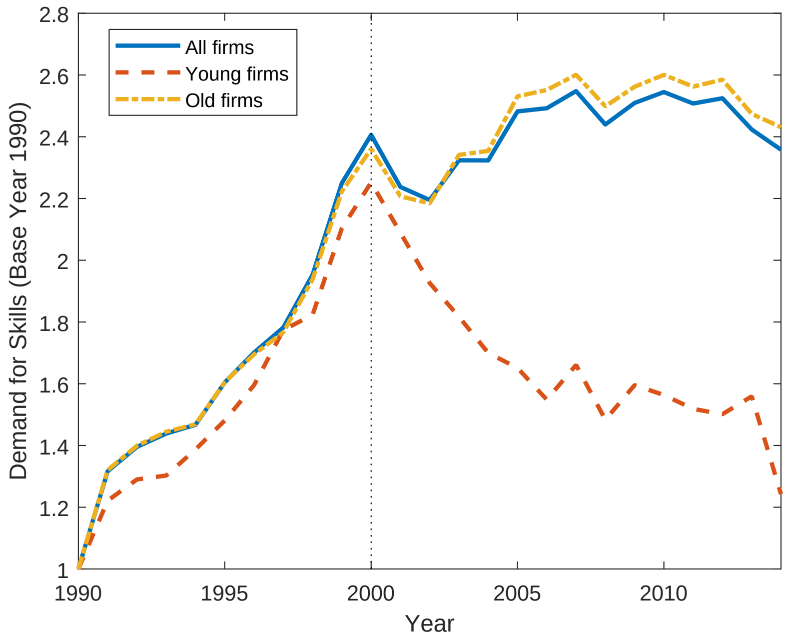


Figure 1: Demand for skills by firm age groups in the high-tech sector

Notes: The blue solid line shows the demand for skills for the high-tech sector as a whole. The red dashed line shows the demand for skills for young firms (less than 5 years old) and the yellow dash-dot line shows the same measure for old firms. The levels at 1990 are normalized to 1 and the elasticity of substitution between high and low skilled labor is 1.62.

high-tech sector resonates with the work of Beaudry et al. (2016) and Valletta (2018), who find a flattening of the wage premium and a reversal of the demand for skills and cognitive tasks for the U.S. economy as a whole. Figure 1 also breaks down the over all sector demand into the demands of young and old firms respectively. While the series for old firms tracks the sectoral trend closely, the demand for skills growth from young firms dropped significantly. Note that the choice of elasticity of substitution is innocuous here. The figure shows the measures assuming an elasticity of 1.62, following Autor et al. (2008), but the significant flattening pattern is robust to elasticities of substitution in the commonly used range of 1 to 3.

The significant slow down of the growth of demand for skills shown in Figure 1 is striking in that it suggests some fundamental change regarding skill demand may have occurred in the high-tech sector post-2000. Given the highly innovative nature of this sector and the importance of human capital to innovation and growth, changes in demand for skills may reflect changes in the underlying innovation process that influence productivity growth.

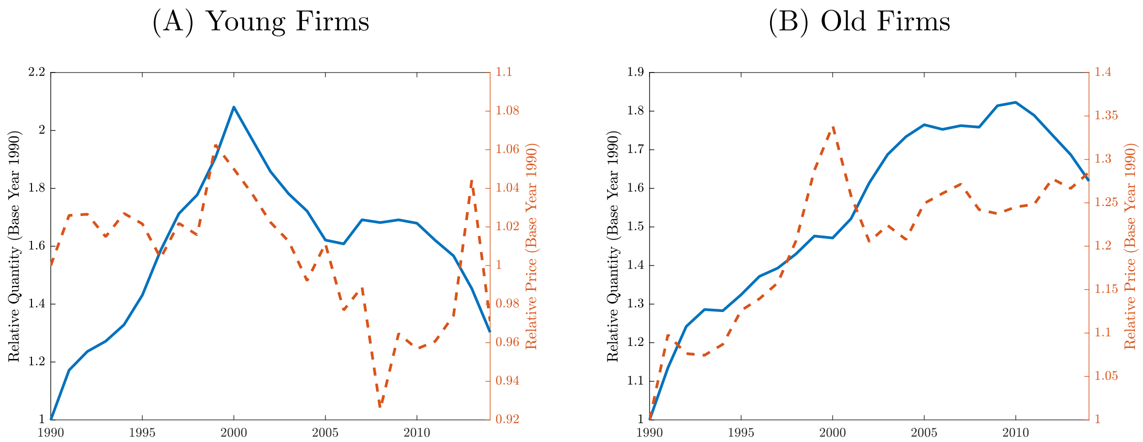


Figure 2: Relative quantity and relative price

Notes: Panel (A) shows the evolution of the relative quantity (solid line, LHS) and relative price (dash line, RHS) of skilled to unskilled labor in young firms (less than 5 year old). Panel (B) shows the evolution of the relative quantity (solid line, LHS) and relative price (dash line, RHS) of skilled to unskilled labor in old firms. The levels in 1990 are normalized to 1.

I further examine the relative price and relative quantity components underlying the estimated demand for skills. Panel (A) of Figure 2 shows that the skill intensity in young firms has been declining since 2000. The declining quantity was accompanied by a decline in the relative price of skill in young firms, indicating a drop in skill demand from young firms. Panel (B) of Figure 2 shows the same series for old firms. I can see that the skill premium flattened post-2000, reflecting anemic demand for skills.

The definition of skill deserves some discussion. The fundamental measure of the skill of an individual is the amount of human capital that individual processes. Education has long been used as a proxy for human capital, although some studies challenged this idea by noting the differences between education and occupation (job task).[8] That line of research argues that human capital is only relevant to the extent it is required by the task a worker is performing. This line of thinking is particularly helpful in analyzing labor market dynamics such as the interaction between technology, offshoring and the structure of wages. Arguable, however, the difference is less of a concern when we focus on a narrowly defined sector where the relationship between education and tasks is more stable. Moreover, the time span I am focusing on is much shorter compared to the literature emphasizing the importance of tasks. Nonetheless, I provide an occupation (task) based demand for skills measure in Appendix B using publicly available data, and

[8]For exmaple, Autor et al. (2010), David and Dorn (2013), among others.

the results are consistent with the what I find in the LEHD using education to proxy for skills.

The sector-level patterns inspire us to dive deeper into individual firms' skill accumulation and look at how that has changed over time. The next section discusses the details.

2.3 Life Cycle of Skill Accumulation among High-Tech Firms

The unique combination of firm and worker information in LEHD allows me to track how firms accumulate skill over time. Before looking into skill accumulation over the firm's life cycle, it is useful to think again about the measurement of skill.

The literature on skill demand following Katz and Murphy (1992) emphasizes that in order to tract aggregate relative quantities and prices, one must adjust for the composition of the pool of workers so that the measured quantity of skill is comparable over time. The same intuition holds when we want to track the quantity of skill a firm possesses over time. To make such a composition adjustment, I convert the number of workers to the equivalent "efficiency units" by multiplying the quantity of workers by a conversion coefficient that is specific to the demographic group the worker belongs to. Demographic groups are defined based on gender, age, education and experience. The conversion factor is defined in the same way as in Autor et al. (2008), as the average relative wage in that group over the entire period from 1990 to 2014.

The life cycle pattern of skill accumulation is calculated as a firm's skill intensity (efficiency units of high-skilled labor divided by efficiency units of low-skilled labor) of that firm at age a relative to age 0.

Figure 3 shows the life cycle pattern of skill intensity of high-tech firms. It is noticeable that high-tech firms accumulate skills rapidly when they are young (less than 5 years old) and that the speed of accumulation slows down. Firms have a relatively stable skill intensity when they mature.

I further examine how this life cycle pattern changes over time. To do so, I first split

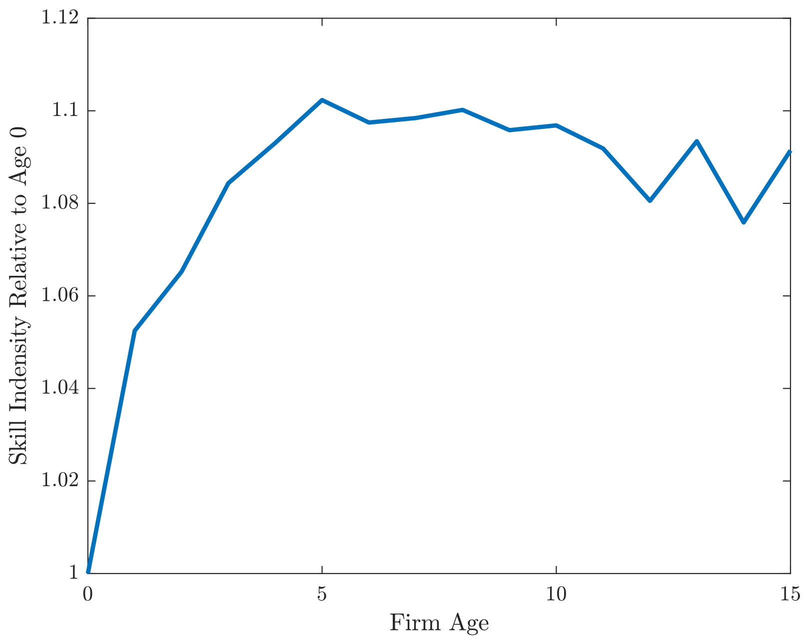


Figure 3: The life cycle of skill accumulation for high-tech firms

Notes: This is the average life cycle of high-tech firms in the sample. We plot the ratio of high skilled labor in efficiency units to low skilled labor in efficiency units relative to age 0. Firms do not need to present for 15 years in order to be included.

the sample firms into cohorts defined by year of entry, and plot the skilled and unskilled labor relative to age 0 for those cohorts. Figure 4 shows the results. Panel (A) of figure 4 shows the high-skilled labor relative to age 0 as the cohorts age. The difference between cohorts entering into the economy before and after 2000 is significant. Cohorts entering before 2000 almost double their level of skilled labor by age 5, but cohorts entering after 2000 increase their skilled labor by less than 50 percent. On the other hand, the differences between the life cycle patterns of unskilled labor are less significant for cohorts entering before and after 2000, as shown in Panel (B) of figure 4.

Finally, I run a fixed effect regression to estimate the life cycle pattern of skill, controlling for firm and time fixed effects. Specifically, I run the following regression:

$$\Delta \ln E^{S,U}_{i,a,t} = \alpha_i + \beta_t + \gamma_a + \epsilon_{a,i,t} \tag{1}$$

for the period before and after 2000 respectively. $E^{S,U}_{i,a,t}$ are the efficiency units of skilled or unskilled labor for firm i with age a at time t. Δ is the change relative to age 0.

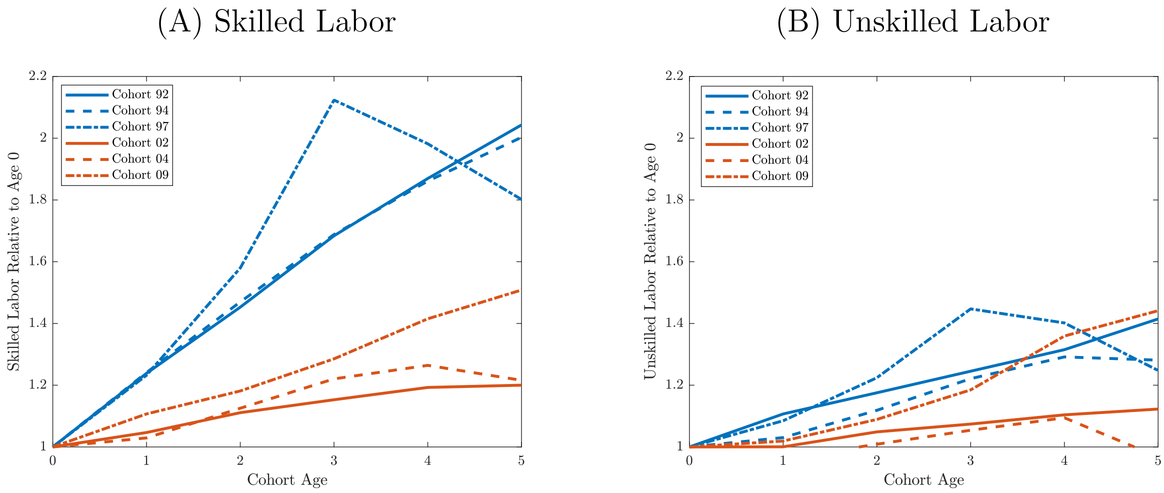


Figure 4: Life cycle of skill for different cohorts

Notes: Panel A shows skilled labor relative to age 0 for cohorts enter the economy at different times. Blue indicates cohorts entering before 2000 and red indicates cohorts entering after 2000. Panel B shows unskilled labor relative to age 0 for cohorts entering the economy at different times. Blue indicates cohorts entering before 2000 and red indicates cohorts entering after 2000.

