Household Debt, Unemployment, and Slow Recoveries*

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Abstract

This paper analyzes the effect of endogenous unemployment risk on the dynamics of recovery from a liquidity trap. In a liquidity trap, an adverse demand shock raises unemployment and produces a period of slow hiring. Slow hiring further reduces demand, both for standard precautionary reasons and because credit conditions endogenously worsen, reducing households’ ability to borrow and consume. The magnitude of this amplification depends on the persistence of the initial shock. Following a permanent shock that requires a period of deleveraging, initial employment falls 3.4 times more than in a complete markets benchmark. In addition, unemployment risk slows the pace of deleveraging because high unemployment tends to increase the dispersion of household debt holdings. I decompose the aggregate Euler equation into terms capturing the effect of binding borrowing constraints (deleveraging), and idiosyncratic consumption volatility (precautionary). I find that amplification occurs primarily due to higher precautionary saving by unconstrained households. Multiple equilibrium paths exist, and which one the economy follows depends on household expectations and the policy rule adopted by the central bank after the economy exits the trap. I find that forward guidance can be very powerful, because endogenous unemployment risk increases the responsiveness of aggregate consumption to future economic conditions.


Keywords: Liquidity Trap, Slow Recovery, Debt Distribution, Precautionary Saving.

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1 Introduction

In the wake of the 2007 – 2009 recession, the U.S. experienced a period of persistently high unemployment with short-term interest rates at the zero lower bound. Several researchers have linked the severity of the recession and the slow recovery of hiring to weak consumer demand, possibly due to high levels of household debt entering the recession.\(^1\) In this paper I offer an alternative hypothesis for persistent weak demand: that slow hiring itself weakens demand, amplifying shocks and producing self-fulfilling slow recoveries.

Slow hiring weakens demand for two reasons. First, slow hiring implies longer spells of unemployment, which prompt greater saving by households for standard precautionary reasons. Second, high unemployment endogenously tightens the financial constraints of households because longer expected spells of unemployment increase the incentives for households to default on their debt. When the economy is demand-constrained, as in a liquidity trap, the increase in desired saving lowers demand, causing employment to fall further.

To explore this mechanism, I develop a model in which households face idiosyncratic unemployment risk. Employed households save and unemployed households dissave in order to smooth consumption over unemployment spells. Unemployed households face a borrowing constraint, set to prevent default. As in a standard New Keynesian model, prices are sticky and the levels of output and employment are determined by demand. Unlike in a standard model, however, the level of employment and the rate of hiring affect demand through the precautionary and credit channels described above.

I consider a persistent demand shock in this setting, i.e. a shock that lowers desired consumption for a given interest rate and job-finding rate, but does not affect the hiring incentives of firms. If prices were flexible, the interest rate would fall to raise desired consumption and maintain the rate of hiring that prevailed before the shock. If instead prices are sticky, the central bank will try to replicate the flexible-price equilibrium by lowering the interest rate. However, if the economy is in a liquidity trap, the central bank will not be able to lower the interest rate sufficiently to offset the demand shock, and employment falls instead.\(^2\)

\(^{1}\) Mian and Sufi (2010), and Mian, Rao, and Sufi (2013) argue that high household leverage contributed to the decline in employment and consumption in 2008. Dynan (2012) and Hall (2011) argue that high levels of household debt contributed to the slow recovery since 2008. Reinhart and Rogoff (2009), Hall (2010), and Jordà, Schularick, and Taylor (2013) argue that financial crises are often preceded by increases in leverage and followed by slow recoveries, but for a contrary finding see Romer and Romer (2015).

\(^{2}\) This result is similar to Eggertsson and Woodford (2003), Werning (2011), and Eggertsson and Krugman (2012).
My first main result is that the presence of unemployment risk amplifies the fall of employment following this shock. Since the demand shock is persistent, employment remains low for a period of time. This implies that the hiring rate falls, which reduces demand via the precautionary and credit channels discussed above. This feedback mechanism amplifies the employment effect of the initial demand shock, producing a larger fall in employment than would otherwise occur. I find that the degree of amplification depends greatly on the persistence of the initial shock. Following a temporary shock to household credit access, I calculate that demand (and thus employment) fall about 40% more than in a complete markets comparison case. By contrast, following a permanent credit shock that requires a period of deleveraging, initial employment falls more than 3 times more than in the complete markets benchmark.

To analyze this amplification channel, I present a decomposition of the precautionary term in the aggregate Euler equation into a component deriving from the idiosyncratic volatility of consumption facing households (precautionary effect), and a component deriving from the Euler equation wedge from households that are directly constrained (deleveraging effect). I find that most of the steady-state aggregate precautionary effect is due to precautionary saving by households that are currently unconstrained. This suggests that simplistic measures of precautionary saving in which a high fraction of households are actively constrained are missing an important part of the story. Moreover, most of the endogenous variation in aggregate consumption growth due to the amplification channel derives from the precautionary effect, rather than the deleveraging effect. This suggests that the fraction of constrained households is not a sufficient statistic for the magnitude of precautionary saving effects, and consideration of consumption volatility faced by households throughout the asset distribution is very important to determining the path of aggregate consumption in an incomplete markets model.

My second main result is that unemployment risk can produce a slow recovery of employment following a demand shock. This occurs because high unemployment increases the dispersion of asset holdings, both by increasing saving by employed households, and by increasing the number and length of unemployment spells. Since unemployed households borrow, this produces more households with large debt. Greater asset dispersion depresses demand because poor households close to the borrowing constraint reduce their consumption by more than wealthy households increase theirs. Thus the increase in asset dispersion keeps demand low after the shock has dissipated, producing a slow recovery of employment.

The speed of recovery varies quite a bit depending on the persistence of the initial shock. A temporary credit shock does not produce a slow recovery because the tighter
borrowing constraint facing households causes deleveraging by poor households. Although unemployed households accumulate additional debt, this is offset by the deleveraging so that the total burden of debt falls. This reduced debt burden allows demand to recover quickly when the shock begins to fade. Intuitively, a shock directly to household credit purges bad balance sheets, so that after the initial deleveraging households are able to rapidly increase spending as credit conditions recover. By contrast, a permanent credit shock produces a slow recovery because the high unemployment causes households to accumulate debt, slowing the deleveraging process.

In a similar manner, the initial asset distribution significantly affects the economy’s response to demand shocks. Greater dispersion in assets implies there are more households close to the constraint, which increases the economy’s sensitivity to shocks. This both increases amplification, since the households close to the constraint are forced to deleverage when the constraint tightens following a shock, and slows the recovery of employment because the greater debt burden requires a longer period of deleveraging.

A further result is that there are many equilibrium paths following any particular shock. If households are pessimistic about the rate of recovery of employment following the demand shock, the initial fall in demand will be greater, and a slower recovery will follow. Such a multiplicity of equilibria is common in New Keynesian models, particularly those with a zero lower bound on the nominal interest rate. Typically determinacy is achieved by assuming that the central bank follows a conditional interest rate policy that rules out undesirable equilibria. However, in a liquidity trap the central bank cannot lower interest rates, and so may not be able to rule out slumps generated by self-fulfilling pessimism about the rate of recovery. Thus any demand shock that pushes the economy into a liquidity trap could bring about a persistent slump.

Not only may pessimistic expectations alter the dynamics of the economy following a demand shock, they may constitute a shock by themselves. If households anticipate a period of reduced hiring, they will reduce their spending. A sufficiently large decrease in consumption would require a negative real interest rate to offset, causing the economy to fall into a liquidity trap and inducing a persistent slump.

These powerful effects of expectations suggest a role for expectations of future policy to offset demand shocks. Several authors, starting with Krugman (1998), have suggested that the central bank can reduce the fall of employment during a liquidity trap through forward guidance, i.e. the promise to keep interest rates low after the economy exits the trap. These low interest rates generate a post-recovery boom in consumption, which

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3See, for example, Bernanke and Woodford (1997), Benhabib, Schmitt-Grohé, and Uribe (2001), Clarida, Gali, and Gertler (2000), and Cochrane (2011).
raises demand during the slump as households smooth their consumption. Forward
guidance is very powerful in my model because the precautionary behavior of uncon-
strained households introduces another forward-looking term into the aggregate Euler
equation. Thus central bank policy that affects future labor market conditions has a signif-
ically greater effect on current demand (and therefore employment) than in a complete
markets model, or an incomplete markets model without endogenous unemployment
risk. However, as emphasized by several authors, engineering such a boom requires the
central bank to commit to following an ex post suboptimal policy once the economy exits
the trap.

Literature. This paper is part of a rapidly growing literature on the dynamics of the
economy in a liquidity trap. The early papers in this literature, Krugman (1998) and Eg-
gertsson and Woodford (2003), were the first to consider the possibility of a binding zero
lower bound on nominal interest rates in a modern context, and suggested the possibility
of forward guidance in substituting for conventional monetary policy in these circum-
stances. Werning (2011) investigates optimal policy in this setting, and clarifies the dy-
namics of a liquidity trap in a simple and elegant model. Cochrane (2011) and Aruoba
and Schorfheide (2013) discuss the problem of determinacy in the presence of a zero lower
bound, and the possibility of deflationary traps, which is related to my finding of multiple
equilibrium paths of recovery.

A number of papers have highlighted the role of household debt and deleveraging
as the cause of low demand leading to a liquidity trap. Eggertsson and Krugman (2012)
first explored this deleveraging channel in the context of a simple model with a zero
lower bound. Hall (2011) develops a similar hypothesis, with particular reference to high
levels of household debt producing a slow recovery of demand. Korinek and Simsek
(2014) study the role of macroprudential regulation in reducing the effects of this delever-
aging channel. Guerrieri and Lorenzoni (2011) investigate the effect of precautionary
saving behavior on the evolution of the distribution of debt holdings during a delever-
aging episode. This paper contributes to this literature by introducing endogenous time-
varying unemployment risk into a model of a liquidity trap and studying the resulting
feedback from high unemployment to weak demand through precautionary saving and
endogenous borrowing constraints.

Several papers have empirically investigated the household debt / aggregate demand
(2013) use detailed county-level data to show that counties with high household leverage
and large declines in house prices before the crisis had larger declines in employment and
output during the crisis. Mian and Sufi (2014) shows that these differential employment declines are driven by the hiring decisions of nontradable good firms, suggesting that the mechanism operates through a demand channel. Dynan (2012) finds that households with high leverage saw larger declines in consumption in 2007 – 2009, despite a smaller decline in net worth, indicating the existence of a household credit channel rather than a wealth channel. Baker (2014) finds that households with higher levels of debt adjust their consumption more in response to changes in income, suggesting a higher marginal propensity to consume for highly indebted households.

Several authors have recently offered alternative models of aggregate demand channels, many involving matching in product or labor markets. Kocherlakota (2012) analyzes a so-called incomplete labor market model, in which the real interest rate is set by the central bank and the labor supply condition may not hold. This formulation is similar to a New Keynesian model with fixed prices, except that in a standard New Keynesian model the labor demand condition is dropped instead of labor supply. Chamley (2014) investigates the possibility of self-fulfilling precautionary demand for savings in a model with money and bonds, in which the economy may become stuck in a low demand saving trap, or may converge very slowly towards full employment. Michaillat (2012) argues that matching frictions are insufficient to explain unemployment and develops a model of job rationing during recessions. Michaillat and Saez (2013) develop a model of aggregate demand with matching frictions in both labor and goods markets, and show that tightness in one market affects tightness in the other, generating an aggregate demand channel for employment fluctuations. I view these approaches as complementary to the New Keynesian formulation of aggregate demand used in this paper.

