Employment and Financial Stability:
Dual Goals of Capital Flow Management*
(Job Market Paper)

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Abstract
I study optimal capital flow management in a small open-economy DSGE model with two frictions: downward nominal wage rigidity and a price-dependent collateral constraint. Wage rigidity introduces an aggregate demand externality under fixed exchange rates and the collateral constraint introduces a pecuniary externality. I provide an analytical characterization of the optimal capital flow management measures and show how they mitigate the externalities. Specifically, I find that the optimal policy in this economy is a prudential tax that discourages capital inflows when the risk of financial crisis is high or when wage is increasing, and it is a stimulative subsidy that encourages capital inflows when unemployment is high and the risk of financial crisis is low. Using quantitative methods and standard calibration, I show that the optimal state-contingent capital inflow tax and even a non-state-contingent flat tax can significantly reduce unemployment and prevent financial crises, hence ultimately improving welfare. These results are of particular relevance for members of a currency union or emerging economies with an exchange rate peg.

JEL codes: E2, F32, F4, H21
Keywords: capital flow management, macroeconomic stability, financial stability, aggregate demand externality, pecuniary externality.

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1 Introduction

Capital flow management, once seen as the enemy of free trade and open market, is becoming increasingly popular among policy makers. The International Monetary Fund has recently endorsed capital flow management as a part of the toolkit for safeguarding macroeconomic and financial stability (IMF, 2012). This change in perspective among policy makers is a response to increasingly volatile capital flows across the world and is supported by advances in academic research. Specifically, the literature on optimal capital flow management can be divided into two streams: one focuses on reducing financial stability risks (see e.g. Korinek 2010); the other focuses on macroeconomic stabilization (see e.g. Farhi and Werning, 2013a and Schmitt-Grohé and Uribe, 2013a).

This paper brings together the two streams of research by studying how capital flow management can address both macroeconomic and financial stability risks. It is important to consider both risks simultaneously since there is a trade-off between macroeconomic and financial stability. A restriction on capital inflows that aims to reduce financial stability risk, for instance, could decrease aggregate demand and eventually worsen unemployment. Moreover, it is relevant to study capital flow management with dual objectives since some countries, such as the euro area member states, do not have an independent monetary policy for macroeconomic stabilization.

To offer normative guidance for designing capital flow management with dual objectives, I study a small open economy model with a price-dependent collateral constraint and a downwardly-rigid wage. On one hand, the collateral constraint binds when the collateral value falls relative to the level of debt. Once the constraint binds, capital inflows would stop, resulting in financial crises. Since the collateral value is determined endogenously, financial crises are endogenous in the model. On the other hand, the downward wage rigidity restricts wage from falling during economic downturns, causing unemployment to rise. Since wage is determined endogenously, unemployment is also endogenous in the model.

A laissez-faire economy suffers from an aggregate demand externality as in Schmitt-Grohé and Uribe (2013a) and Farhi and Werning (2012, 2013b). The aggregate demand externality arises because private agents fail to internalize the impact of their individual consumption choices on the labor market. More specifically, they increase consumption excessively during booms, raising the nominal wage and the risk of future unemployment. On the contrary, agents reduce consumption excessively during busts, driving down labor demand and causing unemployment to rise immediately. Therefore the laissez-faire economy has a high unemployment rate as a result of the aggregate demand externality.

1For example, Brazil imposed tax on foreign inflow (IOF tax) in 2009, and changed the tax policy actively throughout the financial crisis. See Table 9 in Appendix A.

2See average gross capital flows by region Figure 7 in Appendix A.
A laissez-faire economy also suffers from a pecuniary externality as in Korinek (2010). The pecuniary externality arises because private agents fail to internalize the positive impact of their wealth on the asset price. More specifically, they carry too much debt into a financial crisis. Had they borrowed less during tranquil times, the asset price would have been higher during the financial crisis, and the financial crisis would be less severe. Therefore the laissez-faire economy suffers from frequent severe financial crises as a result of the pecuniary externality.

I provide an analytical solution for the optimal capital flow management in this environment by solving a constrained social planner’s problem. The planner faces the same set of constraints as private agents, but he is able to internalize the impacts of individual actions on the asset price and the labor market. I provide an explicit formula for the optimal capital inflow tax (or subsidy) that decentralizes the constrained planner’s allocations. Moreover, I show that the optimal tax can be broken down into three components: one corrects the aggregate demand externality; the other corrects the pecuniary externality; the third accounts for the interaction of the two externalities. The first term is counter-cyclical: it is a prudential tax when the economy has full employment and it is a stimulative subsidy when the economy has unemployment. The sum of the second and the third terms is a prudential tax when the probability of a binding collateral constraint in the next period is positive.

Using a novel graphical framework, I illustrate the impacts of a prudential capital inflow tax on unemployment and financial stability in a laissez-faire economy. The inflow tax decreases aggregate demand and lowers debt by increasing the cost of borrowing. A lower level of debt reduces the probability of a financial crisis in the next period. If the initial laissez-faire economy suffers from unemployment, then the lower aggregate demand increases unemployment. Therefore, there is a trade-off between employment and financial stability: the inflow tax improves financial stability at the cost of higher unemployment. Thus, a prudential inflow tax is optimal if the welfare gain of better financial stability outweighs the cost of higher unemployment. Otherwise, a simulative inflow subsidy is optimal. However, if the initial laissez-faire economy has full employment, then the lower aggregate demand decreases the wage level, reducing future unemployment. Therefore, inflow tax is optimal since it improves both financial stability and future employment.

Using quantitative analysis, I show that the optimal capital flow management is able to reduce both unemployment and financial stability risk significantly compared to the laissez-faire economy. I calibrate the model to the Spanish economy and show that the optimal capital inflow tax lowers average unemployment by 2%, reduces the frequency of severe financial crisis by 6%, and improves permanent consumption by 1% compared to the levels in the laissez-faire economy. Moreover, I find that simple tax rules also lead to significant improvements in the laissez-faire economy. For example, a rule that imposes a 4% inflow tax when unemployment rate is below 2% and no tax when unemployment is above 2% improves permanent consumption by more than 0.5%.

This paper builds on a large literature on pecuniary externalities (see e.g. Korinek, 2010; Bianchi and Mendoza, 2010; Jeanne and Korinek, 2010; Bianchi, 2011; Korinek, 2011a,b; Jeanne and Korinek, 2013; Davila, 2014). These papers examine the inefficiencies that arise from a price-dependent collateral constraint and derive the optimal policies that could mitigate the externality. My paper contributes to this literature by combining it with an aggregate demand externality and examining the optimal capital policy that corrects both externalities. In addition, my paper also adds to both literatures by providing a novel graphical framework for conducting comparative statics.

In particular, the downward nominal wage rigidity in my paper builds on the work of Schmitt-Grohé and Uribe (2011, 2012, 2013a,b). These authors investigate the inefficiencies that arise from the wage rigidity and the optimal policies that can be used to mitigate these inefficiencies. However, their analysis of the optimal inflow tax is numerical in nature. In this paper, I provide an analytical characterization of the inefficiency and an explicit solution for the optimal inflow tax. These analytical results contribute to a better understanding of the ways in which the optimal policy depends on the structural parameters of the economy as well as generating additional insights into the nature of the aggregate demand externality.

This paper also relates to a growing literature that studies the optimal capital control policies in the presence of both nominal and financial frictions. Farhi and Werning (2012, 2013b), for instance, discuss a model with nominal rigidity and risk premium shocks. Davis and Presno (2014) present a similar model with nominal rigidity and a price-dependent collateral constraint. Since these models do not account for uncertainty, their analysis of the optimal capital control policy is limited to one-time unanticipated shocks. This study contributes to this literature by studying optimal capital control policy under uncertainty.

Ottonello (2014) studies exchange rate policy during financial crises with a model that is similar to the one presented in this paper. His work is complementary to mine as I focus on capital flow management. My main contribution is to provide an analytic description of the optimal capital controls in such an environment and compare the optimum with the laissez-faire equilibrium in which both aggregate demand externality and pecuniary externality are present. In Ottonello’s model, flexible exchange rates undo the aggregate demand externality but lead to a trade-off between unemployment and credit access during financial crisis. By contrast, when the exchange rate is fixed, I show that higher unemployment always leads to less credit access so there is no conflict between the dual goals when choosing the level of optimal capital flow management during financial crisis.

The rest of the paper proceeds as follows: in Section 2, I set up the model and define the laissez-faire equilibrium. In Section 3, I solve a constrained social planner’s problem,
characterize the externalities associated with the frictions, and derive a formula for the optimal capital inflow tax. Section 4 presents a graphical framework for comparative statics. In Section 5, I conduct a quantitative analysis and present the numerical results. Section 6 concludes.

2 Model

Consider a DSGE model of a small open economy with a tradable good and a non-tradable good. The economy consists of two types of agents: a unit mass of identical households and a unit mass of identical non-tradable goods producers. There are two sources of exogenous shocks: a stochastic country-specific interest rate $r$ and a stochastic tradable endowment $y_T$.

2.1 Household

The representative household has lifetime welfare function

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t).
$$

(1)

where $\mathbb{E}_0$ is the expectation function at $t = 0$. The period utility function follows the CRRA form with the inter-temporal elasticity of substitution of $\frac{1}{\sigma}$:

$$
u(c_t) = \frac{c_t^{1-\sigma}-1}{1-\sigma}, \quad \sigma > 0.
$$

The aggregate consumption $c_t$ is an Armington-type CES aggregator with the elasticity of substitution of $\xi$ between tradable consumption $c^T_t$ and non-tradable consumption $c^N_t$

$$
c_t = \left[ a(c^T_t)^{1-\frac{1}{\xi}} + (1-a)(c^N_t)^{1-\frac{1}{\xi}} \right]^{\frac{\xi}{\xi-1}}, \quad \xi < 1, a \in (0, 1).
$$

(2)

I assume the intra-temporal elasticity of substitution is greater than the inter-temporal elasticity of substitution

$$
\frac{1}{\sigma} < \xi,
$$

(3)

so that the marginal utility of $c^T_t$, denoted as $u_T(t)$, is decreasing in $c^N_t$.

In each period, the representative household is endowed with $y^T_t$ units of tradable goods and one unit of labor. Let tradable goods be the numeraire. The household sells labor at real wage $w_t$ in the labor market and takes labor demand $h_t$ as given. The
household also receives dividend $\pi_t$ from the non-tradable producers. In the international financial market, the household has access to a one-period non-state-contingent bond denominated in tradable goods with country-specific real interest rate $r_t$. Therefore the household’s period budget constraint is

$$c_t^T + p_t^N c_t^N + d_{t-1} = y_t^T + w_t h_t + \pi_t + \frac{d_t}{(1+r_t)},$$

where $d_t$ is the real outstanding debt due in period $t+1$, $p_t^N$ is the relative price of non-tradable goods in units of tradable goods, which is also the real exchange rate.

The household is subject to a collateral constraint imposed by international lenders that the outstanding debt can not exceed a fraction $\kappa$ of the household income:

$$d_t \leq \kappa(y_t^T + w_t h_t + \pi_t)$$

The household’s problem is to choose stochastic process $\{c_t^T, c_t^N, d_t\}_{t=0}^\infty$ to maximize expected lifetime welfare (I) subject to the budget constraint (IV) and the collateral constraint (V) while taking $\{p_t^N, y_t^T, w_t, h_t, \pi_t, r_t\}_{t=0}^\infty$ and the initial outstanding debt $d_0$ as given.

The household’s optimality conditions are

$$p_t^N = \frac{1-a}{a} \left( \frac{c_t^T}{c_t^N} \right)^{\frac{1}{\xi}}$$

$$u_T(t) = (1 + r_t)\beta E_t u_T(t + 1) + (1 + r_t)\lambda_t^{CC}$$

where $\lambda_t^{CC}$ is the multiplier on the collateral constraint (V).

Equation (VI) defines the real exchange rate as a function of tradable and non-tradable consumption. (VII) is the household’s Euler equation. It sets the household’s marginal utility of an additional unit of tradable consumption to its marginal cost. The cost of the higher tradable consumption includes both the lower future consumption and a deterioration of today’s collateral constraint as more debt is necessary to finance the higher tradable consumption.