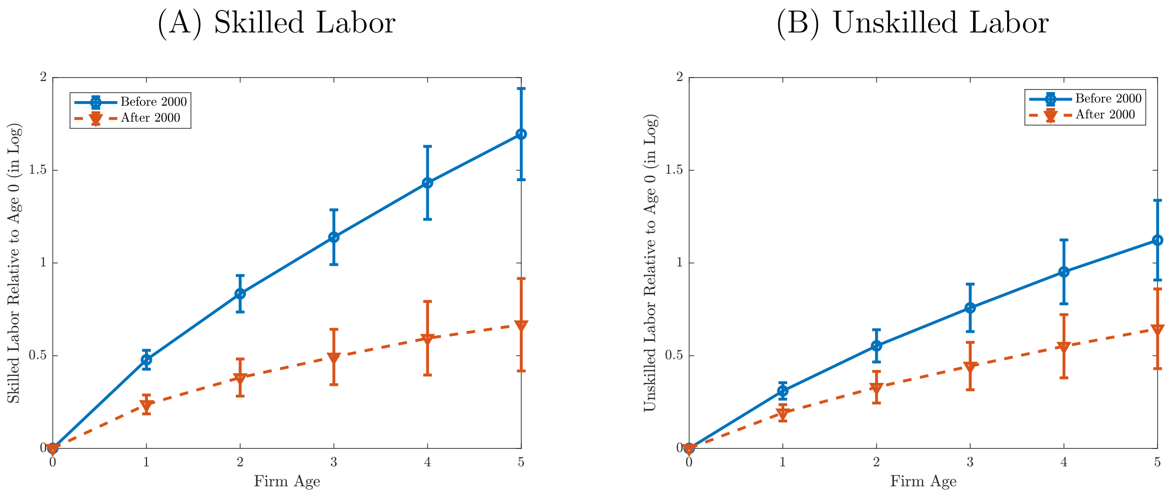


Figure 5: Fixed effects estimates

Notes: Panel (A) shows the fixed effects estimates of the life cycle pattern of skilled labor accumulation for periods before and after 2000 respectively. Panel (B) shows the fixed effects estimates of the life cycle pattern of unskilled labor accumulation for periods before and after 2000 respectively. Error bars indicate 90% confidence intervals.

Figure 5 plots the estimates of γ_a in equation 1, along with 90% confidence intervals. Panel (A) suggests that firms accumulate their skilled labor much more rapidly before 2000 than in the later period. The confidence intervals of the two estimates do not overlap, suggesting that differences between life cycle patterns before and after 2000 are statistically significant. Panel (B) plots the life cycle patterns for unskilled labor. There is also a significant flattening of the life cycle patterns for unskilled labor, but the gap is much smaller than that of unskilled labor. Taking Panel (A) and (B) together, we can see that the skilled labor accumulation in high-tech firms slowed down considerably post-2000.

3 The Model

In this section, I introduce the theoretical framework and characterize the stationary balanced growth equilibrium.

3.1 Final Good Production

The economy has a representative firm that combines intermediate inputs to produce a final good, according to the following production function:

$$\ln Y_t = \int_{j \in \Omega_t} \ln y_{jt} dj, \tag{2}$$

where y_{jt} is the input of intermediate good j at time t, and $\Omega_t \in [0, 1]$ is the set of active product lines at time t. $\mathcal{M}_t$ is the measure of Ω_t and can be smaller than 1. The reason why there can be inactive product lines will be made clear later. The final good is used for consumption.

For each intermediate good j, the final good producer can choose from N_j versions of that good, where the total amount of input j satisfies

$$y_{jt} = \sum_{k=1}^{N_j} x_{jt}^k, \tag{3}$$

where x_{jt}^k is the amount of version k of intermediate good j at time t. Following the standard assumption in the literature (Grossman and Helpman (1991)), I assume that different versions are perfectly substitutable, which implies that the final good producer will use the version with the lowest price.

The optimization problem of the final good producer gives

$$p_{jt}y_{jt} = \frac{1}{\mathcal{M}_t} P_t Y_t, \tag{4}$$

which further implies that

$$\ln P_t = \ln \mathcal{M}_t + \frac{1}{\mathcal{M}_t} \int_{j \in \Omega_t} \ln p_{jt} dj, \tag{5}$$

We choose the final good as the numeraire, i.e. $P_t = 1$.

3.2 Intermediate Good Production

Intermediate good (product) j is produced by the monopolist who has the leading-ledge technology in that product line. A firm can own multiple product lines and produce multiple intermediate goods simultaneously. Firms in the intermediate goods sector hire both skilled and unskilled labor. I assume that after paying a fixed cost of l^f in skilled labor, a firm has access to a linear production technology of the following form:

$$y_{jt} = A_{jt} l_{jt}^u, \tag{6}$$

where l_{jt}^u is the number of unskilled workers employed for producing this good, and A_{jt} is the leading-edge technology of firm f on this product line j. The marginal cost of production is therefore w_t^u / A_{jt}.

I assume that the firm producing version k of intermediate good j has productivity A_{jt}^k and engages in Bertrand monopolistic competition as often assumed in the endogenous technical change literature, where the firm with the lowest marginal cost wins the whole market and sets its price equal to the marginal cost of its closest follower.[9] Assume the

[9] Grossman and Helpman (1991),Lentz and Mortensen (2008),Ates and Saffie (2016), among others.

productivity of the most productive firm of good j is $A_{jt} = \max_k A^k_{jt}$ at time t, with that of the closest follower being $\hat{A}_{jt} = \max_{l, A^k_{jt} \neq A_{jt}} A^k_{jt}$. Due to the Bertrand competition, the equilibrium price of good j is

$$p_{jt} = \frac{1}{\hat{A}_{jt}} w^u_t. \tag{7}$$

3.3 Firm Heterogeneity and the Innovation Process

Firms in the intermediate goods sector engage in both product and process innovation. Product innovations enable firms to acquire new products, and process innovations can further affect the technology advantage of a firm compared to its closest competitors.

Firms improve product qualities through process innovations. Denote the quality improvement $A_{jt}/\hat{A}_{jt}$ by q_{jt}, with $q_{jt} > 1$. I assume that q_{jt} is determined by the amount of skilled labor used in the product line:

$$q_{jt} = q(l^s_{jt}) = \frac{1}{1 - \eta_0 \times (l^s_{jt})^{\eta_1}}, \tag{8}$$

where $\eta_0, \eta_1 \in [0, 1]$. l^s is net of the fixed cost l_f. Intuitively, when the firm doesn't have any skilled labor, i.e. $l^s_{jt} = 0$, it cannot improve upon the current technology. The size of improvement is an increasing function of skilled labor.

The introduction of η_1 deserves some discussion. I assume this specific form so that the uniqueness of the equilibrium can be ensured through certain regularity conditions.

I model product innovations following Klette and Kortum (2004) where the number of product lines of a firm of age a, $n_{a,t}$, changes through a creative-destruction process. The likelihood of success is heterogeneous across firms, depending on the amount of skilled workers per product line of a firm, and the number of product lines owned by the firm. The latter can be considered as a proxy of the knowledge capital of that firm.

Product innovations are undirected. For a firm that owns $n_{a,t}$ product lines, it receives $n_{a,t}$ iid innovation shocks. Each shock follows a Bernoulli distribution with the success probability $\lambda(l^s_{a,t})$, in which $\lambda(\cdot)$ is an increasing and concave function, and $l^s_{a,t}$ denotes the skilled labor per product line. I assume a parsimonious form $\lambda(l^s_{a,t}) = \lambda_0 (l^s_{a,t})^\theta$

with $\theta < 1$.

Denote the equilibrium destruction rate in the economy as μ_t. Conditional on the survival of a firm (with respect to exogenous destruction which will be introduced shortly), a product line with skilled labor $l^s_{a,t}$, in the next period, there are following three cases:

$$\text{The number of product lines} = \begin{cases} 2 & \text{with probability } \lambda(l^s_{a,t})(1-\mu_t) \\ 1 & \text{with probability } (1-\lambda(l^s_{a,t}))(1-\mu_t) + \lambda(l^s_{a,t})\mu_t \\ 0 & \text{with probability } (1-\lambda(l^s_{a,t}))\mu_t \end{cases} .$$

The expected number of product lines is therefore

$$E[n_{a+1,t+1}] = 1 + \lambda(l^s_{a,t}) - \mu_t. \tag{9}$$

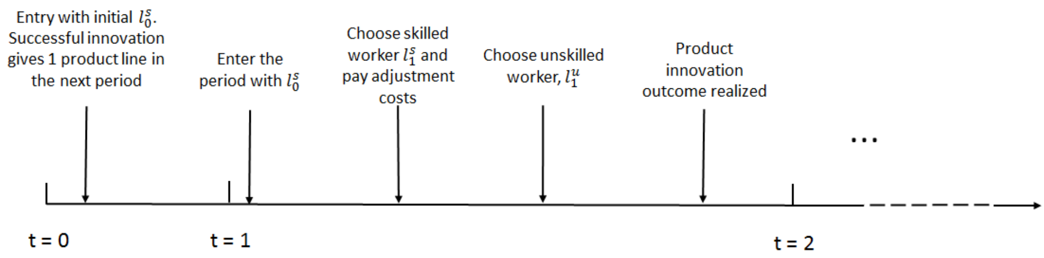
As $l^s_{a,t}$ may change over time for a given firm, I assume it affects the technology advantage of a firm relative to its closest competitors also through process innovation. I assume that the technology advantage is always $q(l^s_{a,t})$. This implies that the technology advantage may decline when $l^s_{a,t}$ is lower, which may be justified as a loss of knowledge due to an imperfect handover process.

Firms need to pay adjustment costs when adjusting their skilled labor. I assume that the amount of skilled labor is the same across product lines owned by a firm and the nominal adjustment cost a firm faces is defined as

$$\phi_t(n_{a,t}, l^s_{a-1,t-1}, l^s_{a,t}, w^s_t) \triangleq \frac{\varphi}{2} w^s_t n_{a,t} l^s_{a-1,t-1} \left[\frac{l^s_{a,t} - (1-\delta) l^s_{a-1,t-1}}{l^s_{a-1,t-1}} \right]^2, \tag{10}$$

where δ is an exogenously given separation rate of skilled labor, and φ determines the difficulty of adjusting skilled labor. I can alternatively assume the adjustment cost is on the total skilled labor per product line, but as I will show later, defining adjustment costs based on the amount of skilled worker per product line will help simplify the optimization problems of firms.

Figure 6: Timeline



3.4 Optimal Decisions of Intermediate Goods Producers

The timeline of the model is illustrated by figure 6. I solve the optimization problem of intermediate goods producers in two steps. In the first step, I express the amount of unskilled labor as a function of the amount of skilled labor per product line. In the second step, I solve the optimal choice of the amount of skilled labor per product line.

Consider a product line j owned by a firm of age a. Given the amount of skilled labor per product line $l^s_{a,t}$, the firm chooses unskilled labor of the product line l^u_{jt} by maximizing operating profits:

$$B_{a,j,t} = \max_{\{l^u_{jt}\}} \Big(p_{jt} y_{jt} - w^u_t l^u_{jt} \Big) \tag{11}$$

$$\text{s.t. } y_{jt} = A_{jt} l^u_{jt} \tag{12}$$

$$y_{jt} = \frac{Y_t}{M_t p_{jt}} \tag{13}$$

$$p_{jt} = \frac{w^u_t}{\tilde{A}_{jt}}. \tag{14}$$

The firm's optimal choice of unskilled labor is

$$l^u_{jt} = \frac{Y_t}{M_t q_{jt} w^u_t}, \tag{15}$$

and this implies the operating profit $B_{a,j,t}$ is given by

$$B_{a,j,t} = \frac{Y_t}{M_t}(1 - \frac{1}{q_{jt}}) = \frac{Y_t}{M_t}\eta_0 (l^s_{a,t})^{\eta_1}. \tag{16}$$

Therefore, all products owned by the firm have the same operating profit $Y_t \eta_0 (l^s_{a,t})^{\eta_1}$, which is only a function of $l^s_{a,t}$, denoted as $B_t(l^s_{a,t})$.

3.5 Entry and Exit

I assume firms' entry decisions following Acemoglu et al. (2017). A new firm hires l^s_0 skill labor and needs to pay a fixed cost to enter into the economy. After paying the fixed cost, the entrant will have access to the innovation technology $\lambda^E(\cdot)$. A successful innovation enables an entrant to own one product line at the beginning of the next period.

