Cost-Benefit Analysis of Leaning Against the Wind: Are Costs Always Larger Than Benefits, and Even More So with a Less Effective Macroprudential Policy?*

Lars E.O. Svensson
Stockholm School of Economics, IMF, CEPR, and NBER

First draft: June 2015
This draft: September 1, 2015

Abstract

“Leaning against the wind” (of asset prices and credit booms) (LAW), that is, a somewhat tighter monetary policy and a higher policy interest rate, has costs in terms of a weaker economy with higher unemployment and lower inflation. It has been justified by possible benefits in terms of a lower probability and severity of a future financial crisis. A worse macro outcome in the near future is then considered to be an acceptable tradeoff for a better expected macro outcome further into the future. But a crisis can come any time, and the cost of a crisis is higher if initially the economy is weaker due to previous leaning against the wind. LAW thus has an additional cost in the form of a higher cost of a crisis when a crisis occurs. With this additional cost, for existing empirical estimates, the costs of LAW exceed with a large margin the possible benefits from a lower probability of a crisis. Furthermore, empirically a lower probability of a crisis is associated with lower real debt growth. But if monetary policy is neutral in the long run, it cannot affect real debt in the long run. Then, if a higher policy rate would result in lower debt growth and a lower probability of a crisis for a few years, this is followed by higher debt growth and a higher probability of a crisis in the future. This implies that the accumulated benefits over time of LAW are close to zero. But even if monetary policy is assumed to be non-neutral and permanently affect real debt, empirically the benefits are still less than the costs. Finally, perhaps somewhat surprisingly, less effective macroprudential policy, with resulting higher probability, severity, or duration of a crisis, can be shown to increase the costs of LAW more than the benefits, thus further strengthening the strong case against LAW.

JEL Codes: E52, E58, G21

*I thank Vivek Arora, Helge Berger, Olivier Blanchard, Lael Brainard, Giovanni Dell’Ariccia, Stanley Fischer, Dong He, Olivier Jeanne, Michael Kiley, Stefan Láséen, David López-Salido, Tommaso Mancini Griffoli, William Nelson, Bengt Petersson, Rafael Portillo, Damiano Sandri, David Vestin, José Viñals, and participants in seminars at the Federal Reserve Board and Bank of Canada for helpful discussions and comments. I also thank Nakul Kapoor for research and editorial assistance. The views expressed in this paper are those of the author and do not necessarily represent those of the IMF or IMF policy.
1 Introduction

By “leaning against the wind” (of asset prices and credit booms) I here mean a monetary policy with a somewhat higher policy interest rate than what is justified by just stabilizing inflation around an inflation target and unemployment around its estimated long-run sustainable rate. Leaning against the wind has obvious costs in terms of a weaker economy with higher unemployment and lower inflation. It has been justified as a way of reducing the probability and severity of a future financial crisis (Bank for International Settlements (2014), Olsen (2015), Sveriges Riksbank (2013)). A somewhat worse macro outcome in the near future is then considered to be an acceptable tradeoff for a better expected macro outcome further into the future. But a crisis can come any time, and the cost of a crisis is higher if initially the economy is weaker. If the unemployment rate is higher when a crisis occurs, the unemployment rate during the crisis will be higher, which increases the cost of a crisis. Leaning against the wind thus not only has cost in terms of a weaker economy if no crisis occurs; it has an additional cost in terms of a higher cost of a crisis if a crisis occurs. With this additional cost of leaning against the wind, for existing empirical estimates, the cost of leaning against the wind can be shown to exceed, with a substantial margin, the benefit from a lower probability of a crisis.

Furthermore, empirically the channel through which a higher policy rate might reduce the probability of a crisis is through lower real debt growth. According to existing empirical estimates, the probability of a crisis depends on the growth rate of real debt during the previous few years (Schularick and Taylor (2012)). If a higher policy rate reduces real debt growth, it might therefore reduce the probability of a crisis. However, there are three important limitation of this channel.

First, if monetary policy is neutral in the long run, it cannot affect real debt in the long run. Therefore, even if a higher policy rate would reduce real debt growth and thereby the probability of a crisis for a few years, if there is no permanent effect on the real debt level, a lower debt growth and probability of a crisis will be followed by a higher debt growth and probability, and the average and accumulated debt growth and probability would not be affected over a longer period. The probability of a crisis would just be shifted between different periods.

Second, as discussed in Svensson (2013a), the effect on real debt of a higher policy rate is likely to be small and could be of either sign. The stock of nominal debt has considerable inertia. Higher interest rate may reduce housing prices and at given loan-to-value ratios reduce the growth of new mortgages. But only a fraction of the stock of mortgages is turned over each year. Furthermore,
even if a higher policy rate slows down the rate of growth of nominal mortgages, it also slows down
the rate of growth of the price level. Thus, both the numerator and the denominator of real debt
is affected in the same direction by the policy rate, making the effect on the ratio smaller. And
if the price level is affected more or quicker than the stock of debt, real debt will rise rather than
fall. Indeed, the “stock” effect may dominate over the “flow” effect for several years. The effect
on the debt-to-GDP ratio of a higher policy rate is even more likely to be small or of the opposite
sign, because then not only the price level but also real GDP enter in the denominator, and the
growth of both are slowed down by a higher policy rate. Several recent papers have indeed found
empirical evidence supporting that a higher policy rate increases the debt-to-GDP ratio (Alpanda
and Zubairy (2014), Gelain, Lansing, and Natvik (2015), and Robstad (2014)).

Third, the empirical relation between previous real debt growth and the probability of a crisis
is of course a reduced-form and correlation result. The underlying determinants of the probability
of a financial crisis are the nature and magnitude of the shocks to the financial system and the
resilience of the system. The latter depends on such things as the strength of balance sheets of
borrowers and lenders, the quality of assets, the amount of loss-absorbing capital, the quality of
lending standards, the degree of liquidity and of maturity transformation, the amount of risk-taking
and speculation, and so on. The former depend on, among other things, possible overvaluation and
riskiness of assets. The extent to which higher real debt growth increases the probability of a crisis
depends on to what extent it is “bad” credit growth that is related to things such as lower lending
standards, higher loan-to-value ratios, speculation, overvaluation of assets, and so on, rather than
“good” credit growth related to financial deepening and developments that does not weaken the
financial system. With better data on the underlying determinants of the nature and magnitude
of shocks and the resilience of the system, it should be possible to assess the probability of a crisis
without relying on aggregate real debt growth. Given the list of underlying determinants of the
probability of a crisis, it is also rather clear that the policy rate is unlikely to have any systematic
impact on most or any of them, and that micro- and macroprudential policy is much more likely
to have such an impact.

In this paper, I will take into account the first limitation, the implication of long-run neutrality
of monetary policy, but I will also consider the result of non-neutrality and possible permanent
effects on real debt of monetary policy. As for the second and third limitations, I will simply take
existing empirical estimates as given and see what follows from them, thus arguably stacking the
cards somewhat in favor of leaning against the wind.1

The existing small literature that has tried to quantify the costs and benefits of leaning against the wind has mainly considered a two-period setup where a higher policy rate has a cost in terms of higher unemployment in the first period and a benefit in terms of a lower probability of a crisis in the second period (Kocherlakota (2014), Svensson (2014), Svensson (2015), and Ajello, Laubach, Lopez-Salido, and Nakata (2015)).2 By assumption there is no possibility of a crisis in the first period, and by assumption a crisis in the second period would start from an initial situation when unemployment equals its long-run sustainable rate.

The two-period framework is an over-simplification. By disregarding the possibility of a crisis in the first period and by assuming that a crisis in the second period occurs when the unemployment rate initially equals its long-run sustainable rate, it disregards that a crisis could come any time and that leaning against the wind increases the cost of a crisis by causing it to start from a higher unemployment rate. Thus it understates the cost of leaning against the wind. Furthermore, by assuming that there is only one period for which the probability of a crisis can be affected, it disregards the consequences of the long-run neutrality of monetary policy and the resulting property that then the probability of a crisis is shifted between periods but the sum of the probabilities remains the same. Thus it overstates the benefit of leaning against the wind.

Given these simplifications, Svensson (2014) and Svensson (2015) nevertheless show that, given empirical estimates and reasonable assumptions, the benefit of leaning against the wind in terms of an expected lower future unemployment rate due to a lower probability of a crisis is tiny compared to the cost of a higher unemployment rate the next few years because of a higher policy rate. Ajello, Laubach, Lopez-Salido, and Nakata (2015) shows that a tiny amount of leaning against the wind may be justified, corresponding to a few basis points increase in the policy rate, but that extreme assumptions are needed to justify more significant leaning against the wind.3

---

1 Another possible benefit of a higher policy rate might be a smaller increase in the unemployment rate in a crisis. According to the empirical results of Flodén (2014), for OECD countries, a higher household debt-to-income rate before the recent financial crisis is associated with a somewhat lower increase in unemployment during the crisis. If a higher policy rate reduces the debt-to-income or debt-to-GDP ratios, a higher policy rate might this way reduce the cost of the crisis. However, according to Flodén (2014), the impact of the initial debt-to-income rate on the crisis increase in the unemployment rate is very small (and not significant for the OECD countries for which housing prices fell during the crisis). Furthermore, as noted, the effect of the policy rate on the debt-to-income ratio is apparently quite small, often not statistically significant from zero, and, according to both theoretical and empirical analysis, a higher policy rate probably increases rather than decreases the debt-to-GDP ratio. This means that there is hardly theoretical or empirical support for the idea that this channel would provide any benefit from leaning against the wind. Nevertheless, the empirical importance of this possible channel is examined in appendix D.

2 Leaning against the wind has been discussed in more general terms in, for instance, Evans (2014), Laseén, Pescatori, and Turunen (2015), Smet (2013), Stein (2014), Svensson (2013b), Williams (2015), Woodford (2012), and Yellen (2014).

3 The early and innovative contribution of Kocherlakota (2014), expressing the value of reducing the probability of a crisis to zero in terms of an unemployment-gap equivalent, is discussed in appendix E.
An exception to this two-period framework is the dynamic approach and analysis of Diaz Kalan, Laséen, Vestin, and Zdzienicka (2015) in a quarterly model, where the probability of a crisis varies over quarters and the cost and benefit of leaning against the wind are accumulated over time. The present paper follows that approach and uses a multi-period quarterly model.

The preliminary results of Diaz Kalan, Laséen, Vestin, and Zdzienicka (2015) indicate that the cost dominates over the benefit during the first few years but that the cost is about equal to benefit over a longer period. However, as far as I can see in the preliminary version of the paper, the cost of leaning against the wind is underestimated because of the assumption that a crisis has a fixed cost, thereby disregarding that the cost of a crisis depends on the initial state of the economy, which in turn depends on the amount of leaning against the wind. It is as if a crisis is assumed to result in a 5 percent unemployment gap regardless of whether the initial unemployment gap is zero or 3 percent. Furthermore, it is not clear to me whether or not long-run neutrality of monetary policy is taken into account and therefore not clear whether the paper overstates the benefit or not.

The new elements in the present paper are (i) to take into account that the cost of a crisis depends in the initial state of the economy, which in turn depends on the amount of leaning against the wind that has preceded the crisis, (ii) to derive the effect on the policy rate on the probability of a crisis, taking into account that this probability depends both on the probability of a crisis start and the duration of a crisis, (iii) to derive the expected marginal cost and marginal benefit of leaning against the wind, in order to assess whether leaning against or with the wind is justified, (iv) to take into account and assess the role of monetary neutrality, (v) to assess whether more or less effective macroprudential policy affects the case for leaning against the wind, in the context of examining how a higher probability and/or severity of a crisis affects the marginal cost and benefit of leaning against the wind.

Section 2 examines the effect of leaning against the wind on the expected future unemployment rate, taking the possibility of a crisis into account. This is a generalization of the previous two-period analysis in Svensson (2014) and Svensson (2015). Section 3 examines the effect of leaning against the wind on expected future quadratic losses and derives the corresponding marginal cost and benefit of leaning against the wind, to assess whether the optimal policy is to lean against or with the wind. The sensitivity of the results to the initial state of the economy and to the magnitude of the policy-rate effect on the expected non-crisis unemployment rate is also reported. Section 4 examines the frequently made argument that leaning against the wind is justified if there is a less effective macroprudential policy. A less effective macroprudential policy is assumed to increase the
probability of a crisis, the severity of a crisis, or the duration of a crisis. This section thus examines whether such changes makes leaning against the wind more or less costly. This way this section also provides some sensitivity analysis of my results. Section 5 provides additional sensitivity analysis by examining whether monetary non-neutrality with a permanent effect on real debt changes the results. Sections 2-5 uses estimates from Schularick and Taylor (2012) of the effect of real debt growth on the probability of crisis with data for 14 countries for 1870–2008. Section 6 shows that recent IMF staff estimates with data for 35 advanced countries for 1970-2012 give similar results. Section 7 summarizes the conclusions. Appendices A-H provide further details and extensions.

2 The effect of leaning against the wind on expected future unemployment

This section examines the effect of leaning against the wind, that is, a somewhat higher policy rate, on the expected future unemployment rate in an economy, taking the possibility of a crisis into account. This is in line with the approach in Svensson (2014) and Svensson (2015), but extends it from a two-period framework to a multi-period quarterly framework.

Let $u_t$ denote the unemployment rate in quarter $t$. Assume that, in each quarter $t$, there are two possible states in the economy, non-crisis and crisis. In a crisis, the unemployment rate is higher by a fixed magnitude, the crisis increase in the unemployment rate, $\Delta u > 0$. Let $u^b_t$ and $u^c_t$ denote the quarter-$t$ non-crisis and crisis unemployment rates, respectively. They then satisfy

$$u^c_t = u^b_t + \Delta u > u^b_t. \quad (2.1)$$

Let $q_t$ denote the probability of a crisis starting in (the beginning of) quarter $t$, meaning that the unemployment rate increases by $\Delta u$ and equals the crisis unemployment rate, $u^c_t$, during quarter $t$. Assume that a crisis has a fixed duration of $n$ quarters, so if a crisis starts in (the beginning of) quarter $t$ it ends in (the beginning of) quarter $t + n$. Thus, if a crisis starts in quarter $t$, the unemployment rate equals the crisis unemployment rate for the $n$ quarters $t, t + 1, ..., t + n - 1$.