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4I have also solved the model using the alternative collateral constraint:

$$\frac{d_t}{1+r_t} \leq \kappa(y_t^T + w_t h_t + \pi_t)$$

Most results continue to hold except that the laissez-faire equilibrium allocations are independent of the state variable $r_t$ when the collateral constraint binds. Then higher interest rate does not lead to more deleveraging or lower consumption during crisis.
2.2 Non-tradable Goods Producer

I assume there is a unit mass of identical firms that produces non-tradable goods using labor as the only input. The firms are price-takers in both the input and output markets. Therefore, a representative firm’s problem is to choose \( h_t \) in each period to maximize profit

\[
\pi_t = \max_{h_t} p_t^N f(h_t) - w_t h_t
\]

where \( f(h_t) \) is the Cobb-Douglas production function with factor of labor equals to \( \alpha \):

\[
f(h_t) = h_t^\alpha.
\]

The firm’s optimal labor demand decision sets the marginal cost of labor equal to its marginal revenue:

\[
w_t = \alpha p_t^N h_t^{\alpha - 1}
\]

(8)

2.3 Wage Rigidity

I follow Schmitt-Grohé and Uribe (2013a) by imposing a downwardly rigid constraint on nominal wages

\[
W_t \geq \gamma W_{t-1}
\]

so that nominal wage \( W_t \) can never fall below a fraction \( \gamma \) of the last period’s nominal level \( W_{t-1} \). I assume the economy has fixed exchange rate, then the nominal rigidity becomes real rigidity:

\[
w_t \geq \gamma w_{t-1}
\]

(9)

Henceforth, I refer to (9) as the wage rigidity constraint and \( \gamma w_{t-1} \) as the wage floor.

In the labor market, labor demand must not exceed labor supply

\[
h_t \leq 1
\]

(10)

Last, the complementary slackness condition must be satisfied

\[
(1 - h_t)(w_t - \gamma w_{t-1}) = 0
\]

(11)

The complementary slackness condition guarantees at least one of inequalities (4) and (11) must hold at equality. So when the wage rigidity constraint binds, there is unemployment. And when the wage rigidity constraint is slack, there is full employment.

\[\text{It also guarantees the uniqueness of the equilibrium. Consider when (11) is not imposed as an equilibrium condition, then there can be an infinite number of equilibria when the full employment wage (the wage that clears the labor market at full employment) is larger than the wage floor: every wage equal or larger than the full employment wage is an equilibrium.}\]
2.4 Market Clearing

Finally the non-tradable goods market must clear:

\[
c_t^N = f(h_t)
\]  
(12)

2.5 Laissez-faire Equilibrium

**Definition 1 (Definition of the Laissez-faire Equilibrium)** Given the stochastic shocks \( \{y_t^T, r_t\}_{t=0}^{\infty} \) and the initial outstanding debt \( d_{-1} \), the laissez-faire equilibrium consists of the stochastic process \( \{c_t^T, c_t^N, h_t, p_t^T, y_t, d_t, \lambda_t^{CC}\}_{t=0}^{\infty} \) that satisfy (6) - (17):

\[
d_t = (1 + r_t) \left( c_t^T + d_{t-1} - y_t^T \right) 
\]  
(13)

\[
\lambda_t^{CC} \geq 0 
\]  
(14)

\[
d_t \leq \kappa(y_t^T + p_t^N y_t^N) 
\]  
(15)

\[
\lambda_t^{CC} \left[ \kappa(y_t^T + p_t^N y_t^N) - d_t \right] = 0
\]  
(16)

To simplify notation, I drop the time subscripts in the rest of the paper. I denote variables in the subsequent period with superscript \(^\prime\), the variables in the previous period with subscript \(_{-1}\), and variables associated with tradable goods and non-tradable goods with subscript \( T \) and \( N \).

2.6 Unemployment and Aggregate Demand

**Proposition 1 ( Tradable Consumption and Unemployment)** The economy is in full employment if and only if \( c_T \geq \widehat{c}_T \), where

\[
\widehat{c}_T = \left( \frac{a}{1-a} \frac{\gamma}{\gamma w_{-1}} \right)^{\xi}.
\]  
(17)

If \( c_T \) is below \( \widehat{c}_T \), then unemployment increases in \( w_{-1} \) and decreases in \( c_T \).

**Proof** First prove that if \( c_T \geq \widehat{c}_T \) then \( h = 1 \) by contradiction.

Suppose \( h < 1 \) when \( c_T \geq \widehat{c}_T \). Then \( w = \gamma w_{-1} \). Substitute (8) and (12) into (8):

\[
w = \alpha^\frac{1-a}{a} (c_T)^{1/\xi} (h)^{a-\alpha/\xi-1}
\]  
(18)

Substitute \( c_T \geq \widehat{c}_T \), \( h < 1 \), and \( (17) \) into \( (18) \) gives \( w > \gamma w_{-1} \), which contradicts with \( w = \gamma w_{-1} \). Therefore \( h = 1 \) when \( c_T \geq \widehat{c}_T \).
Next prove that if \( h = 1 \) then \( c_T \geq \hat{c}_T \). If \( h = 1 \) then \( w > \gamma w_{-1} \). Substitute \( h = 1 \) and \( w > \gamma w_{-1} \) into (18) then

\[
\alpha^{1-a} (c_T T)^{1/\xi} \geq \gamma w_{t-1}.
\]

Rearranging terms gives \( c_T \geq \hat{c}_T \). Therefore, \( h = 1 \) if and only if \( c_T \geq \hat{c}_T \).

Given what I have proved above, if \( c_T < \hat{c}_T \), then \( h < 1 \). When \( h < 1 \), \( w = \gamma w_{-1} \). Then (18) becomes

\[
h = \left[ \alpha \left( \frac{1-a}{\alpha} \right) \left( \frac{1}{\gamma w_{-1}} \right) \right]^{\xi/\xi-\alpha+1} (c_T T) \frac{1}{\xi-\alpha+1} (c_T T) \frac{1}{\xi-\alpha+1}
\]

(19)

So \( \frac{dh}{dc_T} > 0 \) and \( \frac{dh}{dw_{-1}} < 0 \) when \( c_T < \hat{c}_T \).

Proposition 11 shows that unemployment is determined by the previous period’s real wage and the aggregate tradable consumption at the equilibrium. Higher \( w_{-1} \) pushes up the real wage floor and increases the cost of labor, and producers respond by decreasing employment. \( c_T \) should be interpreted as the aggregate demand for the tradable goods. Lower aggregate demand for tradable goods would lead to lower aggregate demand for the non-tradable goods at the initial \( p_N \). Then \( p_N \) must fall to clear the incipient excess supply. The non-tradable producers respond to the lower price by decreasing employment, and hence unemployment increases. Intuitively, the economy suffers from unemployment when the aggregate demand is insufficient (less than \( \hat{c}_T \)), and lower the aggregate demand, the worse the unemployment is.

\( \hat{c}_T \) is the full-employment threshold of aggregate tradable consumption. If the aggregate tradable consumption is above the threshold, the economy has full employment. Then higher demand would not reduce unemployment. Instead, it leads to higher real wage, which increases unemployment in the subsequent period if future aggregate demand is insufficient.

There is a trade-off between financial stability and employment in the model. On the one hand, lower debt reduces the probability of a binding collateral constraint in the next period, decreasing the financial stability risk. On the other hand, the lower debt implies lower aggregate consumption, increasing unemployment when the aggregate demand is insufficient. Households do not internalize their individual actions on the aggregate variables, therefore the trade-off between financial stability and employment might be inefficient in the laissez-faire equilibrium, providing a rationale for capital flow management that takes both objectives into account.
3  Optimal Capital Flow Management

3.1  Constrained Planner’s Problem

A constrained planner directly sets allocations \( c, h, \) and \( d, \) but it is subject to the same set of constraints the private agents face. Moreover, the planner is subject to the laws of supply and demand in the goods market and labor market, so it must take the market determination of real exchange rate and real wage as given. The planner differs from the private agents in its ability to internalize the impacts of allocations on the price and wage levels. More specifically, the constrained planner’s problem is expressed in the following recursive form:

\[
V_{CP}(w_{-1}, d_{-1}, y_T, r) = \max_{c_T, h, d} u(c_T, f(h)) + \beta E V_{CP}(w(c_T, h), d, y_T', r') \tag{20}
\]

subject to

\[
h \leq 1 \tag{21}
\]

\[
y_T + \frac{d}{1+r} - c_T - d_{-1} = 0 \tag{22}
\]

\[
d \leq \kappa [y_T + p_N(c_T, h) f(h)], \tag{23}
\]

\[
w(c_T, h) \geq \gamma w_{-1} \tag{24}
\]

\[
[w(c_T, h) - \gamma w_{-1}] (1 - h) = 0 \tag{25}
\]

where

\[
p_N(c_T, h) = \frac{1-a}{a} \left( \frac{c_T}{f(h)} \right)^{1/\xi} \tag{26}
\]

and

\[
w(c_T, h) = \alpha \frac{1-a}{a} (c_T)^{1/\xi} h^{\alpha - a/\xi - 1}. \tag{27}
\]

The planner takes the resource constraints (21) and (22), the collateral constraint (23), the wage rigidity constraint (24), the complementary slackness condition (25), the real exchange rate function (26) and the real wage function (27) from the decentralized economy as given. Since the wage rigidity depends on the previous period’s wage, therefore \( w_{-1} \) is a state variable in the planner’s problem.

**Definition 2 (Constrained-Optimal Allocations)** The constrained-optimal allocations consist of allocation rules \( \{ c_{CP}^{T}(w_{-1}, d_{-1}, y_T, r), c_{CP}^{N}(w_{-1}, d_{-1}, y_T, r), d_{CP}(w_{-1}, d_{-1}, y_T, r) \} \) that solve the recursive optimization problem of the constrained planner.

Henceforth, I use superscript \( LF \) and \( CP \) to denote the laissez-faire equilibrium allocations and the constrained-optimal allocations respectively.
3.2 Aggregate Demand and Pecuniary Externalities

Following the literature, I call the laissez-faire equilibrium *constrained-inefficient* when it deviates from the constrained-optimal allocations. Below, I first focus on the constrained-inefficiency associated with the wage rigidity by letting the collateral constraints be slack:

**Proposition 2 (Aggregate Demand Externality)** When the collateral constraints are slack, the laissez-faire equilibrium $c_T^{LF}$ and $d_T^{LF}$ are lower than the constrained-optimal levels if $\Phi_T^{CP} > \beta(1+r)\bar{\Phi}_T^{CP'}$, and higher if $\Phi_T^{CP} < \beta(1+r)\bar{\Phi}_T^{CP'}$, where

$$\Phi_T = u_N f'(h) \frac{dh}{dc_T} - \gamma/\beta \bar{\lambda}_w \frac{dw}{dc_T}.$$  

**Proof** When collateral constraints are slack, the constrained planner’s Euler equation can be expressed as

$$u_T + \Phi_T = \beta(1+r)\bar{\Phi}(u_T' + \Phi_T').$$

See Appendix B for detailed derivation. Then

$$u_T^{CP} \leq \beta(1+r)\bar{\Phi} u_T^{CP'} \text{ if } \Phi_T^{CP} \geq \beta(1+r)\bar{\Phi}_T^{CP'}.$$  

On the other hand, the households in the laissez-faire economy set

$$u_T^{LF} = \beta(1+r)\bar{\Phi} u_T^{LF'}.$$  

Therefore when $\Phi_T^{CP} > \beta(1+r)\bar{\Phi}_T^{CP'}$, the constrained-optimal consumption must be higher than the laissez-faire equilibrium levels, and hence the debt is also higher. And the opposite is true when $\Phi_T^{CP} < \beta(1+r)\bar{\Phi}_T^{CP'}$.  

The intuition behind Proposition 2 is that the constrained planner would allocate more tradable consumption (compared to the private agents) to the period when it can improve the unemployment the most. $\Phi_T$ is the marginal benefit of net unemployment improvement from higher tradable consumption. When the present consumption could improve unemployment the most, households fail to internalize the impact of their individual consumption in the labor market, so they under-consume and under-borrow relative to the constrained planner, and by Proposition 1 the laissez-faire equilibrium unemployment rate must also be worse.

It follows from Proposition 1 that $c_T$ affects both contemporaneous and future labor markets. Therefore $\Phi_T$ consists of two components: a contemporaneous component $u_N f'(h) \frac{dh}{dc_T}$ and an inter-temporal component $-\gamma/\beta \bar{\lambda}_w \frac{dw}{dc_T}$.