I assume that the fixed cost is equal to ξ units of skilled labor and $l^s_0 - \xi$ units of skilled labor are involved in the innovation process. The free entry condition gives

$$\max_{l^s_0} \left\{ \lambda^E(l^s_0 - \xi)\mathbb{E}\frac{V(l^s_0)}{1+r} - w^s l^s_0 \right\} = 0, \tag{17}$$

where $\lambda^E(l) = \lambda^E_0 l^\theta$.

The number of entrants will adjust so that the destruction rate μ will ensure the free entry condition holds.

Firm faces an exogenous destruction rate of ν. Firms exit the economy when their number of product lines decreases to zero or was hit by the exogenous destruction shock.

3.6 Value Functions

Now I turn to the second step. Adjustment costs of the skilled labor imply that the amount of skilled labor per product line is a state variable. Firms choose the optimal level of skilled labor by solving the following Bellman equation:

$$\begin{aligned} V_t(n_{a,t}, l^s_{a-1,t-1}) = \max_{\{l^s_{a,t} > l_f\}} \Big\{ & n_{a,t} B_t(l^s_{a,t}) - n_{a,t} w^s_t l^s_{a,t} - \phi_t(n_{a,t}, l^s_{a-1,t-1}, l^s_{a,t}, w^s_t) \\ & + \frac{1-\nu}{1+r}\mathbb{E}\big[V_{t+1}(n_{a+1,t+1}, l^s_{a,t})\big] \Big\}. \end{aligned} \tag{18}$$

The form of adjustment costs further yields that the value function is a linear function of the number of product lines $n_{a,t}$. Define $v_t(l^s_{a-1,t-1}) = V_t(n_{a,t}, l^s_{a-1,t-1})/n_{a,t}$.

LEMMA 1: The Bellman equation (18) can be simplified to

$$v_t(l^s_{a-1,t-1}) = \max_{\{l^s_{a,t}>l_f\}} \Big\{ B_t(l^s_{a,t}) - w^s_t l^s_{a,t} - \frac{\varphi}{2} w^s_t l^s_{a-1,t-1} \Big[\frac{l^s_{a,t} - (1-\delta) l^s_{a-1,t-1}}{l^s_{a-1,t-1}} \Big]^2 \\ + (1-\nu) \frac{1+\lambda(l^s_{a,t}) - \mu_t}{1+r} v_{t+1}(l^s_{a,t}) \Big\}. \quad (19)$$

Proof: It is straightforward to prove the linearity of V in n using guess and verify.

3.7 Household

There is a representative household who maximizes the life time utility

$$\sum_{t=0}^{\infty} \beta^t \ln \big[C_t - A_t \tau (L^s_t)^{\chi} \big], \quad (20)$$

where C_t is the consumption of the final good at time t. $\chi > 1$ and $\frac{1}{\chi-1}$ denotes the Frisch elasticity. As will be defined later, A_t is the aggregate productivity and $A_t = Y_t / L^u_t$. Note that A_t changes endogenously in response to innovation processes of firms. I choose this endogenous scaling by following Ates and Saffie (2016), which provides a justification for this assumption through home production. This assumption is useful for simplifying the analysis of the balanced growth path later.

The representative household owns L^u units of unskilled labor, which is a constant over time, and supplies L^s_t units of skilled labor, which will be supplied elastically based on the wage of the skilled labor w^s_t.

The budget constraint of the household is:

$$C_t = w^s_t L^s + w^u_t L^u + \Pi_t,$$

where Π_t is the aggregate profit, and

$$\Pi_t = \int_{j \in \Omega_t} \pi_{jt} dj, \quad (21)$$

in which

$$\pi_{jt} = B_t(l^s_{jt}) - w^s_t l^s_{jt} - \frac{\phi_t(n_{i,t}, l^s_{i,t-1}, l^s_{i,t}, w^s_t)}{n_{i,t}},$$

and i denotes the firm that owns the product line j in period t, and $l^s_{i,t}$ denotes the average amount of skilled labor owned by firm i in period t.

Aggregate profit as a share of total output is

$$\zeta_t = \frac{\Pi_t}{Y_t} = \frac{1}{Y_t}\int_{j\in\Omega_t} B_t(l^s_{jt}) - w^s_t l^s_{jt} - \frac{\phi_t(n_{i,t}, l^s_{i,t-1}, l^s_{i,t}, w^s_t)}{n_{i,t}} dj.$$

The first order condition of the household optimization problem implies that the following equation holds for the aggregate supply of skilled labor:

$$w^s_t = A_t \tau \chi (L^s_t)^{\chi-1}. \tag{22}$$

3.8 Aggregate Growth Dynamics

As the final good is the numeraire,

$$\begin{aligned} 0 = \ln P_t &= \ln M_t + \frac{1}{M_t}\int_{j\in\Omega_t} \ln p_{jt} dj \\ &= \ln M_t + \frac{1}{M_t}\int_{j\in\Omega_t} (\ln w^u_t - \ln \tilde{A}_{jt}) dj, \end{aligned}$$

which implies that

$$\ln w^u_t = \frac{1}{M_t}\int_{j\in\Omega_t} \ln \tilde{A}_{jt} dj - \ln(M_t). \tag{23}$$

As individual product line's unskilled labor is given by equation (15), I can define the aggregate level of unskilled labor as

$$L^u_t = \frac{Y_t}{M_t w^u_t}\int_{j\in\Omega_t} \frac{1}{q_{jt}} dj, \tag{24}$$

which yields

$$
\begin{aligned}
\ln Y_t &= \ln w_t^u + \ln M_t + \ln L_t^u - \ln(\int_{j\in\Omega_t} \frac{1}{q_{jt}} dj), \\
&= \frac{1}{M_t}\int_{j\in\Omega_t} \ln(\tilde{A}_{jt}) dj + \ln L_t^u - \ln(\int_{j\in\Omega_t} \frac{1}{q_{jt}} dj).
\end{aligned}
$$

Define the aggregate productivity

$$
A_t = Y_t / L_t^u, \tag{25}
$$

and A_t satisfies

$$
\begin{aligned}
\ln A_t =& \frac{1}{M_t}\int_{j\in\Omega_t} \ln \tilde{A}_{jt} dj - \ln(\int_{j\in\Omega_t} \frac{1}{q_{jt}} dj) \\
=& \frac{1}{M_t}\int_{j\in\Omega_t} \ln A_{jt} dj - \frac{1}{M_t}\int_{j\in\Omega_t} \ln q_{jt} dj - \ln(\int_{j\in\Omega_t} \frac{1}{q_{jt}} dj).
\end{aligned}
$$

3.9 Equilibrium

Since the amount of skilled labor per product line is the only state variable, it is the same across firms upon entry within the same cohort, and therefore always the same across firms within the same cohort over time. This implies that $l_{a,t}^s$ and $l_{a,t}^u$ are functions of age in period t.

Denote the measure of product lines owned by firms of age a at the beginning of period t as $\Lambda_{a,t}$, and the measure of entrants in period t as $\Lambda_{0,t}$.

A *competitive equilibrium* is defined as prices $\{\{p_{jt}\}_{j\in\Omega_t},\ w_t^s,\ w_t^u\}$ and choices $\{Y_t$, $\{y_{jt}^D\}_{j\in\Omega_t}$, $\{y_{j,t}^S\}_{j\in\Omega_t}$, $\{l_{a,t}^s\}_{a=0,1,\ldots}\}$, $\{l_{a,t}^u\}_{a=1,\ldots}\}$, C_t, $L_t^s\}$, profit Π_t, destruction rate μ_t, and the distribution of products across age cohorts $\{\Lambda_{a,t}\}_{a=0,1,\ldots}$, such that

1. $\{C_t, L_t^s\}$ solve the decision problem of households (20) taking w_t^s, w_t^u and Π_t as given, in which Π_t satisfies equation (21).
2. Y_t and $\{y_{jt}^D\}$ solve the representative final goods producer's problem (41), taking $\{p_{jt}\}_{j\in\Omega_t}$ as given.

3. $\{y^S_{jt}, l^u_{a,t}\}$ solve the intermediate goods producer of age a's problem (15), taking $Y_t, p_{j,t}$, and $l^s_{a,t}$ as given, if the product line j belongs to the cohort aged a in period t.

4. $\{l^s_{a,t}\}_{a=0,1,\ldots}$ solve the Bellman equation (19), taking w^s_t and μ_t as given.

5. The free entry condition $\max_{l^s_{0,t}} \left\{\lambda^E(l^s_{0,t} - \xi)\frac{v_t(l^s_{0,t})}{1+r} - w^s_t l^s_{0,t}\right\} = 0$ is satisfied, in which v_t satisfies the Bellman equation (19).

6. Market clearing conditions hold:

$$C_t + \sum_{a=1}^{\infty} \Lambda_{a,t} \frac{\phi_t(n_{a,t}, l^s_{a,t-1}, l^s_{a,t}, w^s_t)}{n_{a,t}} = Y_t \tag{26}$$

$$y^D_{j,t} = y^S_{jt} \tag{27}$$

$$\sum_{a=1}^{\infty} \Lambda_{a,t} l^u_{a,t} = L^u \tag{28}$$

$$\sum_{a=0}^{\infty} \Lambda_{a,t} l^s_{a,t} = L^s_t, \tag{29}$$

where L^s_t satisfies equation (22).

7. The distribution of products across age cohorts evolves in an internally consistent way:

$$\Lambda_{1,t+1} = \Lambda_{0,t}\lambda^E(l^s_{0,t} - \xi) \tag{30}$$

$$\Lambda_{a+1,t+1} = \Lambda_{a,t} \times \big(1 + \lambda(l^s_{a,t}) - \mu_t\big)(1 - \nu) \tag{31}$$

8. The equilibrium destruction rate μ_t satisfies:

$$\mu_t = \lambda(l^s_{0,t} - \xi)\Lambda_{0,t} + \sum_{a=1}^{\infty} \lambda(l^s_{a,t})\Lambda_{a,t}, \tag{32}$$

where M_t is the share of product lines alive at the beginning of period t, and this equation holds because I assume that the research is undirected.

To simplify the analysis, I add an assumption that due to technological spillovers, the productivity of idle production line is on average the same as that of active product line

Assumption 1 $\frac{1}{M_t}\int_{j\in\Omega_t}\ln A_{jt}dj = \frac{1}{1-M_t}\int_{j\notin\Omega_t}\ln A_{jt}dj$.

In this paper, I study the *balanced growth path* or the *steady state* of the equilibrium.

Denote $\tilde{X} = X_t/A_t$ for a variable X that is growing at the same rate as A_t, and $X = X_t$ for a variable that is a constant at the steady state. In particular, the following variables are growing at the same rate as A_t: $\tilde{Y} = Y_t/A_t$, $\tilde{w}^s = w_t^s/A_t$, $\tilde{w}^u = w_t^u/A_t$, $\tilde{v} = v_t/A_t$, $\tilde{B}(l_a^s) = B^t(l_{a,t}^s)/A_t$. Variables that are constants at the steady state include: $\mu = \mu_t$, $l_a^s = l_{a,t}^s$, $L^s = L_t^s$, $\Lambda_a = \Lambda_{a,t}$,$M_t = M$ and $PS = PS_t$, in which $a \in \{0, 1, ..., +\infty\}$.

LEMMA 2: Along the balanced growth path,

$$M = \frac{\mu}{1-(1-\mu)(1-\nu)}. \tag{33}$$

Proof: See Appendix F.