Let $p_t$ denote the probability of the economy being in a crisis in quarter $t$. If a crisis lasts $n$ quarters, the probability of being in a crisis equals the probability that a crisis started in any of the last $n$ quarters, including the current quarter $t$, that is, in any of the quarters $t - n + 1, t - n + 2,$

---

4 If a crisis occurs in quarter $t$, the increase $\Delta u$ in the unemployment rate will in reality not occur within the quarter but over the next few quarters. For simplicity, the increase is nevertheless assumed to occur within the quarter.
..., $t$. Then the probability of a being in a crisis in quarter $t$ satisfies

$$p_t = \sum_{\tau=0}^{n-1} q_{t-\tau}. \quad (2.2)$$

In the rest of the paper, I will refer to $p_t$ as the *probability of a crisis* in quarter $t$ and to $q_t$ as the *probability of a crisis start* in quarter $t$.\(^5\)

It follows that the quarter-$t$ unemployment rate, $u_t$, will equal the non-crisis unemployment rate, $u^n_t$, with probability $1 - p_t$ and the crisis unemployment rate with probability $p_t$. The unemployment rate in quarter $t \geq 1$ that is expected in quarter 1, the *expected unemployment rate*, is then given by

$$E_1 u_t = (1 - p_t)E_1 u^n_t + p_t E_1 u^c_t = (1 - p_t)E_1 u^n_t + p_t (E_1 u^n_t + \Delta u) = E_1 u^n_t + p_t \Delta u, \quad (2.3)$$

where $E_1$ denotes the expectations held in quarter 1. The expected future unemployment rate equals the *expected non-crisis unemployment rate*, $E_1 u^n_t$, plus the increase in the *expected unemployment rate* due to the possibility of a crisis, $p_t \Delta u$, which term I will call the *crisis increase in the expected unemployment rate*.

What is then the effect of a higher policy rate on the expected future unemployment rates? Let $\bar{i}_1$ denote a constant policy rate during quarters 1–4, so the policy rate in quarter $t$, $i_t$, satisfies $i_t = \bar{i}_1$ for $1 \leq t \leq 4$. Consider the effect on the expected future unemployment rate of increasing the policy rate during quarters 1–4. By (2.3), it is given by the derivative

$$\frac{dE_1 u_t}{d\bar{i}_1} = \frac{dE_1 u^n_t}{d\bar{i}_1} + \Delta u \frac{dp_t}{d\bar{i}_1}. \quad (2.4)$$

It consists of the effect on the expected non-crisis unemployment rate, $dE_1 u^n_t/d\bar{i}_1$, and the effect on the crisis increase in the expected unemployment rate, $dp_t/d\bar{i}_1$.\(^6\) Let us examine these in turn.

### 2.1 The effect of the policy rate on the expected non-crisis unemployment rate

The effect on the policy rate on the expected non-crisis unemployment rate is just the standard impulse response of the unemployment rate to an increase in the policy rate. As an example, I use the impulse response in the Riksbank’s main model, the DSGE model Ramses, shown in

---

\(^5\) I am grateful to Stefan Laséen and David Vestin for alerting me to the fact that the expression (2.2) is a linear approximation to the probability of a crisis. A more thorough treatment is to model the dynamics of the probability of a crisis as a Markov process, as discussed in appendix A. For the parameter range used here, the linear approximation slightly exaggerates the probability of a crisis but simplifies the derivation of the effect of the policy rate on the probability of a crisis.

\(^6\) Here I am abstracting from the possible effect of the policy rate on the crisis increase in the unemployment rate, $d\Delta u/di_t$. It is examined separately in appendix D, where it is shown that the effect can be of either sign but is so very small that it can be disregarded.
Figure 2.1: The effect on the expected non-crisis unemployment rate of a 1 percentage point higher policy rate during quarters 1–4; deviations from baseline. (Source: Sveriges Riksbank.)

Figure 2.1. The grey line shows an increase in the policy rate of 1 percentage points during quarters 1–4 ($\Delta \bar{i}_1 = 1$ percentage point) and then a return to the baseline level. The red line shows the corresponding deviation of the unemployment rate from the baseline level ($\Delta E_1 u^n_t$). The unemployment rate increases above the baseline level to about 0.5 percentage points in quarter 6 and then slowly falls back towards the baseline level. Under the assumption of approximate linearity, I can take this effect on the expected future non-crisis unemployment rates as the derivative with respect to the policy rate $\bar{i}_1$ of the expected future non-crisis unemployment rate,

$$\frac{dE_1 u^n_t}{d\bar{i}_1} = \frac{\Delta E_1 u^n_t}{\Delta \bar{i}_1} = \Delta E_1 u^n_t \quad \text{for } t \geq 1, \quad (2.5)$$

where $\Delta E_1 u^n_t$ is given by figure 2.1.

Thus, we have determined the first term in (2.4). It remains to determine the second term, that is, the product of the crisis increase in the unemployment rate and the effect on the probability of a crisis of the policy rate. For the crisis increase in the unemployment rate, I use the same assumption as in a crisis scenario discussed in Sveriges Riksbank (2013), that the crisis increase in the unemployment rate is 5 percentage points ($\Delta u = 5$ percentage points). It remains to determine $dp_t/d\bar{i}_1$, the effect of the policy rate on the probability of a crisis in quarter $t \geq 1$.

---

7 The figure shows the impulse response of Ramses for the unemployment rate that was reported by Riksbank Deputy Governor Karolina Ekholm in Ekholm (2013). It is the same response as the one reported to alternative policy-rate paths for quarters 1–12 in Sveriges Riksbank (2014b).
2.2 The effect of the policy rate on the probability of a crisis

In order to determine the effect of the policy rate on the probability of a crisis, \( p_t \), I will use that the probability of a crisis depends on the probability of a crisis start, \( q_t \), in the \( n \) quarters before and including quarter \( t \) according to (2.2), that the probability of a crisis start may depend on real debt growth, and that real debt growth may depend on the policy rate.

2.2.1 The effect of real debt on the probability of a crisis start

According to Schularick and Taylor (2012), the probability of a crisis start depends on the growth rate of real debt. Schularick and Taylor use annual data for 14 developed countries for 1870–2008 and estimate the annual probability of a crisis as a function of annual debt growth lagged 1–5 years. I use their estimates of the coefficients in their main logit regression, Schularick and Taylor (2012, table 3, column (5)), in a quarterly variant of their equation,

\[
q_t = \frac{1}{4} \frac{\exp(X_t)}{1 + \exp(X_t)},
\]

where

\[
X_t = -3.89 - 0.398 g_{t-4} + 7.138^{***} g_{t-8} + 0.888 g_{t-12} + 0.203 g_{t-16} + 1.867 g_{t-20},
\]

(2.6)

numbers within parenthesis are robust standard errors,\(^8\)

\[
g_t = \frac{\left( \sum_{\tau=0}^{3} d_{t-\tau}/4 \right) / \left( \sum_{\tau=0}^{3} d_{t-4-\tau}/4 \right)}{4} - 1,
\]

(2.7)

and \( d_t \) is the level of real debt in quarter \( t \).\(^9\) That is, \( g_t \) is the annual growth rate of the average annual real debt level. Schularick and Taylor (2012, p. 1046) report a marginal effect on the annual probability of a crisis start over all lags equal to 0.30, implying the summary result that 5 percent lower real debt in 5 years reduces the probability of a crisis by about 0.3 percentage points per year. That is, it reduces the quarterly probability \( q_t \) by 7.5 basis points.\(^{10,11}\)

---

\(^8\) One, two, and three stars denote significance at the 10, 5, and 1 percent level, respectively. The five lags are jointly significant at the 1 percent level.

\(^9\) More precisely, what I call real debt is in Schularick and Taylor (2012) total bank loans, defined as the end-of-year amount of outstanding domestic currency lending by domestic banks to domestic households and nonfinancial corporations (excluding lending within the financial system).

\(^{10}\) The linear regression in Schularick and Taylor (2012, table 3, column (1)) implies a corresponding somewhat higher marginal effect of 0.4. This explains the summary result that I have used in Svensson (2014) and Svensson (2015): 5 percent lower real debt in 5 years reduces the annual probability of a crisis start by about 0.4 percentage points. In figure 2.2, real debt decreases by 0.25 percent in 5 years. Then the summary result implies that the annual probability of a crisis decreases by about 0.25 \( \times 0.4/5 = 0.02 \) percentage points, which is the summary result that I have used in Svensson (2014) and Svensson (2015).

\(^{11}\) A full 1 percentage point reduction of the annual real debt growth for 5 years actually reduces the annual probability of a crisis start by 0.288 percentage points rather than 0.30 percentage points, because of the curvature.
However, we notice that the coefficients in (2.6) are not uniform, so the summary result strictly only applies for uniform annual real debt growth during 5 years. If real debt growth fluctuates, the dynamics of the probability of a crisis start is more complicated, as in the dynamic approach of Diaz Kalan, Laséen, Vestin, and Zdzienicka (2015). In particular, we see that annual real debt growth lagged 2 years, $g_{t-8}$, has by far the largest coefficient in (2.6). Thus, annual real growth lagged two years is the major determinant of the probability of a crisis start.\footnote{In one specification, Schularick and Taylor (2012, table 7, column (22)) includes the debt-to-GDP ratio as an explanatory variable. Appendix G shows that including this variable makes the effect of the policy rate on the probability of a crisis start and the probability of a crisis only marginally larger and does not affect any conclusions.}

2.2.2 The effect of the policy rate on real debt, real debt growth, the probability of a crisis start, and the probability of a crisis

Given the effect on the probability of crisis start of real debt growth in (2.6), it remains to determine the effect of the policy rate on real debt growth.

As an example, I use the Sveriges Riksbank (2014a) estimate of the effect on the level of real household debt, $d_t$, of a 1 percentage point higher policy rate during 4 quarters, shown as the red line in figure 2.2.\footnote{The Schularick and Taylor (2012) estimates refer loans to both households and nonfinancial corporations, whereas the estimates in Sveriges Riksbank (2014a) refer to loans to households only. I assume that this difference does not affect the conclusions.} Real debt falls relative to the baseline level by 1 percentage in two years and then rises back and reaches the baseline level again in about 8 years.\footnote{As discussed in Svensson (2014) and Svensson (2015), there is a wide 90 percent probability band around the red line, and the effect is not significantly different from zero and could be of the opposite sign.} Because monetary policy is neutral, there is no long-run effect on real debt.

We can interpret the red line as showing the derivative of real debt $d_t$ with respect to the policy rate $\bar{i}_1$, $d(d_t)/d\bar{i}_1$, for $t \geq 1$, where furthermore $d(d_t)/d\bar{i}_1 \approx 0$ for $t \geq 32$.

The yellow line in figure 2.2, shows the resulting effect on real debt growth $g_t$, the annual growth rate of the average annual real debt level defined by (2.7). Because the real debt level first falls and then rises back to the baseline level, real debt growth will first fall below the baseline growth rate and then rise above the baseline growth rate. Thus, lower real debt growth rates are followed by higher real debt growth rates. Importantly, because there is no effect of the policy rate on real debt in the longer run, there is no effect on the average growth rate over a longer period.\footnote{Schularick and Taylor (2012, table 7, column (22)) reports the result of a model specification that includes debt to GDP as an explanatory variable. The coefficient is significantly different from zero, but as discussed in detail in...}
Figure 2.2: The effect on real debt, the average annual real debt growth, the probability of a crisis start in quarter, and the probability of being in a crisis in quarter of a 1 percentage point higher policy rate during quarters 1–4; deviations from baseline. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

We can interpret the yellow line as showing the derivative of the annual real debt growth $g_t$ with respect to the policy rate $i_1$, $dg_t/di_1$ for $t \geq 1$, where furthermore

$$\sum_{t=1}^{40} \frac{dg_t}{di_1} \approx 0.$$  

The blue line in figure 2.2 shows the resulting dynamics of the probability of a crisis start for each quarter, $q_t$, that follows from (2.6). Because annual real debt growth lagged two years is the main determinant of the probability of a crisis start and the annual real debt growth falls below the baseline and has a negative peak (of about $-0.8$ percentage points per year) in quarter 6, the probability of a crisis will fall below the baseline and have a negative peak (at $-0.04$ percentage points) about two years later, in quarters 14 and 15. Furthermore, annual real debt growth rises above the baseline in quarter 12, which causes the probability of a crisis start to rise above the baseline and have a positive peak (of 0.013 percentage points, barely visible) about 2 years later. Thus, these results imply that an increase in the policy rate actually, after about five years, increases the probability of a crisis start above the baseline. The increase in the policy rate shifts the probability of a crisis start between quarters, first reducing it and then increasing it. But importantly, because the average effect over time on real debt growth is zero, the average effect appendix G, it is so small that it has a very small impact on the probability of a crisis start and the probability of a crisis. I therefore disregard that effect here.
over time on the probability of a crisis start is also zero.

We can hence interpret the blue line as showing the derivative \( dq_t/d\tilde{t}_1 \) for \( t \geq 1 \), with

\[
\sum_{t=1}^{40} \frac{dq_t}{d\tilde{t}_1} \approx 0.
\]

The green line in figure 2.2 shows the dynamics of the probability of a crisis, \( p_t \). According to (2.2), that probability depends on the sum of all the probabilities of a crisis start, \( q_t \), during the last \( n \) quarters, the duration of a crisis. I assume that the duration of a crisis is \( n = 8 \) quarters, so that a crisis implies that the unemployment rate is 5 percentage points higher during the 8 quarters, corresponding to 10 point-years of higher unemployment. Thus, the green line shows an 8-quarter moving sum of the blue line. It has a negative peak of about \(-0.23\) percentage points in quarter 18 and then rises back to zero and turns positive from quarter 25. It is still positive in quarter 40 but will eventually fall to zero.\(^{16}\)

The green line can be interpreted as showing the derivative of the probability of a crisis with respect to the policy rate, \( dp_t/d\tilde{t}_1 \) for \( t \geq 1 \). Furthermore,

\[
\sum_{t=1}^{40} \frac{dp_t}{d\tilde{t}_1} \approx 0. \tag{2.8}
\]

Thus, the higher policy rate reduces the probability somewhat after 3 years and increases it after 6 years, but without any accumulated and average effect over the 40 quarters.

### 2.3 The effect of the policy rate on the expected future unemployment rate

Given the effect of the policy rate on the probability of a crisis \( dp_t/d\tilde{t}_1 \) from figure 2.2, the assumption that the crisis increase in the unemployment rate \( \Delta u \) is 5 percentage points from Sveriges Riksbank (2013), and the effect of the policy rate on the non-crisis expected unemployment rate \( dE_1u_t^2/d\tilde{t}_1 \) from figure 2.1, we can compute the effect of the policy rate on the expected unemployment rate \( dE_1u_t/d\tilde{t}_1 \) according to (2.4). It is shown in figure 2.3.