Since this paper’s primary mechanism operates through precautionary saving by households, it is related to the empirical on precautionary saving behavior. Carroll and Samwick (1997) and Carroll and Samwick (1998) find that households that face greater income uncertainty hold more wealth, and estimate that precautionary saving accounts for 39 – 46% of household asset holdings. Gourinchas and Parker (2002) find that young households target a buffer of precautionary wealth, and estimate a coefficient of relative risk aversion of 0.5 – 1.4. Parker and Preston (2005) also find a significant and strongly countercyclical precautionary saving motive, that is similar in magnitude to the interest-rate motive. Carroll, Slacalek, and Sommer (2012) find that a significant portion of the consumption decline since 2008 was attributable to precautionary effects.

A few papers have considered the interaction of precautionary savings and endogenous unemployment risk in the presence of incomplete markets. Challe et al. (2014) develop a model that combines these ingredients with sticky prices, and show that varia-
tions in precautionary saving over the business cycle amplify employment fluctuations. 
Ravn and Sterk (2013) likewise study the effect of unemployment risk on demand, al-
though their focus is on an exogenous increase in labor market mismatch increasing long-
term unemployment. Caggese and Perez (2013) study a model with unemployment risk
and credit constraints facing both firms and households. They show that precautionary
behavior by households and firms can interact to generate a negative demand externality
that significantly increases the volatility of employment over the business cycle.

The papers cited above generate amplification due to precautionary saving behavior.
This paper sheds additional light on this mechanism, showing that it operates through
a depressed job-finding rate following a persistent demand shock. It also demonstrates
the critical role of the asset distribution in determining these dynamics, which the papers
cited above do not study. This paper also differs by focusing on the liquidity trap case,
by studying the role of expectations in generating multiple equilibrium paths of recovery,
and by studying the role of endogenous credit conditions facing consumers.

2 Households

The model is set in continuous time with a single non-storable consumption good. There
are three types of agents: households, final-good firms, and intermediate-good firms. I
first discuss the household problem, and then turn to the rest of the model.

There is a measure 1 of households, indexed by \( i \in [0, 1] \). Household \( i \) has expected
lifetime utility

\[
E_0 \left[ \int_0^\infty e^{-\rho t} u(c_i(t)) dt \right]
\]

where \( u(\cdot) \) is a standard utility function, and \( \rho \) is the household subjective discount rate.
I assume throughout that \( u(c) \) exhibits constant relative risk aversion \( \gamma \), i.e. \( u(c) = \frac{c^{1-\gamma}}{1-\gamma} \)
for \( \gamma \neq 1 \) and \( u(c) = \log(c) \) for \( \gamma = 1 \).

2.1 Income Process

Households receive a flow of nonlabor income \( e(t) \), which is identical across all house-
holds, and is assumed to be constant over time. This income allows households to carry
debt while maintaining positive consumption. Household \( i \) receives a flow of labor in-
come \( w(t)h_i(t) \), where \( h_i(t) \in \{0, 1\} \) is household \( i \)'s employment status at time \( t \), and
\( w(t) \) is the wage of employed workers.\(^4\)

Since households suffer no disutility from labor, all households would like to work, implying an inelastic labor supply at \( n = 1 \). This would imply full employment in a Walrasian labor market, but here there is a hiring cost that produces equilibrium unemployment. Employment and unemployment shocks arrive with hiring probability \( p(t) \) and separation (job loss) probability \( s(t) \) per unit time. These are flow probabilities, i.e. the job-finding rate \( p(t) \) means that the probability that a worker who is unemployed at time \( t \) finds a job during the interval \([t, t + dt]\) is approximately \( p(t) \cdot dt \), which becomes exact as \( dt \to 0 \). Likewise, the total probability that an unemployed household will become employed at least once during the interval \([t_0, t_1]\) is

\[
1 - e^{-\int_{t_0}^{t_1} p(t)dt}
\]

and likewise for job separation probabilities, with \( s(t) \) in place of \( p(t) \). With a constant hiring rate \( p \), the length of an unemployment spell has an exponential distribution, with expected value \( 1/p \).

Household \( i \) enters period \( t \) with net assets \( a_i(t) \), and can save or borrow at net interest rate \( r(t) \). Assuming no default, household assets evolve according to

\[
\dot{a}_i(t) = r(t)a_i(t) + e(t) + w(t)h_i(t) - c_i(t)
\]  

(1)

where \( \dot{a}_i(t) \) is household \( i \)'s net saving.

Households face a time-varying borrowing constraint, which they take as exogenous. This borrowing limit takes the form of a lower bound on asset holdings \( a(t) \), such that asset holdings of household \( i \) must satisfy

\[
a_i(t) \geq a(t)
\]

at every time \( t \). The borrowing constraint satisfies \( a(t) < 0 \), so that it is always possible for households to carry some debt.\(^5\)

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\(^4\)It is common in models of the labor market that unemployed households receive some income, which is generally interpreted as unemployment benefits. One could interpret \( e(t) \) in my formulation as unemployment benefits, and \( w(t) + e(t) \) as the wage.

\(^5\)Technically, employed households could face a looser borrowing constraint than unemployed households. However, assuming a single borrowing constraint rules out default following job loss. In any case, the borrowing constraint generally does not bind for employed households.
2.2 Household Problem Under Repayment

There is no aggregate uncertainty, so that the paths of all variables are known at time 0 except for the employment status and asset holdings of individual households, \( h_i(t) \) and \( a_i(t) \). Since households are identical except for asset holdings and employment status, we need only derive household decisions at each point \( (h, a, t) \).

Let \( V(a, t) \) be the value function of an employed household in period \( t \) with assets \( a \), and let \( U(a, t) \) be the value function of an unemployed household. Employed households choose current consumption to maximize the Hamiltonian

\[
\rho V(a, t) = \max \left\{ u(c) + V_a(a, t) \cdot \dot{a}_e + V_t(a, t) + s(t) \left( U(a, t) - V(a, t) \right) \right\}
\]

where \( \dot{a}_e = r(t)a + w(t) + e(t) - c \), and where \( V_a \) and \( V_t \) denote the partial derivatives of the value function. I denote the optimal consumption decision rule by \( c_e(a, t) \), which implies the saving decision rule \( \dot{a}_e(a, t) = r(t)a(t) + w(t) + e(t) - c_e(a, t) \).

Unemployed households similarly choose current consumption to maximize the Hamiltonian

\[
\rho U(a, t) = \max \left\{ u(c) + U_a(a, t) \cdot \dot{a}_u + U_t(a, t) + p(t) \left( V(a, t) - U(a, t) \right) \right\}
\]

where \( \dot{a}_u = r(t)a + e(t) - c \). I again denote the optimal consumption decision rule by \( c_u(a, t) \), which implies saving decision rule \( \dot{a}_u(a, t) = r(t)a(t) + e(t) - c_u(a, t) \).

The optimal consumption choices \( c_e(a, t) \) and \( c_u(a, t) \) satisfy

\[
u'(c_e(a, t)) = V_a(a, t)
\]
\[
u'(c_u(a, t)) = U_a(a, t)
\]

for \( a > a \). Intuitively, the shadow value of wealth equals the marginal utility of consumption, since under optimal consumption unconstrained households are indifferent between consuming or saving the marginal unit of wealth.

When \( a = a \), the borrowing constraint may bind. In this case, \( c \) is chosen such that \( \dot{a} = \dot{a} \), where \( \dot{a} \neq 0 \) may hold as the borrowing constraint tightens and loosens with changing macroeconomic conditions. Then the optimality conditions satisfy

\[
u'(c_e) \geq V_a(a)
\]
\[
u'(c_u) \geq U_a(a)
\]
Intuitively, when the constraint binds the household would like to borrow more, but is prevented from doing so by the constraint. This implies that the marginal utility from consumption is strictly greater than the marginal utility from saving, i.e. $u'(c) > V_a$.

The costate equations of the Hamiltonian at $a > a$ are

$$(\rho + s - r) \lambda = \lambda_a \dot{a}_e + \lambda_t + s \kappa$$

$$(\rho + p - r) \kappa = \kappa_a \dot{a}_u + \kappa_t + p \lambda$$

where $\lambda = V_a$ and $\kappa = U_a$ are the costate variables.

Let $\dot{\lambda} = \lambda_a \dot{a}_e + \lambda_t$ and $\dot{\kappa} = \kappa_a \dot{a}_u + \kappa_t$, so that $\dot{\lambda}(a, t)$ and $\dot{\kappa}(a, t)$ are the instantaneous rates of change of the costate variables of employed and unemployed households, respectively, when these households do not change employment status.\footnote{This is an abuse of notation, since $\dot{c}$ refers to the rate of change of consumption by a particular household, rather than the evolution of the household decision rule defined at a particular point $(a, t)$.} Using these terms, we can express the costate equations as

$$-\frac{\dot{\lambda}}{\lambda} = r - \rho + s \left( \frac{\kappa - \lambda}{\lambda} \right)$$

$$-\frac{\dot{\kappa}}{\kappa} = r - \rho + p \left( \frac{\lambda - \kappa}{\kappa} \right)$$

### 2.3 Household Euler Equation

When $a > a$, we can express the costate equations in terms of the consumption decision rules. First taking the expression in terms of $\dot{\lambda}$, and observing that $-\dot{\lambda}/\lambda = \gamma \dot{c}_e/c_e$, where $\dot{c}_e(a, t) = \frac{\partial}{\partial a} c_e(a, t) + \frac{\partial}{\partial t} c_e(a, t)$ is the instantaneous rate of change of consumption of an employed household that does not lose its job, we find that $\dot{c}_e$ satisfies

$$\frac{\dot{c}_e}{c_e} = \gamma^{-1}(r - \rho) + \gamma^{-1}s \left( \frac{c_u - \gamma}{c_e - \gamma} - 1 \right)$$

(6)

This is a form of the continuous time consumption Euler equation. When $s = 0$, there is no unemployment risk, and the expression simplifies to the familiar consumption Euler equation under certainty:

$$\frac{\dot{c}}{c} = \gamma^{-1}(r - \rho)$$

(7)

This implies an increasing path of consumption when $r > \rho$, and a decreasing path when $r < \rho$. This captures the intertemporal substitution of consumption in response to the interest rate. A low interest rate prompts households to shift consumption towards
the present, implying a low growth rate of consumption, whereas a high interest rate prompts households to reduce current consumption in order to save more, which implies a higher rate of consumption growth.