The contemporaneous component captures the direct impact of $c_T$ on present unemployment. The impact is positive when unemployment is positive ($c_T < \bar{c}_T$), and it
is zero when the economy has full employment \((c_T \geq \hat{c}_T)\). When the unemployment is positive, an extra unit of \(c_T\) reduces unemployment \((by \frac{dh}{dw})\) and increases non-tradable consumption \((by f'(h) \frac{dh}{dw})\). Hence welfare is improved \((by u_N f'(h) \frac{dh}{dw})\).

The inter-temporal component captures the indirect impact of \(c_T\) on future unemployment through wage. The impact is positive when the economy has full employment and zero when unemployment is positive. When the economy is in full employment, an extra unit of \(c_T\) increases \(w\) \((by \frac{dw}{dc_T} \) which equals to \(w_T\)), deteriorating next period’s wage rigidity constraint by pushing up the real wage floor \((by \gamma w_T)\), and hence decreases the welfare \((by \beta \gamma w_T E[\lambda'_w])\). When there is unemployment, \(\frac{dw}{dc_T}\) is zero since \(w = \gamma w_{-1}\), so the inter-temporal component also is zero.

The laissez-faire equilibrium is constrained-inefficient because the private agents fail to internalize neither of the two channels that the aggregate tradable consumption affects the labor market, which is the nature of the aggregate demand externality embodied in the wage rigidity.\(^6\)

\[
\Phi_T = \begin{cases} 
  u_N \frac{f(h)}{c_T} \frac{1}{\xi(1/\alpha-1)+1} & \text{when } c_T < \hat{c}_T \\
  -\beta \gamma \frac{\alpha}{\xi} \frac{1-a}{a} \left[ \frac{c_T}{f(h)} \right]^{\frac{1}{\alpha}} \frac{h^{-1} E[\lambda'_w]}{\xi} & \text{when } c_T \geq \hat{c}_T 
\end{cases}
\]

\(\frac{u_N f(h)}{c_T}\) is decreasing in \(f(h)\). So the higher the unemployment, the higher is the marginal benefit of higher \(c_T\) from improving unemployment. And by \((19)\), unemployment is increasing in \(\gamma\). Therefore, when the aggregate tradable consumption is insufficient, the aggregate demand externality term \(\Phi_T\) is increasing in \(\gamma\). And when the aggregate tradable consumption is insufficient, \(\Phi_T < 0\) and it is decreasing in \(\gamma\). So the size of the aggregate demand externality term \(\Phi_T\) is always increasing in \(\gamma\). Moreover, it is clear from the expression \((29)\) that the size of the aggregate demand externality term \(\Phi_T\) is decreasing in \(\xi\).

Next I let the wage rigidity constraints be slack and focus on the inefficiency associated with the financial friction only. Since the results are standard in the literature (see e.g. Korinek, 2010, and Bianchi, 2012), I keep the discussion brief.

\(^6\)The literature has focused on the inter-temporal channel. For example, Schmitt-Grohé and Uribe (2013a) explain the nature of the externality associated with the wage rigidity as “the excessive expansion of private absorption in response to favorable shocks, causing inefficiently large increases in real wages”. The authors describe the inefficiency in the laissez-faire economy as “over-borrowing”. However, there could be excessive decline of private absorption (in response to adverse shocks) as well, causing inefficiently large increases in unemployment. In other words, there could be either “under-borrowing” or “over-borrowing” in the laissez-faire equilibrium compared to the constrained-optimal allocations due to the wage rigidity.
When the present collateral constraint binds, the laissez-faire equilibrium allocations are generally identical to the constrained-optimal levels.\footnote{When the present collateral constraint binds, the laissez-faire allocations are generally identical to the constrained-optimal levels. Once the collateral constraint binds, the planner would go through the same deleveraging process as private households do. The result holds even when the wage rigidity constraints bind. One could rationalize this result by observing that government policies are usually ineffective in stopping sudden stops.\footnote{Governments still try to stop or alleviate these deleveraging process despite they usually do not work. For example, the “fragile five” introduced various measures, including raising interest rates and capital controls, when international investors took capital out with the expectation of higher U.S. interest rates due to the Fed’s tapering of Q.E.}} Once the collateral constraint binds, the planner would go through the same deleveraging process as private households do. The result holds even when the wage rigidity constraints bind. One could rationalize this result by observing that government policies are usually ineffective in stopping sudden stops. For the rest of the discussion in this subsection, I assume the present collateral constraint is slack.

When there is a positive probability of a binding collateral constraint in the next period, the laissez-faire tradable consumption and debt levels are higher compared to the constrained-optimal levels. Inefficiency arises due to the private agent’s failure to internalize the benefit of higher future tradable consumption from relaxing the future collateral constraint. The higher tradable consumption relaxes the collateral constraint by boosting the real household income. Let $\Psi(c_T, h) = \kappa [p_N(c_T, h)f(h) + y_T]$, and $\Psi_T$ denotes the first derivatives of $\Psi$ with respect to $c_T$. Then $\lambda_{CC}\Psi_T$ measures the marginal benefit of tradable consumption from relaxing the collateral constraint.

The collateral constraint also depends on the level of employment. Denote $\Psi_h$ as the first derivative of $\Psi$ with respect to $h$. Then $\Psi_h < 0$ since $\xi < 1$. Ottonello’s (2014) model has a “unemployment-credit access trade-off” because $\Psi_h < 0$. However, my model has no such trade-off because the partial equilibrium effect $\Psi_h$ is dominated by a general equilibrium effect. In my model, $c_T$ must decrease when unemployment increase (see Proposition\footnote{Finally, I consider the case when both constraints bind in the following proposition: Proposition 3 (Dual Externalities) When both constraints bind, the laissez-faire allocations are determined by the same set of equations: (21)-(27) with (28) at equality. The household is also subject to the constraint on Euler equation $u_T \geq \beta(1 + r)E^T u_T^{CP'}$, while the planner is not. However, the planner almost always chooses the allocations that satisfy the household’s Euler equation constraint unless the debt deflation process is extremely severe, in which case it would increase the leverage instead of deleveraging.}, so the general equilibrium effect of an extra unit of unemployment on the collateral is $\lambda_{CC}[\Psi_h - \Psi_T \frac{dc_T}{dh}]$, which is less than or equal to 0.\footnote{In Ottonello’s model the general equilibrium effect is 0 because his planner sets optimal nominal exchange rates, which undo the the wage rigidity and hence the aggregate demand externality is non-existent.} In Ottonello’s model the general equilibrium effect $\frac{dc_T}{dh}$ is 0 because his planner sets optimal nominal exchange rates, which undo the the wage rigidity and hence the aggregate demand externality is non-existent.}

Finally, I consider the case when both constraints bind in the following proposition:

\textbf{Proposition 3 (Dual Externalities)} When both constraints bind, the laissez-faire allocations are determined by the same set of equations: $\Psi(c_T, h) = \kappa [p_N(c_T, h)f(h) + y_T]$, and $\Psi_T$ denotes the first derivatives of $\Psi$ with respect to $c_T$. Then $\lambda_{CC}\Psi_T$ measures the marginal benefit of tradable consumption from relaxing the collateral constraint.
equilibrium allocations $c_T^{LF}$ and $d_T^{LF}$ are lower than the constrained-optimal levels if

$$
\Phi_T - \beta(1 + r)\mathbb{E}\Phi_T' > \beta(1 + r)\mathbb{E}\left[\lambda_{CC}'\left(\Psi_T' + \Psi_{h/dc_T}'\right)\right],
$$

(30)

and higher otherwise. (30) is evaluated at the constrained planner’s allocations.

**Proof** When $\lambda_{CC} = 0$ and $\mathbb{E}\lambda_{CC}' > 0$, the constrained planner’s Euler equation becomes

$$
u_T + \Phi_T = \beta(1 + r)\mathbb{E}\left[\nu_T' + \Phi_T' + \lambda_{CC}'\left(\Psi_T' + \Psi_{h/dc_T}'\right)\right].
$$

(31)

When (31) holds,

$$u_{CP}^{T} < \beta(1 + r)\mathbb{E}u_{CP}'^{T}.$$

Compare (31) to the private agent’s Euler equation (28), the constrained-optimal level of $c_T$ must be higher than the laissez-faire level, so the debt level must also be higher. When the unemployment is positive, higher $c_T$ also implies higher $h$. The opposite is true when the inequality of (31) reverses.

Proposition 3 encapsulates the optimal trade-off between the macroeconomic and financial stability. If the marginal welfare gains from the better employment outweighs the marginal welfare loss of the higher financial stability risk, then planner increases $c_T$. Otherwise the planner decreases $c_T$.

The left hand side of (31) measures the marginal effect of higher $c_T$ on welfare through its impact on unemployment. The marginal welfare effect could be either positive or negative: it is positive when the present unemployment is more severe than the expected future unemployment; it is negative when the expected future unemployment is more severe than the present unemployment. When the marginal welfare effect is positive, there is a trade-off between macroeconomic and financial stability: higher $c_T$ improves the labor market but increases the financial stability risk. When it is negative, there is no trade-off between macroeconomic stability and financial stability: higher $c_T$ deteriorates both unemployment and financial stability.

The right hand side of (31) is the marginal welfare loss of higher $c_T$ from deteriorating the financial stability. The RHS of (31) has two terms since lower $c_T'$ affects the collateral function $\Psi(c_T', h')$ through both input arguments. The first term $\Psi_T'$ is positive since higher $c_T'$ bids up the asset price. The second term $\Psi_{h/dc_T}'$ is negative when $c_T' < \hat{c}_T'$, since lower $c_T'$ lowers $h'$, which improves the asset price. The sum of the two terms can be written as

$$\Psi_T + \Psi_{h/dc_T}' = \left\{
\begin{array}{ll}
\frac{\alpha}{a} \frac{1-a}{\xi} \frac{1}{a} \left[\frac{c_T}{f(h)}\right]^{1-1} & \text{when } c_T < \hat{c}_T \\
\frac{1-a}{a} \left[\frac{c_T}{f(h)}\right]^{1-1} \left[\frac{1}{\alpha + \xi (1-a)}\right] & \text{when } c_T \geq \hat{c}_T
\end{array}
\right.
$$

(32)
Therefore the pecuniary externality term \( \mathbb{E} \left[ \lambda_{CC} \left( \Psi'_T + \Psi'_h \frac{dh}{dc'_T} \right) \right] \geq 0 \). So lower debt always leads to an improvement in financial stability. And it is clear from the expressions in (32) that the pecuniary externality term is increasing in \( \kappa \) and decreasing in \( \xi \).

### 3.3 Optimal Capital Inflow Tax

A capital inflow tax could be used to correct the constrained-inefficiencies in the laissez-faire economy. Letting \( \tau \) be a tax charged on debt, then the Euler equation in a regulated decentralized equilibrium is:

\[
 u_T = (1 + r)\mathbb{E}_t u'_T + (1 + r)\lambda^{CC}
\]

**Proposition 4 (Optimal Capital Inflow Tax)** The constrained-optimal allocations can be implemented in the decentralized economy by imposing tax \( \tau \) (subsidy if \( \tau < 0 \)) on debt and rebating tax revenue back to households as lump sum transfer:

\[
 \tau = \frac{\beta(1 + r)\mathbb{E} \left[ \Phi_T^{CP'} + \lambda^{CC} \Psi_T^{CP'} + \lambda^{CP'} \Psi'_h \frac{dh^{CP'}}{dc''_T} \right] - \Phi_T^{CP}}{\beta(1 + r)\mathbb{E}_T^{CP'}}
\]

where (33) is evaluated at the constrained-optimal allocations.

**Proof** See Appendix C.

When the collateral constraint is not binding in the constrained-optimal allocations, the optimal capital inflow tax corrects the constrained-inefficiencies in the laissez-faire equilibrium. When the collateral constraint binds, the optimal capital inflow tax is zero as the constrained-optimal allocations are identical to the laissez-faire equilibrium.