The following equations hold in the steady state:

$$\tilde{Y} = L^u$$

$$\tilde{B}_a = L^u\eta_0(l_a^s)^{\eta_1}$$

$$\tilde{w}^s = \tau\chi(L^{s,s})^{\chi-1}$$

$$\tilde{w}^u = \frac{1}{M}\sum_{a=1}^{\infty}\Lambda_a\frac{1}{q(l_a^s)}$$

$$\zeta = \frac{1}{L_u}\sum_{a=1}^{\infty}\Lambda_a\times\left(\tilde{B}_a - \tilde{w}^s l_a^s - \frac{\varphi}{2}\tilde{w}^s l_{a-1}^s\Big[l_a^s-(1-\delta)l_{a-1}^s l_{a-1}^s\Big]^2\right)$$

$$\begin{aligned}\tilde{v}(l_{a-1}^s) = \max_{l_a^s}\Big\{&\tilde{B}_a - \tilde{w}^s l_a^s - \frac{\varphi}{2}\tilde{w}^s l_{a-1}^s\Big[\frac{l_a^s-(1-\delta)l_{a-1}^s}{l_{a-1}^s}\Big]^2\\ &+\frac{1-\nu}{1+r}(1+\lambda(l_a^s)-\mu)\tilde{v}(l_a^s)\Big\}\end{aligned}$$

$$\max_{l_0^s}\Big\{\lambda^E(l_0^s-\xi)\frac{\tilde{v}(l_0^s)}{1+r} - \tilde{w}^s l_0^s = 0\Big\}$$

$$\Lambda_1 = \Lambda_0\lambda^E(l_0^s-\xi),$$

$$\Lambda_{a+1} = \Lambda_a \times (1-\nu)\big(1+\lambda(l_a^s)-\mu\big)$$

$$\mu = \lambda^E(l_0^s - \xi)\Lambda_0 + (1-\nu) \times \sum_{a=1}^{\infty} \lambda(l_a^s)\Lambda_a$$

$$L^s = \sum_{a=0}^{\infty} \Lambda_a l_a^s$$

$$L^u = \sum_{a=1}^{\infty} \Lambda_a l_a^u.$$

The steady state growth rate of aggregate productivity is

$$g_A = \ln \frac{A_t}{A_{t-1}} = \Lambda_1(\ln q(l_0^s)) + \sum_{a=1}^{\infty} \Lambda_a(1-\nu)\lambda(l_a^s)\ln(q(l_a^s)).$$

3.10 The Optimal Choices of Skilled Labor

I solve the steady state equilibrium in three steps. In the first step, I solve the optimal decisions $\{l_a^s, \tilde{v}(l_a^s)\}$ as functions of $\{\tilde{w}^s, \tilde{w}^u, \mu\}$. In the second step, I determine μ based on the entry condition. Finally, I use market clearing conditions to solve for equilibrium prices $\tilde{w}^s$ and $\tilde{w}^u$. In this section, I discuss the first step and presents conditions under which there exists an unique skilled labor life cycle solution to the model.

I conjecture that the number of skilled worker per product will asymptotically approach a constant level $\bar{l}$, where $\bar{l}$ satisfies conditions (34) (35), which are derived by letting $\bar{l} = l_a^s = l_{a+1}^s$ and solving the Bellman equation for firm aged a. (Details please revert to the derivation of (49) in the appendix).

$$\varphi \tilde{w}^s \delta^s = \frac{d\tilde{B}^s(\bar{l})}{d\bar{l}} - \tilde{w}^s + \frac{1+\lambda(\bar{l})-\mu}{1+r}\frac{\varphi \tilde{w}^s}{2}(1-(1-\delta)^2) + \frac{\lambda'(\bar{l})}{1+r}v^s(\bar{l}) \tag{34}$$

$$\frac{r+\mu-\lambda(\bar{l})}{1+r}v^s(\bar{l}) = \tilde{B}^s(\bar{l}) - \tilde{w}^s\bar{l} - \frac{\varphi}{2}\tilde{w}^s\bar{l}\delta^2, \tag{35}$$

where $\tilde{B}^s(\bar{l}) = L^u\eta_0\bar{l}^{\eta_1}$, $d\tilde{B}^s(\bar{l})/d\bar{l} = \eta_1 L^u \eta_0(\bar{l})^{\eta_1 - 1}$.

Plugging (35) into (34), we have:

$$\varphi\tilde{w}^s\delta - \frac{d\tilde{B}^s(\bar{l})}{d\bar{l}} + \tilde{w}^s - \frac{1+\lambda(\bar{l})-\mu}{1+r}\frac{\varphi\tilde{w}^s}{2}(2\delta-\delta^2) = \frac{\lambda'(\bar{l})}{r-\lambda(\bar{l})+\mu}\Big(\tilde{B}^s(\bar{l}) - \tilde{w}^s\bar{l} - \frac{\varphi}{2}\tilde{w}^s\bar{l}\delta^2\Big). \tag{36}$$

Note that the upper bound for $\bar{l}$, $\bar{l}_{max}$, is given by

$$\bar{l} \leq \bar{l}_{max} = \Big(\frac{r+\mu}{\lambda_0}\Big)^{\frac{1}{\theta}}.$$

The intuition for $\bar{l}_{max}$ is as follows: the maximum innovation rate of firms should not exceed $r+\mu$, With the assumption that the maximum innovation rate of firms does not exceed μ, the higher the innovative capacity of a firm is, the fewer skilled workers there should be, to avoid violating the condition that the maximum innovation rate should be smaller than $r+\mu$.

To ensure the existence and uniqueness of $\bar{l}$, in the economy, I have Proposition 1, with its proof can be found in appendix (E).

PROPOSITION 1: The existence and uniqueness of $\bar{l}$. Under the following conditions, there is a unique solution of $\bar{l}$

$$\eta_1 < \theta \tag{37}$$

$$\theta + \eta_1 < 1 \tag{38}$$

$$\Big[\frac{B_0}{(1+\frac{\varphi}{2}\delta^2)\tilde{w}^s - l_f}\Big] \times \big(\frac{r+\mu}{\lambda_0}\big)^{\frac{\eta_1-1}{\theta}} < 1 \tag{39}$$

$$\frac{\delta\varphi(2-\delta)}{2+\varphi\delta^2} < \frac{(1-\eta_1)(1-\theta)(1+r)}{(1+\theta-\eta_1)(r+\mu)} \tag{40}$$

Proof: See appendix E.

In the quantitative analysis later, I will restrict the parameters in the range such that these conditions hold. I describe the computation algorithm in Appendix G.

4 Quantitative Analysis

4.1 Calibration

The calibration strategy consists of two parts. First, for standard parameters, I determine their value outside of the model based on the standard practice in the literature. Second, for other parameters, I calibrate them such that the balanced growth path of the model matches several features of the high-tech sector between 1990 and 2000.

I choose the elasticity of successful innovation with respect to R&D as 0.5, following Acemoglu et al. (2018). Interest rate is set to be 0.02, similar to those used by Acemoglu et al. (2018) and Akcigit and Kerr (2018). The Frisch elasticity χ is determined to be 1.455, which comes from Mendoza (1991) and is also standard in the literature. Depreciation rate of human capital captures not only a depreciation of knowledge stock of a firm, but also a separation of skilled workers from firms. I select its value to be 0.1, the same as a typical choice in the literature for the depreciation rate of capital. The results are not sensitive to a smaller value of this depreciation rate. Table 1 summarizes the parameters calibrated outside of the model.

Other parameters are determined inside the model by matching model outcomes with their data counterparts. For the purpose of our analysis, it is key to match the ones governing skilled labor accumulation and distribution, including the life cycle of skilled labor accumulation, the payroll share of skilled to unskilled labor and the distribution of the skilled labor among different firm age groups. I design the calibration strategy to make sure our model covers those key aspects.

I determine process innovation capacity parameters η_0 and η_1 by matching the average payroll shares of skilled to unskilled labor and young firm employment share in skilled labor in LEHD from 1990 to 2000. These two data targets are sensitive to process innovation, because it affects the skill accumulation over firm age through the marginal return of hiring an additional skill worker. For young employment share, in both the model outcome and the data counterpart, I use the employment share of skilled workers across firm age. The results are not sensitive to using all workers rather than skilled

Table 1: Parameters Determined Outside the Model

Parameter	Symbol	Value	Sources/Data Targets
Parameters from Other Studies			
Innovation elasticity	θ	0.50	Acemoglu et al. (2018)
Discount rate	r	0.02	Acemoglu et al. (2018), Akcigit and Kerr (2018)
Frisch elasticity	χ	1.455	Standard
Depreciation rate of human capital	δ	0.1	Standard

Table 2: Parameters Determined Inside the Model

Parameter		Value
A. Data target: payroll share of skill to unskilled workers and young employment share of skilled labor		
Scaling of quality improvement	η_0	0.31
Payroll sensitivity to skilled labor	η_1	0.30
B. Data target: life-cycle of skilled labor		
Skilled labor adjustment cost	φ	1
C. Data targets: distribution of skilled labor across firm age, entry, and exit rates		
Incumbent innovation intensity	λ_0	0.01
Entrant innovation intensity	λ_E	0.0074
Fixed cost	l^f	1.45
Entry cost	ξ	0.45
Exogenous destruction rate	ν	0.005

workers.

The adjustment cost intensity parameter φ is chosen to match the life cycle growth of skilled labor in LEHD for 1990-2000. Lifecycle growth of skill, which, as explained earlier, is defined as the efficiency units of skilled labor at age a relative to that at age 0. I look at the growth from age 0 to age 5 for an average high-tech firm between 1990 and 2000, which is taken from the coefficient of the fixed effects regression in equation (1).

Entrant and incumbent innovation capacity λ_E and λ_0, fixed operation costs l_f, fixed cost of entry ξ, and exogenous destruction rate ν are jointly chosen by matching the skilled labor distribution across firm age groups and firm exit rates computed from LEHD for 1990-2000.

Table 3: Data Targets and Model Counterparts

Data Target	Data	Model
A. Data target: payroll share of skilled to unskilled workers and young employment share		
The ratio of non-production to production worker compensation	2.9	2.9
Share of skilled labor in firm age 0-4	9.6%	9.8%
B. Data target: life-cycle of skilled labor		
Age 5 to Age 0 ratio of skill intensity	5.5	5.5
C. Data targets: distribution of skilled labor across firm age, entry, and exit rates (LEHD 90-14)		
Share of skilled labor in firm age 5-7	5.4%	7.2%
Share of skilled labor in firm age 8-10	5.4%	6.7%
Share of skilled labor in firm age 11-15	8.9%	9.9%
Exit rate (small/young)	9.9%	6.5%
Exit rate (small/old)	8.8%	6.4%

Table 2 summarizes the value of parameters and Table 3 shows the model performance.

4.2 Quantitative Results

To highlight the importance of skilled labor accumulation as a channel for frictions to affect long-term productivity growth, I compare two exogenous changes in the model parameters to see how they affect aggregate productivity growth. The first is an increase in the adjustment cost of changing the stock of skilled labor of a firm, and the second is a decline in the entrant innovation capacity. The former represents frictions which affect all incumbent firms and the latter is a way to capture frictions facing by entrants only. I will show how the endogenous variables: young firm employment share, life cycle of skilled labor accumulation, demand for skills, and productivity growth, respond under the two different types of shocks.

I discipline the change in adjustment costs and entry innovation capacity so that young firm employment shares decrease from 9.6% in the pre-2000 period to 6.2% by 2014.[10] The life cycle of skilled labor accumulation is un-targeted. To match the decline

[10]These are the young firm employment shares in terms of the skilled labor. Young firm are those less

in young firm employment shares, in the first experiment, entrant innovation capacity declines from 0.0074 to 0.0065, other things equal. In the second experiment, the skilled labor adjustment cost parameter increases from 1 to 10, while keeping other parameters unchanged. To proxy entry costs, I consider a decline in the entrant innovation capacity, as recent studies do suggest that the innovation production function may have changed over time (Bloom et al. (2017), Fernald and Jones (2014)). The results are broadly consistent if I study rising fixed costs of entry instead.

I show that, rising frictions in skilled labor adjustment can generate decline in young firm employment share and demand for skills, consistent with data, while declining entry costs can not generate a decline in demand for skills. I further show that a rise in skilled labor adjustment costs which generates a decline in young firm employment share consistent with data would imply a 75 basis point decrease in the productivity growth in the high-tech sector.

4.2.1 Skilled Labor Distribution by Firm Age

The employment distribution across firm age is sensitive to both entrant innovation capacity and skilled labor adjustment costs. Figure 7 shows that with a decline in young firm innovation capacity, employment shares of firms aged between 0-4, 5-7, 8-10 and 10-15 all decline. Similarly, increasing adjustment costs also shifts the skilled labor distribution towards old firms.

While both entrant innovation capacity and adjustment costs shift the distribution towards the old firms, they operate through different channels. Decreasing entrant innovation capacity lowers entry rate from 2.9% to 1.9%. This reduction in the entry rate brings down young firm employment shares. Increasing adjustment costs, on the other hand, has limited impact on the entry margin. Entry rate decreases by only 30 basis points to 2.6%. The main channel through which adjustment costs affect young employment share is the intensive margin. Facing higher adjustment costs, both young and old firms would hire less skilled labor but the reduction is more significant for young firms,

than 5 years old. 9.6% is the 10-year average between 1990 and 2000, and 6.2% is the young employment share in 2014.