The red line shows the effect on the expected non-crisis unemployment rate, the same line as in figure 2.1. The blue line shows the effect on the expected unemployment rate. It hardly differs from the effect on the non-crisis unemployment rate. The reason is that the effect on the crisis increase in the expected unemployment rate, \( \Delta u dp_t/d\tilde{t}_1 \), is very small compared to the effect on the expected non-crisis unemployment rate. It is shown as the green line, in basis points, measured along the

\(^{16}\) Note that the Schularick and Taylor estimates in (2.6) has a relatively large coefficient (although not significant) on the annual real growth rate lagged 5 years, meaning that the probability of a crisis start and the probability of a crisis are still affected by the higher real debt growth 5-6 years earlier.
Figure 2.3: The effect on the expected unemployment rate and the expected non-crisis unemployment rate of a 1 percentage point higher policy rate during quarters 1–4; deviations from baseline. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

right vertical axis. As we have noticed in figure 2.2, the largest effect on the probability occurs in quarter 18, when $dp_{18}/d\bar{d}_1$ is $-0.23$ percentage points. This means that the term $\Delta u dp_t/d\bar{d}_1 = -0.0023 \cdot 5 = -0.0116$ percentage points = $-1.16$ basis points, is quite small compared to the effect on the expected non-crisis unemployment rate in quarter 18, $dE_1u_{18}^n/d\bar{d}_1 = 0.16$ percentage points = 16 basis points. And from quarter 25 the effect of the policy rate on the probability of a crisis continues to be very small, but positive.

Furthermore, because the accumulated and average effect on the probability of a crisis over the 40 quarters is approximately zero, the accumulated effect on the expected unemployment rate is approximately equal to the effect on the expected non-crisis unemployment rate,

$$\sum_{t=1}^{40} \frac{dE_1u_t}{d\bar{d}_1} = \sum_{t=1}^{40} \frac{dE_1u_t^n}{d\bar{d}_1} + \Delta u \sum_{t=1}^{40} \frac{dp_t}{d\bar{d}_1} \approx \sum_{t=1}^{40} \frac{dE_1u_t^n}{d\bar{d}_1}.$$

In figure 2.3, the accumulated effect on the expected non-crisis unemployment rate is 6.9 point-quarters, whereas the accumulated effect on the expected crisis increase in the unemployment rate is only $-0.03$ point-quarters. The area under the red and the blue curves are approximately equal for a horizon of 40 quarters.

In summary, the effect of the policy rate on the expected future unemployment rate is the sum of the effect on the expected non-crisis unemployment rate and the effect on crisis increase in the expected unemployment rate, the product of the probability of a crisis and the crisis increase in the
unemployment rate. The latter effect is very small, because a higher policy rate has only a modest decreasing effect on the probability of a crisis for a few years. Furthermore, after a few years effect is a small increase. Because the accumulated effect on the probability of a crisis is approximately zero, by the long-run neutrality of monetary policy, there is no accumulated effect of the policy rate on expected crisis increase in the unemployment rate.\footnote{This zero long-run effect is strictly true only under the assumption of the probability being a linear function of debt growth. But the effects of nonlinearities, for instance from a logistic model of the probability of a crisis, will be of second order under these small changes and will hardly change the conclusions. Furthermore, the logistic function (2.6) is slightly convex in the range of the relevant real debt growth rates (see figure 4.1 below), meaning that any increased variability in real debt growth rates caused by the higher policy rate will increase the average probability of a crisis, but very slightly so.}

According to these results, it is simply not true that a higher unemployment rate in the near future can be traded for a lower expected unemployment rate further into the future. Instead, leaning against the wind increases the expected unemployment rate both in the near future and further into the future.

3 The effect on expected future quadratic losses of leaning against the wind

In order to assess whether leaning against the wind is justified or not, it is not sufficient to only look at the expected future unemployment rate. The marginal welfare loss from a higher unemployment rate is larger the more the initial unemployment rate exceeds its desirable level, something that is captured by a quadratic loss function. In this section I therefore examine whether or not leaning against the wind is justified when gains and losses are measured by a quadratic loss function.

Let $u^*_t$ denote the benchmark unemployment rate, by which I mean the unemployment rate resulting from optimal flexible inflation targeting when the possibility of a crisis is disregarded and the probability of a crisis hence set to zero, $p_t \equiv 0$ for $t \geq 1$. The benchmark unemployment rate is here assumed to depend on exogenous shocks (see appendix C for details). Let $\tilde{u}_t$ denote the unemployment gap, the gap between the unemployment rate and the benchmark unemployment rate,

$$\tilde{u}_t \equiv u_t - u^*_t,$$

and let $\tilde{u}^n_t \equiv u^n_t - u^*_t$ and $\tilde{u}^c_t \equiv u^c_t - u^*_t$ denote the non-crisis and crisis unemployment gaps, respectively.
Introduce the expected intertemporal loss,
\[
\mathbb{E}_1 \sum_{t=1}^{\infty} \delta^{t-1}L_t = \sum_{t=1}^{\infty} \delta^{t-1}\mathbb{E}_1 L_t, \tag{3.2}
\]
where \(\delta\) denotes a discount factor and satisfies \(0 < \delta < 1\) and the quarter-\(t\) loss function, \(L_t\), is a simple quadratic loss function of the unemployment gap,
\[
L_t = (\bar{u}_t)^2. \tag{3.3}
\]

Thus, the expected intertemporal loss consists of the sum of discounted expected future losses. Let me examine the expected quarter-\(t\) loss, \(\mathbb{E}_1 L_t\). It can be expressed as
\[
\mathbb{E}_1 L_t = \mathbb{E}_t (\bar{u}_t)^2 = (1 - p_t)\mathbb{E}_1 (\bar{u}_t^n)^2 + p_t\mathbb{E}_1 (\bar{u}_t^c)^2 = (1 - p_t)\mathbb{E}_1 (\bar{u}_t^n)^2 + p_t\mathbb{E}_1 (\bar{u}_t^n + \Delta u)^2, \tag{3.4}
\]
where I have used that
\[
\bar{u}_t^c = \bar{u}_t^n + \Delta u. \tag{3.5}
\]
Thus, the expected quarter-\(t\) loss can be seen as the probability-weighted expected loss in a non-crisis, \(\mathbb{E}_1 (\bar{u}_t^n)^2\), plus the probability-weighted expected loss in a crisis, \(\mathbb{E}_1 (\bar{u}_t^c)^2\).

3.1 When the probability of a crisis is independent of the policy rate: Leaning with the wind

Let me first establish that the possibility of a crisis introduces a strong tendency to lean with the wind, not against. This is easiest to see under the assumption that the probability of a crisis is independent of the policy rate,
\[
\frac{dp_t}{d\bar{u}_t} = 0 \text{ for } t \geq 1. \tag{3.6}
\]
Then we can treat \(p_t\) as an exogenous variable in (3.4).

We will need an estimate of the probability of a crisis. The sum of the coefficients in (2.6) and the reported marginal effect of 0.30 by Schularick and Taylor (2012) is consistent with a constant annual probability of a crisis start equal to 3.2 percent.\(^\text{18}\) This corresponds to a crisis start on average every 31 years. A constant annual probability of a crisis start of 3.2 percent implies a corresponding constant probability of a crisis start in a given quarter, denoted \(q\), equal to \(3.2/4 = 0.8\) percent. Furthermore, as mentioned I have assumed that a crisis lasts 8 quarters (\(n = 8\)).

\(^{18}\) See appendix B for details.
Conditional on no crisis in quarter 1, for a given $q$ and $n$, the probability of a crisis in quarter $t$ is then, according to (2.2),

$$p_t = \begin{cases} 
0 & \text{for } t = 1, \\
(t - 1)q > 0 & \text{for } 2 \leq t \leq n, \\
nq > 0 & \text{for } t \geq n + 1.
\end{cases} \quad (3.7)$$

Thus, $p_t$ rises linearly from 0 in quarter 1 to its steady-state value $p = nq$ in quarter $n + 1$. With $n = 8$ quarters and $q = 0.8$ percent, $p_t$ this rises linearly from 0 in quarter 1 to $p = 6.4$ percent in quarter 9 and then stays at 6.4 percent, as shown in figure 3.1.\(^{19}\)

Let me next examine the quarter-$t$ expected loss for quarters beyond 9, such that the probability of a crisis is constant and equal to $p = 6.4$ percent. Furthermore, let me use that

$$E_1(\tilde{u}_t^n)^2 = (E_1\tilde{u}_t^n)^2 + \text{Var}_1\tilde{u}_t^n,$$

$$E_1(\tilde{u}_t^n + \Delta u)^2 = (E_1\tilde{u}_t^n + \Delta u)^2 + \text{Var}_1\tilde{u}_t^n,$$

where $\text{Var}_1\tilde{u}_t^n$ denotes the variance of $\tilde{u}_t^n$ conditional on information available in quarter 1. Then I can write the quarter-$t$ expected loss (3.4) as

$$E_1L_t = (1 - p)E_1(\tilde{u}_t^n)^2 + pE_1(\tilde{u}_t^n + \Delta u)^2$$

$$= (1 - p)(E_1\tilde{u}_t^n)^2 + p(E_1\tilde{u}_t^n + \Delta u)^2 + \text{Var}_1\tilde{u}_t^n. \quad (3.8)$$

---

\(^{19}\) As mentioned in footnote 5, (3.7) is a linear approximation to a Markov process for the probability of a crisis. As shown in appendix A and figure A.1, for the relevant Markov process, the probability of a crisis can be shown to rise from zero in quarter 1 to 6.2 percent in quarter 9 and then converges to 6.0 percent in quarter 16.
Figure 3.2: The probability-weighted quadratic (dashed) and marginal (solid) non-crisis loss (blue), crisis loss (red), and total loss (black) as a function of the expected non-crisis unemployment gap (under the assumption that the probability of a crisis is 6.4 percent and the crisis increase in the unemployment rate is 5 percent)

Under the assumption of a linear relation between the policy rate and the expected non-crisis unemployment gap as well as additive shocks, the conditional variance $\text{Var}_t \tilde{u}_n^t$ is independent of policy. Let me therefore focus on the first two terms in (3.8), the probability-weighted non-crisis loss $(1 - p)(E_1 \tilde{u}_t^n)^2$ and the probability-weighted crisis loss $p(E_1 \tilde{u}_t^n + \Delta u)^2$ (both exclusive of the corresponding conditional-variance term).

In figure 3.2, the blue dashed line shows the probability-weighted non-crisis loss,

$$(1 - p)(E_1 \tilde{u}_t^n)^2 = 0.936(E_1 \tilde{u}_t^n)^2,$$

as a function of the expected non-crisis unemployment gap, $E_1 \tilde{u}_t^n$. It has a minimum for $E_1 \tilde{u}_t^n = 0$, corresponding to point A. The blue solid line shows the corresponding probability-weighted marginal non-crisis loss (with respect to an increase in the expected non-crisis unemployment gap),

$$\frac{d(1 - p)(E_1 \tilde{u}_t^n)^2}{dE_1 \tilde{u}_t^n} = 0.936 \cdot 2E_1 \tilde{u}_t^n.$$

It is zero where the probability-weighted non-crisis loss has a minimum, for $E_1 \tilde{u}_t^n = 0$, and has a positive slope of 1.872.

Under the assumption that the probability of a crisis is zero, the non-crisis loss is the only loss that matters, and the optimal policy is to set the expected non-crisis unemployment gap equal to
zero. But if the probability of a crisis is positive, the probability-weighted crisis loss has to be taken into account.

The red dashed line shows the probability-weighted crisis loss,

\[ p(E_1 \tilde{u}_t^n + \Delta u)^2 = 0.064(E_1 \tilde{u}_t^n + 5)^2, \]

where I have used that the crisis increase in the unemployment rate is 5 percent. The probability-weighted crisis loss has a minimum for \( E_1 \tilde{u}_t^n = -5 \) percentage points, and is upward-sloping for the range of expected non-crisis unemployment gaps shown in the figure. For \( E_1 \tilde{u}_t^n = 0 \), the probability-weighted crisis loss is \( 0.064 \cdot 5^2 = 1.61 \), corresponding to point C in the figure. The red solid line shows the corresponding probability-weighted marginal crisis loss,

\[ \frac{dp(E_1 \tilde{u}_t^n + \Delta u)^2}{dE_1 \tilde{u}_t^n} = 0.064 \cdot 2(E_1 \tilde{u}_t^n + 5). \]

The probability-weighted marginal crisis loss is zero for \( E_1 \tilde{u}_t^n = -5 \) and positive and equal to \( 0.064 \cdot 2(5) = 0.64 \) for \( E_1 \tilde{u}_t^n = 0 \), and it has a positive slope of 0.128.

The black dashed line shows the total (quarter-\( t \) expected) loss (exclusive of the conditional-variance term),

\[ (1 - p)(E_1 \tilde{u}_t^n)^2 + p(E_1 \tilde{u}_t^n + \Delta u)^2 = 0.936(E_1 \tilde{u}_t^n)^2 + 0.064(E_1 \tilde{u}_t^n + 5)^2, \]

the vertical sum of the blue and red dashed lines. The black solid line shows the corresponding marginal loss,

\[ \frac{d(1 - p)(E_1 \tilde{u}_t^n)^2 + p(E_1 \tilde{u}_t^n + \Delta u)^2}{dE_1 \tilde{u}_t^n} = 2(E_1 \tilde{u}_t^n + p\Delta u) = 2(E_1 \tilde{u}_t^n + 0.32). \] (3.9)

For \( E_1 \tilde{u}_t^n = 0 \), the total loss is \( 0.064 \cdot 5^2 = 1.61 \), corresponding to point C, and the marginal loss is 0.64, corresponding to point B. It is obvious from the figure that this is not a minimum for the total loss.

The minimum for the total loss occurs where the marginal loss is zero, for which \( E_1 \tilde{u}_t^n = -0.32 \) percentage points, corresponding to point D. Then the total loss is 1.50, corresponding to point E. The gain, the reduction in total loss from point C to point E is 0.11 = 0.32\(^2 \), thus equivalent to the negative of the loss of increasing the unemployment rate by 0.32 percentage points from its optimal level.

Thus, if the probability of a crisis is zero, it is optimal to set the expected non-crisis unemployment gap equal to zero. If the probability of a crisis is positive, it is optimal to reduce the non-crisis
unemployment gap below zero. That is, it is optimal to lower the policy rate and thus lean with the wind.

We can see this in a different way. We can rewrite the expected quarter-t loss as the sum of the squared expected unemployment gap and the conditional variance of the unemployment gap, the first equality in the equation,

\[
E_1 L_t = E_1 (\tilde{u}_t)^2 = (E_1 \tilde{u}_t)^2 + \text{Var}_1 \tilde{u}_t = (E_1 \tilde{u}_t)^2 + \text{Var}_1 \tilde{u}_t^n + p_t(1 - p_t)(\Delta u)^2. \tag{3.10}
\]

The second equality in (3.10) uses that the conditional variance of the unemployment gap is the sum of the conditional variance the non-crisis unemployment gap, \(\text{Var}_1 \tilde{u}_t^n\), and the variance of a binomial distribution, \(p_t(1 - p_t)(\Delta u)^2\), because the unemployment gap is the sum of the non-crisis unemployment gap and a binomial random variable that takes the value \(\Delta u\) with probability \(p_t\) and the value 0 with probability \(1 - p_t\).\(^{20}\)

If the policy rate has no effect on the probability of a crisis, the variance terms in (3.10) are independent of policy.\(^{21}\) Then the marginal loss with respect to an increase in the expected unemployment gap satisfies,

\[
d\frac{E_1 L_t}{E_1 \tilde{u}_t} = \frac{d(E_1 \tilde{u}_t)^2}{dE_1 \tilde{u}_t} = 2E_1 \tilde{u}_t,
\]

and the optimal policy is to set the marginal loss and thereby the expected unemployment gap equal to zero,

\[
E_1 \tilde{u}_t = E_1 \tilde{u}_t^n + p_t \Delta u = 0.
\]

This implies setting the expected non-crisis unemployment gap equal to the minus the probability-weighted crisis increase in the unemployment rate,

\[
E_1 \tilde{u}_t^n = - p_t \Delta u < 0.
\]

Once seen, this is completely obvious. If there is a positive probability of a crisis, the expected unemployment gap is greater than the expected non-crisis unemployment gap. It is optimal to set the expected unemployment gap equal to zero; hence it is optimal to set the expected non-crisis unemployment gap below zero. Leaning with the wind is the obvious policy in this case. The amount of leaning with the wind is a non-crisis unemployment gap equal to \(-0.32\) percentage points instead of zero.