\( \tau \) can be broken down into three terms: \( \tau^{AD} \) corrects for the aggregate demand externality; \( \tau^{FS} \) corrects for the pecuniary externality; \( \tau^{AD,FS} \) corrects for the interaction between the two externalities.

\[
 \tau^{AD} = \frac{\beta(1 + r)\mathbb{E}_T^{CP'} \Phi_T^{CP} - \Phi_T^{CP}}{\beta(1 + r)\mathbb{E}_T^{CP'}},
\]

\[
 \tau^{FS} = \frac{\mathbb{E} \left[ \lambda^{CP'} \Psi_T^{CP'} \right]}{\mathbb{E}_T^{CP'}},
\]

\[
 \tau^{AD,FS} = \frac{\mathbb{E} \left[ \lambda^{CP'} \Psi_T^{CP'} \frac{dh^{CP'}}{dc''_T} \right]}{\mathbb{E}_T^{CP'}}.
\]
If the wage rigidity is the only source of friction in the model (e.g. Schmitt-Grohé and Uribe, 2013a), then the optimal capital inflow tax simplifies to $\tau^{AD}$. If the price-dependent collateral constraint is the only source of friction in the model (e.g. Korinek, 2010), then the optimal capital inflow tax simplifies to $\tau^{FS}$.

$\tau^{AD}$ is counter-cyclical: it is negative when the present unemployment is high and positive when the economy has full employment. In other words, it is a stimulative subsidy (to encourage capital inflows) during economic downturns, and it is a prudential tax (to discourage capital inflows) during booms.

Since the size of $\Phi_T$ is increasing in $\gamma$ and decreasing in $\xi$, a higher $\gamma$ or lower $\xi$ implies a more volatile $\tau^{AD}$. Intuitively, a higher $\gamma$ and a lower $\xi$ increase the marginal effect of tradable consumption on both employment and wage. Therefore, they require a larger prudential tax during booms and a larger stimulative subsidy during recessions.

$\tau^{FS}$ is prudential tax and it is acyclical. $\tau^{FS}$ is positive when the probability of a binding collateral constraint in the next period is greater than zero, otherwise it is zero. The collateral constraint is more likely to bind in the next period when the debt level is higher. Therefore, $\tau^{FS}$ is a prudential policy to discourage capital inflows when the debt is high. Moreover, since debt could increase during both booms and recessions, $\tau^{FS}$ is acyclical.

The interaction term $\tau^{AD,FS}$ partially offsets the prudential tax $\tau^{FS}$. A binding collateral constraint usually leads to unemployment, so $\tau^{AD,FS} < 0$ when $\tau^{FS} > 0$. Then the sum $\tau^{FS} + \tau^{FS,AD} < \tau^{FS}$. By (32), $\tau^{FS} + \tau^{FS,AD} \geq 0$, and $\tau^{FS} + \tau^{FS,AD}$ is increasing in $\kappa$ and decreasing in $\xi$. Intuitively, higher $\kappa$ and lower $\xi$ increases the marginal effect of tradable consumption on collateral. Therefore, they require a larger prudential tax when the debt level is high.

Finally, when there is a trade-off between macroeconomic and financial stability, the optimal policy is a prudential tax on inflows when the financial stability is the dominant concern, and it is a stimulative subsidy on inflows (or a tax on outflow) if the macroeconomic stability is the dominant concern.

---

$^{10}$ $\tau^{FS}$ could in fact be pro-cyclical. The collateral constraint is more likely to bind when the economy is hit by an adverse shock in the next period. Given the persistence of shocks, the risk of a future binding collateral constraint is higher when the economy is hit by an adverse shock in the present period compared to a favorable shock (assuming they lead to the same debt level). Therefore, $\tau^{FS}$ is higher when the economy is hit by an adverse shock.
4 Graphical Analysis

In this section, I first provide a graphical framework of the theoretical model, and then I use it to analyze the impacts of the introduction of an inflow tax in the laissez-faire economy.

4.1 Graphical Framework: $EE, WR$ and $CC$ Curves

The graphical framework focuses on the decentralized partial equilibrium in a given period. The partial equilibrium consists of two endogenous variables $c_T$ and $h$ for given $\{d_{-1}, w_{-1}, \tau, r, y_T, c'_T, h'\}$, and the equilibrium is determined by three curves.

Figure 1: Graphical Framework: $EE, WR$ and $CC$ Curves

Note: $EE$ is defined in (34), $WR$ is defined in (35), $CC$ is defined in (36).

4.1.1 $EE$ Curve

The first curve captures the inter-temporal optimality at the equilibrium. Since it is derived from the Euler equation, I denote it as $EE$ and define it below.
**Definition 3 (EE Curve)** Given \( \{r, \tau, c'_T, h'\} \), EE is the collection of all the positive employment-consumption bundles \((h, c_T)\) that satisfy:

\[
(c_T)^{-1/\xi} [c(c_T, h^\alpha)]^{-\sigma+1/\xi} = (1 + r)(1 + \tau) c'_T \left[ c(c'_T, f(h')) \right]^{-\sigma+1/\xi}.
\]

(34)

It is clear from (34) that EE is downward-sloping for given \( \{r, \tau, c'_T, h'\} \) under assumption (3) as in Figure II. When the collateral constraint does not bind, hence \((h, c_T)\) lies on EE. When the collateral constraint is binding, the equilibrium lies below EE. Therefore the equilibrium never lies above EE curve.

EE shifts toward the origin when \( r \) or \( \tau \) increases, or when \( c'_T \) or \( h' \) decreases. Intuitively, households would like to consume more tomorrow and less today when the domestic real interest rate is higher. And the effective domestic real gross interest rate faced by the households is \((1 + r)(1 + \tau)\). Therefore today’s aggregate demand goes down when \( r \) or \( \tau \) increases, and the lower aggregate demand is represented by the inward shift of EE.

### 4.1.2 WR Curve

The second curve captures the nominal friction in the model and it is derived from the downward nominal wage rigidity constraint, so I denote it as WR and define it below:

**Definition 4 (WR Curve)** Given \( w_{-1} \), WR is the set of all the positive employment-consumption bundles \((h, c_T)\) that satisfy

\[
h = \begin{cases} 
\left( \frac{1-a}{a} \left( \frac{1}{\gamma w_{-1}} \right) \right)^{\frac{\xi}{\xi-a\xi+\alpha}} c'_T^{\frac{1}{\xi-a\xi+\alpha}} c_T & \text{when } c_T \leq \tilde{c}_T \\
1 & \text{when } c_T > \tilde{c}_T.
\end{cases}
\]

(35)

WR consists of two segments: an upward sloping segment when \( c_T < \tilde{c}_T \) and a vertical segment when \( c_T \geq \tilde{c}_T \). The wage rigidity constraint binds and there is involuntary unemployment on the upward sloping segment, while the wage rigidity constraint is slack and the economy is in full employment on the vertical segment. See the Proof of Proposition II for a detailed derivation of (35).

The kink point \((\tilde{c}_T, 1)\) and the upward sloping segment of WR shift down if \( w_{-1} \) decreases. Intuitively, a lower \( \gamma w_{-1} \) relaxes the downward wage rigidity, hence it should be easier for the economy to reach full employment. That is, the minimum tradable consumption required to reach full employment must be lower. Thus, the kink point and the upward-sloping segment of WR are also lower.
4.1.3 CC Curve

The third curve in the graphical framework captures the financial friction. Since it is derived from the collateral constraint, so I denote it as CC. It is defined below.

**Definition 5 (CC Curve)** Given \( \{r, y_T, d_{-1}\} \), CC is the set of all the positive employment-consumption bundles \( (h, c_T) \) that satisfy

\[
(1 + r)(c_T - y_T + d_{-1}) = \kappa \left[ y_T + \frac{1 - \alpha}{\alpha} c_T^{1/\xi} (h^\alpha)^{(1-1/\xi)} \right].
\] (36)

CC slices the \( (h, c_T) \) space into two regions. The collateral constraint is not satisfied at allocations in the shaded area to the right of CC where the real household income is too low to secure the level of debt needed to finance the consumption, so the household must deleverage.\(^\text{11}\) Henceforth I refer to the shaded area as the deleveraging area. The collateral constraint is satisfied at equality on CC where the real household income is just high enough to secure the debt. And the collateral constraint is slack to the left of CC where the real household income is more than necessary to secure the debt. (36) is derived by substituting (3) and (13) into the collateral constraint (15) at equality.

CC is \( U \)-shaped with the opening facing to the right as illustrated in Figure 1. Denote the inflection point \( (h^{\text{inflow}}, c_T^{\text{inflow}}) \), they are given by

\[
h^{\text{inflow}} = \left[ \frac{\kappa(1-\alpha)}{\alpha(1-\xi)} \right] \frac{\xi}{(1-\xi)} \left( c_T^{\text{inflow}} \right)^{\frac{1}{\alpha}}
\]

\[
c_T^{\text{inflow}} = \frac{(1+r)(y_T - d_{-1}) + \kappa y_T}{(1-\xi)(1+r)}.
\]

The debt deflationary process is the extremely severe on the upward sloping segment of CC since one unit reduction in \( c_T \) decreases the collateral by more than it decreases the debt.

Following the literature (see e.g. Korinek and Mendoza, 2014), I focus on the equilibrium space below \( c_T^{\text{inflow}} \) by assuming moderate values for \( \kappa \) and \( \xi \). As a result, the planner would not intervene during financial crisis and choose to go through the same deleveraging process as the households.

CC shifts toward the origin when \( d_{-1} \) or \( r \) increases, or when \( y_T \) decreases. Intuitively, the collateral constraint must be binding at more sets of \( (h, c_T) \) when the initial debt outstanding or the real interest rate increases, or the real income that can be used to secure debt decreases. So the deleveraging area must expand and it follows that CC must shift toward the origin.

\(^\text{11}\)The statement depends on the assumption that \( \xi < 1 \). If \( \xi > 1 \), then the deleveraging area is the area to the left of CC.
4.2 Determination of the Partial Equilibrium

The partial equilibrium is determined jointly by three curves. The intersection of $EE$ and $WR$ is the equilibrium point when it lies outside the deleveraging area. Otherwise, the intersection of $WR$ and $CC$ that lies below $EE$ is the equilibrium point. See Appendix D for the proof of the existence and the uniqueness of the partial equilibrium.

**Figure 2: Four Cases of Equilibrium**

A. Neither constraint binds.

B. Downward wage rigidity constraint binds.

C. Collateral constraint binds.

D. Both constraints bind.

Note: The deleveraging area is indicated by the shaded area. Equilibrium is at point $A$ in panels A and B. And equilibrium is at point $B$ in panels C and D.

Figure 2 shows four cases of equilibrium that differ by the constraints that bind. In panels A and B, the equilibrium is determined by the intersection of $EE$ and $WR$, point $A$, since it lies outside the deleveraging area. The collateral constraint is slack at the equilibrium in both panels. The downward wage rigidity is slack in Panel A since the equilibrium point lies on the full employment segment of $WR$, and the wage rigidity constraint binds in panel B, since point $A$ lies on the involuntary segment of $WR$. 

20
In panels C and D, point $A$ is inside the deleveraging area. Therefore, the equilibrium is determined by the intersection of $CC$ and $WR$ that lies below $EE$, point $B$, and the collateral constraint binds at the equilibrium in both panels. The wage rigidity constraint is slack in Panel C since point $B$ lies on the full employment segment of $WR$, and the wage rigidity constraint binds in Panel D since its point $B$ is on the involuntary unemployment segment of $WR$.

### 4.3 Impacts of Capital Inflow Tax in a Laissez-faire Economy

Below I consider the introduction of an inflow tax in the laissez-faire economy and evaluate its static impact in the present period and the dynamic effect in the subsequent period using the graphical framework.

#### 4.3.1 Within-period Impacts of Capital Inflow Tax

Figure 3 plots the impact of the introduction of a positive $\tau$ in laissez-faire economies. As in Figure 2, there are four cases of laissez-faire economies depending on the binding constraints. Let the solid lines represent curves in the laissez-faire economy.

A small inflow tax shifts $EE$ out to $EE'$. When the collateral constraint binds at the laissez-faire equilibrium (see panels C and D), a small outward shift of $EE$ has no real impact. When the collateral constraint is slack at the laissez-faire equilibrium (see panels A and B), the equilibrium moves from point $A$ to $A'$. Therefore the inflow tax reduces $c_T$ and hence $d$. And when the wage rigidity constraint binds (panel B), the inflow tax also reduces $h$. $w$ decreases in panel A but not in panel B.

If a large inflow tax is introduced, then a slack wage rigidity constraint could become binding, which causes unemployment. Consider full employment laissez-faire economies in panel A and C, a large inflow tax causes large downward shift in $CC$, then the decentralized economies move from panel A to B, and from panel C to D.