When \( s > 0 \), households face unemployment risk. The term \( \gamma^{-1} s \left( \frac{c_u^{-\gamma}}{c_e^{-\gamma}} - 1 \right) \) is positive because \( c_u < c_e \), implying a faster rate of consumption growth corresponding to greater saving. This term does not perfectly capture the precautionary motive, however, because \( \dot{c}_e \) only contains changes in consumption for households that remain employed, whereas we also should consider changes in consumption due to job loss. In order to isolate the precautionary saving effect, we separate this expression into a precautionary term related to the volatility of future consumption, and a term corresponding to the expected level of future consumption. To do this, we simply add the expected change in consumption due to job loss to each side of (6) to obtain:

\[
\dot{c}_e + s \left( \frac{c_u - c_e}{c_e} \right) \frac{c_e}{E[e/c]} = \gamma^{-1} (r - \rho) + s \left[ \gamma^{-1} \left( \frac{c_u^{-\gamma}}{c_e^{-\gamma}} - 1 \right) - \left( 1 - \frac{c_u}{c_e} \right) \right] \tag{8}
\]

We may likewise express the Euler equation for unemployed households as:

\[
\frac{\dot{c}_u}{c_u} + p \left( \frac{c_e - c_u}{c_u} \right) = \gamma^{-1} (r - \rho) + p \left[ \gamma^{-1} \left( \frac{c_e^{-\gamma}}{c_u^{-\gamma}} - 1 \right) - \left( 1 - \frac{c_e}{c_u} \right) \right] \tag{9}
\]

Equations (8) and (9) correspond more precisely to our idea of an Euler equation. They give expected consumption growth as a function of intertemporal substitution and a precautionary saving term. Let

\[
T(x) = \gamma^{-1} \left( x^{-\gamma} - 1 \right) - (1 - x) \tag{10}
\]

so that the the precautionary term for employed households is \( sT(c_u/c_e) \), and for unemployed households is \( pT(c_e/c_u) \). Thus the precautionary motive is a simple function of the percentage change in consumption due to a change of employment status, times the probability of this change. Since \( T(x) = 0 \) at \( x = 1 \), the precautionary motive approaches zero as \( c_u \to c_e \) — intuitively, households would not be concerned about unemployment spells if they could perfectly smooth their consumption. The derivative of \( T \) is \( T'(x) = (x^{\gamma+1} - 1)/x^2 \). This is strictly negative for \( x \in (0, 1) \), and strictly positive for \( x > 1 \). Therefore \( T(x) \) obtains a unique minimum at \( x = 1 \) on the interval \( (0, \infty) \), so that the precautionary motive term is always positive.
To gain a little intuition for the precautionary motive term, we can take a second-order Taylor expansion around \( x = 1 \). Then the precautionary motive term is approximately

\[
T(x) \approx \frac{1}{2} (1 + \gamma) \times (1 - x)^2
\]

This expression has a simple intuition. Taking the problem of a currently employed household, let \( dc \) be the change in consumption of this household over a small time interval \( dt \). Then for \( dt \) small, \( dc \) behaves like a binary random variable with probability distribution

\[
dc = \begin{cases} 
\dot{c}_e \cdot dt & \text{with probability } 1 - sdt \\
\dot{c}_u - \dot{c}_e & \text{with probability } sdt
\end{cases} \tag{11}
\]

We used a similar concept above when we observed that \( E[\ddot{c}] = [\dot{c}_e + s (\dot{c}_u - \dot{c}_e)] \) in equation (8). Since \( \ddot{c} = dc/c \), we can multiply by \( dt \) to obtain \( E[dc] = [\dot{c}_e + s (\dot{c}_u - \dot{c}_e)] dt \), which gives the expectation of the random variable \( dc \). We can likewise compute the variance, which, again neglecting higher order terms, is:

\[
Var(dc) = s (\dot{c}_u - \dot{c}_e)^2 dt
\]

or in terms of the rate of change in consumption, \( s(\dot{c}_u - \dot{c}_e)^2 = Var(dc/dt) \cdot dt \).\(^7\)

By analogy to the Euler equation, we are interested in the variance of the growth rate of consumption, \( dc/c \). Thus we divide \( dc \) by \( c \), which at time \( t \) is a known constant. Then we see that the Taylor expansion of the precautionary motive term for employed households is just:

\[
sT \left( \frac{\dot{c}_u}{\dot{c}_e} \right) \approx \frac{1 + \gamma}{2} \times Var \left( \frac{dc/dt}{c} \right) dt \tag{12}
\]

The term \( 1 + \gamma = -\frac{cu''(c)}{u''(c)} \), and so \( 1 + \gamma \) is the relative prudence of the utility function, as defined by Kimball (1990). Thus the precautionary saving term is just (one-half) prudence times a term proportional to the variance of the consumption growth rate. This yields a very intuitive expression for the Euler equation:

\[
E \left[ \frac{dc/dt}{c} \right] \approx \frac{1}{\gamma} (r - \rho) + \frac{1}{2} (1 + \gamma) \cdot Var \left( \frac{dc/dt}{c} \right) \cdot dt \tag{13}
\]

\(^7\)Note that this implies that as \( dt \to 0 \), so that \( dc/dt \to \dot{c} \), the variance of \( dc/dt \) approaches infinity. This reflects the possibility of discrete jumps in consumption following employment shocks.
2.4 Aggregate Euler Equation

We can aggregate the Euler equations of individual households to obtain an aggregate Euler equation. The household Euler equations (8) and (9) can be written as

\[ E \left[ \dot{c} \right] = \gamma^{-1} (r - \rho) c + q T \left( \frac{c_{-h}}{c} \right) \cdot c \]

where \( T(x) \) is the precautionary motive term (10), \( q \) is the probability of an employment transition, and \( c_{-h} \) is consumption if the household switches employment status.

There is also a mass of constrained households, who are unemployed at asset level \( a \). If they remain unemployed, their consumption grows at \( \dot{c} = -\ddot{a} \), but if they become employed, which happens with probability \( p dt \) in time interval \( dt \), they will increase their consumption by amount \( c_e(a) - c_u(a) \). Therefore their expected consumption growth is:

\[ E \left[ \dot{c}_a \right] = p \left( c_e(a) - c_u(a) \right) - \ddot{a} \]

The expected rate of change of aggregate consumption \( \dot{C} \) is just the weighted average of the expected changes of consumption of individual households. Moreover, because there is no aggregate risk in the economy, the actual growth of aggregate consumption equals its expectation. Therefore, letting \( \chi \) be the share of households that are constrained, aggregate consumption growth satisfies

\[ \frac{\dot{C}}{C} = \frac{(1 - \chi) \gamma^{-1} (r - \rho)}{\text{interest-rate substitution}} + \underbrace{T(\sigma^2_C)}_{\text{precautionary saving}} + \frac{\chi \left( p \Delta c(a) - \ddot{a} \right)}{\text{constrained households}} \] (14)

where \( \Delta c(a) = c_e(a) - c_u(a) \) is the increase in consumption when a constrained household finds a job, and

\[ T(\sigma^2_C) = \int \left[ m_e c_e C \cdot sT \left( \frac{c_u}{c_e} \right) + m_u c_u C \cdot pT \left( \frac{c_e}{c_u} \right) \right] \] (15)

is the consumption-weighted average of the precautionary saving terms of individual households.\(^8\)

We can derive a more intuitive expression for aggregate consumption growth using the second-order Taylor approximation of \( T(x) \) given above. Let \( \sigma^2_e(a, t) \) and \( \sigma^2_u(a, t) \) be

\(^8\)\( m_e(a, t) \) and \( m_u(a, t) \) give the mass of employed and unemployed households, respectively, with assets \( a \) at time \( t \). The integral is taken over assets \( a > a \), meaning that it excludes the point mass of constrained unemployed households households with \( a = a \). Since employed households are not constrained, there is a negligible mass of employed households with \( a = a \).
the consumption growth volatility facing a particular employed or unemployed household with assets $a$ at time $t$. Then, as we showed above, $sT(c_u/c_e) \approx \left(\frac{1+\gamma}{2}\right)\sigma_u^2$ and $pT(c_e/c_u) \approx \left(\frac{1+\gamma}{2}\right)\sigma_u^2$. Then we can write the aggregate consumption Euler equation as:

$$\frac{\dot{C}}{C} \approx (1 - \chi) \gamma^{-1} (r - \rho) + \left(\frac{1+\gamma}{2}\right)\sigma_C^2 + \chi \left(\frac{p\Delta c(a) - \ddot{a}}{C}\right)$$

(16)

where $\sigma_C^2 = \int_a \left[m_e\sigma_e^2 C + m_u\sigma_u^2 C\right]$ is the consumption-weighted average variance of the growth rate of consumption facing unconstrained households.

Observe that the interest-rate term in (14) and (16) is multiplied by the share of unconstrained households $1 - \chi$. This reflects that, whereas unconstrained households balance consumption volatility and intertemporal substitution, constrained households are restricted to simply consume their current income. The result is that future interest rates are discounted at the rate $1 - \chi$, and for this reason I call (14) the **discounted** aggregate Euler equation. This discounting reflects that binding constraints on some households make the Euler equation less forward-looking, as emphasized by McKay, Nakamura, and Steinsson (2015). However, (14) makes clear that while incomplete markets reduce the sensitivity of aggregate consumption to future interest rates, they also introduce another forward-looking term through precautionary responses to future consumption volatility. Thus whether incomplete markets make the Euler equation more or less forward-looking overall depends on the relative strength of these effects. For instance, if a relatively low fraction of households are constrained at any point in time, but idiosyncratic consumption volatility varies considerably with the business cycle and with monetary policy, then an incomplete markets model may be more forward-looking than a complete markets model.

### 2.5 Importance of the aggregate Euler equation

The aggregate Euler equation is critical to the dynamics of a New Keynesian model of the liquidity trap. To understand the source of this model’s dynamics, it is useful to contrast the Euler equation above to the standard Euler equation under certainty:

$$\frac{\dot{C}}{C} = \gamma^{-1} (r - r^*)$$

(17)

$r^*$ is often called the “natural rate of interest”, which under complete markets is equal to the rate of time preference. In the simplest model, the path of output is completely determined by the path of this gap. With no capital, the path of consumption determines the paths of output and employment. Under the maintained assumption that the economy
returns to its steady state in the long run, the path of consumption growth rates will then also determine the path of employment.

In this setting, a stylized way of capturing a demand shock is as a fall in the natural rate of interest \( \rho \). If the central bank succeeds in setting \( r = \rho \), the only effect will be a fall in the interest rate — in this case the central bank succeeds in fully offsetting the demand shock. However, if \( r \) does not fall enough, e.g. because of the zero lower bound on nominal interest rates, then \( r > \rho \) will result. This implies a positive growth rate of aggregate output, and under the assumption of convergence to the steady state this likewise implies a large current fall in output.

Rearranging (14) slightly, we can express our aggregate Euler equation in terms of a “natural rate of interest” as well:

\[
\gamma \frac{\dot{C}}{C} = r - \left[ \rho - \gamma T(\sigma_C^2) - \gamma \chi \mu \right]
\]

Natural rate of interest

(18)

where \( \mu = (\bar{a} + p \Delta c(\bar{a})) / C - \gamma^{-1} (r - \rho) \) is the wedge due to binding constraints.