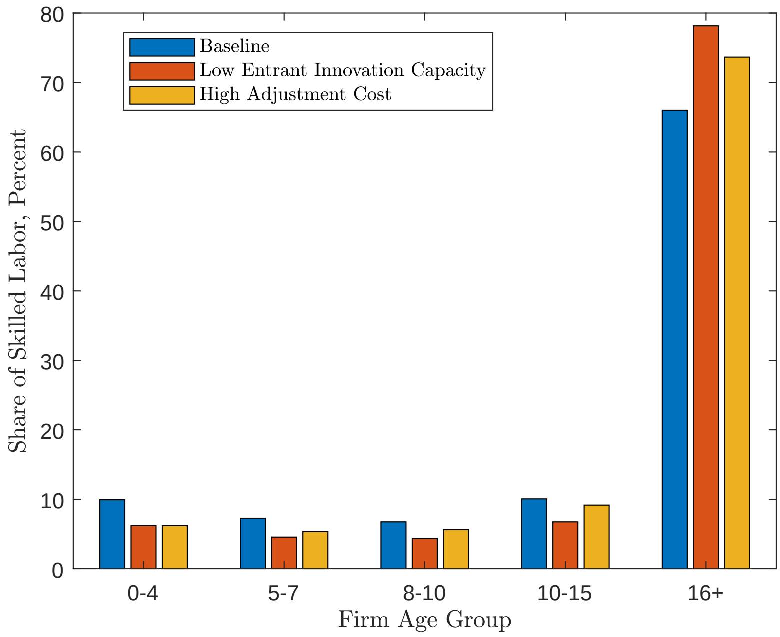


Figure 7: Distribution of skilled labor across firm age groups under different scenarios

Notes: Blue bars (left) is the baseline. Red bars(center) represent the share of skilled labor owned by a particular firm age group under low entrant innovation capacity. Yellow bars (right) represent the share of skilled labor owned by a particular firm age group under high adjustment costs. Both lowering entrant innovation capacity and increasing skilled labor adjustment costs shift the skilled labor distribution towards old firms.

as they have higher incentive to hire skilled labor. The difference between entrant innovation capacity and adjustment costs can also be seen from Figure 7, where the gap between the base case the low entrant innovation capacity case is similar for firms in age groups 0-4, 5-7, 8-10 and 10-15, but the gap is larger for firms in the younger age groups in the case of high adjustment cost.

4.2.2 The Lifecycle of Skilled Labor Accumulation

Declining entrant innovation capacity and rising skilled labor adjustment costs have opposite impacts on the life cycle of skilled labor accumulation, as is shown in Figure 8. Declining entrant innovation capacity induces incumbents to hire more skilled labor. The intuition is that the marginal return of hiring an additional skilled worker increases with the expected probability of survival which increases with a decline in the entrant innovation capacity. Equilibrium destruction rate reduces to 6.1% from 6.8% when I lower the entrant innovation capacity.

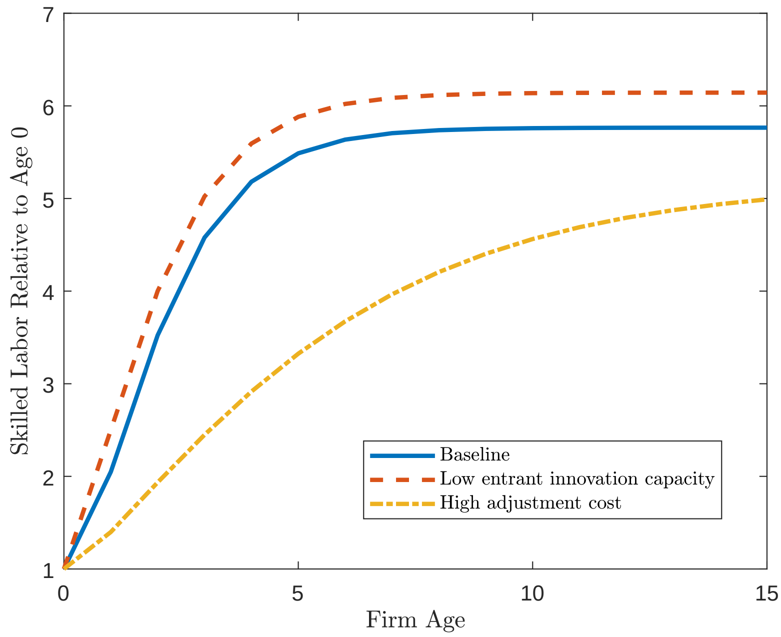


Figure 8: Life cycle of skill intensity under different scenarios

Notes: This figure plots the skilled labor relative to age 0 under different scenarios. Solid blue line is the baseline case. Red dash line is the life cycle under lower entrant innovation capacity and yellow dash-dot line is the life cycle under higher skilled labor adjustment costs. Entrant innovation capacity and skilled labor adjustment costs have different impacts on the life cycle of skilled labor accumulation.

Rising adjustment costs clearly discourage firms from hiring skilled workers. The marginal cost of hiring an additional unit of skilled labor increases for both young and old firms. Even though the destruction rate also decreases (from 6.8% to 6.4%), the marginal benefit is not sufficiently large to offset the costs. Hence in the equilibrium, all firms hire few skilled labor and the life cycle of skilled labor accumulation flattens, as shown in Figure 8.

One may wonder if the flattened life cycle under higher adjustment costs is simply a manifest of higher initial level of skilled labor, as entrants may choose to increase their initial stock of skilled labor to avoid to pay higher adjustment costs later on. We show that this effect is minimum. Initial stock of the skilled labor only increases slightly from 2.4 to 2.7, and the stock of skilled labor decreases for incumbents of all ages (for example, the stock of skilled labor for an age 5 firm decreases from 18.3 to 10.6).

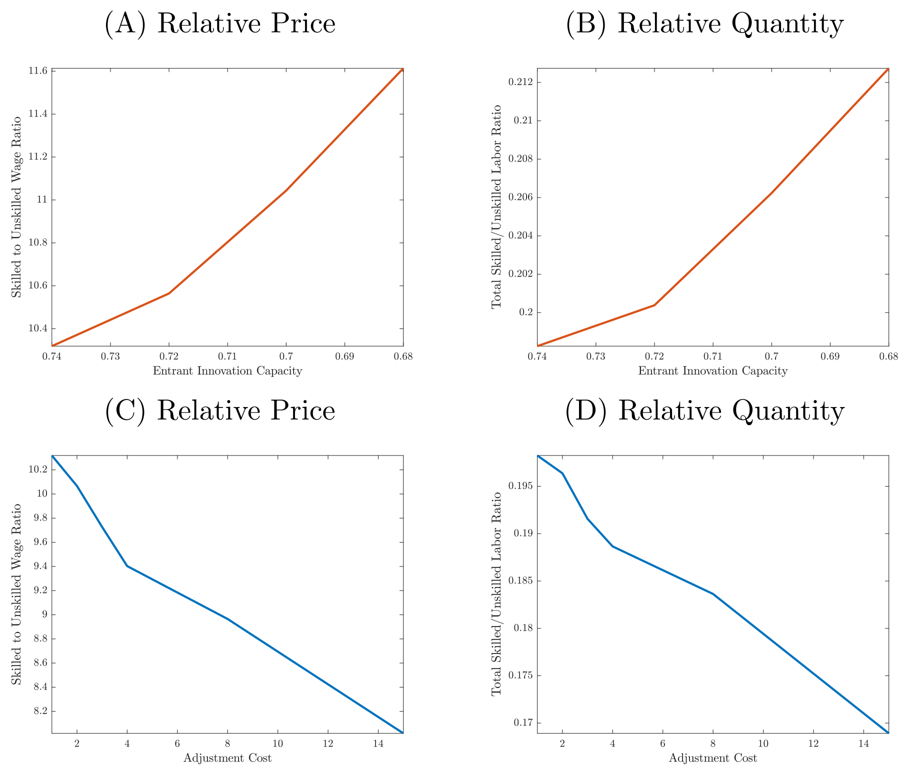


Figure 9: Relative price and quantity under different scenarios

Notes: Panel (A) shows the high to low skilled labor wage ratio when entrant innovation capacity declines. Panel (B) shows the ratio of aggregate high to low skilled labor when entrant innovation capacity declines. Panel (C) shows the high to low skilled labor wage ratio when skilled labor adjustment costs increase. Panel (D) shows the ratio of aggregate high to low skilled labor when skilled labor adjustment costs increase.

4.2.3 Aggregate Demand for Skills

Aggregate demand for skills respond differently to declining entrant innovation capacity and rising adjustment costs. A decline in entrant innovation capacity leads to a strong rise in aggregate demand for skills, which is measured as the compensation ratio between high and low skilled labor, from 3.0 to 3.6. The increase in demand for skills is the response to the reduced equilibrium destruction rate which increases the survival rate and hence the expected value of incumbents. An increase in the skilled labor adjustment costs, on the other hand, leads to a significant decline in the aggregate demand for skills which drops to 2.4 from 3.0.

I further breakdown demand for skills into relative price and quantity, as shown in Figure 9. A decline in entrant innovation capacity increases both the relative price (wage) and relative quantity of skilled to unskilled labor. An increase in skilled labor adjustment

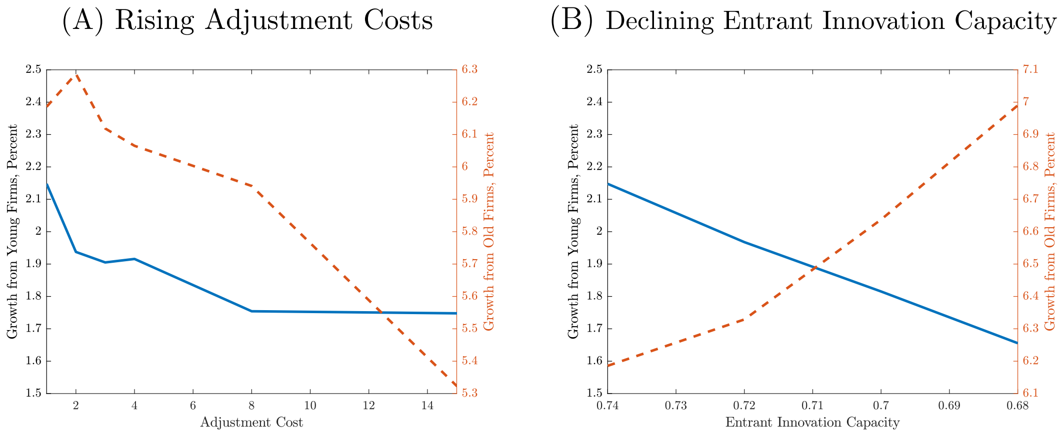


Figure 10: Sources of Growth

Notes: Panel (A) shows the growth from young firms (solid line, LHS) and old firms (dash line, RHS) under rising skilled labor adjustment costs. Panel (B) shows the growth from young firms (solid line, LHS) and old firms (dash line,RHS) under decreasing entrant innovation capacity. Young firms are those less than 5 years old.

costs, on the other hand, decreases both relative price and quantity of skilled to unskilled labor.

4.2.4 Implications for Productivity Growth

I now look at the implications for productivity growth under the two different scenarios. When skilled labor adjustment costs increase, productivity growth decrease by 75 basis points. On the other hand, when entrant innovation capacity declines, aggregate productivity actually increases by 55 basis points. The counterintuitive productivity gain is a result of the increased demand for skills, which increases the stock for skilled labor for incumbent firms and hence produces higher innovation success rate and high productivity growth.

The effects can be better seen from Figure 10 where I decompose the aggregate productivity growth into growth from young firms (including entrants) and growth from old firms. Panel A shows the growth components under rising adjustment costs. We can see that when adjustment costs are higher, both young and old firms' growth rates decline. Panel B shows the growth components under decreasing entrant innovation capacities. When entrant innovation capacity decreases, the growth from entrants declines. But such decline is offset by the increase in the growth rate of incumbents whose innovation success

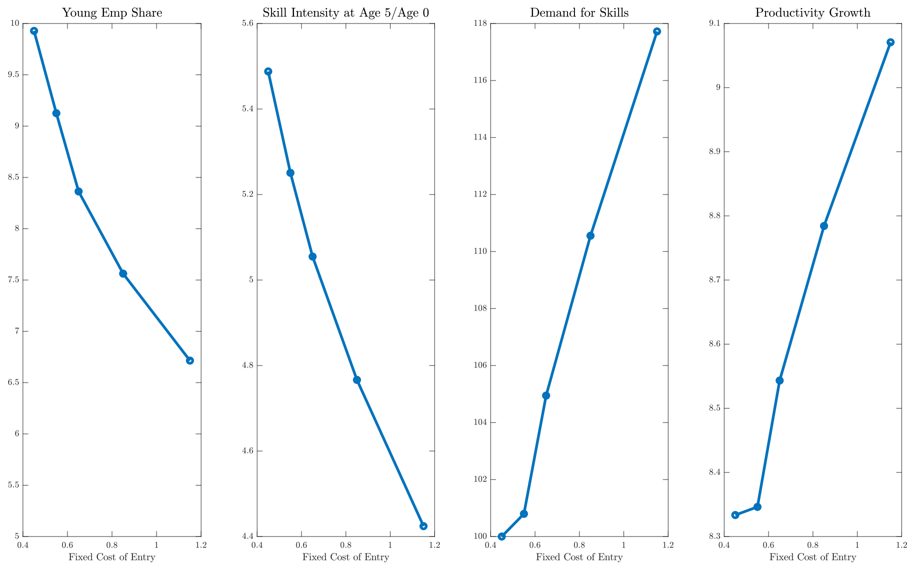


Figure 11: Effects of fixed cost of entry

Notes: We show how young firm employment share, life cycle of skilled labor accumulation, demand for skills, and productivity growth response to a change in the fixed cost of entry.

rate increase as they now facing a lower threat of being destructed and hire more skilled labor.