To conclude, a small inflow tax decreases tradable consumption and debt levels when the collateral constraint is slack at the laissez-faire equilibrium, it also decreases wage if the wage rigidity constraint is also slack at the laissez-faire equilibrium, and it increases unemployment when the wage aridity binds at the laissez-faire equilibrium. A large inflow tax tightens the wage rigidity constraint, and decreases tradable consumption, debt, real wage, and employment.

The inflow tax in the present period could have dynamic effects in the next period through the lower debt level and real wage, which I discuss below separately.

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12 The inflow tax also has general equilibrium effect: a lower debt implies a higher $c'_T$, which partially offsets some of the inward shift in $EE$. 

21
Figure 3: Impacts of A Small Capital Inflow Tax in Laissez-faire Economy

A. Neither constraint binds.

B. Downward wage rigidity constraint binds.

C. Collateral constraint binds.

D. Both constraints bind.

Note: The deleveraging area is indicated by the shaded area. The solid lines represent the laissez-faire economy. The dash lines represent the impacts of the inflow tax. Superscript ' indicates the new equilibrium points under the inflow tax.

4.3.2 Impacts of Lower Initial Debt

Figure 4 shows the impact of a small decrease in $d_{-1}$ in the decentralized economies. Lower $d_{-1}$ shifts $CC$ outward to $CC'$. When the collateral constraint is slack at the initial equilibrium (panels A and B), there is no real impact. When the collateral constraint binds at the initial equilibrium (panels C and D), $c_T$ increases since the constrained households are able to borrow more. When the wage constraint also binds (panel D), $h$ also increases because of the aggregate demand externality.\footnote{The smaller $d_{-1}$ also have general equilibrium effect: $c_T'$ should increase. Therefore $EE$ shifts outward, leading to higher $c_T$ in panel A and B, and higher $h$ in panel B.}

If the decrease in $d_{-1}$ is large, then a binding collateral constraint could become slack. Consider financially constrained economies in panels C and D, a large outward
Figure 4: Comparative Statics: $d_{-1} \downarrow$

A. Neither constraint binds.

B. Downward wage rigidity constraint binds.

C. Collateral constraint binds.

D. Both constraints bind.

Note: The deleveraging area is indicated by the shaded area. The solid lines represent the initial economy. The dash lines represent the impacts of the lower $d_{-1}$. Superscript ' indicates the new equilibrium points.

The lower $w_{-1}$ also have general equilibrium effect in panel B: lower $d$ increases $c_T'$, which shifts $EE$ out, offsetting some of the decrease in $c_T$ and increasing $h$ by even more.

\[14\]
Figure 5: Comparative Statics: $w_{-1} \downarrow$

A. Neither constraint binds.

B. Downward wage rigidity constraint binds.

C. Collateral constraint binds.

D. Both constraints bind.

Note: The deleveraging area is indicated by the shaded area. The solid lines represent initial economy. The dash lines represent the impacts of the lower $w_{-1}$. Superscript ' indicates the new equilibrium points.

If the decrease in $w_{-1}$ is large, then a binding wage rigidity constraint could become slack. Consider economies with unemployment in panels B and D, a large downward shift of $\text{WR}$ move the economies from panel B to A, and from D to C, where the economies would be in full employment at the new equilibrium.

4.3.4 Dynamic Impacts of Capital Inflow Tax in a Laissez-faire Economy

Table I summarizes the impacts of the introduction of a small inflow tax on employment and collateral constraint in the laissez-faire economies for different combinations of binding constraints. Table II omits the case with a binding collateral constraint in the present period because a small inflow tax has no real impact in the laissez-faire economy.
Table 1: Impacts of A Small Capital Inflow Tax on Employment and Collateral Constraint

<table>
<thead>
<tr>
<th>Present Period</th>
<th>Next Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC slack, WR slack.</td>
<td>No Impact.</td>
</tr>
<tr>
<td>CC slack, WR slack.</td>
<td>h ↑.</td>
</tr>
<tr>
<td>CC' slack, CC' binds.</td>
<td>CC' relaxes.</td>
</tr>
<tr>
<td>CC' slack, WR' binds.</td>
<td>h ↑.</td>
</tr>
<tr>
<td>CC' binds, WR' slack.</td>
<td>h ↓.</td>
</tr>
<tr>
<td>CC' binds, WR' binds.</td>
<td>h ↑.</td>
</tr>
</tbody>
</table>

Note: WR denotes the wage rigidity constraint. CC denotes the collateral constraint. h denotes employment. Superscript ‘ denotes the variables in the next period. ↑ denotes an increase in employment and ↓ denotes a decrease in employment.

The first row of Table 1 shows that an inflow tax improves either future unemployment or financial stability or both in the laissez-faire economy when both constraints are slack in the present period and one of the constraints binds in the future period. The second row of Table 1 shows the benefits of an inflow tax on future employment and financial stability persists, but at a cost of higher current unemployment. Therefore, an inflow tax is desirable when the present unemployment is low. Since an inflow subsidy has the opposite effect as an inflow tax, an inflow subsidy is desirable when the present unemployment is high.

5 Quantitative Analysis

This section solves the model numerically and quantifies the welfare improvements of capital flow management.

5.1 Parameter Calibration

I calibrate the model at annual frequency using Spain’s data from 1980-2013. The sample period starts in 1980 because of the data availability. More specifically, β and κ are calibrated so that the net international investment position (NIIP) to GDP ratio...
Table 2: Baseline Parameter Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.92</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.83</td>
<td>Intra-temporal Elasticity of substitution</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.32</td>
<td>Share of income that is used as collateral for debt</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.96</td>
<td>Degree of downward wage rigidity</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>Inverse of inter-temporal elasticity of consumption</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.75</td>
<td>Labor share in the non-tradable goods sector</td>
</tr>
<tr>
<td>$a$</td>
<td>0.31</td>
<td>Share of tradable</td>
</tr>
</tbody>
</table>

Note: Parameters are calibrated to the Spanish Economy at annual frequency.

and the frequency of sudden stops in the simulated laissez-faire economy match with those in the Spanish data. Since the NIIP data are only available from 1992, I construct the NIIP from 1980-1991 using net flow data. The average NIIP-GDP ratio is -28% in Spain from 1980-2013. Since the NIIP equals to the debt level in the model, the time discount factor $\beta$ is calibrated to 0.92 so the simulated debt-GDP ratio is 28% in the simulated laissez-faire economy.

Following Forbes and Warnock (2012), I define sudden stops as episodes with gross inflows exceeding one standard deviation, where gross inflows are the sum of net portfolio investment liabilities, other liabilities, and net foreign direct investment. The frequency of sudden stops is 6% during the sample period in Spain, which is similar to the average level found in a panel study by Eichengreen et al. (2006). $\kappa$ is calibrated to 0.32 so the frequency of sudden stops is 6% in the simulated laissez-faire economy.

---

16 To focus on private liability, I exclude official loans from NIIP by deducting (net) “Other Investment, Liabilities, General Government (Excludes Exceptional Financing)”. The exclusion of official loans lowers NIIP significantly. The NIIP reported in IFS is -98.2% of GDP in 2013. After I exclude official loans, the number becomes -72% of GDP. See Figure 8 in Appendix A for a comparison of the reconstructed NIIP and reported NIIP. In reality, a large proportion of the increase in foreign liabilities during the financial crisis was in the form of official loans to the government. See Figure 1 in Appendix A for a comparison of household’s debt and public debt levels.

17 The exchange rate regime of Spain could be not characterized as a fixed exchange rate during the entire sample period as the currency depreciated against Deutsch Mark numerous times before 1998 (see Bacchetta 1997). Fortunately, the debt-GDP ratio in the simulated laissez-faire economy when the wage rigidity is dropped (equivalent to having an optimal flexible exchange rate) is also about 28%.

18 Since Forbes and Warnock (2012) use quarterly data, they use the two standard deviation cutoff line. In contrast, I use annual data, so I lower the cutoff line to one standard deviation like in Eichengreen et al. (2006).

19 I find two episodes of sudden stops in the Spanish data: one during the Mexico Peso crisis of 1994 (also known as the Tequila crisis) and the other during the height of the Euro crisis of 2010. Using a panel dataset of 24 emerging economies spanning from 1980-2003, Eichengreen et al. (2006) find the average frequency of sudden stops to be 5.5%.
\( \xi \) and \( \gamma \) are calibrated to values found in the empirical literature. The empirical literature finds \( \xi \) to be between 0.40 and 0.83.\(^{20}\) Since a lower \( \xi \) implies a more severe debt-deflationary process, I set \( \xi \) at the upper bound 0.83 as a conservative estimate of the debt deflationary process. Schmitt-Grohé and Uribe (2013a) find \( \gamma \) to be between 0.99 and 1.02 using quarterly data from Argentina and Europe. I use the lower bound 0.99 as a conservative estimate of the wage rigidity. And since my model has annual frequency, then \( \gamma \) is set to 0.96.

The rest of the parameter calibrations are all standard in the small open economy DSGE literature: the inter-temporal elasticity of substitution is set to be 0.45 with \( \sigma = 2 \); the share of labor input \( \alpha \) is 0.75; and \( \alpha \) is set at 0.31 so the share of tradable consumption is about one third of the aggregate consumption.

The model has two sources of exogenous shocks: a tradable endowment \( y^T_t \) and a country-specific interest rate \( r_t \). I assume that the shock \( (y^T_t, r_t) \) follows a bivariate AR(1) process. Estimating the AR(1) process using annual data in Spain from 1980-2013, the coefficients are:

\[
\begin{bmatrix}
\ln y^T_t \\
\ln \frac{1+r_t}{1+r}
\end{bmatrix} =
\begin{bmatrix}
0.66 & -0.22 \\
-0.03 & 0.90
\end{bmatrix}
\begin{bmatrix}
\ln y^T_{t-1} \\
\ln \frac{1+r_{t-1}}{1+r}
\end{bmatrix} + u_t
\]

where \( r = 0.048 \) and \( u_t \) is a random variable vector of 2 by 1 with normal distribution \( N(\emptyset, \Sigma_u) \),

\[
\Sigma_u =
\begin{bmatrix}
0.000855 & -0.000007 \\
-0.000007 & 0.000132
\end{bmatrix}
\]

The unconditional standard deviations of \( \ln(y^T_t) \) and \( r_t \) are 4.93 percent and 3.05 percent respectively.\(^{21}\)

I approximate the bivariate AR(1) process using a discrete method. I first generate the shock process for 50 million periods. Then I discretize the continuous shock space using three grid points for \( y^T \) and three grid points for \( r \), and compute the transition probabilities of going from one state to the other.\(^{22}\) The discretized shock process closely


\(^{21}\)The tradable GDP is computed as the sum of the value added in the tradable sectors. The definition tradable sector is standard. The tradable sector includes: industry excluding building and construction and agriculture, forestry and fishery. The real interest rate time series is computed as the difference between annualized interest rate on the 10 government bond and the inflation rate computed from the Euro area’s GDP deflator.

\(^{22}\)One grid point is at the mean, a second point is 1.5 unconditional standard deviation above the mean, and a third point is 1.5 unconditional standard deviation below the mean. I further reduce the 9 states to 7 states by eliminating the states with the stationary probability smaller than 1%. The states that are dropped are the pairs of a positive \( y^T \) (or \( r \)) shock combined with an adverse \( r \) (or \( y^T \)) shock. The most adverse shocks and the most positive shocks are both included in my discrete states. I also approximate the bivariate AR(1) process using Tauchen’s method (86) and it produces similar
replicates the bivariate AR(1) process and the unconditional standard deviations.

5.2 Quantitative Results

The dynamic stochastic model must be solved using a global method because of non-linearities in the decision rules and value functions caused by the inequality constraints—the collateral constraint and the wage rigidity constraint. The constrained-optimal allocations are solved using value function iteration while the laissez-faire allocations are solved using policy function iteration. The value function and the policy functions are defined over a discrete state-space of four state variables \((w_{-1}, d_{-1}, y_T, r)\). The Appendix E provides a detailed description of the solution methods.

5.2.1 Means and Standard Deviations of the Simulated Economies

To see the dynamic impact of the capital flow management, I simulate both a laissez-faire economy and the economy with optimal capital flow management for 1000,000 periods. The laissez-faire economy is simulated using the laissez-faire decision rules. The economy with optimal capital flow management is simulated using the decision rules solved from the constrained planner’s problem. Table 3 reports the means and standard deviations of the key variables in the simulations.