\(^{20}\) The conditional covariance between the non-crisis unemployment gap and a crisis start is assumed to be zero.

\(^{21}\) If the conditional variance terms are independent of policy, Certainty Equivalence holds, and it is sufficient to focus on the conditional means of the relevant variables. When the conditional variance terms depend on policy, as when the probability of a crisis depends on the policy rate, Certainty Equivalence no longer holds and optimal policy also has to take into account the effect on the conditional variance terms.
In summary, we see that there is a strong tendency towards some leaning with the wind rather than against. Only if the policy rate has a sufficiently strong negative effect on the probability of a crisis can leaning against the wind be justified. Let me now examine further whether it possible that leaning against the wind might be justified.

3.2 The marginal cost, marginal benefit, and net marginal cost of leaning against the wind

Thus, I want to determine whether the optimal policy is to set the non-crisis unemployment gap above or below zero when the policy rate has an effect on the probability of a crisis, \( dp_t/di_1 \neq 0 \), \( t \geq 1 \). Let me hence consider the initial situation when the expected non-crisis unemployment gap is equal to zero for all quarters,

\[
E_1 \bar{u}_t^n = 0 \quad \text{for} \quad t \geq 1. \tag{3.11}
\]

In this initial situation, let me then examine whether increasing the policy rate increases or reduces the intertemporal loss. This means to examine the derivative of the intertemporal loss function with respect to the policy rate during quarters 1–4, the marginal expected loss from increasing the policy rate,

\[
\frac{d}{di_1} E_1 \sum_{t=1}^{\infty} \delta^{t-1} L_t = \sum_{t=1}^{\infty} \delta^{t-1} \frac{dE_1 L_t}{di_1}. \tag{3.12}
\]

If this marginal expected loss from increasing the policy rate is negative, it is optimal to raise the policy rate and increase the expected future unemployment gaps above zero, and thus lean against the wind. If the marginal expected loss is positive, it is optimal to lower the policy rate and reduce the expected future unemployment gaps below zero, and thus lean with the wind.

The marginal expected loss is equal to the discounted sum of the derivatives of expected future quarterly losses, the future quarter-\( t \) marginal expected losses. Let me examine the marginal expected loss for a given quarter \( t \), starting from the expression (3.4) for the expected quarter-\( t \) loss and taking the derivative with respect to the policy rate,

\[
\frac{dE_1 L_t}{di_1} = 2(E_1 \bar{u}_t^n + p_t \Delta u) \frac{dE_1 u_t^n}{di_1} + [(\Delta u)^2 + 2\Delta u E_1 \bar{u}_t^n] \frac{dp_t}{di_1}, \tag{3.13}
\]

where I have used that \( E_1(\bar{u}_t^n + \Delta u)^2 - E_1(\bar{u}_t^n)^2 = (\Delta u)^2 + 2\Delta u E_1 \bar{u}_t^n \). I have also assumed sufficient linearity, such that the derivatives \( dE_1 u_t^n/di_1 \) and \( dp_t/di_1 \) are independent of the non-crisis unemployment gap.
If the expected non-crisis unemployment gap is zero, (3.11), the derivative (3.13) simplifies to

$$\frac{dE_1L_t}{d\tilde{i}_1} = 2p_t \Delta u \frac{dE_1u^n_t}{d\tilde{i}_1} + (\Delta u)^2 \frac{dp_t}{d\tilde{i}_1}. \tag{3.14}$$

The first term in (3.13) and (3.14) is the product of the above-examined quarter-\(t\) marginal loss of increasing the expected non-crisis unemployment gap, (3.9) and the effect of the policy rate on the non-crisis unemployment rate that we examined above. The crucial term \(p_t \Delta u\) enters because an increase in non-crisis unemployment gap increases the crisis unemployment gap, and, as we have seen in figure 3.2, the probability-weighted marginal loss of increasing the crisis unemployment gap is positive also when the expected non-crisis unemployment gap is zero.

This term in the marginal expected loss of leaning against the wind, the expected marginal loss from an increase in the crisis unemployment gap, has been disregarded by the previous literature on the costs and benefits of leaning against the wind (Svensson (2014), Svensson (2015), Ajello, Laubach, Lopez-Salido, and Nakata (2015), and Diaz Kalan, Laséen, Vestin, and Zdziejicka (2015)). But it is a crucial component in assessing the whether or not leaning against the wind is justified.\(^{22}\)

The third term in (3.13) is the marginal expected loss from a change in the probability of a crisis. It consists of the loss increase in a crisis multiplied by the effect of the policy rate on the probability of a crisis. If the expected non-crisis unemployment gap is zero, the loss increase in a crisis is given by \((\Delta u)^2\). If a policy-rate increase reduces the probability of a crisis, the second term in (3.14) is negative. Thus, the sign of the quarter-\(t\) marginal expected loss (3.14) depends on whether the marginal expected loss from a change in the probability of a crisis is negative and of a sufficiently large magnitude to dominate the positive expected marginal loss of increasing the crisis unemployment gap.

In order to examine this more closely, let me identify the left side of (3.13), the marginal expected loss from a policy-rate increase \(dE_1L_t/d\tilde{i}_1\), with the quarter-\(t\) net marginal cost, \(\text{NMC}_t\), of leaning against the wind. The first two terms on the right side of (3.13) can be identified with the quarter-\(t\) marginal cost, \(\text{MC}_t\), and the negative of the second term on the right side with the quarter-\(t\) marginal benefit, \(\text{MB}_t\), that is,

$$\text{NMC}_t(E_1\tilde{u}_t^n) = \text{MC}_t(E_1\tilde{u}_t^n) - \text{MB}_t(E_1\tilde{u}_t^n), \tag{3.15}$$

\(^{22}\) Appendix F looks more closely at the case of a fixed cost of a crisis, more precisely when the crisis unemployment rate reaches \(\Delta u\) regardless of the non-crisis unemployment gap. This stacks the cards in favor of leaning against the wind. Nevertheless, the optimal amount of leaning against the wind is very small and involves an increase of the policy rate in quarters 1–4 of only a few basis points (in line with the results of Ajello, Laubach, Lopez-Salido, and Nakata (2015)). The reduction in the probability of a crisis and the net reduction in the expected intertemporal loss are completely insignificant.
Figure 3.3: The marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap equals zero. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

where

\[
MC_t(E_1 \tilde{u}_t^n) = 2[E_1 \tilde{u}_t^n + p_t \Delta u] \frac{dE_1 u_t^n}{d\bar{i}_1},
\]

\[
MB_t(E_1 \tilde{u}_t^n) = [(\Delta u)^2 + 2\Delta u E_1 \tilde{u}_t^n](- \frac{dp_t}{d\bar{i}_1}).
\]

When the expected non-crisis unemployment gap is zero, (3.11), we can write

\[
NMC_t(0) = MC_t(0) - MB_t(0),
\]

where

\[
MC_t(0) = 2p_t \Delta u \frac{dE_1 u_t^n}{d\bar{i}_1},
\]

\[
MB_t(0) = (\Delta u)^2(- \frac{dp_t}{d\bar{i}_1}),
\]

where (3.19) is the first term in (3.14) and (3.20) is the negative of the second term in (3.14).

3.3 Leaning with or against the wind?

Given this, the marginal cost, the marginal benefit, and the net marginal benefit in (3.18)-(3.20) are shown for each quarter 1–40 in figure 3.3. The red line in figure 3.3 shows the marginal cost (3.19). From quarter 9, when \(p_t\) is constant, it is proportional to \(dE_1 u_t^n/d\bar{i}_1\) (the red line in figure
2.1) and positive. For quarter 1–8, the marginal cost is affected by the fact that $p_t$ is increasing, giving it a sharper and later peak than $dE_1u^n_t/d\bar{d}_1$. The green line in figure 3.3 shows the marginal benefit (3.20). It is proportional to $-dp_t/d\bar{d}_1$ (the green line in figure 2.2). The blue line shows the net marginal cost (3.18), the difference between the red and the green lines in the figure.

Importantly, from (2.8) we know that accumulated effects of the policy rate on the probability of a crisis is approximately zero. This means that the undiscounted sum of the marginal benefits (3.20) is approximately zero,
\[
\sum_{t=1}^{40} MB_t(0) \approx 0. \tag{3.21}
\]
This implies that the undiscounted sum of the net marginal costs is approximately equal to the undiscounted sum of the marginal costs,
\[
\sum_{t=1}^{40} NMC_t(0) = \sum_{t=1}^{40} MC_t(0) - \sum_{t=1}^{40} MB_t(0) \approx \sum_{t=1}^{40} MC_t(0) > 0. \tag{3.22}
\]

Discounting the sums will not affect this result much, so it is clear that, for (3.11), the intertemporal expected loss is increasing in the policy rate. This means that leaning with the wind is indeed justified, not leaning against.\footnote{We can look more closely at quarter 18, when the marginal benefit is the largest. Because for that quarter, $dp_{18}/\bar{d}_1 = -0.23$ percentage points, and we have $\Delta u = 5$ percentage points, by (3.20), $MB_{18}(0) = 0.0023 \cdot 5^2 = -0.058$. But for quarter 18, $dE_{18}u^n_1/d\bar{d}_1 = 0.16$ and, by (3.20), $MC_{18}(0) = 2 \cdot 0.064 \cdot 5 \cdot 0.16 = 0.10$, still larger than $MB_{18}(0)$.}

Furthermore, at a closer look, the assumption about monetary neutrality and the resulting negative marginal benefit in later years do not seem essential for rejecting leaning against the wind. From figure 3.3, it is apparent that if the marginal benefit would be zero instead of negative beyond quarter 24, the conclusions would not change. This is also the case if we would disregard the positive marginal cost beyond quarter 24 and only consider marginal cost and benefit up to quarter 26. The role of monetary neutrality and non-neutrality is further examined in section 5.

### 3.4 The sensitivity to the initial state of the economy

The above examination is for an initial situation of a zero expected non-crisis unemployment gap, (3.11). Some advocacy for leaning against the wind seems to recommend it more or less regardless of the initial state of the economy (for instance, Bank for International Settlements (2014)). But an initial positive expected non-crisis unemployment gap – an initially weaker economy – dramatically strengthens the case against leaning against the wind.

\footnote{A particular constrained-optimal policy is examined in appendix H.}
Figure 3.4: The marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap is positive and equals 0.25 percentage points for all quarters. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

In figure 3.4, the dashed lines show the marginal cost, marginal benefit, and net marginal benefit of leaning against the wind when the expected non-crisis unemployment gap is positive and equal to a modest 0.25 percentage point for all quarters, whereas the solid lines show these variables when the expected non-crisis unemployment gap equals zero (as in figure 3.3). With a non-zero expected unemployment gap, the marginal cost is given by (3.16) rather than (3.19). This modest expected non-crisis unemployment gap has a substantial impact on the marginal cost (because in (3.16) the term $E_1 \tilde{u}_t^n = 0.25$ is of a similar order of magnitude as the term $p_t \Delta u$ (which rises from 0 in quarter 1 to 0.32 percentage points in quarter 9). The marginal benefit is given by (3.17) rather than (3.20), but the expected non-crisis unemployment gap has a quite small impact on it (because in (3.17) the term $2 \Delta u E_1 \tilde{u}_t^n = 2.5$ is small relative to $(\Delta u)^2 = 25$). The net marginal cost therefore shifts up substantially and the case against leaning against the wind gets even stronger.

Assuming that initially the expected non-crisis unemployment gap is positive and the same for all future quarters allows us to examine the impact on the marginal cost and benefit at all quarters. A more realistic initial situation is arguably when the expected unemployment gap is positive for the first few quarters and approaches zero in later quarters. From figure 3.4 we realize that we still get a substantial increase in the marginal cost of leaning against the wind if, for instance, the expected non-crisis unemployment gap is positive and equal to 0.25 percentage points for only the
Figure 3.5: The effect of a reduction in the policy-rate effect on the expected non-crisis unemployment gap from the baseline (solid lines) by a half (dashed lines) on the marginal cost and the net marginal cost of leaning against the wind, when the expected unemployment gap is zero. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

first 8 quarters and then falls and equals zero from quarter 12 and onwards, in which case the dashed lines for the marginal cost and net marginal cost are equal to the solid lines from quarter 12 onwards.

Since leaning against the wind is not justified with a zero initial expected unemployment gap, it is of course even less justified for an initial positive expected unemployment gap. The marginal and net marginal cost of leaning increase substantially with a higher initial expected unemployment gap.

3.5 The sensitivity to the effect of the policy rate on the non-crisis unemployment gap

The marginal cost and net marginal cost of leaning against the wind depend by (3.16) on the initial expected unemployment gap \( (E_1 \tilde{u}_n) \), the probability of a crisis \( (p_t) \), the crisis increase in the unemployment gap \( (\Delta u) \), and the effect of the policy rate on the expected non-crisis unemployment gap \( (dE_1 \tilde{u}_n / d\tilde{t}_1) \). The sensitivity to the initial expected unemployment gap has been examined in section 3.4. The sensitivity to the probability of a crisis and the crisis increase in the unemployment gap will be examined in section 4. Here I look at the sensitivity to the effect of the policy rate increase on the expected non-crisis unemployment gap.
As a baseline, I have used the Riksbank estimate of the effect shown in figure 2.1. In figure 3.5, the dashed lines show the marginal cost and net marginal cost when baseline policy-rate effect on the expected non-crisis unemployment gap is reduced to a half of the Riksbank estimate, whereas the solid lines show the baseline case. The marginal cost shifts down by a half, the marginal benefit (3.20) is not affected, and the net marginal cost shifts down with the marginal cost but remains substantially positive, except in quarters 18–21 where it is slightly negative. Clearly, leaning against the wind is still not justified.