This expression for the natural rate differs in two respects from the complete markets benchmark. First, it contains a precautionary term arising from idiosyncratic consumption volatility due to incomplete markets. If all households were unconstrained, so that \( \chi = 0 \), this would be the only difference. However, as long as there is a non-trivial mass of households at the constraint, these households will mechanically consume their liquid wealth every period. Thus there are two departures from the Euler equation under certainty, corresponding to the precautionary motive of households responding to consumption volatility, and a further reduction of spending by households who are currently constrained.

What are the consequences of these changes for the model’s dynamics? The first consequence is to lower the steady state of interest to \( r < \rho \). In (18), note that both precautionary terms are positive, implying a higher rate of consumption growth than under certainty. In steady state, we need \( \dot{C} = 0 \), and so we need \( r < \rho \). Further, the magnitude of \( \rho - r \) is growing in the degree of income risk in the steady state. By analogy to the Euler equation under certainty, we can speak of the natural rate of interest as the path of \( r \) that achieves \( \dot{C} = 0 \).

While a lower natural rate of interest might seem a mere curiosity, one implication is that the zero lower bound is potentially a greater problem. For a given rate of time

\[ \text{Since income risk and the share of constrained households are endogenous, there may not be a unique natural rate of interest at each point in time. But we can still speak of a natural path of interest rates, and for ease of terminology I will refer to the current rate on one such path as the “natural rate of interest”.} \]
preference, the natural rate of interest is lower, and thus \( r \geq 0 \) is a tighter constraint. Moreover, an increase in steady state income risk or a tightening of borrowing constraints will lower the natural rate still further, pushing the economy closer to \( r = 0 \).

Perhaps more interesting are the consequences for the dynamic response of consumption to shocks. Any shock that temporarily raises income risk or tightens borrowing constraints will raise the growth rate of consumption and therefore lower current output. Since both events typically happen during a recession, this is a generic amplification channel as long as demand shocks cannot be fully offset by monetary policy. Moreover, there is the potential for additional feedback effects, since this fall in demand will typically raise income risk and tighten borrowing constraints further. This is the amplification channel developed in this paper.

### 2.6 Household saving rules

Above we discussed household decision rules in terms of consumption decision rules. We can equivalently represent household decision rules in terms of the rate of asset accumulation for employed and unemployed households, denoted by \( \dot{a}_e \) and \( \dot{a}_u \) respectively. As implied by the discussion above, the rate of asset accumulation is decreasing in asset holdings \( a \), because as households gain wealth, their precautionary saving motive declines (in the limit as \( a \) becomes arbitrarily large, \( c_u \to c_e \) and the smoothing motive goes to zero).

Assuming \( r < \rho \), which will hold in equilibrium, there is a maximum target asset level \( \bar{a} \) for employed households that satisfies

\[
s \left[ \left( \frac{c_u(\bar{a})}{c_e(\bar{a})} \right)^{-\gamma} - 1 \right] = \rho - r
\]

This is the asset level at which the precautionary motive to accumulate assets is offset by the impatience of consumers relative to the interest rate. At any asset level \( a < \bar{a} \), employed households accumulate assets towards the target.

Figure 1 represents the saving decision rules of employed and unemployed households with different levels of asset holdings.\(^{10} \) Unemployed households choose to dissave until they reach the borrowing constraint \( \underline{a} \), at which point they choose \( \dot{a} = 0 \). Employed households choose to accumulate assets up to their target level of assets \( \bar{a} \). For both employed and unemployed households, \( \dot{a} \) is strictly decreasing in asset holdings \( a \). These

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\(^{10}\)This figure depicts decision rules at the steady state defined in section 4, using the parameters discussed in section 3.4.
are general features of the household decision rules.

An important feature of the household decision rules is that the desired rate of asset accumulation is convex in asset holdings. This is particularly pronounced for unemployed households, since they rapidly decrease borrowing as their total debt approaches the borrowing constraint. This convexity implies that an increase in dispersion of asset holdings will raise aggregate desired savings, or equivalently will decrease desired aggregate consumption. This implies that the distribution of asset holdings is an important determinant of aggregate demand. Since the curvature of the saving rule is greater for unemployed households close to the borrowing limit, the share of households with very low wealth (close to being constrained) is of particular importance. A corollary is that the determination of the borrowing constraint is also of great importance.

The saving decision rule of households depicted in Figure 1 matches some features of the data. First, it shows that highly indebted unemployed households have a higher marginal propensity to consume out of wealth than less indebted households. This matches the finding of Mian, Rao, and Sufi (2013) that highly levered households have a higher marginal propensity to consume out of changes in housing wealth, and that this is governed by credit access. Also, the difference between the saving of unemployed and employed households decreases greatly as households become indebted, implying that high-debt households reduce their consumption much more when they become unemployed. This matches the findings of Baker (2014) that the consumption of highly
indebted households is more sensitive to changes in their income.

Nevertheless, the saving behavior implied by the model does not perfectly match the stylized facts found in the data. In particular, the decision rule implies that saving rates are decreasing in wealth, whereas they are increasing in the data, as documented by Dynan, Skinner, and Zeldes (2004) and others. More precisely, Figure 1 shows that saving rates conditional on employment status are decreasing in wealth. However, because employed households are on average wealthier and have greater saving than unemployed households, in the steady state of the baseline calibration the correlation between wealth and saving is effectively 0. Still, there is no positive relation as documented in the data.

### 2.7 Borrowing Constraint

I assume that indebted households make the decision to repay or default at every point in time $t$. If a household defaults, it suffers a fixed utility penalty $D$ and is able to borrow again immediately. Thus the value function of an unemployed household that repays at time $t$ is $U(a, t)$, whereas its value function under default is $U(0, t) - D$, and so an unemployed household will repay if and only if

$$U(a, t) \geq U(0, t) - D$$

Since the value function $U(a, t)$ is strictly increasing in $a$, there is a unique threshold level of assets below which unemployed households default. I assume that lenders set the borrowing constraint equal to this threshold level $a(t)$, which is implicitly defined by

$$U(a(t), t) = U(0, t) - D$$  \hspace{1cm} (19)

There is a similarly-defined default threshold for employed households defined by $V(a^e, t) = V(0, t) - D$. However, this default threshold will be lower than that of unemployed households, and employed households will choose to accumulate assets at $a$. Thus we need only worry about the borrowing constraint of unemployed households, since no household will ever hold assets below this level. Moreover, since employed households will never choose to hold assets $a < a^e$ and unemployed households cannot choose to do so, no default will occur in equilibrium.

Since the cost of default is fixed, endogenous variation in the borrowing constraint is driven by changes in the benefit from defaulting. This benefit is $U(a, t) - U(0, t)$, which
we can write as $\int_{a}^{0} U_a da$. Thus the borrowing constraint is defined by

$$\int_{\underline{a}(t)}^{0} \kappa(a, t) da = D$$

where $\kappa(a, t) = U_a(a, t)$ is the marginal value of wealth at asset level $a$. We can write this is more intuitively as $|a| \cdot \bar{\kappa} = D$, where $\bar{\kappa}$ is the average marginal value of wealth over the interval $[a, 0]$. Thus the benefit of defaulting is the increase in wealth from defaulting times the average value of wealth.

Holding $a$ fixed, any increase in the marginal value of wealth will raise the benefit of defaulting. This will prompt an increase in $a$, i.e. a tightening of the borrowing constraint. Intuitively, if wealth is more valuable, there is a greater benefit to defaulting, and so lenders must tighten the borrowing constraint to prevent defaults. One of the chief determinants of the marginal value of wealth for unemployed households is the hiring probability in the near future. When the hiring probability is high, households expect that they will not be unemployed for very long, and so choose a relatively high level of consumption in anticipation of their greater future wealth. This higher consumption implies a lower marginal value of wealth, since $\kappa = c_u^{\gamma}$. This is the mechanism by which high unemployment causes tighter borrowing constraints, since higher unemployment implies a lower rate of job-finding.
Figure 2 depicts \( a(p) \) for various levels of the job-finding rate \( p \).\(^{11}\) The figure illustrates that borrowing constraints can vary substantially with the hiring probability. From the beginning of 2008 to the end of 2009, the quarterly job-finding rate in the U.S. fell from about 1.9 to 0.75, which according to Figure 2 would tighten the borrowing constraint by over 20%.

3 Production, Monetary Policy, and Equilibrium

Now that we have discussed the household problem, I describe the rest of the model and define the equilibrium.

3.1 Firms

Final goods are produced from intermediate goods using a Dixit-Stiglitz aggregation technology

\[
Y = \left( \int_i y_i^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}
\]

where \( \varepsilon > 1 \) is the elasticity of substitution between inputs. Consider the problem of a representative final good firm that purchases intermediate goods from intermediate good producers at prices \( p_i \). The aggregator firm chooses inputs \( y_i \) to maximize profits

\[
\Pi = P \left( \int_i y_i^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_i p_i y_i
\]

which yields optimality condition

\[
y_i = Y \left( \frac{p_i}{P} \right)^{-\varepsilon} \quad (20)
\]

Assuming zero profits, the aggregate price level is

\[
P = \left( \int_i p_i^{-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}.
\]

The level of output is not pinned down by the price distribution, but is instead determined by the aggregate demand for final goods. Thus the final good producer takes aggregate demand as given, and purchases intermediate goods according to (20) to meet this demand. Since in equilibrium all intermediate good prices will be identical, the purchase of intermediate goods will satisfy \( y_i = Y \).\(^{12}\)

\(^{11}\)Parameters for this case are given in section 3.4.
\(^{12}\)All intermediate good prices will be identical because we will only consider the extremal cases of perfectly flexible or perfectly fixed prices. If prices were partially flexible, we would have price dispersion,
Intermediate Good Firms. There is a measure 1 of identical intermediate good firms that face demand function (20). Firm $i$ with $n_i$ employees produces output

$$y_i(t) = z(t)n_i(t) - C_i(t)$$

where $C_i(t)$ is the hiring cost paid by the firm for new workers, which takes the form of a loss in output.

Firms perceive equilibrium wage $w_e(t)$ and per-worker hiring cost $\vartheta(t)$. Firms lose workers at an exogenous hazard rate of $s$ and the firm must pay the hiring cost on its gross hires $\dot{n} + sn$. Thus the total hiring cost is:

$$C_i(t) = \vartheta(t) \cdot [\dot{n}_i(t) + s(t)n_i(t)]$$

I consider firm behavior under two pricing regimes: fixed prices and fully flexible prices. The fixed price regime will be the baseline case and may be interpreted as a demand-constrained environment with anchored inflation expectations. This seems a good approximation to the circumstances that prevailed in the US and Europe in 2008 – 2014. The flexible price case is useful mainly as a benchmark for optimal behavior, and will be used to define the central bank policy target. I assume that the central bank would like to replicate the flexible price benchmark.\textsuperscript{13}

Fixed Prices. Suppose that firms’ prices are fixed at $p(t) = 1$, which implies that the aggregate price level is $P(t) = 1$. As is standard in New Keynesian models, firms are assumed to meet the demand that they face at this price. Thus firm $i$ chooses a path of $n_i(t)$ that satisfies

$$d_i(t) = y_i(t)$$

(21)

where $d_i(t)$ is demand for the firm’s output.