Table 3 shows that the optimal capital flow management significantly reduces the average unemployment rate, increases aggregate consumption, and decreases the volatilities in both unemployment rate and aggregate consumption. The unemployment is almost 2% lower under optimal capital flow management \(\tau\) compared to the level in the laissez-faire economy, and the aggregate consumption is 1% higher. The lower average unemployment rate implies higher average non-tradable consumption, which contributes to the higher aggregate consumption under the optimal \(\tau\).

Table 3 also shows that the optimal capital flow management stabilizes capital flows. The volatility of current account under optimal \(\tau\) is less than half the level in the laissez-faire economy. The lower volatility in current account is consistent with the lower volatility in trade balance and debt. These results are in stark contrast to the ones found by Schmitt-Grohé and Uribe (2013a). In their model, the optimal capital flow management induces higher volatilities in capital flows, tradable balance, and debt compared to the laissez-faire economy. My results differ from theirs because of the inclusion of the collateral constraint in my model. In absence of the collateral constraint,

\footnote{The lower and upper bounds of \(w_{-1}\) and \(d_{-1}\) are set so all the simulated \(w\) and \(d\) are well above the lower bounds and well below the upper bounds. I use 600 grid points for state variable \(d_{-1}\), 60 grid points for state variable \(w_{-1}\), and 7 grid points for the shock process \((y_T, r)\). More grid points are allocated to \(d_{-1}\) than \(w_{-1}\) because the non-linearity in value functions and the policy functions are found to be more severe in \(d_{-1}\).}
Table 3: Means and Standard Deviations

<table>
<thead>
<tr>
<th></th>
<th>Laissez-faire</th>
<th>Optimal $\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Consumption</td>
<td>0.98 (0.04)</td>
<td>0.99 (0.02)</td>
</tr>
<tr>
<td>Unemployment Rate (%)</td>
<td>1.96 (4.62)</td>
<td>0.07 (0.64)</td>
</tr>
<tr>
<td>Tradable Consumption</td>
<td>0.96 (0.07)</td>
<td>0.96 (0.05)</td>
</tr>
<tr>
<td>Real Wage</td>
<td>1.62 (0.11)</td>
<td>1.60 (0.10)</td>
</tr>
<tr>
<td>Current Account</td>
<td>0.00 (0.05)</td>
<td>0.00 (0.02)</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>0.04 (0.05)</td>
<td>0.04 (0.03)</td>
</tr>
<tr>
<td>Outstanding Debt</td>
<td>0.93 (0.05)</td>
<td>0.81 (0.04)</td>
</tr>
<tr>
<td>GDP</td>
<td>3.12 (0.22)</td>
<td>3.13 (0.17)</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>2.15 (0.15)</td>
<td>2.13 (0.13)</td>
</tr>
<tr>
<td>Capital Inflow Tax (%)</td>
<td>0.00 (0.00)</td>
<td>3.63 (2.98)</td>
</tr>
</tbody>
</table>

Note: The standard deviations are reported in the parenthesis. Optimal $\tau$ refers to the economy with optimal capital flow management. Means and standard deviations are computed using the simulated data.

when unemployment rate is high, the planner would encourage a large capital inflow to increase the aggregate consumption and reduce unemployment. However, when households are subject to a collateral constraint, a large inflow increases the probability of a binding collateral constraint in the next period that increases the financial stability risk. In other words, the presence of pecuniary externality restrains the planner’s use of current account to reduce unemployment.

On average, the optimal $\tau$ is an inflow tax rate of 3.63%, and it decreases the average debt by 3% of the GDP. However, $\tau$ is not always positive. $\tau$ is negative—a stimulative inflow subsidy or outflow tax—when the unemployment rate is high. The stimulative inflow subsidy or outflow tax has an average rate of 1.76%, and it happens 12% of the time. Thus, for the vast majority of the time, $\tau$ is an inflow tax.

5.2.2 Correlations with GDP

Table 4 reports correlations between GDP and some key variables in the simulated economies along with observed correlations in the Spanish data. Spain can be characterized as a laissez-faire economy, and the correlations in the laissez-faire model are similar to the observed levels in the Spanish economy. Thus, the model takes into account observed business cycle moments in Spain reasonably well.

Table 4 shows that the laissez-faire economy suffers from pro-cyclical unemployment.

\[24\text{When the collateral constraint binds, } \tau \text{ is zero.}\]
Table 4: Correlation with GDP

<table>
<thead>
<tr>
<th></th>
<th>Model Laissez-faire</th>
<th>Model Optimal $\tau$</th>
<th>Data Spain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Consumption</td>
<td>0.86</td>
<td>0.97</td>
<td>0.92</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>-0.60</td>
<td>-0.21</td>
<td>-0.83</td>
</tr>
<tr>
<td>Trade Balance-GDP</td>
<td>-0.72</td>
<td>-0.34</td>
<td>-0.77</td>
</tr>
<tr>
<td>Current Account-GDP</td>
<td>-0.51</td>
<td>0.22</td>
<td>-0.78</td>
</tr>
</tbody>
</table>

Note: The correlations are computed using the simulated data and the empirical Spanish data.

and pro-cyclical capital flows. The optimal policy could reduce the pro-cyclicality by leaning against the wind. The negative correlations between unemployment and GDP and between the current account (as a percent of GDP) and GDP imply that the unemployment rate would rise and capital inflows would decrease during economic downturns, both of which amplify the boom and bust cycle fluctuations. Under optimal $\tau$, unemployment becomes less pro-cyclical so it falls by a lesser degree during economic downturns, and capital inflows become counter-cyclical so they increase during economic downturns, both of which help to smooth out the boom and bust cycles. These changes are all due to the counter-cyclicity of the optimal $\tau$: $\tau$ increases when GDP increases, and it decreases when $r$ increases. The correlation between $\tau$ and $r$ is $-0.8$ (not reported in the table).

5.2.3 Boom and Bust Cycle

To see how the counter-cyclical capital controls stabilize the economy over time, I plot a typical boom and bust cycle in the laissez-faire economy and compare it to a typical boom and bust cycle under the optimal capital flow management in Figure 6. A typical boom and bust cycle is computed by taking the average over all the boom and bust cycles that satisfy the following criteria: both shocks are at their mean values in period 0; $r$ is 1.5 standard deviations below its mean in period 8 and 1.5 standard deviations above its mean in period 12; $y_T$ is equal to or above its mean in period 8 and equal to or below its mean in period 12. This definition of a boom and bust cycle aims to capture a long period of cheap credit followed by a sudden rise in the cost of borrowing.

Figure 6 shows that the cheap credit leads to an inflow of capital and a rapid build-up of debt in the laissez-faire economy. The capital inflow finances an expansion of tradable consumption, which pushes up the real wage during a boom.\footnote{Despite of the rising wage, the unemployment is positive during boom. This is because the graphs...
Figure 6: Boom and Bust Cycle

Note: The boom and bust cycle is computed by taking the average over all the boom and bust cycles that satisfy the following criteria: both shocks are at their mean values in period 0; $r$ is 1.5 standard deviations below its mean in period 8 and 1.5 standard deviations above its mean in period 12; $y_T$ is equal to or above its mean in period 8 and equal to or below its mean in period 12.

As long as one of the 4000 boom and bust cycles has a boom period with a binding wage rigidity constraint, the average unemployment would be positive despite the wage rigidity constraint being slack on average.

of borrowing increases, capital inflows come to a sudden stop. Households are forced to go through a deleverage process by cutting back consumption. The high real wage that is set during the boom causes the unemployment rate to rise. Moreover, the contraction in aggregate demand exacerbates the unemployment.

are plotted from the averages of 4000 unique boom and bust cycles. As long as one of the 4000 boom and bust cycles has a boom period with a binding wage rigidity constraint, the average unemployment would be positive despite the wage rigidity constraint being slack on average.
Under optimal capital flow management, capital inflows are restricted during booms, so the level of debt does not increase. Tradable consumption still expands but by less than the levels in the laissez-faire economy. As a result, the increase in the real wage is also more moderate. When the cost of borrowing increases, the optimal policy leans against the wind—a stimulative inflow subsidy is introduced to encourage inflows. Unlike the laissez-faire case, the economy does not experience a sudden stop in capital inflows because the debt is relatively low. Therefore, the contraction in consumption is smaller, alleviating unemployment.

Figure 6 also illustrates the trade-off between macroeconomic and financial stability. During the bust, despite the fact that unemployment rate is positive, the planner withdraws the stimulative policy once the outstanding debt is above 28% of GDP, and imposes a prudential tax instead. If the planner had continued with the stimulative policy, the unemployment rate would have been lower, but the debt would have risen to a risky level. More specifically, when the debt gets too high, the collateral constraint would bind if the economy is hit by an adverse shock in the next period, and capital inflows would come to a sudden stop.

5.2.4 Frequency and Severity of Sudden Stops

Table 5: First Moments During Sudden Stops

<table>
<thead>
<tr>
<th></th>
<th>Laissez-faire</th>
<th>Optimal $\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sudden Stops Frequency (%)</td>
<td>5.96</td>
<td>0.07</td>
</tr>
<tr>
<td>Aggregate Consumption</td>
<td>0.86</td>
<td>0.87</td>
</tr>
<tr>
<td>Unemployment Rate (%)</td>
<td>15.15</td>
<td>13.09</td>
</tr>
<tr>
<td>Current Account</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>0.14</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Note: The moments are computed using the simulated data.

Table 5 reports the frequency and severity of sudden stops in the simulated economies. I consider sudden stops because they are episodes of severe financial crises. Following the literature (see e.g. Bianchi, 2012), a sudden stop episode is defined as one with a binding collateral constraint and a current account reversal that is greater than one standard deviation.

Sudden stops are rare events: they happen 6% of the time in the laissez-faire economy, and they almost never happen under the optimal capital flow management. Once a sudden stop happens, the cost of welfare is equally high in both the laissez-faire economy and under the optimal capital flow management: the aggregate consumption decreases by 12%, and unemployment rates increase by 13%. Therefore, most of the
welfare gains from better financial stability under capital flow management comes from the lower frequency of severe financial crises such as sudden stops.

5.2.5 Frequency of Unemployment and Binding Collateral Constraint

Table 6: Joint Probability Distribution

<table>
<thead>
<tr>
<th></th>
<th>Laissez-faire</th>
<th>Optimal $\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Risk of a Binding Collateral Constraint</td>
<td>No Risk of a Binding Collateral Constraint</td>
</tr>
<tr>
<td>Unemployment</td>
<td>24.3%</td>
<td>2.5%</td>
</tr>
<tr>
<td></td>
<td>7.0%</td>
<td>0.5%</td>
</tr>
<tr>
<td>Full Employment</td>
<td>60.7%</td>
<td>2.5%</td>
</tr>
<tr>
<td></td>
<td>8.0%</td>
<td>94.5%</td>
</tr>
</tbody>
</table>

Note: The probabilities are computed using the simulated data. A period is defined to have risk of a binding collateral constraint if the collateral constraint binds when the economy is hit by the worst possible shock in the subsequent period.

Table 6 reports the joint probability distributions of unemployment and the risk of a binding collateral constraint in the simulated economies. I define a period to have the risk of a binding collateral constraint if the collateral constraint binds when the economy is hit by the worst possible shock in the subsequent period. So the first cell 24.3% means that the laissez-faire economy spends 24.3% of the time in states with unemployment in the current period and a positive probability of binding collateral constraint in the next period. Then one can compute the frequency of unemployment by summing up the numbers in the first row, and compute the frequency of having financial stability risk by summing up the numbers in the first column.

Table 6 shows that the optimal capital flow management significantly reduces the frequency of unemployment and the frequency of having a positive probability of binding collateral constraint in the next period. The frequency of unemployment decreases from 31% in the laissez-faire economy to only 3% under the optimal capital flow management. In addition, the frequency of having a positive probability of binding collateral constraint decreases from 85% to only 5%. Moreover, the laissez-faire economy spends only 8% of the time in the best state—with full employment and no financial stability risk—and the frequency increases to 94.5% under the optimal capital flow management.
Table 6 also shows that the trade-off between macroeconomic and financial stability is relevant in the model. The laissez-faire economy spends a quarter of the time in states with both financial stability risk and unemployment. In these states, an inflow tax reduces financial stability risk at the cost of higher unemployment.