4 Does less effective macroprudential policy strengthen the case for leaning against the wind?

A common view is that macroprudential policy should provide the first line of defence of financial stability but that monetary policy may have a role as a second line of defence, in case macroprudential policy is not sufficiently effective. In line with this view, one might ask whether less effective macroprudential policy might strengthen the case for leaning against the wind. Let me examine this issue in the present framework.

What would more effective macroprudential policy imply in the present framework? Such macroprudential policy would in general imply more resilience of the financial system to shocks, for instance through stronger balance sheets with more loss-absorbing capital. This might reduce the probability of a crisis start, $q_t$. It might also reduce the severity of a crisis, in the sense of implying a smaller crisis increase in the unemployment rate, $\Delta u$, and/or a shorter duration of a crisis, $n$.

Correspondingly, less effective macroprudential policy might increase the probability of a crisis, as well as implying a larger crisis increase in the unemployment rate and/or a longer duration of a crisis. Consequently, in order to assess whether less effective macroprudential policy provides a case for leaning against the wind, I examine how the marginal cost, marginal benefit, and thus the net marginal cost of leaning against the wind shifts, if the probability of crisis start is higher, the increase in the unemployment rate is larger, or the duration of a crisis is longer. This way I also conduct some further sensitivity analysis of my results.

4.1 A higher probability of a crisis start

Let me first examine the consequences of a higher probability of a crisis start. So far I have assumed an annual probability of a crisis start of 3.21 percent (corresponding to a crisis start on average
Figure 4.1: The annual probability of a crisis start (percent) as a function of real debt growth. (Source: Schularick and Taylor (2012) and own calculations.)

every 31 years), which for the estimates in (2.6) is consistent with a steady annual growth rate of real debt of 5 percent. Let me now consider an increase in the annual probability of a crisis start by 1 percentage points to 4.21 percent (corresponding to a crisis start on average every 24 years). This is consistent with an annual steady growth rate of 7.9 percent. Thus, we might think of less effective macroprudential policy resulting in a higher real debt growth, which in turn increases the probability of a crisis start. Figure 4.1 shows how the annual probability of a crisis start depends on a steady annual real debt growth.

In figure 4.2, dashed lines show the marginal cost, marginal benefit, and net marginal cost from leaning against the wind for the higher annual probability 4.21 percent of a crisis start, to be compared with the solid lines for the baseline case of an annual probability of 3.21 percent. A higher probability of a crisis start $q_t$ leads by (2.2) to a higher probability of a crisis $p_t$. We see in (3.19) that the marginal cost of leaning against the wind is proportional to $p_t$. A higher $p_t$ thus shifts up the marginal cost from the solid to the dashed red line in the figure.

What is the effect on the marginal benefit? A higher steady growth rate of real debt will increase the marginal effect of steady real debt growth on the probability of, because the logistic function with the estimates in (2.6) is convex for growth rates in this range (see figure 4.1). This will increase the magnitude of the effect of the policy rate on the probability of a crisis start and on the probability of a crisis, $-dp_t/d\bar{\iota}_1$. We see in (3.20) that the marginal benefit is proportional to $-dp_t/d\bar{\iota}_1$, so this will increase the magnitude of the marginal benefit and shift it from the solid
Figure 4.2: The effect of an increase in the annual probability of a crisis start from 3.21 percent (solid lines) to 4.21 percent (dashed lines) on the marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

to the dashed green line in the figure. We see in figure 4.2 that the net effect on the net marginal cost is a significant increase in the net marginal cost, from the solid to the dashed blue line.

It follows that the discounted net marginal cost increases. Thus, less effective macroprudential policy, to the extent that it leads to higher real debt growth and a higher probability of a crisis start, does not strengthen the case for leaning against the wind; it strengthens the case against leaning against the wind.

4.2 A larger crisis increase in the unemployment rate

In figure 4.3, the dashed lines show the marginal cost, marginal benefit, and net marginal cost for a larger the crisis increase $\Delta u$ in the unemployment rate of 6 percentage points, to be compared with the solid lines for the baseline case of a crisis increase in the unemployment rate of 5 percentage points. We see in (3.19) and (3.20) that the marginal cost is linear in $\Delta u$ and the marginal benefit is quadratic in $\Delta u$. Thus, the magnitudes of the marginal cost and marginal benefit increase with $\Delta u$. We see that the net effect is an increase of the net marginal cost except around quarter 19 where the marginal benefit increases slightly more than the marginal cost.

It follows that, also in this case, the sum of discounted net marginal costs increases. Less effective macroprudential policy, to the extent that it implies a larger crisis increase in the unemployment
Figure 4.3: The effect of an increase in the crisis increase in the unemployment rate from 5 (solid lines) to 6 percentage points (dashed lines) on the marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

...rate, again strengthens the case against leaning against the wind.

### 4.3 A longer duration of a crisis

Finally, in figure 4.4, dashed lines show the marginal cost, marginal benefit, and net marginal cost for a longer crisis duration of \( n = 12 \) quarters, to be compared with the solid lines for the baseline crisis duration of 8 quarters. A longer duration means by (2.2) that the probability of a crisis, for the linear approximation used here, is the sum over a few more previous quarterly probabilities of a crisis start, implying a shift to the right of the marginal cost and marginal benefit of leaning against the wind.\(^{25}\) As a result, the net marginal cost shifts up, except around quarter 24, where the marginal benefit increases slightly more than the marginal cost. The sum of discounted net marginal costs increases. Thus, to the extent that less effective macroprudential policy increases the crisis duration, it again strengthens the case against leaning against the wind.

Overall, for this section’s intuitive assumptions about the consequence of a less effective macroprudential policy, such a less effective policy consistently further strengthens the already strong case against leaning against the wind. The presumption that a less effective macroprudential pol-

\(^{25}\) For the linear approximation (2.2), the probability of a crisis increases from zero in quarter 1 to 9.6 percent in quarter 13 and then stays at 9.6 percent. For the relevant Markov process discussed in Appendix A, the probability of a crisis increases from zero in quarter 1 to 9.1 percent in quarter 13 and then converges to 8.8 percent in quarter 20.
Figure 4.4: The effect of an increase in crisis duration from 8 (solid lines) to 12 quarters (dashed lines) on the marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

Non-neutral monetary policy: a permanent effect on real debt

Monetary neutrality implies that monetary policy has no effect on real debt in the long run, and therefore no effect on average and accumulated real debt growth over a longer period. Thus there is no effect on average and accumulated probabilities of a crisis over a longer period. One might think that, if monetary policy would be non-neutral and would have a permanent effect on the real debt level, this might strengthen the case for leaning against the wind.

Thus, in order to examine this, assume that the effect of the policy rate on real debt is permanent. More precisely, suppose that real debt permanently stays down at its maximum deviation from the baseline in figure 2.2 (−1.03 percent), from quarter 8 onwards, as shown in figure 5.1. As seen in the figure, there is a large and persistent, but not permanent, reduction in the probability of a crisis. As seen in figure 5.2, the marginal benefit is larger and more persistent. Nevertheless, this marginal benefit is not sufficient to prevent the net marginal cost from being positive and the discounted sum of the net marginal cost to be positive and large. Thus leaning against the wind still has a large positive net marginal cost, and leaning with the wind remains a better policy.
Figure 5.1: For a permanent effect on real debt, the effect on real debt, the average annual real debt growth, the probability of a crisis start in quarter, and the probability of being in a crisis in quarter of a 1 percentage point higher policy rate during quarters 1–4; deviations from baseline. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

Figure 5.2: For a permanent effect on real debt, the marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap is zero. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)
Figure 5.3: For a 5.8 times larger effect on the probability of a crisis start than the baseline and a permanent effect on real debt, the effect on real debt, the average annual real debt growth, the probability of a crisis start in quarter, and the probability of being in a crisis in quarter of a 1 percentage point higher policy rate during quarters 1–4; deviations from baseline. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

5.1 How much larger an effect on the probability of a crisis start is needed to justify leaning against the wind?

Under the assumption of a permanent effect on real debt of a 1 percentage point higher policy rate during quarters 1–4, the accumulated marginal benefit in figure 5.2 is positive and equal to 0.64. The accumulated marginal cost is 3.71, about 5.8 times larger. We then realize that, for leaning against the wind to be justified, the effect on the policy rate on the probability of a crisis must be more than 5.8 times larger than the estimates in (2.6). If the largest coefficient, 7.138, on the three-year lag of the annual growth rate, would be two standard deviations larger, it would be 12.4, that is 1.74 times larger than 7.138. This is very far from 5.8 times larger. The dashed lines in figures 5.3 and 5.4 show the case when the effect on the probability of a crisis is 5.8 larger and the accumulated marginal cost and marginal benefit are equal.

Clearly, even under the extreme assumption of a large permanent effect on real debt, we need an extreme assumption on the effect of the policy rate on the probability of a crisis for the accumulated net marginal cost not being positive but zero.
Figure 5.4: For a 5.8 times larger effect on the probability of crisis start than the baseline and a permanent effect on real debt, the marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap is zero. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

6 Results for a dataset of Laeven and Valencia (2012)

So far I have used the estimates in Schularick and Taylor (2012) for their dataset covering 14 developed countries for 1870–2008. IMF staff have used a dataset of Laeven and Valencia (2012) to estimate the quarterly probability of a crisis start for banking crises in 35 advanced countries 1970–2011.\textsuperscript{26} The equation and estimates are

\[ q_t = \frac{\exp(X_t)}{1 + \exp(X_t)}, \]

where\textsuperscript{27}

\[ X_t = -5.630^{***} - 5.650^{*} g_t + 4.210 g_{t-4} + 12.342^{**} g_{t-8} - 5.259 g_{t-12}. \]

As in the Schularick and Taylor (2012) estimates, the annual growth rate of the average annual debt lagged two years is the major determinant of the probability of a crisis start, \( q_t \). For 5 percent steady real debt growth, the annual probability of a crisis start is 1.89 percent, approximately equal to the frequency of crises starts in the sample. It implies a crisis start on average every 53 years. The corresponding constant quarterly probability of a crisis start, \( q \), is thus about 0.47 percent. The coefficients in (6.1) sum to 5.64, implying that the marginal effect on the annual probability

\textsuperscript{26} I am grateful to Damiano Sandri for several discussions about the estimates.

\textsuperscript{27} One, two, and three stars denote significance at the 10, 5, and 1 percent level, respectively.
of a crisis start over all lags is equal to 0.11, implying the summary result that 1 percentage point lower steady real debt growth reduces the annual probability of a crisis by about 0.1 percentage points.

Figure 6.1 shows the resulting effect of the policy rate on the probability of a crisis start and of a crisis. Comparing with the previous figure 2.2 for the Schularick and Taylor (2012) estimates, we see that now the effect on the probability of a crisis start \( \frac{dq_t}{d\tilde{f}_1} \), the blue line) fluctuates more. As a result, the effect on the probability of a crisis \( \frac{dp_t}{d\tilde{f}_1} \), the green line) also fluctuates more: first it increases relative to the baseline before it falls to a negative peak of -0.27 percentage point in quarter 17, after which it increases and reaches a positive peak of 0.15 percentage points in quarter 26, after which it finally falls to zero in quarter 40. It lacks the long positive tail that shows in figure 2.2. The accumulated effect on the probability of a crisis is of course still approximately zero, (2.8).

Figure 6.2 shows the resulting effect on the expected future unemployment rate, which is still very similar to the effect on the expected future non-crisis rate. Clearly, a higher policy rate increases the expected unemployment rate at all horizons. The effect on the expected unemployment rate does not provide any support for leaning against the wind.

Figure 6.3 shows the corresponding marginal cost, marginal benefit, and net marginal cost
Figure 6.2: The effect on the expected unemployment rate and the expected non-crisis unemployment rate of a 1 percentage point higher policy rate during quarters 1–4; deviations from baseline. (Source: IMF staff estimates, Sveriges Riksbank, and own calculations.)

Figure 6.3: The marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap equals zero. (Source: IMF staff estimates, Sveriges Riksbank, and own calculations.)
according to (3.18)-(3.20). The marginal cost depends on \( p_t \), which in this case, by (3.7) increases from 0 in quarter 1 to 1.89 percent in quarter 9, after which it stays at 1.89 percent. As a result, the marginal cost has a lower peak and is smaller than in the previous figure 3.3. Compared with the previous figure, the marginal benefit now fluctuates more. In quarters 17-19, it is equal to the marginal cost, making the net marginal cost equal to zero for those quarters. However, around quarter 8 and, in particular, around quarter 26, the marginal benefit is negative and adds to the net marginal cost. The sum of the marginal benefits over the 40 quarters is \(-0.02\) percentage points and thus very close to zero (so the sum of the net marginal costs over the 40 quarters is approximately equal to the sum of the marginal costs, as in (3.22)). Again, the net marginal cost of leaning against the wind is clearly positive, implying that leaning with the wind, not against, is justified.

7 Conclusions

The conclusions from this analysis are quite strong:

For existing empirical estimates and reasonable assumptions, the marginal cost of leaning against the wind is much higher than the marginal benefit. Thus, leaning against the wind is not justified. If anything, a modest leaning with the wind, in the sense of a somewhat lower policy rate, is justified.

The main component of the marginal cost of leaning against the wind is the marginal cost of increasing the crisis unemployment gap. Leaning against the wind increases both the non-crisis and the crisis unemployment gaps. Even if the initial non-crisis unemployment gap is zero, in which case the marginal cost of increasing the non-crisis unemployment gap is zero, the crisis unemployment gap is not zero, and the marginal cost of increasing the crisis unemployment gap is positive.

The main component of the marginal benefit is the reduction in the expected cost of a crisis due to a possibly lower probability of a crisis from a higher policy rate. For existing empirical estimates and channels, this possible effect of the policy rate on the probability of a crisis is too small to match the marginal cost of a higher policy rate.

The main empirical channel through which the policy rate might reduce the probability of a crisis is via an effect on the growth rate of real debt. But if monetary policy is neutral in the long run, there is no effect on the accumulated real debt growth over the longer run. A possible lower real debt growth rate and a lower probability of a crisis for a few years is then followed by a higher
growth rate and a higher probability of a crisis in later years. The probability of a crisis is shifted between periods, but there is no effect on the average and accumulated probability of a crisis over the longer run. Then neither is there any effect on the average and accumulated marginal benefit over the longer run.

Even if monetary policy would be non-neutral and able to reduce the real debt level and thereby the accumulated debt growth in the longer run, so the accumulated marginal benefit would be positive in the longer run, empirically the marginal benefit is still too small to match the marginal cost. For leaning against the wind to be justified, the effect of the policy rate on the probability of a crisis must be so large as to be completely unrealistic.

It is sometimes argued that leaning against the wind is justified if macroprudential policy is less effective. But if macroprudential is less effective and this results in a crisis being more likely, being deeper, or having a longer duration, the marginal cost of a crisis increases more than the marginal benefit, making the case against leaning against the wind even stronger.

These results are robust for an alternative dataset of Laeven and Valencia (2012) with more recent data and more countries than the baseline Schularick and Taylor (2012) dataset used.