4.2.5 Discussion

I first discuss the effect of fixed costs of entry. The analysis above considers entrant innovation capacity as a proxy for entry costs. While fixed costs of entry is another form and entry costs which lead to broadly consistent implications as entrant innovation capacity. Figure 11 shows the results. Increasing fixed costs of entry lowers entry rate and decrease the young firm employment share. Meanwhile, lowered threat from entrants increases the probability of survival of incumbents and their demand for skills. In the equilibrium, the increase in growth from incumbents is more than offset by lower entrant growth, similar to the effects of lower entrant innovation capacity. The difference comes from the implications for life cycle growth. When fixed costs of entry increase, firms need to hire a higher amount of skilled labor in order to entry which leads to a slower growth of skilled labor post-entry.

I then discuss the effects when lowering the innovation capacity for entrants and

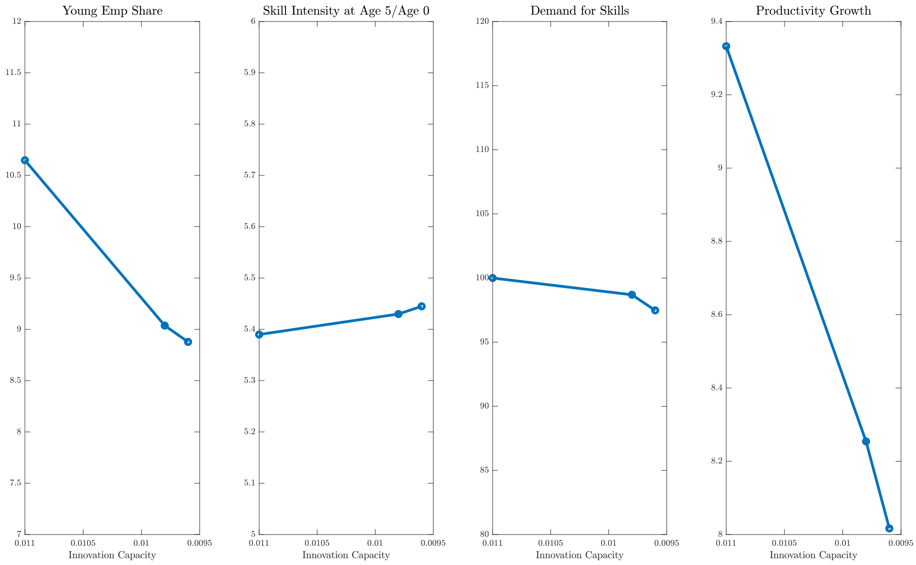


Figure 12: Effects of lower innovation capacities for both entrants and incumbents

Notes: We show how young firm employment share, life cycle of skilled labor accumulation, demand for skills, and productivity growth response to a change in innovation capacity.

incumbents together while keeping the relative strength of entrant and incumbent innovation capacity constant. Figure 12 shows the results. Intuitively, decreasing innovation capacity will lower aggregate growth. In fact, in this case, both the life cycle of skilled labor and demand for skills remain unchanged and the growth of entrants and incumbents decline purely as a result of declined innovation capacity. Young firm share declines slightly in this case. Since when innovation capacity declines, both entrants and incumbents have lower success rate of product innovation. Incumbents process innovation advantage now become higher as they have higher stock of skilled labor to conduct process innovation (as reflected in the quality improvement/mark up function equation 8).

Finally, note that in our analysis the life cycle pattern I got under rising adjustment frictions is still above the post-2000 life cycle from data. If there are factors which further bring down firms' demand for skills, we should expect the productivity growth will be further hampered.

5 Concluding Remarks

This paper studies the drivers behind declining business dynamism and the implications for productivity growth. Using a longitudinal worker-firm matched dataset which covers the universe of U.S. private sector jobs, I document that accompanying the decline of young firm activities is a decline in the growth of demand for skills in the high-tech sector post-2000. Moreover, I document that underlying the decline in aggregate demand for skills is a flattening of the life cycle of skilled labor accumulation among high-tech firms.

I develop an endogenous growth firm dynamics model to study the joint evolution of young firm activities and demand for skills. I incorporate adjustment costs to the standard framework so that the model can be consistent with the micro level patterns of firms' life cycle.

I show that rising frictions in skilled labor adjustment could reconcile the empirical patterns we observed in the data, i.e. a decline in the young firm activity share, a flattening of the life cycle of firms' skilled labor accumulation and a decline in aggregate demand for skills. Moreover, aggregate productivity growth is hampered in this case, as firms accumulate a lower stock of skilled labor facing higher adjustment costs. By calibrating the model to the high-tech sector, I find that rising adjustment frictions could lead to a 75 basis points decrease in productivity growth in the high-tech sector.

The model also suggests that the productivity gain from reallocating skilled labor from old to young firms is not always guaranteed, as higher equilibrium destruction rate will discourage incumbents from hiring skilled labor. This channel offsets the gain from reallocation. Admittedly, in the current model, young firms are equally innovative as old firms which may leave interesting post-entry dynamics unexplored. Future research may want to include richer post-entry dynamics to better assess the relative strength of the two offsetting channels in the equilibrium.

Finally, even though my focus has been on the high-tech sector, the methodology is by no means restrictive to this sector alone. Aggregate patterns have suggested that the decline in demand for skills may exist more broadly outside the high-tech sector and it would be interesting to explore its connection to the declining business dynamism and productivity growth in the broader U.S. economy.

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Appendices

A Definition of the High-Tech Sector

We define high-tech based on the shares of workers in the STEM occupations of science, technology, engineering, and math, based on the methodology developed by Heckler (2005). High-tech sector in our analysis includes the following 14 four-digit NAICS industries, similar as Decker et al. (2018).

Table 4: High-Technology Industries

NAICS	Industry
Information and Communications Technology (ICT) High-Tech	
3341	Computer and peripheral equipment manufacturing
3342	Communications equipment manufacturing
3344	Semiconductor and other electronic component manufacturing
3345	Navigational, measuring, electromedical, and control instruments manufacturing
5112	Software publishers
5161	Internet publishing and broadcasting
5179	Other telecommunications
5181	Internet service providers and Web search portals
5182	Data processing, hosting, and related services
5415	Computer systems design and related services
Miscellaneous High-Tech	
3254	Pharmaceutical and medicine manufacturing
3364	Aerospace product and parts manufacturing
5413	Architectural, engineering, and related services
5417	Scientific research-and-development services

B An Occupation-Based Measure of Demand for Skills

In this section, I present a demand for skills measure for the high-tech sector based on occupation using public available datasets.

The datasets used are 1980, 1990 and 2000 Decennial Census and ACS from 2005 to 2016 from IPUMS USA. We stack three consecutive years' ACS to obtain sample for the middle years (2006, 2009, 2012, 2015). The industry information in Census and ACS

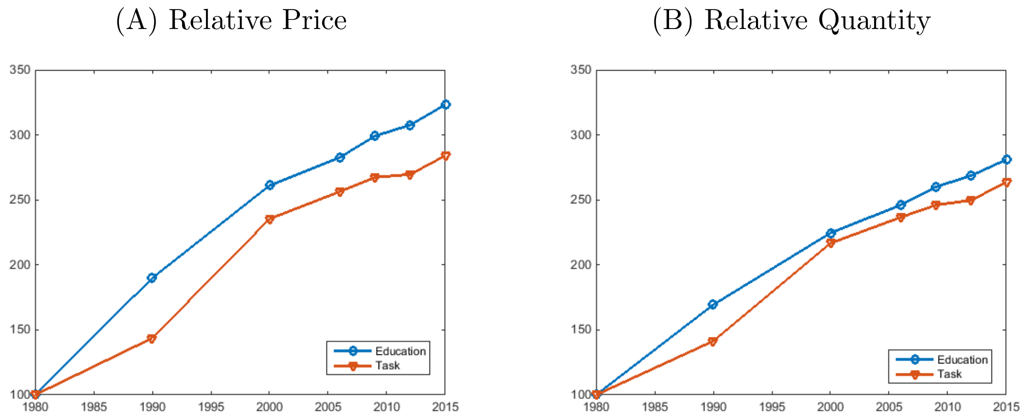


Figure 13: Relative price and quantity under occupation based skill measure

Notes: This figure plots the relative price and quantity of skill under alternative definitions of skill. Panel (A) compares the relative price series. Panel (B) compares the relative quantity series.

are not ideal. We manually construct a crosswalk from Census and ACS industry codes to NAICS2002 NAICS and keep the high-tech industries defined in Appendix A.

The advantage of Census and ACS is that they keep the occupation information for each individual. To define high and low skilled workers, I integrate the task score for each occupation from David and Dorn (2013) and define high-skilled workers as workers whose occupations have a higher-than-average abstract task score. Our measure is robust to other classification of high and low skilled workers.

Figure 13 shows the relative price and quantity series for alternative definitions of skill (eduction and task). The skill premium flattened at 2000 and the flattening can be seen under both measures of skill.

C Final Good Producer's Problem

The optimization problem of the final good producer is

$$\max_{\{y_{jt},Y_t\}} \left(P_t Y_t - \int_{j\in\Omega_t} p_{jt} y_{jt} dj\right) \tag{41}$$

$$\text{s.t.} \quad \ln Y_t = \frac{\int_{j\in\Omega_t} \ln y_{jt} dj}{M_t},$$

The Lagrangian function of the problem is

$$\mathcal{L} = P_t Y_t - \int_{j\in\Omega_t} p_{jt} y_{jt} dj - \lambda(\ln Y_t - \int_{j\in\Omega_t} \ln y_{jt} dj),$$

Solving this problem yields:

$$p_{jt} y_{jt} = \frac{P_t Y_t}{M_t}, \tag{42}$$

which further implies that

$$\ln P_t = \ln M_t + \frac{1}{M_t} \int_{j\in\Omega_t} \ln p_{jt} dj, \tag{43}$$

We choose the final good as the numeraire, i.e. $P_t = 1$.

D Analysis of the Bellman Equation

$$\tilde{v}(l^s_{a-1}) = \max_{\{l^s_a\}} \Big\{ \tilde{B}(l^s_a) - \tilde{w}^s(l^s_a + l_f)$$
$$-\frac{\varphi}{2}\tilde{w}^s(l^s_{a-1} + l_f)\Big[\frac{(l^s_a + l_f) - (1-\delta)(l^s_{a-1} + l_f)}{l^s_{a-1} + l_f}\Big]^2 + (1-\nu)\frac{1 + \lambda(l^s_a) - \mu}{1+r}\tilde{v}(l^s_a)\Big\}. \tag{44}$$

Redefine $\hat{r} = (r+\nu)/(1-\nu)$

The FOC wrt $l^s_{a,t}$ gives:

$$\tilde{B}'(l^s_a) - \tilde{w}^s - \varphi\tilde{w}^s\Big[\frac{l^s_a + l_f}{l^s_{a-1} + l_f} - (1-\delta)\Big] + \frac{1 + \lambda(l^s_a) - \mu}{1+\hat{r}}\tilde{v}'(l^s_a) + \frac{\lambda'(l^s_a)}{1+\hat{r}}\tilde{v}(l^s_a) = 0 \tag{45}$$

The envelope condition gives:

$$\tilde{v}'(l^s_a) = \frac{\varphi w^s}{2}\Big[\frac{(l^s_{a+1} + l_f)^2}{(l^s_a + l_f)^2} - (1-\delta)^2\Big]. \tag{46}$$

Equation (45) and (46) give that along the balanced growth path,

$$\varphi\tilde{w}^s\Big[\frac{l_a^s+l_f}{l_{a-1}^s+l_f}-(1-\delta)\Big]=\tilde{B}'(\bar{l}_a^s)-\tilde{w}^s+ \frac{1+\lambda(l_a^s)-\mu_t}{1+\hat{r}}\frac{\varphi\tilde{w}^s}{2}\Big[\frac{(l_{a+1}^s+l_f)^2}{(l_a^s+l_f)^2}-(1-\delta)^2\Big]+\frac{\lambda'(l_a^s)}{1+\hat{r}}\tilde{v}(l_a^s). \quad (47)$$