5.2.6 Welfare Analysis of Capital Flow Management

The optimal capital flow management requires frequent policy adjustments to shocks. However, frequent policy changes are unrealistic because of political constraints (see Eichengreen and Rose, 2014). Therefore, I consider two simple capital control rules. The first one (simple rule A) is a flat inflow tax of 4%. The second one (simple rule B) is contingent on the unemployment rate: a 4% inflow tax if the unemployment rate is below 2%; and no tax otherwise. The 4% inflow tax rate is the average tax rate imposed under the optimal capital flow management when \( \tau > 0 \). Simple rule B differs from A by taking both financial and macroeconomic stability into account.

To quantify the welfare gains of capital flow management, I follow the literature by computing the percentage increases in the permanent consumption when capital controls are introduced in the laissez-faire economy. The permanent consumption is a function of the state variables, and it is defined as:

\[
C_{Perm}^i(d_{-1}, w_{-1}, y_T, r) = [(1 - \beta)(1 - \sigma)V^i(d_{-1}, w_{-1}, y_T, r) + 1]^{\frac{1}{1-\sigma}}
\]

where \( i \) denotes the regime type, which can be laissez-faire (LF), optimal capital flow management, simple rule A, or simple rule B. \( V^i(d_{-1}, w_{-1}, y_T, r) \) is the household’s value function under regime \( i \). Thus, the percentage increases in the permanent consumption when the capital control rule \( \tau_i \) introduced in the laissez-faire economy is

\[
\Delta C_{Perm}^{\tau_i}(d_{-1}, w_{-1}, y_T, r) = \frac{C_{Perm}^{\tau_i}(d_{-1}, w_{-1}, y_T, r)}{C_{ Perm}^{LF}(d_{-1}, w_{-1}, y_T, r)} - 1 \tag{39}
\]

Table 7 reports the minimum, maximum, and weighted averages of \( \Delta C_{Perm}^{\tau_i}(d_{-1}, w_{-1}, y_T, r) \) at the non-zero-measure initial states of the laissez-faire economy’s ergodic distribution for different capital control rules. The weighted average is computed by weighing the permanent consumption increase at each state by its ergodic density.

Table 6 shows that capital flow management leads to significant welfare improvement. The optimal (state-contingent) capital controls increase the permanent consumption by close to 1% on average. The simple rules are also effective, increasing permanent consumption by 0.5% on average.

Table 6 also shows the importance of taking both macroeconomic and financial stability into account. Simple rule B outperforms simple rule A. Moreover, simple rule A
Table 7: Welfare Gains of Capital Flow Management

<table>
<thead>
<tr>
<th></th>
<th>Optimal $\tau$</th>
<th>Simple Rule A</th>
<th>Simple Rule B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighted Average</td>
<td>0.91</td>
<td>0.43</td>
<td>0.53</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.66</td>
<td>-0.43</td>
<td>0.13</td>
</tr>
<tr>
<td>Maximum</td>
<td>1.31</td>
<td>0.73</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Note: All numbers are in percentage of permanent consumption increase and they are computed from the simulated data. Welfare improvements are computed using (39).

decreases in permanent consumption in the worst scenario—the minimum welfare gain is negative. This is because simple rule A only takes financial stability into account: the prudential inflow tax increases unemployment during economic crises, and the loss from higher unemployment is greater than the gains from lower financial stability risk, so the welfare is lower compared to the laissez-faire economy. On the contrary, simple rule B always increases welfare because it takes both macroeconomic and financial stability into account.

5.3 Sensitivity Analysis

This subsection looks at how sensitive the numerical results in the last subsection are to alternative parameter calibrations. In particular, I focus on three key parameters that affect the aggregate demand externality and the pecuniary externality: $\gamma$, $\kappa$, and $\xi$. The size of the aggregate demand externality is increasing in $\gamma$ and decreasing in $\xi$, and the pecuniary externality is increasing in $\kappa$ and decreasing in $\xi$. Though $\kappa$ increases the pecuniary externality when the collateral constraint binds, it also makes the collateral constraint less likely to bind. Thus, the overall effect of $\kappa$ on the welfare could be ambiguous.

Table 8 presents the numerical results for alternative parameter calibrations. As the baseline numerical model is calibrated to the empirical lower bound of $\gamma$ and the empirical upper bound of $\xi$, I consider a higher $\gamma$ of 0.98 and a lower $\xi$ of 0.70 as alternative calibrations. In addition, I consider a higher $\kappa$ of 0.34 so all three parameter changes increase the size of the externalities. To isolate the impact of each parameter, I only change one parameter at a time and keep the other parameters the same as in Table 2.

Table 8 shows the average welfare improvement of the optimal capital flow management becomes is under each alternative calibration compared to the baseline results, indicating larger externalities. More specifically, the higher $\gamma$ and the lower $\xi$ increase the aggregate demand externality, so the unemployment rate becomes higher and more volatile in the laissez-faire economy compared to the baseline results. In addition, the
<table>
<thead>
<tr>
<th>Sudden Stops</th>
<th>Welfare Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment Rate (%)</td>
<td>Weighted Average</td>
</tr>
<tr>
<td>Average Aggregate Consumption (%)</td>
<td>Minimum</td>
</tr>
<tr>
<td>Average Current Account Balance (%)</td>
<td>Maximum</td>
</tr>
</tbody>
</table>

Note: The standard deviations are reported in the parenthesis. Welfare improvements are in percentage of permanent consumption increase, and defined in (23).

### Table 8: Numerical Results: Alternative Parameter Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\gamma = 0.98$</th>
<th>$\xi = 0.34$</th>
<th>Laissez-Faire Optimal $\tau$</th>
<th>Laissez-Faire Optimal $\tau$</th>
<th>Laissez-Faire Optimal $\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamma</td>
<td>0.98</td>
<td>0.34</td>
<td>0.98</td>
<td>0.34</td>
<td>0.98</td>
</tr>
<tr>
<td>Sudden Stop Frequency</td>
<td>5.6%</td>
<td>0.1%</td>
<td>5.3%</td>
<td>0.1%</td>
<td>5.1%</td>
</tr>
<tr>
<td>Avg. Aggregate Consumption (%)</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>Avg. Unemployment Rate (%)</td>
<td>17.77</td>
<td>13.91</td>
<td>16.09</td>
<td>14.71</td>
<td>16.68</td>
</tr>
<tr>
<td>Avg. Current Account Balance (%)</td>
<td>0.11</td>
<td>0.10</td>
<td>0.12</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>Avg. Trade Balance (%)</td>
<td>0.14</td>
<td>0.14</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
</tbody>
</table>

### Means and Standard Deviations

<table>
<thead>
<tr>
<th>Means and Standard Deviations</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment Rate (%)</td>
<td>3.24 (5.39)</td>
<td>0.13 (0.91)</td>
</tr>
<tr>
<td>Aggregate Consumption</td>
<td>0.99 (0.05)</td>
<td>0.98 (0.09)</td>
</tr>
<tr>
<td>Outstanding Debt</td>
<td>0.98 (0.05)</td>
<td>0.98 (0.05)</td>
</tr>
<tr>
<td>Current Account</td>
<td>0.00 (0.04)</td>
<td>0.00 (0.04)</td>
</tr>
<tr>
<td>Capital Inflow Tax (%)</td>
<td>3.69 (3.24)</td>
<td>3.62 (3.02)</td>
</tr>
</tbody>
</table>

Note: The standard deviations are reported in the parenthesis. Welfare improvements are in percentage of permanent consumption increase, and defined in (23).
higher $\kappa$ and lower $\xi$ increase the pecuniary externality, so the consumption contraction and unemployment rise in the laissez-faire economy during sudden stops are 1% higher than those in the baseline results.

Moreover, Table 8 shows the average optimal capital inflow tax is higher and it requires larger adjustments to shocks (higher volatility) when $\gamma$ is higher and when $\xi$ is lower.

6 Conclusion

This paper conducts a normative analysis of optimal capital flow management in a small open economy with both financial and nominal frictions. The laissez-faire economy suffers from frequent financial crises and high unemployment because of the aggregate demand and pecuniary externalities. I show that the size of the aggregate demand externality is increasing in the wage rigidity parameter ($\gamma$), and decreasing in the intra-temporal elasticity of substitution between tradable and non-tradable goods ($\xi$). In addition, I show that the pecuniary externality is increasing in the share of income that can be used as collateral ($\kappa$), and decreasing in the intra-temporal elasticity of substitution between tradable and non-tradable goods ($\xi$).

I provide an explicit solution to the optimal capital flow management problem in the form of a tax on international capital inflows. I show that the optimal policy can be broken down into three components: one corrects the aggregate demand externality, the second corrects the pecuniary externality, and the third accounts for the interaction of both externalities. The first component is counter-cyclical: it is a prudential tax when the economy has full employment; it is a stimulative subsidy when the economy has unemployment. The second and third components combined represent a prudential tax when the probability of a binding collateral constraint is positive.

By calibrating the model to the Spanish economy, I show that capital flow management could lead to significant welfare improvements in the decentralized economy by reducing both the frequency of financial crises as well as the level and the volatility of unemployment. In addition, I show that simple inflow tax rules that take into account both financial and macroeconomic stability could lead to significant welfare improvements in the decentralized economy.

Although this paper focuses on the optimal capital flow management, the results have implications that are applicable to the design of other financial regulations. The bulk of the current discussion on financial regulations has focused on their roles in reducing financial stability risk. However, financial regulations could lead to unintended macroeconomic instability if they only focus on financial stability. I argue that countries could use independent and effective monetary policies to correct for these unintended
macroeconomic consequences. However, if the countries do not have independent monetary policies at their disposal (e.g. the euro area member states) or if they are in a liquidity trap, then regulators must take into account the impact of financial regulation on macroeconomics stability. Therefore, the analyses on the optimal capital flow management with dual objectives presented in this paper could be relevant and useful for designing other types of financial regulations.
References


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Appendix

Appendix A. Additional Tables and Figures

Table 9: Brazil Capital Controls: IOF Tax Since 2009

<table>
<thead>
<tr>
<th>Date</th>
<th>Capital Controls</th>
</tr>
</thead>
<tbody>
<tr>
<td>19-Oct-09</td>
<td>2% IOF entry tax on portfolio inflows (equities &amp; fixed income)</td>
</tr>
<tr>
<td>4-Oct-10</td>
<td>4% IOF entry tax on fixed income inflows</td>
</tr>
<tr>
<td>18-Oct-10</td>
<td>6% IOF entry tax on fixed income inflows</td>
</tr>
<tr>
<td>29-Mar-11</td>
<td>6% IOF entry tax on foreign loans with maturity below 1 year</td>
</tr>
<tr>
<td>6-Apr-11</td>
<td>6% IOF entry tax on foreign loans with maturity below 2 years</td>
</tr>
<tr>
<td>29-Feb-12</td>
<td>6% IOF entry tax on foreign loans with maturity below 3 years</td>
</tr>
<tr>
<td>9-Mar-12</td>
<td>6% IOF entry tax on foreign loans with maturity below 5 years</td>
</tr>
<tr>
<td>14-Jun-12</td>
<td>Tax on foreign loans with maturity above 2 years drops to 0%.</td>
</tr>
<tr>
<td>05-Jun-13</td>
<td>Tax on foreign loans with maturity above 1 year drops to 0%.</td>
</tr>
<tr>
<td>05-Jun-14</td>
<td>Tax on foreign loans with maturity above 180 days drops to 0%.</td>
</tr>
</tbody>
</table>

Source: Financial Times, Haver

Table 10: Data Source

<table>
<thead>
<tr>
<th>Data</th>
<th>Source</th>
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</thead>
<tbody>
<tr>
<td>Capital flows</td>
<td>International Financial Statistics (IFS)</td>
</tr>
<tr>
<td>Net International Investment Position</td>
<td>IFS</td>
</tr>
<tr>
<td>GDP by Sector</td>
<td>Annual Macro-Economic Database (AMECO)</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>IFS</td>
</tr>
<tr>
<td>GDP Deflator</td>
<td>AMECO</td>
</tr>
<tr>
<td>Current Account</td>
<td>IFS</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>World Economic Outlook</td>
</tr>
<tr>
<td>Household Debt</td>
<td>Financial Soundness Indicators</td>
</tr>
<tr>
<td>Government Debt</td>
<td>World Development Indicators</td>
</tr>
</tbody>
</table>
**Figure 7:** Gross Capital Inflows (% GDP) to Countries by Region

Note: Data from IMF’s International Financial Statistics.