Thus at every point in time, I assume that firms set employment $n(t)$ equal to demand. This may imply an instantaneous adjustment of aggregate employment at time $t = 0$, as initial employment $n(0)$ jumps to the equilibrium path. However, I assume that the path of demand $d(t)$ and therefore of employment $n(t)$ is continuous and differentiable for $t > 0$, so that $\dot{n}(t)$ and $p(t)$ are defined for all $t \geq 0$.\textsuperscript{14} I further assume that firms will be

13This corresponds to setting the interest rate equal to the “natural rate”. In a standard New Keynesian model, such a policy is optimal since it produces zero inflation and a zero output gap. In the present model this policy is suboptimal, because there are frictions in the labor and capital markets. Nevertheless, this assumption provides a simple and intuitive policy target.

14$\dot{n}(0)$ is defined as the right-derivative of $n(t)$ at $t = 0$, i.e. as the limit of $\dot{n}(t)$ as $t \to 0$. 

21
able to hire sufficient workers to meet the demand they face, which will be true in all the cases we consider.

I assume that wages are a constant markdown of labor productivity, \( w(t) = \varphi z(t) \) for \( \varphi \in (0, 1) \). This implies that wages are unaffected by fluctuations in demand, although they respond to changes in productivity.\(^{15}\) Under this assumption, firm \( i \)'s flow profits are

\[
\pi_i(t) = (1 - \varphi) z(t) n_i(t) - \theta(t) \cdot [\dot{n}_i(t) + s n_i(t)]
\]

(22)

where \( \theta(t) \) is the hiring cost, and total hires are \( \dot{n}(t) + s n(t) \) since firms must hire a flow of \( s \cdot n(t) \) workers to replace workers lost to exogenous separation. I assume that households all own equal shares of firms, and that net profits are paid to households every period, so that nonlabor income is \( e = \overline{e} + \pi. \)\(^{16}\)

**Flexible Prices.** Now suppose that firms may adjust their prices and choose any \( (p_i, y_i) \) consistent with (20). Further suppose that the government provides a production subsidy to firms, financed by a lump-sum tax, chosen to exactly offset the incentive for underproduction due to market power. If the government sets the production subsidy equal to \( \tau = \frac{1}{\varepsilon - 1}, \) then the firm problem is isomorphic to maximizing discounted flow profits (22).\(^{17}\)

Suppose that firms discount flow profits at the same rate as households \( \rho. \) Firms choose the path of hiring to maximize discounted profits

\[
\int_0^\infty e^{-\rho t} \pi_i(t) dt
\]

Let \( J(n, t) \) be the value function of a firm at time \( t \) with employment \( n. \) Then firms choose hires \( \dot{n} \) to maximize the Hamiltonian

\[
\rho J(n, t) = \max_{\dot{n}} \{ (1 - \varphi) zn - \theta (\dot{n} + sn) + J n \dot{n} + J_I \}
\]

\(^{15}\)Many researchers consider sticky wages to be a promising mechanism to produce greater responses of employment to shocks (e.g., Hall (2005), Pissarides (2009), and Galí (2011)). However, in the present model sticky wages are not necessary to produce employment volatility because under fixed prices the path of employment is determined by demand regardless of the incentives for job creation. The main consequence of fixed wages is to simplify the central bank policy rule.

\(^{16}\)In the baseline calibration, steady state profits amount to just 0.25% of GDP, so they make little difference to the analysis.

\(^{17}\)Specifically, firm revenue net of taxes is \( R = (1 + \tau) PY^{\frac{1}{\varepsilon}} (zn)^{\frac{\varepsilon - 1}{\varepsilon}} - \tau PY, \) and the marginal revenue product of labor is \( R_n = \frac{\varepsilon - 1}{\varepsilon} (1 + \tau) PY^{\frac{1}{\varepsilon}} (zn)^{\frac{\varepsilon - 1}{\varepsilon}} \frac{1}{n}. \) In a symmetric equilibrium \( (P = 1 \) and \( Y = zn), \) this is equivalent to \( R = zn \) and firms perceive \( R_n = z. \)
The firm optimality condition is \( J_n = \vartheta \), which implies that firms hire workers until the marginal value of an additional worker is equal to the hiring cost.\(^{18}\) The costate equation of the Hamiltonian is \( (\rho + s) J_n = (1 - \phi) z + J_n \). Combining these, we obtain the flexible price job creation condition

\[
(\rho + s) \vartheta = (1 - \phi) z + \dot{\vartheta} \tag{23}
\]

Equation (23) has a straightforward interpretation. When \( \dot{\vartheta} = 0 \), so that the cost of hiring workers is constant over time, it says that the cost of hiring must equal the present discounted value of profits that the firm will receive from this worker, i.e. \( \vartheta = \frac{(1 - \varphi)z}{\rho + s} \). Hiring behavior is altered somewhat when the cost of hiring is expected to change over time. For example, if \( \dot{\vartheta} < 0 \) so that hiring is becoming less expensive over time, then firms have an incentive to defer hiring until the future when it is less costly. They will do so until they drive down the hiring cost sufficiently that they are indifferent between hiring the marginal worker today, or hiring the worker in the future at a somewhat lower cost.

Under a few assumptions, (23) becomes quite simple. First suppose that the discount rate of firms \( \rho \), the separation rate \( s \), and productivity \( z \) are constant over time.\(^{19}\) Further assume that the economy will eventually reach a steady state equilibrium with interior employment (i.e. \( n \in (0,1) \)). Finally, suppose that the hiring cost \( \vartheta \) is a simple increasing function of the job-finding probability of households \( p \). Then (23) implies a constant job-finding rate \( p^\star \) that satisfies \( \vartheta(p^\star) = \frac{(1 - \varphi)z}{\rho + s} \).

If we use our baseline functional form \( \vartheta(p) = \psi p^\alpha \), this becomes\(^{20}\)

\[
p^\star = \left( \frac{1}{\psi} \frac{(1 - \varphi)z}{\rho + s} \right)^{1/\alpha} \tag{24}
\]

We can write (24) more intuitively in its implicit form as

\[
\psi(p^\star)^\alpha = (1 - \varphi) \times \frac{z}{\rho + s} \times \text{PDV output}
\]

---

\(^{18}\)By assumption, firms are small and do not internalize that their hiring decisions affect the aggregate hiring cost.

\(^{19}\)If the firm discount rate were not fixed, cyclical variations in firm discount rates could significantly affect hiring incentives. For an analysis of such effects, see Hall (2014). Since the present paper is chiefly concerned with the determination of demand rather than incentives for job creation, I avoid these complications by assuming a fixed discount rate.

\(^{20}\)This functional form is isomorphic to a fixed vacancy-posting cost combined with a Cobb-Douglas matching function, as in Shimer (2005). If the flow cost of maintaining a vacancy is \( c \), and the matching function is \( m = \mu v^b \nu^{1 - b} \), then the cost per worker hired is \( \vartheta = c \mu^{-\frac{1}{b}} \nu^{\frac{1-b}{b}} \).
which makes clear that $p^*$ corresponds to the rate of hiring at which expected discounted profits from the marginal hire equal the hiring cost.

Optimal hiring is given in terms of an optimal job-finding rate instead of an optimal hiring rate because the cost of hiring a worker is increasing in the tightness of the labor market. The standard definition of tightness depends on the aggregate hiring rate relative to the number of unemployed workers, which is equivalent to the job-finding rate of households. Thus when unemployment is high, the optimal rate of hiring is higher because it is less costly to hire workers (we can interpret this as a high rate of filling posted vacancies in a model with explicit search). This works out to a constant job-finding rate perceived by workers.

3.2 Equilibrium under Fixed Prices

Since prices are fixed, there is no inflation and the real interest rate equals the nominal interest rate. Thus the path of interest rates $r(t)$ is set by the central bank. Likewise the paths of non-labor income $e(t)$, labor productivity $z(t)$, and the separation rate $s(t)$ are given.

The law of motion of aggregate employment satisfies

$$\dot{n}(t) = p(t) \cdot (1 - n(t)) - s(t) \cdot n(t)$$

We need a market clearing condition in the asset market. By Walras’ law, if the asset market clears, the goods market clears as well. Let $m_e(a,t)$ and $m_u(a,t)$ be the mass of employed and unemployed households with assets $a$ at time $t$. Thus $m_e$ and $m_u$ are the probability density functions of the asset distribution across households. $m_e$ and $m_u$ are related to employment $n$ by

$$n(t) = \int_a m_e(a,t) da$$
$$u(t) = 1 - n(t) = \int_a m_u(a,t) da$$

In equilibrium, we need aggregate asset holdings at every time $t$ to equal zero. We can express this as

$$\int_a a \cdot m_e(a,t) da + \int_a a \cdot m_u(a,t) da = 0$$

Here $\int_a a \cdot m_e(a,t) da$ is total assets held by employed households, and $\int_a a \cdot m_u(a,t) da$ is total assets held by unemployed households.

When the economy is not in its long-term steady state, the asset distribution will
 evolve over time. The law of motion for the asset distribution is

\[ \dot{m}_e = pm_u - sm_e - \frac{d}{da} (m_e \dot{a}_e) \] (28)

\[ \dot{m}_u = sm_e - pm_u - \frac{d}{da} (m_u \dot{a}_u) \] (29)

where (28) and (29) must be consistent with the law of motion of aggregate labor.

Equations (28) and (29) can be interpreted as flow equations. The term \( m \dot{a} \) is the mass of households at a point in the asset distribution times their rate of asset accumulation. Since the rate of asset accumulation is the “velocity” of that household along the asset dimension, we can interpret this term as the rate of “flow” of households through a point on the asset dimension, moving from lower to higher assets. Then the rate of change of the mass of households in the neighborhood of a point in the asset distribution is the difference between the rate of flow into that neighborhood minus the flow out of that neighborhood. This is equivalent to minus the slope of the flow rate along the asset dimension. For example, if \( \frac{d}{da} (m \dot{a}) < 0 \), so that the rate of flow is decreasing in assets in the neighborhood of a point in the asset distribution, then the flow into that neighborhood is greater than the flow out of that neighborhood, and so the mass of households in that neighborhood is increasing.

**Definition 1** (Equilibrium). Given a path of \( \{z, e, r, s, D\} \) and initial asset distribution \( m_e(a, 0) \) and \( m_u(a, 0) \), an equilibrium is a path of \( \{m_e, m_u, V, U, c_e, c_u, \dot{a}_e, \dot{a}_u, w, e, \pi, p, g, n, u\} \) that satisfies (1) - (19), (22), and (25) - (29).

### 3.3 Equilibrium Determinacy and Central Bank Policy

Definition 1 specifies an equilibrium for a given path of exogenous variables and initial conditions. However, it leaves open the question of what path of interest rates the central bank sets, and whether a unique equilibrium exists for a given interest rate path. Our benchmark assumption is that the central bank would like to set interest rates in order to replicate the flexible price job-finding rate \( p^* \) defined by (24). We work out the implications of this assumption for equilibrium determinacy.²¹

²¹This policy rule is equivalent to setting the interest rate equal to the Wicksellian natural rate. This would be optimal if the only friction were sticky prices. However, in this model the flexible-price equilibrium is not Pareto optimal because of the presence of incomplete markets. We may interpret this policy rule as a central bank that limits itself to short-run stabilization, and does not seek to correct inefficiencies arising from long-term structural features of the economy.
**Perfectly flexible interest rates.** First suppose that there are no restrictions on the path of interest rates that the central bank can select. Then the central bank will always be able to hit its policy target, and we know that the job-finding rate satisfies \( p = p^* \). However, without further assumptions this does not specify an equilibrium: the initial level of employment \( n(0) \) can take on any value, which together with constant job-finding rate \( p = p^* \) will imply a particular path of employment. These together will imply a path of the interest rate \( r \) that is consistent with this path of employment, i.e. in which the path of demand exactly equals the path of output implied by this path of employment.