Let $\bar{a}$ be the age when $l_{\bar{a}}^s=\bar{l}$, then I can solve (47) backwardly as:

$$\varphi\tilde{w}^s\Big[\frac{l_{\bar{a}-j}^s+l_f}{l_{\bar{a}-j-1}^s+l_f}-(1-\delta)\Big]=\tilde{B}'(\bar{l}_{\bar{a}-j}^s)-\tilde{w}^s+ \frac{1+\lambda(l_{\bar{a}-j}^s)-\mu_t}{1+\hat{r}}\frac{\varphi\tilde{w}^s}{2}\Big[\frac{(l_{\bar{a}+1-j}^s+l_f)^2}{(l_{\bar{a}-j}^s+l_f)^2}-(1-\delta)^2\Big]+\frac{\lambda'(l_{\bar{a}-j}^s)}{1+\hat{r}}\tilde{v}(l_{\bar{a}-j}^s). \quad (48)$$

$\bar{l}$ and $\tilde{v}(\bar{l})$ should satisfy:

$$\varphi\delta\tilde{w}^s=\tilde{B}'(\bar{l})-\tilde{w}^s+\frac{1+\lambda(\bar{l})-\mu}{1+\hat{r}}\frac{\varphi\tilde{w}^s}{2}(2\delta-\delta^2)+\frac{\lambda'(\bar{l})}{1+\hat{r}}\tilde{v}(\bar{l}), \quad (49)$$

$$\frac{\hat{r}+\mu-\lambda(\bar{l})}{1+\hat{r}}\tilde{v}(\bar{l})=\tilde{B}(\bar{l})-\tilde{w}^s(\bar{l}+l_f)(1+\frac{\varphi}{2}\delta^2). \quad (50)$$

E Existence and Uniqueness of $\bar{l}$

This section shows under certain conditions, there exist a unique value of $\bar{l}$ as the solution to (49). To begin with, note that $B(\bar{l})=B_0\bar{l}^{\eta_1}$, where $B_0=L^u\eta_0$. Equation (49) can be written as:

$$\varphi w^s\delta-B'(\bar{l})+w^s-\frac{1+\lambda(\bar{l})-\mu}{1+\hat{r}}\frac{\varphi w^s}{2}(2\delta-\delta^2)=\frac{\lambda'(\bar{l})}{r-\lambda(\bar{l})+\mu}\Big(B(\bar{l})-w^s\bar{l}-\frac{\varphi}{2}w^s\bar{l}\delta^2\Big). \quad (51)$$

Redefine $\bar{l}=l^L-l_f$, then I have

$$\varphi w^s\delta-B'(\bar{l})+w^s-\frac{1+\lambda(\bar{l})-\mu}{1+\hat{r}}\frac{\varphi w^s}{2}(2\delta-\delta^2)=\frac{\lambda'(\bar{l})}{r-\lambda(\bar{l})+\mu}\Big(B(\bar{l})-w^s(\bar{l}+l_f)-\frac{\varphi}{2}w^s(\bar{l}+l_f)\delta^2\Big)$$

$\Longrightarrow$

$$w^s(1+\varphi\delta)+\frac{\varphi}{2}(\delta^2-2\delta)\frac{1+\lambda_0\bar{l}^\theta-\mu}{1+\hat{r}}+\frac{\lambda_0\theta\bar{l}^{\theta-1}}{\hat{r}+\mu-\lambda_0\bar{l}^\theta}w^2(\bar{l}+l_f)$$
$$+\frac{\varphi}{2}w^s\delta^2(\bar{l}+l_f)\frac{\lambda_0\theta\bar{l}^{\theta-1}}{\hat{r}+\mu-\lambda_0\bar{l}^\theta}$$
$$=\left[\eta_1+\frac{\lambda_0\theta\bar{l}^\theta}{\hat{r}+\mu-\lambda_0\bar{l}^\theta}\right]\times B_0\bar{l}^{\eta_1-1}$$

$\Longrightarrow$

$$\left[\frac{1+\varphi\delta}{1+\frac{\varphi}{2}\delta^2}+\frac{\frac{\varphi}{2}(\delta^2-2\delta)}{1+\frac{\varphi}{2}\delta^2}\frac{1+\lambda_0\bar{l}^\theta-\mu}{1+\hat{r}}+\frac{\lambda_0\theta\bar{l}^{\theta-1}(\bar{l}+l_f)}{\hat{r}+\mu-\lambda_0\bar{l}^\theta}\right]\left[\eta_1+\frac{\lambda_0\theta\bar{l}^\theta}{\hat{r}+\mu-\lambda_0\bar{l}^\theta}\right]^{-1}$$
$$=\frac{B_0\bar{l}^{\eta_1-1}}{\left(1+\frac{\varphi}{2}\delta^2\right)w^s} \tag{52}$$

LHS of equation (52) can be written as

$$LHS=1+\left[1-\eta_1+C\cdot(\hat{r}+\mu-\lambda_0\bar{l}^\theta)+\frac{\lambda_0\theta\bar{l}^{\theta-1}l_f}{\hat{r}+\mu-\lambda_0\bar{l}^\theta}\right]\left[\eta_1+\frac{\lambda_0\theta\bar{l}^\theta}{\hat{r}+\mu-\lambda_0\bar{l}^\theta}\right]^{-1}$$
$$=1+\left[1-\eta_1+C\cdot(\hat{r}+\mu-\lambda_0\bar{l}^\theta)-\eta_1\frac{l_f}{\bar{l}}\right]\left[\eta_1+\frac{\lambda_0\theta\bar{l}^\theta}{\hat{r}+\mu-\lambda_0\bar{l}^\theta}\right]^{-1}+\frac{l_f}{\bar{l}}$$
$$=1+\left[1-\eta_1+C\cdot(\hat{r}+\mu-\lambda_0\bar{l}^\theta)\right]\left[\eta_1+\frac{\lambda_0\theta\bar{l}^\theta}{\hat{r}+\mu-\lambda_0\bar{l}^\theta}\right]^{-1}+\frac{l_f}{\bar{l}}\left\{1-\eta_1\left[\eta_1+\frac{\lambda_0\theta\bar{l}^\theta}{\hat{r}+\mu-\lambda_0\bar{l}^\theta}\right]^{-1}\right\}$$

where

$$C=\frac{1}{1+\hat{r}}\frac{\frac{\varphi}{2}(2\delta-\delta^2)}{1+\frac{\varphi}{2}\delta^2}.$$

Let

$$f_1(\bar{l})=\frac{B_0\bar{l}^{\eta_1-1}}{(1+\frac{\varphi}{2}\delta^2)w^s}-\frac{l_f}{\bar{l}}\left\{1-\eta_1\left[\eta_1+\frac{\lambda_0\theta\bar{l}^\theta}{r+\mu-\lambda_0\bar{l}^\theta}\right]^{-1}\right\}$$

$$f_2(\bar{l})=\left[1-\eta_1+C(r+\mu-\lambda_0\bar{l}^\theta)\right]\left[\eta_1+\frac{\lambda_0\theta\bar{l}^\theta}{r+\mu-\lambda_0\bar{l}^\theta}\right]^{-1}+1$$

$f_1(\bar{l})\bar{l}=f_2(\bar{l})\bar{l}$ gives

$$\frac{B_0\bar{l}^{\eta_1}}{(1+\frac{\varphi}{2}\delta^2)w^s}-l_f\left\{1-\eta_1\left[\eta_1+\frac{\lambda_0\theta\bar{l}^\theta}{r+\mu-\lambda_0\bar{l}^\theta}\right]^{-1}\right\}=\frac{(1-\eta_1)\bar{l}+C\bar{l}(r+\mu-\lambda_0\bar{l}^\theta)}{\eta_1+\frac{\lambda_0\theta\bar{l}^\theta}{r+\mu-\lambda_0\bar{l}^\theta}}+\bar{l}, \tag{53}$$

and

$$\frac{B_0\bar{l}^{\eta_1}}{(1+\frac{\varphi}{2}\delta^2)w^s} = \frac{(1-\eta_1)\bar{l} + C\bar{l}(r+\mu-\lambda_0\bar{l}^\theta) - l_f\eta_1}{\eta_1 + \frac{\lambda_0\theta\bar{l}^\theta}{r+\mu-\lambda_0\bar{l}^\theta}} + \bar{l} + l_f \tag{54}$$

Define the LHS of equation (54) to be new f_1 and RHS to be new f_2. We want to find conditions under which $df_1/d\bar{l} < df_2/d\bar{l}$

The LHS of the inequality can be written as

$$\frac{df_1}{d\bar{l}} = \frac{\eta_1}{\bar{l}} f_1 = \frac{\eta_1}{\bar{l}} f_2$$

. We now calculate the RHS of the inequality.

$$\begin{aligned}\frac{df_2}{d\bar{l}} = 1 + &\frac{(1-\eta_1)\bar{l} + C\bar{l}(r+\mu-\lambda_0\bar{l}^\theta) - C\lambda_0\theta\bar{l}^\theta}{\eta_1 + \frac{\lambda_0\theta\bar{l}}{r+\mu-\lambda_0\bar{l}^\theta}}\\ &- \frac{(1-\eta_1)\bar{l} + C\bar{l}(r+\mu-\lambda_0\bar{l}^\theta) - l_f\eta_1}{\left(\eta_1 + \frac{\lambda_0\theta\bar{l}}{r+\mu-\lambda_0\bar{l}^\theta}\right)^2} \times \frac{\lambda_0\theta^2(r+\mu)\bar{l}^{\theta-1}}{(r+\mu-\lambda_0\bar{l}^\theta)^2}\end{aligned}$$

Hence we need to have

$$\begin{aligned}&\frac{\eta_1}{\bar{l}} \times \Big[\frac{(1-\eta_1)\bar{l} + C\bar{l}(r+\mu-\lambda_0\bar{l}^\theta) - l_f\eta_1}{\eta_1 + \frac{\lambda_0\theta\bar{l}}{r+\mu-\lambda_0\bar{l}^\theta}} + l_f\Big]\\ &< \frac{(1-\eta_1)\bar{l} + C\bar{l}(r+\mu-\lambda_0\bar{l}^\theta) - C\lambda_0\theta\bar{l}^\theta}{\eta_1 + \frac{\lambda_0\theta\bar{l}}{r+\mu-\lambda_0\bar{l}^\theta}} + 1 - \eta_1\\ &- \frac{(1-\eta_1)\bar{l} + C\bar{l}(r+\mu-\lambda_0\bar{l}^\theta) - l_f\eta_1}{\left(\eta_1 + \frac{\lambda_0\theta\bar{l}}{r+\mu-\lambda_0\bar{l}^\theta}\right)^2} \times \frac{\lambda_0\theta^2(r+\mu)\bar{l}^{\theta-1}}{(r+\mu-\lambda_0\bar{l}^\theta)^2}\end{aligned}$$

Multiply both sides by $\eta_1 + \frac{\lambda_0\theta\bar{l}}{r+\mu-\lambda_0\bar{l}^\theta}$ and organize the terms, we have

$$\begin{aligned}&\eta_1(1-\eta_1) + \eta_1 C(r+\mu-\lambda_0\bar{l}^\theta) + \eta_1\frac{l_f}{\bar{l}}\frac{\lambda_0\theta\bar{l}^\theta}{r+\mu-\lambda_0\bar{l}^\theta}\\ &< (1-\eta_1) + C(r+\mu-\lambda_0\bar{l}^\theta) - C\lambda_0\theta\bar{l}^\theta + (1-\eta_1)\times\left(\eta_1 + \frac{\lambda_0\theta\bar{l}^\theta}{r+\mu-\lambda_0\bar{l}^\theta}\right)\\ &- \frac{(1-\eta_1)\bar{l} + C\bar{l}(r+\mu-\lambda_0\bar{l}^\theta) - l_f\eta_1}{\eta_1 + \frac{\lambda_0\theta\bar{l}^\theta}{r+\mu-\lambda_0\bar{l}^\theta}}\frac{\lambda_0\theta\bar{l}^\theta}{r+\mu-\lambda_0\bar{l}^\theta} \times \frac{r+\mu}{r+\mu-\lambda_0\bar{l}^\theta}\frac{1}{\bar{l}}\end{aligned}$$

[The rest of the algebra will be added soon.]