To this can be added some results, discussed in detail in appendix F, for the unrealistic assumption of the cost of a crisis being fixed and independent of the initial state of the economy. Then, if the initial non-crisis unemployment gap is zero, the marginal cost of leaning against the wind is zero, whereas the marginal benefit is small but positive. Then some leaning against the wind is optimal. But, for existing empirical estimates, the optimal leaning against the wind is extremely small and correspond to only a few basis points higher policy rate and non-crisis unemployment gap (this is in line with the results of Ajello, Laubach, Lopez-Salido, and Nakata (2015)). The net reduction in the expected loss of a future crisis, the net gain from leaning against the wind, is completely insignificant. This is the case even under the assumption of monetary non-neutrality and a permanent effect on real debt and positive effect on accumulated marginal benefit. Thus, even then any significant leaning of the wind is not justified.

In summary, no support for leaning against the wind is found, even under assumptions strongly biased in favor of it. A minimal policy conclusion is that no leaning against the wind should be undertaken without support from a thorough cost-benefit analysis. Given the strong case against leaning against the wind, the burden of proof should be on its advocates. So far, it seems extremely unlikely that such a cost-benefit analysis would lend any support.

Another policy conclusion is that, when it comes to reducing the probability and/or severity of
a financial crisis, there seems to be no choice but to use other policies than monetary policy, such as micro- and macroprudential policy, housing policy, or fiscal policy. Regarding the potential of micro- and macroprudential policy, results by an IMF team in a soon-to-be-published IMF Staff Discussion Note indicate that 15–22 percent bank capital relative to risk-weighted assets would have been enough to avoid 85 percent of the historical banking crises in the OECD countries since 1970 (Blanchard (2015)); thus, sufficient capital may lead to a dramatic reduction in the probability of a crisis.

References


Appendix

A A Markov process for crisis and non-crisis states

Consider the situation when the probability of a crisis start is $q$ and the duration of a crisis is $n$ quarters. We can model this as a Markov process with $n + 1$ states, where state 1 corresponds to a non-crisis and state $j$ for $2 \leq j \leq n + 1$ corresponds to a crisis in its $(j - 1)$th quarter.\(^\text{28}\)

Let the $(n + 1) \times (n + 1)$ transition matrix be $P = [P_{ij}]$, where $P_{ij} = \Pr(j \mid i)$ is the probability of a transition from state $i$ in quarter $t$ to state $j$ in quarter $t + 1$. The transition probabilities will be zero except for $P_{11} = 1 - q$, $P_{12} = q$, $P_{i,i+1} = 1$ for $2 \leq i \leq n$, and $P_{n+1,1} = 1$. As an illustration, for $n = 3$ the $4 \times 4$ transition matrix is

$$
P = \begin{bmatrix}
1 - q & q & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0
\end{bmatrix}.
$$

Let the row vector $\pi_t = (\pi_{ti})_{i=1}^{n+1}$ denote the probability distribution in quarter $t$, and let $\pi_1 = (1, 0, \ldots, 0)$, corresponding to a non-crisis in quarter 1. Then the probability distribution in quarter $t \geq 1$, conditional on a non-crisis in quarter 1, is given by

$$
\pi_t = \pi_1 P^t,
$$

and the probability of crisis in quarter $t$, $p_t$, is given by

$$
p_t = 1 - \pi_{t1} \quad \text{for } t \geq 1. \quad \text{(A.1)}
$$

Figure A.1 shows the result for the linear approximation (2.2) (as in figure 3.1) and the Markov process (A.1), when $q = 0.8$ percent and $n = 8$ quarters. The probability of a crisis converges to 6.4 percent for the linear approximation and to 6.0 percent for the Markov process. The linear approximation thus exaggerates the probability of a crisis somewhat.

The main advantage with the linear approximation (2.2), is that the effect of the policy rate on the probability of a crisis is easy to calculate. Given the effect on the probability of a crisis start, $dq_t/d\tau_1$ for $t \geq 1$, from figure 2.2, it simply satisfies

$$
\frac{dp_t}{d\tau_1} = \sum_{\tau=0}^{n-1} \frac{dq_t}{d\tau_1}. \quad \text{(A.2)}
$$

\(^{28}\) I am grateful for helpful discussion with Stefan Laséen and David Vestin on the Markov process of crisis and non-crisis states.
Figure A.1: The probability of a crisis in quarter by the linear approximation (2.2) and by the Markov process (A.1) for $q = 0.8$ percent and $n = 8$ quarters.

Figure A.2: The effect on the probability of a crisis for the linear approximation (A.2) and the Markov process (A.3) (for $q = 0.8$ percent and $n = 8$ quarters) of a 1 percentage points higher policy rate during quarters 1–4.
For the Markov process, the calculation is a bit more complicated. Let \( P_t = [P_{t,ij}] \) for \( t \geq 1 \), denote the transition matrix from states in quarter \( t \) to states in quarter \( t+1 \), where \( P_{t-1,11} = 1 - q_t \), \( P_{t-1,12} = q_t \) and \( P_{t-1,ij} = P_{ij} \) for \( (i,j) \neq (1,1), (1,2) \). Furthermore, we can write

\[
\pi_t = \pi_{t-1} P_{t-1} \quad \text{for} \quad t \geq 2.
\]

Then the effect of the policy rate on the probability distribution satisfies

\[
\frac{d\pi_t}{d\delta_1} = \frac{d\pi_{t-1}}{d\delta_1} P_{t-1} + \pi_{t-1} \frac{dP_{t-1}}{d\delta_1} \quad \text{for} \quad t \geq 2,
\]

where \( dP_{t-1,11}/d\delta_1 = 1 - dq_t/d\delta_1 \), \( dP_{t-1,12}/d\delta_1 = dq_t/d\delta_1 \), and \( dP_{t-1,ij}/d\delta_1 = 0 \) for \( (i,j) \neq (1,1), (1,2) \). Then

\[
\frac{dp_t}{d\delta_1} = - \frac{d\pi_{t1}}{d\delta_1} \quad \text{for} \quad t \geq 2. \tag{A.3}
\]

Figure A.2 shows the effect of a higher policy rate on the probability of a crisis, for the linear approximation (A.2) (as in figure 2.2) and the Markov process (A.2). The linear approximation exaggerates somewhat the effect on the probability of a crisis and thus the marginal benefit of leaning against the wind.

## B The logistic function

Consider the logistic function

\[
q = \frac{\exp(a + bg)}{1 + \exp(a + bg)} = \frac{1}{1 + \exp[-(a + bg)]},
\]

where \( q \) is a probability, \( g \) is the steady annual growth rate of real debt and \( a \) and \( b \) are constants. In a logit regression of crises starts on current and lagged annual growth rates of real debt, \( b \) then corresponds to the sum of the coefficients on the lagged annual growth rates. We have the convenient expression for the derivative of \( q \) with respect to \( g \), the marginal effect of steady real debt growth on the probability \( q \),

\[
\frac{dq}{dg} = bq(1 - q).
\]

[To be completed.]
C The simple loss function and a loss function in terms of inflation and unemployment

Assume a quadratic loss function of inflation and unemployment,

\[ L_t^* (\pi_t, u_t) \equiv \pi_t^2 + \lambda (u_t - \bar{u})^2, \]  

(C.1)

where \( \pi_t \) denotes the gap between the inflation rate and and a fixed inflation target in quarter \( t \), and \( u_t - \bar{u} \) is the gap between the unemployment rate \( u_t \) in quarter \( t \) and the long-run sustainable unemployment rate \( \bar{u} \). Assume a simple Phillips curve,

\[ \pi_t = z_t - \gamma (u_t - \bar{u}), \]  

(C.2)

where the intercept, \( z_t \), is a stochastic process representing cost-push shocks that cause a tradeoff between achieving an inflation rate equal to the inflation target and an unemployment rate equal to the long-run sustainable rate. A positive (negative) \( z_t \) implies that a zero inflation gap requires a positive (negative) unemployment tap.

By combining (C.1) and (C.2), the loss function incorporating the Phillips curve can be written

\[ L_0^t ((u_t - \bar{u}); z_t) \equiv L_t^* [z_t - \gamma (u_t - \bar{u}), u_t] = [z_t - \gamma (u_t - \bar{u})]^2 + \lambda (u_t - \bar{u})^2 \]

(C.3)

where

\[ u^* (z_t) - \bar{u} \equiv \frac{\gamma z_t}{\gamma^2 + \lambda}. \]  

(C.4)

It follows from (C.3) that \( u^* (z_t) \), given by (C.4), is the unemployment rate that for given \( z_t \) minimizes the loss function (C.1) subject to the Phillips curve (C.2). Furthermore, it is clear that choosing \( u_t \) to minimize the simple quadratic loss function

\[ L_t (u_t; u^*_t) \equiv (u_t - u^*_t)^2, \]  

(C.5)

where \( u^*_t \equiv u^* (z_t) \) is equivalent to choosing \( u_t \) to minimize the loss function \( L_0^t (u_t; z_t) \) incorporating the Phillips curve. I call \( u^*_t \) the benchmark unemployment rate.

A crisis is considered to be a negative demand shock that for a given policy setting increases the unemployment rate by the fixed amount \( \Delta u > 0 \). It is assumed to be independent of \( z_t \) and thus independent of the benchmark unemployment rate.
D The effect of the policy rate on the crisis increase in the unemployment rate

A possible benefit of a higher policy rate might be a smaller increase in the unemployment rate in a crisis. According to Flodén (2014), for the OECD countries, a lower household debt-to-income ratio in 2007 is associated with a lower increase in the unemployment rate during 2007-2012. More precisely, a 1 percentage point lower DTI ratio is associated with a 0.02 percentage point lower increase in the unemployment rate. This is a small effect. It is statistically significant for the sample of all OECD countries but not for the sample of OECD countries during which housing prices fell.

If a higher policy rate would lower the debt-to-income ratio, a higher policy rate might through this channel reduce the severity of a crisis, by reducing the crisis increase in the unemployment rate, \( d\Delta u/d\bar{\delta}_1 < 0 \). Taking such a possibility into account, the effect of the policy rate on the expected unemployment rate, (2.4), will by (2.3) have a third term.

\[
\frac{dE_t u_t}{d\bar{\delta}_1} = \frac{dE_t u^0_t}{d\bar{\delta}_1} + \Delta u \frac{dp_t}{d\bar{\delta}_1} + p_t \frac{d\Delta u}{d\bar{\delta}_1}.
\] (D.1)

Furthermore, the effect of a higher policy rate on the expected quadratic loss, (3.13), will by have an additional term,

\[
\frac{dE_t L_t}{d\bar{\delta}_1} = 2[E_t \tilde{u}^0_t + p_t \Delta u] \frac{dE_t u^0_t}{d\bar{\delta}_1} + [(\Delta u)^2 + 2\Delta u E_t \tilde{u}^0_t] \frac{dp_t}{d\bar{\delta}_1} + 2p_t [\Delta u + E_t \tilde{u}^0_t] \frac{d\Delta u}{d\bar{\delta}_1}.
\]

This leads to an additional term in the marginal benefit, (3.17),

\[
MB_t(E_t \tilde{u}^0_t) = [(\Delta u)^2 + 2\Delta u E_t \tilde{u}^0_t](\frac{dp_t}{d\bar{\delta}_1}) + 2p_t [\Delta u + E_t \tilde{u}^0_t](\frac{d\Delta u}{d\bar{\delta}_1}).
\]

For (3.11), the marginal benefit is then given by

\[
MB_t(0) \equiv (\Delta u)^2(\frac{dp_t}{d\bar{\delta}_1}) + 2p_t \Delta u(\frac{d\Delta u}{d\bar{\delta}_1}).
\] (D.2)

I wrote “if a higher policy rate would lower the debt-to-income ratio,” because it is highly uncertain what the direction is of any effect of the policy rate on the debt-to-income ratio. A higher policy rate may slow down the growth of nominal debt, but it also slows down the growth
of nominal GDP and nominal income. Thus, the growth of both numerator and denominator of
the debt-to-income ratio are reduced. The stock of nominal debt is quite sticky, given the average
length and turnover rates of mortgages. If nominal income is affected faster or stronger, the debt-
to-income ratio will rise rather than fall. Regardless of the direction of the effect, it is likely to be
quite small.

Svensson (2013a) shows that under reasonable assumptions about the determinants of housing
prices and the stock of mortgages, in particular realistic assumptions about mortgage loan length
and mortgage turnover rates, for standard impulse responses for GDP and inflation in Sweden, a
higher policy rate is likely to increase the debt-to-income ratio before it eventually returns to the
baseline ratio. Since then, Alpanda and Zubairy (2014), Gelain, Lansing, and Natvik (2015), and
Robstad (2014) has provided empirical and theoretical support for a higher policy rate increasing
rather than reducing the debt-to-income ratio.

As an example, I use the Sveriges Riksbank (2014a) estimate of the effect on the Swedish
household debt-to-income ratio of a 1 percentage point higher policy rate during four quarters,
the point estimate of which is shown as the red line in figure D.1. It shows the debt-to-income
ratio falling below baseline from a policy-rate increase. However, as discussed in Svensson (2014)
and Svensson (2015), the 90-percent uncertainty band around the estimate is very wide, and the
estimate is not statistically significant and could be of the opposite sign. Nevertheless, for the sake
of the argument, I take it as given and examine the resulting effects.

We see that the largest effect, a negative peak of $-1.4$ percentage points, occurs already after
4 quarters. Given the estimate of Flodén (2014), this would imply that the crisis increase in the
unemployment rate would be $-d\Delta u/d\tilde{u}_1 = 0.02 \cdot 1.4 = 0.28$ percentage points lower, that is, the
 crisis increase would be 4.72 percentage points rather than 5 percentage points. However, after 5
years, in quarter 20, the debt-to-income ratio is only 0.44 percentage points below the baseline,
meaning that the increase in the crisis unemployment rate would be $-d\Delta u/d\tilde{u}_1 = 0.088$ percentage
points less.\footnote{This is the summary results I have used in Svensson (2014) and Svensson (2015).}

Furthermore, to get the effect on the expected unemployment rate, the third term in (D.1),
these numbers should be multiplied by the probability $p_t$ of a crisis in quarter $t$, (3.7). This results
in very small effects, shown as the blue line in figure D.1. The largest effect is a negative peak in
quarter 9, when the probability of a crisis has increased to 6.4 percent. The peak is $-0.12$ basis
points = $-0.0012$ percentage points, which is only a tenth of the largest difference in figure 2.3,
which difference is already very small. Clearly, the effect through this channel on the expected future unemployment rate can be disregarded.

In order to calculate the additional third term in the marginal benefit (D.2), we simply have to multiply this third term in the expected unemployment term (D.1) by $2\Delta u = 10$ percentage point and switch the sign to get the green line in figure D.1. It thus has a positive peak at 0.012 percentage points for quarter 11, which is small relative to the peak of the marginal benefit in figure 3.3.