While many such equilibria exist, and each are consistent with the central bank’s policy target, they intuitively correspond to disruptions in the rate of employment by the central bank. If we suppose that the economy starts at the steady state level of employment \( n(0) = n^* / (p^* + s) \), then a fall in \( n(0) \) would correspond to the central bank raising interest rates and then lowering them over time to maintain a constant job-finding rate. Thus a reasonable equilibrium selection rule is that the central bank will choose to keep employment at its long-run steady state level when it can do so.

**Lower bound on interest rates.** Now suppose that there is a lower bound on the interest rate \( r \) so that only policy paths that satisfy \( r(t) \geq r \) are possible. Then it might not be possible for the central bank to hit its target job-finding rate \( p = p^* \). In particular, if the flexible interest rate equilibrium requires the interest rate to fall below \( r \) at any point, then this equilibrium violates the constraint.

We must now modify the policy rule of the central bank. Instead of assuming a fixed job-finding rate \( p(t) = p^* \) and a variable interest rate, suppose that at every point in time either the central bank has successfully hit its target \( p(t) = p^* \) and the interest rate takes on some value \( r \geq r_L \), or the interest-rate constraint is binding and the central bank cannot hit its target, meaning that \( r = r_L \) and \( p(t) \leq p^* \). Note that the central bank can always kill off an excessive boom by raising interest rates, so we don’t need to worry about \( p > p^* \).

This yields the *constrained policy rule*:

\[
r(t) \geq r \text{ and } p(t) = p^* \quad \text{OR} \quad r(t) = r_L \text{ and } p(t) \leq p^* \quad (30)
\]

The assumption that the central bank will not allow \( p(t) > p^* \) implies a lack of commitment on the part of the central bank. Situations may arise when the lower bound on the interest rate causes a period of low employment and low job-finding, that the central bank could reduce if it could credibly promise to allow \( p(t) > p^* \) after the liquidity trap has concluded. In section 7.1 I will analyze what happens if we allow such forward
guidance.\textsuperscript{22}

Under (30), there are again multiple equilibrium paths. This is analogous to the multiplicity that exists when the interest rate is unconstrained, but in this case it is less reasonable to assume that the central bank can pick any equilibrium that it pleases since its choice of interest rates is constrained.

To fix intuitions, consider the case that the economy experiences an unanticipated adverse demand shock at \( t = 0 \) that causes \( r < r \) in the flexible price equilibrium, i.e. the lower bound on the interest rate binds. Suppose further that the economy permanently exits the liquidity trap at some future point \( T^* \).\textsuperscript{23} Then from the policy rule (30), we know that \( \forall t \leq T^*, r(t) = r \) and \( p(t) \leq p^* \), and \( \forall t \geq T^*, r(t) \geq r \) and \( p(t) = p^* \). Clearly this implies that \( r(T^*) = r \) and \( p(T^*) = p^* \). The period \( t < T^* \) corresponds to the liquidity trap, when hiring is below target and the interest rate is against the zero lower bound. The period \( t \geq T^* \) is after the liquidity trap, when the central bank can achieve its hiring target by setting \( r \geq r \).

If we knew the date of exit from the liquidity trap \( T^* \), this would pin down the equilibrium. However, multiple \( T^* \) are possible. In particular, we can always assume that households expect a later \( T^* \), in which case demand will be lower throughout, and the central bank cannot offset this lower demand by lowering interest rates because of the lower bound.

But note that not all \( T^* \) can be an equilibrium. For instance, suppose that a very large adverse demand shock occurred at \( t = 0 \), and suppose that \( T^* \to 0 \). Then when we enter the post-liquidity trap period, we have \( p(t) = p^* \) forever. Then no matter what initial level of employment \( n(0) \) is chosen, employment will converge to steady state fairly rapidly. If the demand shock has lasted beyond this point (which it could), and since \( r(t) \geq r \) and so cannot be lowered further to boost demand, this will not be an equilibrium for a sufficiently large and persistent demand shock. Thus for any initial demand shock there exists a continuum of equilibria, corresponding to various dates of exit from the liquidity trap, \( T^* \in [\mathcal{T}, \infty] \). A later exit corresponds to a larger initial fall in initial employment \( n(0) \), and a lower path of recovery.

In my baseline experiments, I will assume that the economy follows the equilibrium with the highest path of employment \( n(t) \), which in the demand shock cases corresponds to the smallest \( T^* \) that is consistent with equilibrium. In section 7 I consider the possibility

\textsuperscript{22}This no commitment assumption is similar to the baseline case considered by Werning (2011), who considers a liquidity trap that ends at a fixed time \( T \), and assumes that the central bank implements the no commitment equilibrium for \( t \geq T \).

\textsuperscript{23}This formulation is similar to that used in Werning (2011) and related papers in the liquidity trap literature, except that in my model the date of exit from the liquidity trap \( T^* \) is endogenous.
of worse equilibria, and argue that these correspond to pessimistic expectations about the pace of recovery.

### 3.4 Calibration

I use the following parameters in the baseline calibration:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$e$</td>
<td>0.5</td>
<td>Targets income share about 1/3</td>
</tr>
<tr>
<td>$s$</td>
<td>0.1</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.9843</td>
<td>Achieves $p = 1.35$ target</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
<td>Blanchard and Galí (2010)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.39%</td>
<td>Achieves $r^s = 0.25%$ target</td>
</tr>
<tr>
<td>$D$</td>
<td>3.39</td>
<td>Achieves $a = -4.16$ target</td>
</tr>
<tr>
<td>$\theta(p)$</td>
<td>$\psi p^\alpha$</td>
<td>Blanchard and Galí (2010) (isomorphic to Shimer (2005))</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1</td>
<td>Blanchard and Galí (2010)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.115</td>
<td>Matches job-finding cost in Shimer (2005)</td>
</tr>
</tbody>
</table>

Table 1: Baseline Calibration

The quarterly separation rate of 0.1 corresponds to the 3.3% monthly separation rate reported as the average job separation rate for the US economy in 1951 – 2003 by Shimer (2005). The target steady state job-finding rate for unemployed households is 1.35, or 0.45 on a monthly basis, which is the 1951 – 2003 average reported by Shimer (2005). The wage share of output $\varphi$ is set to achieve this target in the flexible price equilibrium. By way of comparison, the wage share of output in the baseline calibration in Shimer (2005) is about 0.9825, so this number is similar.

The coefficient of relative risk aversion $\gamma = 1$ is a standard choice in business cycle models, i.e. in Blanchard and Galí (2010). It is also the middle of the range of estimates reported by Gourinchas and Parker (2002) based on their analysis of household behavior in the Consumer Expenditure Survey and Panel Study of Income Dynamics. Note that this implies a fairly low level of risk aversion compared to many other papers, such as Guerrieri and Lorenzoni (2011), and thus represents a conservative estimate of the degree of risk aversion among households.

The target steady state interest rate is 0.25% on an annual basis. This target was chosen so that the zero lower bound is effectively binding in the steady state. While this calibr-
tion is mainly chosen for tractability, such an occurrence is not completely unreasonable, given that Japan has experienced zero short-term policy rates for most of the last 25 years, as have the U.S. and Europe since 2008. Some commentators have suggested that a combination of well-anchored inflation expectations, low population growth rates, and slow technological progress have lowered the natural (flexible price) rate of interest to zero or below, a situation known as “secular stagnation.” If they are right, this calibration could be literally correct. Nevertheless, the qualitative results of the model do not depend on this assumption.

The steady state borrowing constraint $a$ in absolute value equals $4.16$ times quarterly labor income, as in Guerrieri and Lorenzoni (2011). This produces an analogous asset distribution across households, but shifted to the left. The shift is because I assume that assets are in zero net supply, so that the distribution is centered at zero, whereas Guerrieri and Lorenzoni (2011) assume positive net aggregate asset holdings of households due to a supply of government bonds.

The hiring cost is assumed to be of the form $\vartheta = \psi p^\alpha$. I choose $\alpha = 1$, which corresponds to $b = \frac{1}{2}$ in the Cobb-Douglas matching function. In addition to being tractable, this form is consistent with estimates of the matching function according to Blanchard and Galí (2010), who use the same functional form. I set $\psi$ so that the hiring cost in the steady state equals that in Shimer (2005).

4 Steady State

For given labor productivity $z$, nonlabor income $e$, and separation rate $s$, the equilibrium conditions, together with the steady state condition $\dot{m}_e = 0$ and $\dot{m}_u = 0$, define steady state equilibrium. In the fixed price case there is a continuum of steady state equilibria, corresponding to different pairs $\{r, p\}$. We may think of each of these as corresponding to different targets of the steady state job-finding rate (and associated employment rate), together with the steady state interest rate that achieves this target. One of these equilibria has job-finding rate $p^*$, which corresponds to the flexible price equilibrium.

For a given job-finding rate $p$ and interest rate $r$, we can define household saving rules $\dot{a}_e$ and $\dot{a}_u$. Together with $\dot{m}_e = 0$ and $\dot{m}_u = 0$, and the relative mass of employed and unemployed households given by $n = p/(p+s)$, these define a stationary distribution of asset holdings.

---

25The concept of secular stagnation was first proposed by Hansen (1939). Eggertsson and Mehrotra (2014) present a model of secular stagnation.

26However, the direct comparison is a bit tricky, because Guerrieri and Lorenzoni (2011) have households that differ in labor productivity.
The steady state asset distribution for the baseline calibration is depicted in Figure 3. The figure depicts the probability mass of households conditional on employment status at each asset level. There is a point mass of unemployed households at the borrowing constraint, but otherwise the asset distribution has a smooth bell shape.

For given \((r, p)\), we can therefore define aggregate desired steady state asset holdings \(A(r, p)\). Since there is no storage or capital in the economy, aggregate total asset holdings must equal zero in equilibrium. Therefore a steady state equilibrium pair \(\{r, p\}\) must satisfy \(A(r, p) = 0\).

The equilibrium for the flexible price job-finding rate \(p^*\) is depicted in figure 4. For fixed \(p\), aggregate steady state asset demand is an upward-sloping schedule in \(r\), which we may think of as the steady state equivalent of the supply of savings.\(^{27}\) A higher interest rate induces households at all wealth levels to save more, shifting out the steady state asset distribution and raising asset demand. The supply of assets is a vertical line at \(A = 0\), and equilibrium is the intersection of the two curves.