F Proof of LEMMA 2

$$
\begin{aligned}
\mu =&\Lambda_1 + \Lambda_1\lambda(l_1^s)(1-\nu) + \Lambda_1\lambda(l_2^s)[1+\lambda(l_1^s)-\mu](1-\nu)^2 + ... \\
&\Lambda_1\lambda(l_3^s)[1+\lambda(l_2^s)-\mu][1+\lambda(l_1^s)-\mu](1-\nu)^3 + ... \\
&\Lambda_1(1-\nu)^{\bar{a}-1}\Pi_{i=1}^{\bar{a}-2}[1+\lambda(l_i^s)-\mu]\lambda(l_{\bar{a}-1}^s) + ... \\
&\Lambda_1(1-\nu)^{\bar{a}}\Pi_{i=1}^{\bar{a}-1}[1+\lambda(l_i^s)-\mu]\frac{\lambda(l_{\bar{a}}^s)}{1-(1-\nu)(1+\lambda(l_{\bar{a}}^s)-\mu)}
\end{aligned} \tag{55}
$$

Define

$$
\begin{aligned}
M =& \Lambda_1 + \Lambda_1(1-\nu)[1+\lambda(l_1^s)-\mu] + \Lambda_1(1-\nu)^2[1+\lambda(l_1^s)-\mu][1+\lambda(l_2^s)-\mu]... \\
&+ \Lambda_1(1-\nu)^{\bar{a}-1}\Pi_{i=1}^{\bar{a}-1}[1+\lambda(l_i^s)-\mu] + \Lambda_1(1-\nu)^{\bar{a}}\Pi_{i=1}^{\bar{a}}[1+\lambda(l_i^s)-\mu]\frac{1}{1-(1-\nu)(1+\lambda(l_{\bar{a}}^s)-\mu)}
\end{aligned}
$$

Now let the kth term in equation (55) be combined with the $(k-1)$th term of $M(1-\mu)(1-\nu)$, we have

$$
\begin{aligned}
\mu + M(1-\mu)(1-\nu) =& \Lambda_1 + \Lambda_1(1-\nu)[1+\lambda(l_1^s)-\mu]... \\
&+ \Lambda_1(1-\nu)^2[1+\lambda(l_1^s)-\mu][1+\lambda(l_2^s)-\mu]... \\
&+ \Lambda_1(1-\nu)^{\bar{a}-1}\Pi_{i=1}^{\bar{a}-1}[1+\lambda(l_i^s)-\mu]... \\
&+ \Lambda_1(1-\nu)^{\bar{a}}\Pi_{i=1}^{\bar{a}}[1+\lambda(l_i^s)-\mu]\frac{1}{1-(1-\nu)(1+\lambda(l_{\bar{a}}^s)-\mu)} \\
&= M,
\end{aligned}
$$

So we can solve

$$
M = \frac{\mu}{1-(1-\mu)(1-\nu)}. \tag{56}
$$

G Algorithm of Solving the Balanced Growth Path

In this section, I describe the computation algorithm.

Step 1: We solve for $\bar{l}$ using equation (36) and calculate $v^s(\bar{l})$ using equation (35).

Step 2: Define the policy function $h(l)$ as the optimal choice of skilled labor per product line when the skilled labor per product line in the end of the last period is equal to l. We solve $v^s(l)$ and $h(l)$ using backward induction. Given $\hat{l}^s$, $h(\hat{l}_s)$, we can solve l^s, $h(l^s)$ and $v^s(l)$ such that $h(l^s) = \hat{l}^s$, using the following equations derived from the Bellman equation:

$$\begin{aligned}\varphi w^{s,s}\frac{(\hat{l}^s - (1-\delta)l^s)}{l^s} &= \frac{dB^s(\hat{l}^s)}{d\hat{l}^s} - w^{s,s} \\ &\quad + \frac{1+\lambda(\hat{l}^s)-\mu}{1+r}\frac{\varphi w^{s,s}}{2}\frac{h(\hat{l}^s)^2 - (1-\delta)^2(\hat{l}^s)^2}{\hat{l^s}^2} + \frac{\lambda'(\hat{l}^s)}{1+r}v^s(\hat{l}^s) \quad (57)\end{aligned}$$

$$v^s(l^s) = B^s(\hat{l}^s) - w^{s,s}\hat{l}^s - \frac{\varphi w^{s,s}}{2}l^s\left[\frac{\hat{l}^s}{l^s} - 1 + \delta\right]^2 + \frac{1+\lambda(\hat{l}^s)-\mu^s}{1+r}v^s(\hat{l}^s) \quad (58)$$

In our current model, for a given optimal choice of l in a certain period, there may not be a solution so that in the long run, the choices converge to $\bar{l}$, and may actually fluctuate around $\bar{l}$. We see our model as an approximation for a continuous time model. We will do backward induction starting from values close to $\bar{l}$ and get policy functions and value functions defined on a grid. Next, I interpolate firms' optimal choices when they do not fall exactly on the grids that are calculated from the backward induction. Specifically, we choose a very small $\epsilon_l > 0$, and let $h(\bar{l} - \epsilon_l) = \bar{l}$ and $v^s(\bar{l} - \epsilon_l) = v^s(\bar{l}) - \epsilon_l dv^s/d\bar{l}$.

Step 3: I determine μ^s as a function of $w^{s,s}$ and ξ, using the entry condition: (17):

$$\max_{l_0^{s,s}}\left\{\lambda^E(l_0^{s,s} - \xi)\frac{v^s(l_0^{s,s})}{1+r} - w^{s,s}l_0^{s,s}\right\} = 0.$$

11

However, when the wage of the skilled worker $w^{s,s}$ is sufficiently high, μ^s may be lower than $\lambda(\bar{l})$, in which case the steady state with a positive entry does not exist.[12] In this case, the steady state will have no entry, $\mu = \lambda(\bar{l})$, and $\bar{l}$ satisfies the following equation:

$$\varphi w^{s,s}\delta - \frac{dB^s(\bar{l})}{d\bar{l}} + w^{s,s} - \frac{1}{1+r}\frac{\varphi w^{s,s}}{2}(2\delta - \delta^2) = \frac{\lambda'(\bar{l})}{r}\Big(B^s(\bar{l}) - w^{s,s}\bar{l} - \frac{\varphi}{2}w^{s,s}\bar{l}\delta^2\Big). \quad (59)$$

Step 4: If there is a positive entry, Λ_0^s is a function of $w^{s,s}$ and ξ. Otherwise, $\Lambda_0^s = 0$.

As we have obtained the optimal choices of firms $l_1^{s,s} = h(l_0^{s,s})$, $l_2^{s,s} = h(l_1^{s,s})$, $\cdots$, $l_{\bar{a}}^{s,s} = \bar{l} - \xi$, we can derive the distribution of firms as a function of $w^{s,s}$ and ξ.

Note that

$$\begin{cases} \Lambda_1^s = \Lambda_0^s \lambda^E(l_0^{s,s} - \xi) \\ \Lambda_a^s = \Lambda_1^s \Pi_{i=1}^{a-1}(1 + \lambda(l_i^{s,s}) - \mu^s), \quad \text{for } 2 \le a \le \bar{a}, \\ \Lambda_a^s = \Lambda_1^s \big(\Pi_{i=1}^{\bar{a}-1}(1 + \lambda(l_i^{s,s}) - \mu^s)\big)\big(1 + \lambda(l_{\bar{a}}^{s,s}) - \mu^s\big)^{a-\bar{a}}, \quad \text{for } a > \bar{a}. \end{cases}$$

[11] The conditions in Proposition 1 require that μ^s should satisfy the following conditions:

$$\mu^s < (1-\eta_1)\frac{1 + \frac{\varphi\delta_s^2}{2}}{\varphi\delta_s}(1+r) - r$$
$$\mu^s < \frac{(1-\eta_1)^2}{\theta}\frac{1 + \frac{\varphi\delta_s^2}{2}}{\varphi\delta_s}(1+r) - r$$
$$\mu^s \ge \lambda_0\Big[\frac{1}{L^u\eta_0}w^{s,s}(1 + \frac{\varphi}{2}(\delta)^2)\Big]^{\frac{\theta}{\eta_1 - 1}} - r$$

[12] This can be proved by noting that $\bar{l}$ and v^s are both decreasing functions of μ. If the term inside the maximization operator is negative, μ needs to decline to make the entry condition holds, and $\bar{l}$ will rise, making it possible to violate $\mu > \lambda(\bar{l})$.

We then calculate Λ_0^s as a function of μ^s based on the following equation:

$$\mu^s = M \times \{\Lambda_0^s \lambda^E(l_0^{s,s} - \xi) + \sum_{a=1}^{\infty} \lambda(l_a^{s,s})\Lambda_a^s(1-\nu)\}$$
$$= M \times \Lambda_0^s \lambda^E(l_0^{s,s} - \xi) \times \Big\{1 + \lambda(l_1^s)(1-\nu) + ...$$
$$\sum_{a=2}^{\bar{a}-1} \lambda(l_a^s)(1-\nu)^a \Pi_{i=1}^{a-1}(1+\lambda(l_i^s) - \mu) + ...$$
$$(1-\nu)^{\bar{a}} \Pi_{i=1}^{\bar{a}-1}[1+\lambda(l_i^s) - \mu] \frac{\lambda(l_{\bar{a}}^s)}{1-(1-\nu)(1+\lambda(l_{\bar{a}}^s) - \mu)}\Big\} \tag{60}$$

Step 5: If there is a positive entry, we solve $w^{s,s}$ from the market clearing condition of the skilled labor (29):

$$L^{s,s} = [\frac{w^{s,s}}{\tau\chi}]^{1/(\chi-1)} = \Lambda_0^s \Big\{ l_0^{s,s} + \lambda^E(l_0^{s,s} - \xi) l_1^{s,s} +$$
$$\sum_{a=2}^{\bar{a}-1} \lambda^E(l_0^{s,s} - \xi)\big(\Pi_{i=1}^{a-1}(1+\lambda(l_i^{s,s}) - \mu^s)\big) l_a^s + \lambda^E(l_0^{s,s} - \xi)\frac{\Pi_{i=1}^{\bar{a}-1}(1+\lambda(l_i^{s,s}) - \mu^s)}{\mu^s - \lambda(\bar{l})}\bar{l}\Big\} \tag{61}$$

If there is no positive entry,

$$L^{s,s} = [\frac{w^{s,s}}{\tau\chi}]^{1/(\chi-1)} = \bar{l}. \tag{62}$$

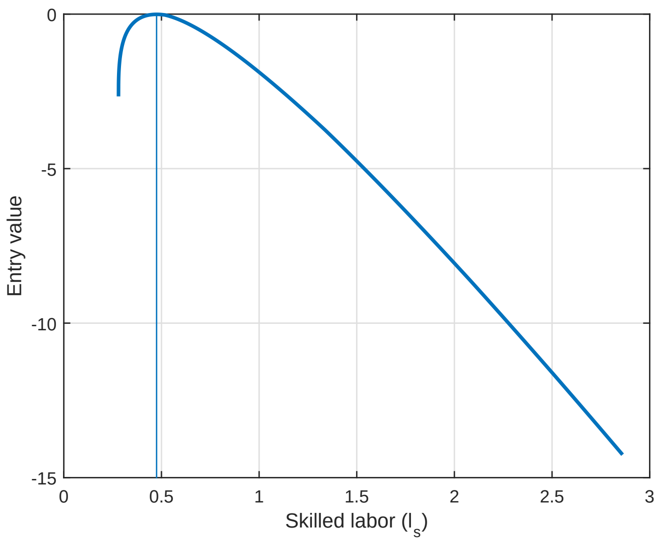
Step 6: If there is positive entry, we determine $w^{u,s}$, using the market clearing condition:

$$w^{u,s} = \sum_{a=1}^{\infty} \frac{\Lambda_a^s}{q(l_a^{s,s})}.$$

If there is no positive entry,

$$w^{u,s} = \frac{1}{q(\bar{l})}.$$

Figure 14: Entry Decision



H Entry Decision

Figure 14 shows how the initial skilled labor is determined in the model: l_0^s is chosen to maximize the entry value. The mass of entry adjust so that the entry value equals zero.

I Firm Size Distribution

Let $F(a, n)$ be the number of firms at age a with n product lines.

- When $a = 1$,

$$F(1, 1) = \Lambda_0 \lambda^E (l_0^s - \xi). \tag{63}$$

- When $a > 1$,

$$F(a+1, n) = \sum_{k=[\frac{n+1}{2}]}^{2^{a-1}} F(a, k)(1-\nu) \sum_{l=0}^{k-[\frac{n+1}{2}]} C_{k-l}^{k} \mu^l (1-\mu)^{k-l} C_{n-(k-l)}^{k} \lambda^{n-(k-l)} (1-\lambda)^{2k-l-n}, \tag{64}$$

for $n \in [1, 2^a]$.

Hence the number of firms with n product lines can be written as:

$$F^n = \sum_{a=1}^{\infty} F(a,n). \tag{65}$$

Firm exit rate:

The share of exiting firms between age k and l can be calculated as:

$$\sum_{a=k}^{l} \sum_{n=1}^{2^{a-1}} F(a,n)(1-\nu)\mu^n(1-\lambda)^n / \sum_{a=k}^{l} \sum_{n=1}^{2^{a-1}} F(a,n) + \nu. \tag{66}$$