It seems clear that the conclusions of this paper are not affected by disregarding this (doubtful) effect of the policy rate on the crisis increase in the unemployment rate.

**E  Kocherlakota on the value of eliminating the possibility of a crisis**

An early and innovative cost-benefit analysis of leaning against the wind is provided by Kocherlakota (2014). He assesses the value of eliminating the probability of a crisis, in the sense of reducing the probability of a crisis to zero. He assumes that the crisis increase in the unemployment rate is 4 percentage points, that the expectation in 2014 of the 2017 unemployment rate is equal to a natural
unemployment rate of 5 percent, and that a crisis would therefore imply that the unemployment rate would reach 9 percent. As an estimate of the upper limit of the probability of a crisis, he then uses the probability of the 2017 unemployment rate exceeding 9 percent that can be inferred from the 2014Q1 Survey of Professional Forecasters. This probability is 0.29 percent. It is considered an upper limit because the unemployment rate could exceed 9 percent for other reasons than a crisis. We immediately notice that this probability is much smaller than the probability of crisis beyond quarter 9 of 6.4 percent used here. Given Kocherlakota’s estimate, the expected loss increase of a crisis is $0.0029 \cdot 4^2 = 0.0464 = 0.22^2$. That is, eliminating the possibility of a crisis is worth an expected non-crisis unemployment gap from zero to only 0.22 percentage points.

With a crisis increase in the unemployment rate of 5 percentage points, as assumed in the present paper, the expected loss from a crisis would be $0.0029 \cdot 25 = 0.0725 = 0.27^2$, in which case it is worth an increase in the expected non-crisis unemployment gap from zero to 0.27 percentage points.

For the baseline assumptions in the present paper, the steady-state probability of a crisis is 6.4 percent, so the expected loss equals $0.064 \cdot 25 = 1.6 = 1.26^2$ under the baseline assumptions. Thus, reducing the probability of a crisis from 6.4 percent to zero is under the baseline assumptions worth an increase in the non-crisis unemployment gap from zero to 1.26 percentage points, a substantial increase.

For the IMF staff estimates discussed in section 6, the steady-state probability of a crisis is 3.78 percent, implying that the expected loss due to the possibility of a crisis is $0.0378 \cdot 25 = 0.945 = 0.97^2$. Then eliminating the possibility of a crisis is worth an increase in the expected non-crisis unemployment gap of 0.97 percentage points, still a substantial increase.

These numbers are much higher than the estimate in Kocherlakota (2014). The main difference is that the estimates of a probability of a crisis that follow from Schularick and Taylor (2012) or the IMF staff estimates, 6.4 and 3.8 percent, respectively, are much higher than the estimate from Survey of Professional Forecasts, 0.29 percent. However, I am not sure that the forecasts in the Survey of Professional Forecasts take the possibility of a crisis into account. If they don’t, they can obviously not be used to infer the professional forecasters’ estimate of a probability of a crisis.
The alternative assumption of a fixed cost of a crisis

So far we have made the natural assumption that (2.1) holds and that hence a crisis means that the unemployment gap increases by \( \Delta u > 0 \). This means that the expected loss in a crisis loss depends on the non-crisis unemployment gap,

\[
E_1(\tilde{u}_t^c)^2 = E_1(\tilde{u}_t^n + \Delta u)^2,
\]

such that a higher (positive) expected non-crisis unemployment gap increases the expected loss in a crisis. It is natural that the cost of a crisis is larger if the economy initially is weaker and thus depends on the non-crisis unemployment gap.

This in turn means, as we have seen, that the net marginal cost of leaning against the wind, the marginal expected loss of increasing the policy rate, is positive also when the non-crisis unemployment gap is zero,

\[
\text{NMC}_t(E_1\tilde{u}_t^n) = \frac{dE_1L_t}{di_1} = \frac{d}{di_1}[(1 - p_t)E_1(\tilde{u}_t^n)^2 + p_tE_1(\tilde{u}_t^n + \Delta u)^2] = 2(E_1\tilde{u}_t^n + p_t\Delta u)\frac{dE_1t^n}{di_1} - [(\Delta u)^2 + 2E_1\tilde{u}_t^n\Delta u](-\frac{dp_t}{di_1}) = \text{MC}_t(E_1\tilde{u}_t^n) - \text{MB}_t(E_1\tilde{u}_t^n),
\]

so

\[
\text{MC}_t(0) = 2p_t\Delta u \frac{dE_1t^n}{di_1}.
\]

However, suppose that we instead make the less natural assumption that a crisis means that the expected unemployment gap does not increase by \( \Delta u \) but reaches \( \Delta u \), regardless of what the expected non-crisis unemployment gap \( u_t^n \) is. That is, the crisis unemployment gap simply satisfies

\[
E_1\tilde{u}_t^c = \Delta u,
\]

regardless of \( E_1\tilde{u}_t^n \). This means that the cost of a crisis is fixed and independent of the non-crisis unemployment gap, in line with the assumption made in Svensson (2014), Svensson (2015), Ajello, Laubach, Lopez-Salido, and Nakata (2015), and Diaz Kalan, Laséen, Vestin, and Zdzienicka (2015).

First, regarding the expected unemployment gap, we note that it is now given by

\[
E_1\tilde{u}_t = (1 - p_t)E_1\tilde{u}_t^n + p_t\Delta u,
\]

and the effect of the policy rate is

\[
\frac{dE_1\tilde{u}_t}{di_1} = (1 - p_t)\frac{dE_1\tilde{u}_t^n}{di_1} + (\Delta u - E_1\tilde{u}_t^n)\frac{dp_t}{di_1}.
\]
Figure F.1: For a fixed cost of a crisis, the effect on the expected unemployment gap and its component of a 1 percentage point higher policy rate during quarters 1–4, when the expected unemployment gap is zero.

Figure F.1 shows the effect on the expected non-crisis unemployment gap ($d\bar{u}_t^n/d\bar{d}_1$, the red solid line), the expected unemployment gap for an exogenous probability of a crisis ($\left(1 - p_t\right)d\bar{u}_t^n/d\bar{d}_1$, the red dashed line), and the expected unemployment gap ($d\bar{u}_t/d\bar{d}_1$, the blue line). The green line shows the difference between the last two, that is, the difference due to the effect on the probability of a crisis, when the expected non-crisis unemployment gap is zero ($\Delta udp_t/d\bar{d}_1$, measured along the right in basis points). The difference is the same as in figure 2.3 and thus equally small.

Second, regarding the effect on the expected quadratic loss, we first note that it is practical to make the innocuous assumption that the conditional variance of the crisis unemployment gap is the same as that of the non-crisis unemployment gap. It follows that the expected loss in a crisis satisfies

$$E_1(\bar{u}_t^c)^2 = (\Delta u)^2 + \text{Var}_1\bar{u}_t^c = (\Delta u)^2 + \text{Var}_1\bar{u}_t^n,$$

and is independent of the expected non-crisis unemployment gap, $E_1\bar{u}_t^n$.

Then the expected quarter-$t$ loss is

$$E_1L_t = (1 - p_t)E_1(\bar{u}_t^n)^2 + p_t[(\Delta u)^2 + \text{Var}_1\bar{u}_t^n]$$

$$= (1 - p_t)(E_1\bar{u}_t^n)^2 + p_t(\Delta u)^2 + \text{Var}_1\bar{u}_t^n,$$

where I have used that $E_1(\bar{u}_t^n)^2 = (E_1\bar{u}_t^n)^2 + \text{Var}_1\bar{u}_t^n$. The quarter-$t$ marginal loss of increasing the
Figure F.2: For a fixed cost of a crisis, the marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap is zero.

policy rate, the net marginal cost of leaning against the wind, is then

$$NMC_t(E_1 \tilde{u}_t^n) = \frac{dE_1L_t}{dt_1} = (1 - p_t)2E_1 \tilde{u}_t^n \frac{dE_1u_t^n}{dt_1} + [(\Delta u)^2 - E_1(\tilde{u}_t^n)^2] \frac{dp_t}{dt_1}. $$

The marginal cost and marginal benefit of leaning against the wind hence satisfy

$$MC_t(E_1 \tilde{u}_t^n) = 2(1 - p_t)E_1 \tilde{u}_t^n \frac{dE_1u_t^n}{dt_1}, $$

$$MB_t(E_1 \tilde{u}_t^n) = [(\Delta u)^2 - E_1(\tilde{u}_t^n)^2](-\frac{dp_t}{dt_1}). $$

Furthermore,

$$MC_t(0) = 0, $$

$$MB_t(0) = (\Delta u)^2(-\frac{dp_t}{dt_1}). $$

That is, if the expected non-crisis unemployment gap is zero, the marginal cost is now zero, not positive as in (3.19, whereas at the marginal benefit is the same as in (3.20) and positive if the probability of a crisis is decreasing in the policy rate.

We also note that the loss increase in a crisis, the term in the squared bracket in (F.5), is decreasing in $(E_1 \tilde{u}_t^n)^2$. It is at its maximum when the expected non-crisis unemployment gap is zero and becomes negative when the expected non-crisis unemployment gap exceeds $\Delta u$, $E_1 \tilde{u}_t^n > E_1 \tilde{u}_t^C = \Delta u$. Of course, in the (unlikely) situation in which the expected non-crisis unemployment gap is greater than the expected crisis unemployment gap, it is better to be in a crisis (under the
Figure F.3: For a fixed cost of a crisis, the marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap is positive and equal to 0.25 percentage points for all quarters.

The maintained assumption that the conditional variance of the crisis unemployment gap is not larger than that of the non-crisis unemployment gap).

Figure F.2 shows the marginal cost, marginal benefit, and net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap is zero the loss in a crisis is fixed. Because the marginal cost is zero, the net marginal cost is simply the negative of the marginal benefit and thus negative for quarters 1–23 and positive for quarter 24 and beyond. Because of the neutrality of money, the accumulated marginal benefit and net marginal cost over a long horizon are approximately zero. However, because the marginal benefit is positive earlier and negative later, the sum of the discounted marginal benefits would be positive, whereas the sum of the discounted marginal costs would be zero. Then a small amount of leaning against the wind would be optimal.

If the expected non-crisis unemployment gap is not zero but positive, by (F.4) the marginal cost of leaning against the wind is no longer zero but positive. In figure F.3, the dashed lines show the marginal cost, marginal benefit, and net marginal cost when the expected non-crisis unemployment gap is 0.25 percentage points for all quarters. The solid lines show the same, when the expected non-crisis unemployment gap is zero for all quarters. There is a substantial increase in the marginal cost but no noticeable change in the marginal benefit (because in (F.5) the term \((E_1 \tilde{u}_t^n)^2 = 0.25^2 = 0.0625\) is so small relative to the term \((\Delta u)^2 = 25\)). Clearly, sum of discounted net marginal costs is positive. This shows that a small positive expected unemployment gap removes
any justification for leaning against the wind, also in the case when the cost of a crisis is fixed. We
realize that this would be the case also if the expected non-crisis unemployment gap would be 0.25
percentage points just for the first 12 quarters (or even for the first 8 quarters) and then zero after.

This indicates that any optimal leaning against the wind is very small, definitely much smaller
than that resulting in an expected non-crisis unemployment gap of 0.25 percentage point. Let me
examine the optimal leaning against the wind further.

F.1 The optimal leaning against the wind

What is the optimal policy? We can simplify this question a bit by just asking, given the above
relations, what is the optimal policy rate \( i_0 \) and the optimal non-crisis unemployment gap, \( \tilde{u}_t \)? That is, if policy is constrained to choosing the same constant policy rate \( \tilde{i}_1 \) for quarters
1–4 and return the policy rate to its baseline level after that, what level of the policy rate \( \tilde{i}_1 \) is then
optimal. Thus, as in appendix H, we are considering a particular constrained optimal policy, not
an unconstrained policy.

Proceeding as in appendix H, by (F.4) and (F.5) we have

\[
MC_t(E_1 \tilde{u}_t^n) = 2(1 - p_t)E_1 \tilde{u}_t^n \frac{dE_1 u_t^n}{d\tilde{i}_1} \\
= 2(1 - p_t)(\frac{dE_1 u_t^n}{d\tilde{i}_1})^2 \tilde{i}_1 \\
= f_t \tilde{i}_1 \quad (F.8)
\]

\[
MB_t(E_1 \tilde{u}_t^n) = [(\Delta u)^2 - E_1 (\tilde{u}_t^n)^2](- \frac{dp_t}{d\tilde{i}_1}) \\
= (\Delta u)^2(- \frac{dp_t}{d\tilde{i}_1}) + (\frac{dE_1 u_t^n}{d\tilde{i}_1})^2 \frac{dp_t}{d\tilde{i}_1} \tilde{i}_1^2 \\
= g_t + h_t (\tilde{i}_1)^2. \quad (F.9)
\]

Given these definitions of \( f_t \), \( g_t \), and \( h_t \) for \( t \geq 1 \), we have

\[
NMC_t(E_1 \tilde{u}_t^n) = f_t \tilde{i}_1 - g_t - h_t (\tilde{i}_1)^2.
\]

We are thus looking for a solution to the quadratic equation

\[
\sum_{t=1}^{40} \delta^{t-1}NMC_t(E_1 \tilde{u}_t^n) = \sum_{t=1}^{40} \delta^{t-1}[\tilde{h}_t (\tilde{i}_1)^2 + f_t \tilde{i}_1 - g_t] = A(\tilde{i}_1)^2 + 2B\tilde{i}_1 + C = 0,
\]

where

\[
A \equiv \sum_{t=1}^{40} - \delta^{t-1}h_t, \quad B \equiv \frac{1}{2} \sum_{t=1}^{40} \delta^{t-1}f_t, \quad C \equiv \sum_{t=1}^{40} - \delta^{t-1}g_t. \quad (F.10)
\]

52
Figure F.4: For a fixed cost of a crisis and a horizon of 24 quarters, the optimal policy rate and the corresponding discounted net marginal cost, expected non-crisis unemployment gap (all left axis), and expected unemployment gap (right axis)

It follows that the optimal \( \tilde{i}_1 \) will be given by one of the two solutions,

\[
\tilde{i}_1 = -\frac{B}{A} \pm \sqrt{\frac{B^2}{A^2} - \frac{C}{A}}.
\]

**F.2 Stacking the cards further in favor of leaning against the wind**

Before solving for the optimal policy, let me stack the cards further in favor of leaning against the wind. Suppose the policymaker only considers a horizon of 24 quarters. That is, the policymaker disregards the negative marginal benefit beyond quarter 24 in figure F.2. Then the discounted marginal benefit is positive while the discounted marginal cost is zero when the expected unemployment gap is zero, and some leaning will be optimal.

Thus, suppose the policy maker solves for the optimal constant quarter 1–4 policy rate \( \tilde{i}_1 \), taking only quarters 1–24 into account. This means solving for the optimal \( \tilde{i}_1 \) when sums in (F.10) run from 1 to 24 instead of from 1 to 40. Then \( A \) and \( B \) will be positive and \( C \) will be negative. The relevant solution in (F.11) is the one with a plus before the square root. The solution is shown in figure F.4, for an annual discount factor of 0.98.