What happens to the steady state if there is an increase in the job-finding rate \(p\)? For households at every asset level and employment status, higher \(p\) implies greater lifetime wealth, and so households increase their consumption, or equivalently decrease their savings. However, a higher steady state \(p\) also implies a higher steady state employment rate

\(^{27}\)This is not a supply curve in the traditional sense, because each point corresponds to a different desired steady state asset distribution. A true aggregate saving curve would depict aggregate desired savings for various interest rates for a fixed asset distribution.
$n$, which will tend to increase desired savings, because employed households save more than unemployed households.

Which effect dominates depends on the current level of $n$. When $n$ is high, an increase in $p$ will decrease desired savings, while the reverse holds for low $n$. The reason is that if the majority of households are employed, an increase in $p$ implies a reduction in the aggregate income risk facing households, which will reduce desired precautionary savings. Conversely, when $n$ is low, and in particular when $n < 0.5$, an increase in $p$ will raise income risk facing households, leading to an increase in desired savings.\footnote{If this is unintuitive, consider what happens at $p = 0$, so that all households are unemployed in equilibrium, and there is zero income risk. Then an increase in $p$ will clearly raise income risk, since it is now positive.} Since in all advanced economies the employment rate is well above 50%, the possibility that an increase in $p$ may raise savings is not empirically relevant, and so I will assume for the remainder that aggregate steady state asset demand is strictly decreasing in $p$.

Figure 5 depicts the result of an increase in the steady state job-finding rate $p$ on the steady state equilibrium. The asset demand curve shifts left because higher $p$ implies lower income risk, which depresses desired savings. This shift in asset demand causes the equilibrium interest rate to rise, implying a positive relationship between steady state $r$ and steady state $n$.

We can equivalently interpret this in terms of the aggregate Euler equation (14). An
increase in \( p \) will lower aggregate income volatility, which lowers aggregate consumption volatility \( T(\sigma_C^2) \). Then \( \dot{C} \) will decrease, and so to achieve \( \dot{C} = 0 \), which must prevail in steady state, \( r \) must rise. Under our earlier definition, this is equivalent to saying that the natural rate of interest has risen.

Since steady state \( r \) is increasing in \( p \), the set of steady state equilibria in the fixed price case comprise an upward sloping schedule in \((r, p)\) space.

5 Dynamics Following a Credit Shock

Suppose that the economy is in the steady state equilibrium for \( t < 0 \), and at \( t = 0 \) experiences an unanticipated shock. In particular, consider a temporary fall in the default penalty \( D \). Since lower \( D \) causes borrowing constraints facing consumers to tighten, we can interpret this as a credit shock.29

---

29Because an unanticipated shock tightens the borrowing constraint instantaneously, some households will violate the constraint. I assume that these households are forced to deleverage very quickly. In particular, I extrapolate these households’ saving rules below \( a \), but place a minimum on their consumption at 0.01.
5.1 Fixed Price Equilibrium

Consider the dynamic path of the economy under fixed prices in a liquidity trap scenario. The key feature of a liquidity trap is that the interest rate cannot fall below some lower limit. The simplest way to capture this is to suppose that the interest rate is fixed at its steady state level.

Figure 6 depicts the path of the economy following a temporary credit shock under the baseline equilibrium selection rule discussed in section 3.2. The particular experiment is that the default penalty $D$ falls discretely at $t = 0$, remains at this lower level for 8 quarters, and then steadily recovers to its steady state level over the next 8 quarters. This corresponds to an exogenous discrete worsening of credit conditions for two years, followed by a recovery in credit conditions over the following two years. The initial fall in $D$ is about 1/3 of its steady state value, which was chosen to produce a fall in initial employment of 5 percentage points, equal to the rise in the unemployment rate in the U.S. between January 2008 and October 2009, i.e. from the official beginning of the recession until the trough of the labor market.

Employment initially falls sharply, and then steadily returns to its steady state level after approximately 10 quarters. These dynamics are driven by consumption demand from households. Following the credit shock, demand falls for two reasons. First, there are some households that are at or below the borrowing constraint, and are thus mechanically forced to reduce their spending, what we might call a forced deleveraging effect.
Second, households throughout the asset distribution face tighter borrowing constraints that might bind in the future. This reduces households’ ability to smooth consumption over unemployment spells, increasing precautionary saving.

The job-finding rate falls and rises in tandem with the employment rate. This occurs because the shock is persistent, and so employment remains depressed for a period of time. This implies a period of weak hiring, and so the job-finding rate falls and remains low during the recovery. As demand recovers, the job-finding rate rises in tandem with employment as firms increase hiring to boost production.

This period of low hiring amplifies the initial fall in demand. This happens both because slow hiring raises income volatility facing households, increasing precautionary savings, and because slow hiring endogenously tightens the borrowing constraint. The latter effect can be seen in the path of the borrowing constraint in Figure 6. The borrowing constraint recovers as the hiring rate increases, well before the default penalty begins to increase. Since both of these factors lower demand, they make the initial fall in employment larger than it would otherwise be, i.e. they act to amplify the initial shock.

5.2 Precautionary Saving versus Deleveraging

As discussed above, demand falls both because constrained households are forced to reduce spending, a deleveraging effect, and because tighter constraints reduce households’ ability to smooth consumption, and so raise precautionary saving by unconstrained households, a precautionary effect. In this section we compute the relative contributions of these two effects to the total fall in demand and therefore employment.

As discussed in section 2.4, the path of demand is governed by the aggregate Euler equation, which can be expressed in terms of the gap between the rate of interest set by the central bank and the natural rate. The fall in employment following the credit shock is therefore equivalent to a fall in the natural rate of interest. We can use equation (14) to decompose the fall in the natural rate into a component due to deleveraging by constrained households, and a component due to the precautionary response to higher consumption volatility.

Figure 7 depicts this decomposition for the baseline credit shock. The natural rate falls sharply to about −3.5% immediately following the shock, and most of this fall is due to a reduction of spending by constrained households. Before the shock, there is an initial mass of households near the constraint (Figure 3). When the constraint tightens these households are forced to deleverage, causing an immediate drop in demand. After this initial deleveraging, which lasts about 2 or 3 quarters, demand remains depressed for a
time, mainly due to the precautionary effect. The precautionary motive accounts for the majority of the reduction in the natural rate for the remainder of the crisis.

By integrating over the contribution of each term to the fall in the natural rate, we can calculate the contribution of each component to the total fall in demand at time $t = 0$. Doing so reveals that 54% of the initial fall in aggregate demand is due to precautionary saving effects, whereas 46% is due to deleveraging. Thus both deleveraging and precautionary behavior contribute to the fall in employment, but the contribution of the precautionary motive is somewhat larger. This suggests that analyses that focus exclusively on the deleveraging behavior of constrained households may miss a big part of the story.

### 5.3 Measure of Amplification

We now turn to determining the magnitude of the amplification from endogenous unemployment risk. To do so, we first compute the flexible price equilibrium following the credit shock. This implies a path of the interest rate that stabilizes output and the job-finding rate. We can take this as a reduced form representation of the initial demand shock in the absence of feedback from endogenous variation in unemployment risk to demand. Comparing this path of interest to the “natural rate” computed in the previous section gives a measure of amplification from endogenous unemployment risk, which we can again decompose into deleveraging and precautionary components.
Flexible Price Benchmark What would happen if the central bank was able to adjust the interest rate in response to the credit shock? In this case the central bank would set $r$ to replicate the flexible price equilibrium. Since the credit shock does not affect the incentives for production, the firm optimality condition (23) implies a constant rate of hiring. Thus employment would remain constant at its steady state level, and the interest rate would adjust to produce the consumption demand necessary to clear the market.

This equilibrium is depicted in Figure 8. The resulting path of the interest rate is shown in the left panel. Since the tightening of the borrowing constraint reduces demand, the interest rate must fall to stimulate spending and stabilize output. Since output remains high, households are able to reduce their debt holdings to a much greater degree, as depicted in the middle panel. Finally, the right panel shows that the borrowing constraint is a little looser under flexible prices, because the improved labor market conditions reduce the endogenous tightening of the constraint.

Comparison to Reduced Form Demand Shock. We can interpret the path of the interest rate in the flexible price case as a sort of natural rate of interest. Since the central bank succeeds in stabilizing output under flexible prices, this is the path of the interest rate that restores demand in the absence of endogenous feedback from labor market conditions. This interpretation allows us to determine the magnitude of the feedback channel from endogenous unemployment risk in the baseline fixed price model. First we take the flexible price interest rate and treat it as a reduced form shock to the natural rate of interest in the standard aggregate Euler equation (17). This path of interest corresponds to the effect on demand of the exogenous credit shock by itself, with no feedback from labor market conditions. We then compute the resulting path of demand arising from this reduced form shock, and compare it to the path of demand from the full model with endogenous income risk. The difference is a measure of the magnitude of amplification.
Figure 9: Reduced form shock (solid) vs. endogenous unemployment risk (dashed).

from this channel.

Figure 9 presents this comparison. The left panel compares the path of the natural rate of interest in each case. Overall, the natural rate falls further and remains lower in the fixed price model. This reflects amplification arising from endogenous unemployment risk: since under fixed prices the central bank does not stabilize output, the job-finding rate falls and income risk increases. This lowers demand, which is reflected in a lower natural rate of interest.

The right panel of Figure 9 compares the path of output implied by the natural rate paths shown in the left panel. The initial fall in employment in the reduced form shock case is 3.54 pct. points, compared to 5 pct. points in the fixed price model. Thus about 30% of the initial fall in employment in the fixed price model is due to the amplification mechanism from endogenous income risk, and the rest is due to the initial exogenous shock. Equivalently, we can say that the employment multiplier from the amplification process is 1.41.

The path of employment overshoots the steady state level somewhat in the reduced form shock case. This is because the forced deleveraging following the financial shock results in high demand after the shock dissipates. This corresponds to a positive reduced form demand shock, leading employment to overshoot.

We can again use the aggregate Euler equation (14) to determine the degree to which the amplification process operates through increased deleveraging by constrained households versus increased precautionary saving by constrained households. We first decompose the rate of interest in the flexible price case into deleveraging and precautionary terms, and then subtract these from the terms computed from the whole model.
Figure 10 depicts the result of this exercise. It reveals that the majority (about 70%) of the additional fall in the natural rate due to amplification is due to an increase in the precautionary term, rather than the deleveraging term. This is intuitive, since the credit shock most directly affects households with low wealth who become constrained, whereas a decrease in the job-finding rate increases the income risk facing households throughout the distribution.

5.4 Dynamics of the Asset Distribution

Given the emphasis placed on household debt in the wake the 2007 – 2009 recession, it is interesting to consider the dynamics of the asset distribution in the wake of the financial shock. Since assets are in zero net supply in the model, there is by construction no change in aggregate net worth. Instead, we are interested in changes in the distribution of assets across households.

The bottom middle panel of Figure 6 above shows the path of the total debt following the baseline temporary credit shock. Here total debt means the sum of all assets held by households with negative net worth. This statistic is therefore a measure of the dispersion of assets held by households.

Total debt falls during the slump because the financial shock causes the borrowing constraint to tighten significantly, forcing low-wealth households to deleverage by reducing their debt holdings. Deleveraging occurs both because households that are highly
indebted and close to the constraint are mechanically forced to reduce their debt, and be-
cause unconstrained households with low wealth become afraid of hitting the constraint.
Because there is a mass of households that become constrained immediately following
the shock, there is an initial burst of deleveraging, but after a few quarters the pace of
deleveraging slows.