**Figure 8:** Spain’s Interest Rate and NIIP

Note: Spread is the difference between Spain’s interest rate and Germany’s interest rate. The left vertical axis provides scales for the real interest rate and the interest rate spread (%). The right vertical axis provides scale for the NIIPs as percentage of GDP (%).
Figure 9: Spain’s Debt by Household and Government as Percentage of GDP

Note: See Table 10 for data source.
Appendix B. Derivations

Constrained Planner’s Problem

\[ L = u(c_T, f(h)) + \beta \mathbb{E} V^CP (w(c_T, h), d, y_T, r') + \lambda_h (1 - h) + \lambda^{CP} (y_T + \frac{d}{1+r} - c_T - d_{-1}) + \lambda^{CP} \Psi(c_T, h) - d] + \lambda_w [w(c_T, h) - \gamma w_{-1}] \]

FOCs:

\[ u_T + \beta w_T \mathbb{E} V^{CP}_w - \lambda^{CP} + \lambda^{CP} \Psi_T + \lambda_w w_T = 0 \]
\[ u_N f'(h) + \beta w_h \mathbb{E} V^{CP}_h - \lambda_h + \lambda^{CP} \Psi_h + \lambda_w w_h = 0 \]
\[ \beta \mathbb{E} V^{CP}_d + \frac{\lambda^{CP}}{1+r} - \lambda^{CP}_{CC} = 0 \]

Envelope theorem gives

\[ V^{CP}_w = -\gamma \lambda_w \]
\[ V^{CP}_d = -\lambda^{CP} \]

Complementary slackness requires

\[ \mathbb{1}_{\lambda_h > 0} + \mathbb{1}_{\lambda_w > 0} = 1 \]

Substitute the first envelope condition into the first FOC:

\[ u_T - \gamma \beta w_T \mathbb{E} \lambda'_w - \lambda^{CP} + \lambda^{CP} \Psi_T + \lambda_w w_T = 0 \] (40)

Substitute the first envelope condition into the second FOC,

\[ u_N f'(h) - \gamma \beta w_h \mathbb{E} \lambda'_w - \lambda_h + \lambda^{CP} \Psi_h + \lambda_w w_h = 0 \] (41)

When \( \lambda_w > 0, \lambda_h = 0 \), then (41) can be rewritten as

\[ -\beta \mathbb{E} \gamma \lambda'_w w_T + \lambda_w w_T = (u_N - \lambda^{CP} \Psi_h) \left( -\frac{w_T}{w_h} \right) \]

Substitute it into (41):

\[ \lambda^{CP} = u_T + \lambda^{CP} \Psi_T + \left( u_N - \lambda^{CP} \Psi_h \right) \left( -\frac{w_T}{w_h} \right), \text{ when } \lambda_w > 0 \] (42)

When \( \lambda_w = 0 \), then (41) can be rewritten as

\[ \lambda^{CP} = u_T - \gamma \beta w_T \mathbb{E} \lambda'_w + \lambda^{CP} \Psi_T, \text{ when } \lambda_w = 0 \] (43)
Notice

\[
\frac{dh}{dc_T} = \begin{cases} \frac{w_T}{w_h} & \text{when } \lambda_w > 0 \\ 0 & \text{when } \lambda_w = 0 \end{cases}
\]

and

\[
\Phi_T = \begin{cases} uf(h) \frac{w_h}{w_T} & \text{when } \lambda_w > 0 \\ -\beta \gamma w_T \mathbb{E}[\lambda_w'] & \text{when } \lambda_w = 0 \end{cases}
\]

so (42) and (43) can be combined and rewritten as

\[
\lambda^{CP} = u_T + \Phi_T + \lambda^{CP}_C \left[ \Psi_T + \Psi_h \frac{dh}{dc_T} \right]
\] (44)

Substitute the second envelope condition into the third FOC,

\[
\lambda^{CP} = (1 + r)\beta \mathbb{E}[\lambda^{CP'}] + (1 + r)\lambda^{CP}_C
\] (45)

Substituting (44) into (45) gives the constrained-optimal Euler equation:

\[
u_T + \Phi_T + \lambda^{CP}_C \left( \Psi_T + \Psi_h \frac{dh}{dc_T} \right) = (1 + r)\beta \mathbb{E} \left[ u_T' + \Phi_T' + \lambda^{CP'}_C \left( \Psi_T' + \Psi_h' \frac{dh'}{dc_T'} \right) \right] + (1 + r)\lambda^{CP}_C
\] (46)
Appendix C. Proofs

Optimal Inflow Tax

**Proof** Since the laissez-faire equilibrium is identical to the constrained-optimal allocations, so I just need to consider the cost when the present collateral constraint is slack. Then household’s Euler equation in the decentralized economy is

\[ u_T = (1 + r)(1 + \tau)\beta \mathbb{E} u'_T \]

Substitute (33) into the household’s Euler equation in the decentralized economy,

\[ u_T + \Phi_T = (1 + r)\beta \mathbb{E} \left[ u'_T + \Phi'_T + \lambda'_C \left( \Psi'_T + \Psi'_h \frac{d h'_T}{d c'_T} \right) \right] \tag{47} \]

which is identical to the constrained planner’s Euler equation. And since the rest of the equilibrium conditions in the decentralized economy are identical to the constrained planner’s problem, then the decentralized equilibrium allocations are identical to the constrained-optimal allocations. \qed
Appendix D. Existence and Uniqueness of Partial Equilibrium

**Lemma 1 (Existence and Uniqueness of the Financially-Unconstrained Equilibrium)**

$EE$ and $WR$ always have an unique intersection.

**Proof** Since $EE$ is downward sloping, and $WR$ is upward sloping or perfectly vertical.

**Lemma 2 (Existence and Uniqueness of the Financially-Constrained Equilibrium)**

When $EE$ and $WR$ intersect inside the deleveraging area, $WR$ and $CC$ always have an unique intersection lying below $EE$.

**Proof** Straightforward by discussing all four cases.

---

**Figure 10: Multiple Equilibria**

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Lemma 1 guarantees the uniqueness of the equilibrium when the collateral constraint is slack, and Lemma 2 guarantees the uniqueness of the equilibrium when the financially-unconstrained equilibrium does not exist. Therefore Lemmas 1 and 2 together guarantee the existence of an equilibrium. However, the two lemmas do not guarantee the uniqueness of the equilibrium as the collateral constraint could be binding at one equilibrium while slack at the other. For example in Figure 10, points $A$, $B$, and $C$ all are equilibria of the model. The equilibrium is financially-unconstrained at point $A$, while it is financially constrained at $B$ and $D$. The economy is at full employment at $A$ and $B$, while there is unemployment at $C$.

The model features multiple equilibria because of the collateral constraint, a feature of the rational expectations and the partial equilibrium. Imagine international

---

\[^{26}\text{Without the wage rigidity, the multiple equilibria are at points } A, B, \text{ and } D \text{ where the full employment line intersects with } EE \text{ and } CC.\]
investors could form different belief of the country’s ability to borrow. At A, the international investors believe the country has ability to repay all the debt (high collateral), therefore the country is financially unconstrained and the consumption and output is high, and the collateral (GDP) ends up being high which confirms the initial positive belief. At C, the international investors believe the country has trouble paying back the debt, therefore the country is borrowing constrained, which lowers both consumption and output and hence the collateral ends up being low which confirms the investor’s initial negative belief.

A and B are not saddle-path stable. The reason is that the future consumption is endogenous and determined by the debt accumulated in the present period. Suppose the EE in Figure 1 is associated with the debt level at C. If the economy consumes at A or B, the debt goes up and future consumption goes down, therefore EE would shift down given τ. And when EE associated with the debt levels at A and B intersect with WR inside the deleverage area, A and B are not dynamically rational expectations. Therefore, I impose the following stringent selection criteria:

**Definition 6 (Equilibrium Selection Criteria)** When there are multiple equilibria, let the unique equilibrium be the one associated with the lowest debt level.

Once I imposed the selection criteria, I have the following definition:

**Definition 7 (Determination of the Partial Equilibrium)** Given \( \{r, \tau, \epsilon, d_1, w_{-1}, c_T', h'\} \), the equilibrium is determined by the lowest intersection of WR and CC that lies below EE. If such point does not exist, then the equilibrium is determined by the intersection of EE and WR.

It then follows from Lemmas 1 and 2 that the equilibrium defined in Definition 7 exists and is unique.
Appendix E. Numerical Solution Method

Constrained Optimal Allocations

I solve the constrained planner’s problem by value function iteration. The value function is defined by (21). The difficult part is to find the feasible set. The feasible set is determined by the state variables \((d_{-1}, w_{-1}, y_T, r)\). Notice for given state variables, I can plot \(WR\) and \(CC\) as in Figure 11. The constrained planner could choose from any point along the bolded line, which is the segment of \(WR\) that is below point \(B\). \(B\) is the lowest intersection of \(CC\) and \(WR\). The allocations at \(B\) are solved from (35) and (36). Therefore, the feasible allocations are determined. The rest is standard.

To summarize: for given state variables \((d_{-1}, w_{-1}, y_T, r)\) and initial guess of value function \(V^0(d_{-1}, w_{-1}, y_T, r)\), solve (21) by choosing allocations from the feasible set—the allocations on \(WR\) that is below \(B\) and using \(V^0(d_{-1}, w_{-1}, y_T, r)\) for computing the expectation of the next period’s value function. A new value function is solved this way. Repeat the process until the value function converges.

**Figure 11**: WR & CC given \((d_{-1}, w_{-1}, y_T, r)\)

Note: The feasible allocations are indicated by the bold lines.
Laissez-faire Equilibrium Allocations

The more challenging problem is to solve for the laissez-faire allocations. I will use policy function iteration with the help of the graphical framework for solving this problem.

Given initial guess of the decision rules $c_0^T(d_{-1}, w_{-1}, y_T, r)$ and $h_0^0(d_{-1}, w_{-1}, y_T, r)$, for any given $(d_{-1}, w_{-1}, y_T, r)$ I redefine $EE$ curve as

$$(c_T)^{-1/\xi} [c(c_T, h^0)]^{-\sigma+1/\xi} = \beta(1 + r)(1 + \tau)E \left[ (c_T^0)^{-1/\xi} [c(c_T^0, f(h^0))]^{-\sigma+1/\xi} \right].$$

(48)

where

$$c_T^0 = c_T^0((1 + r)(y_T - d_{-1} - c_T), w(c_T, h), y_T^I, r')$$

$$h_T^0 = h_T^0((1 + r)(y_T - d_{-1} - c_T), w(c_T, h), y_T^I, r')$$

**Step 0** Use the policy functions of the constrained planner’s problem as the initial guess of policy functions, $c_0^T(d_{-1}, w_{-1}, y_T, r)$ and $h_0^0(d_{-1}, w_{-1}, y_T, r)$.

**Step 1** Solve for the intersection of $EE$ and $WR$, point $A$ from (45) and (48), denote the solution $c_A^T(d_{-1}, w_{-1}, y_T, r)$ and $h_A^0(d_{-1}, w_{-1}, y_T, r)$.

**Step 2** If point $A$ satisfies the collateral constraint (23), then it is the equilibrium and the new policy functions are

$$c^1_T(d_{-1}, w_{-1}, y_T, r) = c_A^T(d_{-1}, w_{-1}, y_T, r)$$

and

$$h^1(d_{-1}, w_{-1}, y_T, r) = h_A^0(d_{-1}, w_{-1}, y_T, r).$$

**Step 3** If point $A$ does not satisfy the collateral constraint, solve for the intersection of $WR$ and $CC$, point $B$ from (53) and (56), denote the solution $c_B^T(d_{-1}, w_{-1}, y_T, r)$ and $h_B^0(d_{-1}, w_{-1}, y_T, r)$. and the new policy functions are

$$c^1_T(d_{-1}, w_{-1}, y_T, r) = c_B^T(d_{-1}, w_{-1}, y_T, r)$$

and

$$h^1(d_{-1}, w_{-1}, y_T, r) = h_B^0(d_{-1}, w_{-1}, y_T, r).$$

Iterate the policy functions $c_T(d_{-1}, w_{-1}, y_T, r)$ and $h(d_{-1}, w_{-1}, y_T, r)$ by repeating steps 1-3 until the policy functions converge.