It is striking how small the optimal leaning is, with a policy-rate increase of only 0.11 percentage points (the grey line). The expected non-crisis unemployment gap (the red line) is only about 0.05 percentage points at its maximum (in quarter 6), thus, very close to zero. The maximum reduction
Figure F.5: For a fixed cost of a crisis and a horizon of 24 quarters, the optimal policy rate and the corresponding discounted net marginal cost, marginal cost, and marginal benefit in the probability of a crisis is only 0.025 percentage points (in quarter 18), a small reduction compared to the steady-state probability of a crisis of 6.4 percent. This optimal leaning of the wind only reduces the expected intertemporal loss is only reduced by 0.07 percent of its baseline value. This is in spite of my having stacked the cards in favor of leaning against the wind, by assuming a fixed cost of a crisis so the marginal cost of leaning is zero at an initial zero expected non-crisis unemployment gap, and by restricting the horizon to 24 quarters so as to maximize the accumulated marginal benefit by disregarding the negative marginal benefit beyond that horizon.

Figure F.5 shows more detail, not only the discounted net marginal cost but also the discounted marginal cost and marginal benefit. The sum of net marginal cost from quarter 1 to quarter 24 is zero. The net marginal cost is composed mainly of the marginal cost up to quarter 12 and of the negative of the marginal benefit from quarter 12 to quarter 24.

Clearly, the main message is that, even if the cards are stacked in favor of leaning against the wind, the optimal leaning against the wind and the reduction in the intertemporal loss is insignificant, and the obvious policy conclusion is that even then, leaning against the wind is not justified and worth bothering about.

**F.3 The “quarterly-optimal” expected non-crisis unemployment gap**

We can illustrate how small the optimal leaning against the wind is in a different way. Consider the quarter-\(t\) marginal cost with respect to an increase in the expected non-crisis unemployment
gap (rather than with respect to an increase in the policy rate, therefore the subindex $u$),

$$MC_{ut}(E_t \tilde{u}_t^n) = 2(1 - p_t)E_t \tilde{u}_t^n,$$

and the corresponding marginal benefit from an increase in the non-crisis unemployment gap,

$$MB_{ut}(E_t \tilde{u}_t^n) = [(\Delta u)^2 - E_t(\tilde{u}_t^n)^2](- \frac{dp_t}{dE_t u_t^n}).$$

Here

$$- \frac{dp_t}{dE_t u_t^n} = - \frac{dp_t/d\tilde{u}_t}{dE_t u_t^n/d\tilde{u}_t}$$

denotes the decrease in the probability of a crisis in quarter $t$ associated with an increase in the expected quarter-$t$ unemployment gap. We can think of this as a measure of the tradeoff between a higher expected non-crisis unemployment rate and a lower probability of a crisis, the marginal transformation of a higher expected unemployment rate into a lower probability of a crisis.

Let the quarterly-optimal expected non-crisis unemployment gap be the unemployment gap that equalizes the quarter-$t$ marginal cost and benefit, that is, the expected non-crisis unemployment gap that is optimal when quarter $t$ is considered in isolation. This is the solution to this second-order equation,

$$2(1 - p_t)E_t \tilde{u}_t^n = [(\Delta u)^2 - E_t(\tilde{u}_t^n)^2](- \frac{dp_t}{dE_t u_t^n}).$$

However, the term $E_t(\tilde{u}_t^n)^2$ will be very small relative to $(\Delta u)^2$ and can be disregarded. (Since this means slightly increasing the marginal benefit, it will slightly increase, and therefore be an upper bound of, the quarterly-optimal expected non-crisis unemployment gap.) Then the quarterly-optimal expected non-crisis unemployment gap is given by

$$E_t \tilde{u}_t^n = \frac{(\Delta u)^2}{2(1 - p_t)}(- \frac{dp_t}{dE_t u_t^n}).$$

(F.12)

Figure F.6 shows for each quarter $1 \leq t \leq 40$ the reduction in the probability of a crisis per increase in the expected non-crisis unemployment gap and the quarterly-optimal expected non-crisis unemployment gap. We see that the most favorable probability-unemployment tradeoff occurs for quarter 20, and equals a probability reduction of 1.6 percentage points for an 1 percentage point expected non-crisis unemployment increase. That is, the probability of a crisis falls by 0.016 for an increase of 1 percentage point in the expected non-crisis unemployment gap. Given this, together with $\Delta u = 5$ percentage points and $p_{20} = 6.4$ percent, the quarterly-optimal expected non-crisis unemployment gap in (F.12) equals a modest 0.22 percentage points for the quarter when it is the largest. Again, it is striking how small the maximum quarterly expected non-crisis unemployment gap is. (Since $0.22^2 \approx 0.05$ is a small fraction of $(\Delta u)^2 = 25$, the above approximation is justified.)
Figure F.6: The reduction in the probability of a crisis per increase in the expected non-crisis unemployment gap and, for a fixed cost of a crisis, the quarterly-optimal expected non-crisis unemployment gap

G The debt-to-GDP term in Schularick and Taylor (2012, table 7, column (22))

Schularick and Taylor (2012, table 7, column (22)) contains a logit regression of the annual probability of a crisis start, where the log of debt-to-GDP ratio is added as an explanatory variable. The coefficient is 1.1 with a standard error of 0.624 and is significant at the 10 percent level. The estimates of the coefficients of the lagged real debt growth rates and of the sum of these coefficients do not change much: the sum is 9.984 rather than 9.697. We can represent this variant as

$$q_t = \frac{1}{1 + \exp(X_t + \gamma h_t)},$$

where $\gamma = 1.1$ and $h_t$ is the log of the debt-to-GDP ratio. The derivative of $q_t$ with respect to $h_t$ is

$$\frac{\partial q_t}{\partial h_t} = \frac{1}{4} \gamma 4q_t (1 - 4q_t) = \gamma q_t (1 - 4q_t).$$

With $q_t = 0.008$ and $\gamma = 1.1$, $\frac{\partial q_t}{dh_t} = 1.1 \cdot 0.008 \cdot 0.968 = 0.0085$. That is, 1 percentage point lower debt-to-GDP ratio lowers the probability of a crisis start by 0.0085 percentage points.

Figure G.1 shows the Riksbank’s estimate of the effect of the policy rate on the debt to GDP ratio (expressed in the percentage change) and the resulting separate effect on the probabilities of a crisis start and of a crisis in each quarter. We see that the effect is very small. In figure
Figure G.1: The effect on the debt-to-income ratio and the separate effect on the probability of a crisis start and of a crisis. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

Figure G.2: The estimates of the policy-rate effect on the unemployment rate, real debt and the debt-to-GDP ratio. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)
G.2, the dashed blue and green lines show the total effect on the probability if a crisis start and the probability of a crisis, when this additional effect via a lower debt-to-GDP ratio is taken into account. We see that the total effect is only marginally larger than the effect via real debt growth only.

H A constrained-optimal policy

What is the optimal policy? We can simplify this question a bit by just asking, given the relations presented in section 2 and 3, what is the optimal policy rate \( \bar{i}_1 \) and the optimal non-crisis unemployment gap, \( \bar{u}_t \equiv u_t^n - u_t^* \)? That is, if policy is constrained to choose the same constant policy rate \( \bar{i}_1 \) for quarters 1–4 and to return the policy rate to its baseline level after that, what level of the policy rate \( \bar{i}_1 \) is then optimal. Thus, we are considering a particular constrained optimal policy, not an unconstrained policy.

This means that we are looking for the policy rate that sets the derivative of the intertemporal loss function (3.2) equal to zero,

\[
\frac{d}{di_1} \sum_{t=0}^{\infty} \delta^{t-1} L_t = 0. 
\]  

(H.1)

We can write (H.1) as

\[
\sum_{t=0}^{\infty} \delta^{t-1} NMC_t(E_1 \bar{u}_t^n) = \sum_{t=0}^{\infty} \delta^{t-1} [MC_t(E_1 \bar{u}_t^n) - MB_t(E_1 \bar{u}_t^n)] = 0, \tag{H.2}
\]

and use that

\[
E_1 \bar{u}_t^n = \frac{dE_1 u_t^n}{di_1} \bar{i}_1 \text{ for } t \geq 1 \tag{H.3}
\]

and where I have normalized the policy rate such that \( \bar{i}_1 = 0 \) for (3.11). That is, I assume that we start from an equilibrium where (3.11) holds and consider deviating from that.

We can write, by (3.16) and (3.17),

\[
MC_t(E_1 \bar{u}_t^n) = 2p_t \Delta u \frac{dE_1 u_t^n}{di_1} + 2 \frac{dE_1 u_t^n}{di_1} E_1 \bar{u}_t^n
\]

\[
= 2p_t \Delta u \frac{dE_1 u_t^n}{di_1} + 2(\frac{dE_1 u_t^n}{di_1})^2 \bar{i}_1
\]

\[
\equiv a_t + b_t \bar{i}_1,
\]

\[
MB_t(E_1 \bar{u}_t^n) = (\Delta u)^2 (-\frac{dp_t}{di_1}) + 2 \Delta u (-\frac{dp_t}{di_1}) E_1 \bar{u}_t^n
\]

\[
= (\Delta u)^2 (-\frac{dp_t}{di_1}) + 2 \Delta u (-\frac{dp_t}{di_1}) \frac{dE_1 u_t^n}{di_1} \bar{i}_1
\]

\[
\equiv c_t + d_t \bar{i}_1.
\]

58
Figure H.1: The constrained-optimal policy rate, discounted net marginal cost, expected non-crisis unemployment gap, and expected unemployment gap. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

this way defining the coefficients $a_t, b_t, c_t,$ and $d_t$ for $t \geq 1$. Then we can write,

$$\text{NMC}_t(\mathbb{E}_1 \tilde{u}_t^n) = (a_t - c_t) + (b_t - d_t) \tilde{i}_1$$

$$\equiv f_t + g_t \tilde{i}_1; \quad (H.4)$$

this way defining the coefficients $f_t$ and $g_t$ for $t \geq 1$.

By (H.2) and (H.4), we are thus looking for the $\tilde{i}_1$ that solves

$$\sum_{t=1}^{\infty} \delta^{t-1} f_t + \sum_{t=1}^{\infty} \delta^{t-1} g_t \tilde{i}_1 = 0,$$

that is,

$$\tilde{i}_1 = - \frac{\sum_{t=1}^{\infty} \delta^{t-1} f_t}{\sum_{t=1}^{\infty} \delta^{t-1} g_t} .$$

Given that I only know the coefficients $f_t$ and $g_t$ for $1 \leq t \leq 40$, I will approximate by just solving for

$$\tilde{i}_1 = - \frac{\sum_{t=1}^{40} \delta^{t-1} f_t}{\sum_{t=1}^{40} \delta^{t-1} g_t} .$$

It turns out that the constrained-optimal policy in this case, with the numbers above and a quarterly discount factor of $\delta = 0.995$ (an annual discount factor of 0.98), is $\tilde{i}_1 = -0.83$ percentage point. That is, given the constraints specified above, the optimal policy is to reduce the policy rate for quarters 1–4 below the baseline by about 0.8 percentage points and then let it return to the baseline, as shown by the grey line in figure H.1.
Figure H.2: The constrained-optimal policy rate, discounted net marginal cost, expected non-crisis unemployment gap, and expected unemployment gap. (Source: IMF staff estimates, Sveriges Riksbank, and own calculations.)

The blue line show the corresponding discounted net marginal cost, $\delta^{t-1}\text{NMC}_t$ for $1 \leq t \leq 40$. By (H.2), they sum to zero for $1 \leq t \leq 40$. That is, the sum of the magnitude of the negative discounted net marginal costs up to quarter 10 equals the sum of the positive discounted net marginal costs from quarter 11.

The yellow and red lines show, respectively, the constrained-optimal (undiscounted) expected unemployment rate taking the possibility of a crisis into account and expected non-crisis unemployment rate; both are shown relative to the expected benchmark unemployment rate. We see that the expected unemployment rate including the possibility of a crisis is below the benchmark unemployment rate up to quarter 10 and above the benchmark from quarter 11. Furthermore, the expected non-crisis unemployment rate is below but approaches the benchmark unemployment rate at quarter 40 and beyond, meaning that the expected unemployment rate taking into account the possibility of a crisis approaches a level 0.32 percentage points above the benchmark unemployment rate.

We may ask why the optimal policy is not to keep the expected unemployment rate including the possibility of a crisis equal to the benchmark far into the future rather than allowing it to exceed the benchmark by a substantial margin. The reason is the severe constraint on policy here, that we are only considering changes in the policy rate during quarters 1–4. By figure 2.1, which shows the derivative $dE_1u^\alpha_t/d\bar{t}_1$, this constraint implies that the derivative is so small for quarters far into
the future that it is not possible to affect the expected non-crisis unemployment rate far into the future without affecting the expected non-crisis unemployment rate in the near future much more. Policy is constrained to scaling the expected non-crisis unemployment rate in figure 2.1 by positive or negative factors. Given this, the best policy is to scale it by \( \tilde{z}_1 = -0.83 \).

As a robustness test, assume that the policymaker would disregard the marginal cost and benefit beyond quarter 24, when the marginal benefit turns negative. This can also be seen as assuming some non-neutrality of monetary policy, such that there is a long-run effect on the real debt level. Here it means solving for the optimal \( \tilde{z}_1 \) when the discounted terms \( f_t \) and \( g_t \) are summed up to quarter 24,

\[
\tilde{z}_1 = - \sum_{t=1}^{24} \delta^{t-1} f_t \bigg/ \sum_{t=1}^{24} \delta^{t-1} g_t .
\]

The constrained-optimal policy rate is then given by \( \tilde{z}_1 = -0.68 \), that is, it consists of lowering the policy rate in quarters 1–4 by 0.68 percent, somewhat less than when the marginal cost and benefit in all quarters up through quarter 40 are taken into account.

Figure H.2 shows the constrained-optimal policy for the IMF staff estimates for the Laeven and Valencia (2012) dataset discussed in section 6. In this case the optimal \( \tilde{z}_1 = -0.51 \), that is, the constrained-optimal policy is to lower the policy rate in quarters 1–4 by 0.51 percentage points, somewhat less than for the Schularick and Taylor (2012) dataset. If the marginal benefit and cost beyond quarter 24 is disregarded, the constrained-optimal policy is to lower the policy rate in quarters 1–4 by somewhat less, 0.40 percentage points.

In summary, regardless of whether or not the marginal cost and benefit of beyond quarter 24 are taken into account, and regardless of whether the datasets of Schularick and Taylor (2012) or Laeven and Valencia (2012) are used, the constrained-optimal policy is to lower the policy rate in quarters 1–4 and thus lean with the wind, not against.