As seen in section 2, the saving decision rule of households is convex in assets. Thus an
increase in the dispersion of asset holdings will tend to raise desired savings and lower
demand, while a decrease in asset dispersion will have the opposite effect. Intuitively,
greater asset dispersion implies more poor households and more wealthy households.
Wealthy households do not spend much more than medium-wealth households, whereas
poor households are close to being constrained and so reduce their consumption greatly
relative to medium-wealth households. Thus the lower asset dispersion following the
credit shock raises demand and strengthens the recovery.

This occurs because the credit shock forces households to improve their balance
sheets, so that after the initial fall in employment their improved asset position allows
them to finance a higher level of consumption during the recovery. This suggests that a
temporary credit shock may produce a faster recovery than other sorts of demand shocks,
because it tends to purge bad balance sheets, enabling a stronger recovery. This is simi-
lar to the old view that recessions served a necessary restorative role by liquidating bad
investments, now applied to households rather than firms.

We can approximate the magnitude of the effect of asset dispersion on demand
through a partial equilibrium exercise. Let \((m^*_e, m^*_u)\) be the asset distribution at time \(t = 0\)
after the initial fall in employment. Then we compute the market clearing level of employ-
ment \(n(t)\) at every point in time using the equilibrium decision rules \((\dot{a}_e(a,t), \dot{a}_u(a,t))\)
from our baseline experiment, but leaving the asset distribution fixed. That is, \(n(t)\) is
implicitly defined by:

\[
\int_a \left[ m^*_e(a) \dot{a}_e(a,t) + m^*_u(a) \dot{a}_u(a,t) - \left( \frac{n(t) - n(0)}{n(0)} \right) m^*_e(a) \left( \dot{a}_e(a,t) - \dot{a}_u(a,t) \right) \right] da = 0
\]

The result is shown in Figure 11. The solid line is the baseline path of recovery, iden-
tical to that shown in Figure 6. The dashed line uses the same decision rules at every
point in time, but holds the asset distribution unchanged at the steady state (initial) dis-
btribution. This results in a significantly slower recovery in demand than in the baseline
case.
6 Recovery from High Debt State

The temporary credit shock examined in section 5 does not seem a good description of what happened to the U.S. economy during the 2007 – 2009 recession. As Figure 6 reveals, a temporary credit shock produces a temporary decline in household debt, followed by a rapid recovery in hiring when the shock dissipates. In contrast, the 2007 – 2009 recession saw a long decline in household debt with a period of slow hiring.

Many theories of the magnitude of the 2007 – 2009 recession and the duration of the recovery have focused on the role of high levels of household debt in the run up to the crisis. For instance, Mian and Sufi (2010) find that counties with higher household leverage in 2006 experienced larger declines in employment. Similarly, Dynan (2012) argues that household leverage remained high well into the recovery and may be responsible for depressed consumption during the recovery. In light of such arguments, it may make more sense to model the events of 2007 – 2009 as a permanent shock to credit standards, that necessitated a transition to a new credit regime. The present model can provide some insight into this process, since it features a full distribution of household asset holdings, and captures the role of endogenous unemployment risk during this transition.30

In this section, we will consider dynamic equilibrium paths where the economy begins at time $t = 0$ with an initial asset distribution with higher total household debt (i.e. greater wealth dispersion). Since the initial asset distribution is no longer the steady state distribution, we can no longer interpret these experiments as an unanticipated shock that

\[30\text{An important caveat is that this model does not include durable goods or housing, which represent a large fraction of household debt.}\]
occurs in the steady state. Instead, we can interpret this case as depicting the result of an unmodeled shock that affects the ability of households to carry debt. For instance, perhaps an unmodeled credit boom (such as a housing bubble) enabled households to accumulate excessive debt. Then there was an unanticipated correction to credit conditions (e.g. a collapse of the housing bubble), and we are considering the transition to a new steady state.

6.1 Fixed Price Equilibrium

Suppose that the economy enters $t = 0$ with greater dispersion of assets than in steady state. The wider asset distribution acts as a demand shock: it produces an initial fall in employment relative to the steady state, followed by a recovery. We can interpret these dynamics as the result of unwinding a great accumulation of debt by some households in some unmodeled past period. This is similar to the circumstances of the U.S. economy in 2009 after the financial crisis, when many more households were highly indebted than during normal times, and a period of deleveraging was necessary.

Suppose the initial asset distribution is a mean-preserving spread of the steady state distribution such that the total debt held by households, i.e. minus the sum of assets held by households with $a < 0$, is 32.4\% greater than in steady state. This corresponds to the increase in the household debt to GDP ratio from 2002Q1 to 2008Q1.\footnote{Here household debt is taken to be the level of total liabilities held by households and non-profits from}
distribution is depicted in figure 12.

We now consider the path of the economy from \( t = 0 \) onward. This path is depicted in Figure 13. The result is a 4% fall in initial employment, followed by a very slow recovery. The recovery is much slower than the one following the credit shock considered in section 5. This is because the wide asset dispersion requires a substantial period of deleveraging, during which high levels of household debt depress demand. The sustained period of low employment induces additional debt accumulation by unemployed households, which decreases the rate of deleveraging, further slowing recovery. Moreover, the period of slow hiring induces precautionary saving by employed households, which reduces demand and further increases the variance of asset holdings. The result is that the dispersion of assets falls only very slowly, leading to a sustained period of low employment.

One benefit of the high-debt recovery case relative to the credit shock scenario is that we can get a sense of the size of the endogenous tightening of borrowing constraints without the complication of an exogenous tightening. Figure 13 shows that the borrowing limit tightens by about 9% relative to the steady state. This is a significant tightening of the borrowing constraint, though much less than what is seen in the credit shock case.

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the Federal Reserve Financial Accounts of the United States. For details of how the mean-preserving spread of assets is calculated, see the appendix.
**Decomposition of Natural Rate.** As in the temporary credit shock case, we can represent the period of weak demand resulting from the initially wide asset distribution as a reduction in the natural rate of interest. We can then decompose the fall in the natural rate into components deriving from deleveraging of constrained households and precautionary behavior by unconstrained households.

Figure 14 depicts this decomposition. It reveals that the great majority of the reduction in the natural rate during the transition from the high debt state is due to the increase in the precautionary motive. The only exception is at the very beginning of the transition, when constrained households are forced to deleverage following the endogenous tightening of the constraint. Overall, 77% of the total reduction in initial consumption demand is attributable to the precautionary motive.

### 6.2 Flexible Price Benchmark

The flexible price equilibrium again provides a useful point of comparison. Figure 15 depicts the equilibrium paths in both the flexible and fixed price models. The left panel depicts the path of the interest rate. In the flexible price model, the interest rate falls initially by about 60 bp to accommodate an initial burst of deleveraging by highly indebted households. The interest rate then quickly recovers to about 10 bp below its steady state value after 5 quarters. It remains slightly depressed for some time, as the economy slowly transitions to its new stationary asset distribution. This indicates that the total reduction...
in demand due to the necessary deleveraging process is not so great, since it requires only a small fall in the interest rate to accommodate.

As the center panel of Figure 15 makes clear, the reduction in total household debt occurs somewhat faster under flexible prices than in the fixed price case. In particular, after 20 quarters excess household debt has fallen by 60% in the flexible price case compared to 47% in the fixed price case. Likewise, after 40 quarters, excess household debt has fallen by 86% in the flexible price case compared to 74% in the fixed price case. This happens because labor market conditions are stabilized under flexible prices, and so households are in a better position to pay down their debts. As shown in Figure 13, under fixed prices employment falls and remains low throughout the transition. Thus a higher fraction of highly indebted households are unemployed. These households do not reduce their debts while they are unemployed, and so high unemployment slows down the pace of deleveraging.

Finally, as the right panel indicates, borrowing constraints do not tighten in the flexible price case. Since there is no exogenous credit shock in this case, the tightening of the borrowing constraint in the fixed price case is entirely due to worse labor market conditions. Since output and hiring are stabilized in the flexible price case, there is no endogenous feedback to credit conditions, and so borrowing constraints fail to tighten. In fact, they loosen slightly, because lower interest rates reduce the cost of repayment.

\subsection{6.3 Measure of Amplification}

We can again take the flexible price interest rate as a reduced form representation of the initial demand shock, absent the feedback from endogenous income risk. The difference between the flexible interest rate path and the natural rate in the full model with fixed prices is a measure of the amplification from this feedback. The left panel of Figure 16
presents both interest rate paths, while the right panel presents the implied paths of employment from the standard aggregate Euler equation (17).

Overall, the natural rate of interest as defined by equation (18) in the model with endogenous income risk falls substantially more than the flexible price path of interest. This is particularly true at $t = 0$, when the natural rate plunges 225 bp, but remains true throughout the recovery. This larger fall in the natural rate implies a similarly larger fall in employment. Initial employment falls nearly four times as much in the model with endogenous unemployment risk than in the complete markets model with the reduced form demand shock. This implies that 70% of the initial fall in aggregate consumption, and 69% of the initial fall in employment, is due to amplification from endogenous income risk. This implies a “multiplier” from the amplification mechanism of 3.38.

In the case of the temporary credit shock in section 5, only 30% of the fall of consumption was due to amplification, and the multiplier was 1.41. Why is the amplification process so much stronger in the deleveraging case than in the case of a credit shock? The answer is that while the direct reduction in initial demand implied by deleveraging is less than in the case of the credit shock, the persistence of the deleveraging shock is much greater. We can see this by comparing the path of employment implied by setting the natural rate of interest equal to the flexible price interest rate in each case. These are the solid lines in Figures 16 and 9. While the initial fall in employment in the temporary credit shock case is 2.2 pct. points greater than in the deleveraging case, the cumulative loss of employment over the entire path of recovery is nearly 5 pct. points greater in the deleveraging case. Thus the deleveraging case implies a greater total total loss of employment, and therefore prompts a greater precautionary saving response from forward-
looking households.

One implication of this result is that it is better to have a sharp and short demand shock, rather than a long and slow shock. Whereas the total initial exogenous reduction in demand may be the same in each case, they imply very different paths of future hiring, and therefore different income risk perceived by households. A short and sharp demand shock implies a large initial fall in employment followed by a rapid recovery. Households anticipate this rapid recovery, and so have limited desire to engage in precautionary saving, leading to limited amplification through endogenous income risk. By contrast, a long and shallow demand shock implies a lengthy period of slow hiring, with higher income risk facing households. This increases precautionary saving by households, and amplifies the fall in demand.

**Decomposition of Amplification.** We can again use equation (14) to decompose the amplification term into a component corresponding to forced deleveraging by constrained households, and a component arising from the precautionary saving behavior of unconstrained households. The result of this exercise is depicted in Figure 17. Unsurprisingly given the discussion above, most (76%) of the amplification is due to precautionary saving by unconstrained households.
Figure 18: High debt distribution with credit shock (solid) vs. no shock (dashed).

6.4 Credit Shock in High Debt Initial State

The analysis above modeled the events of 2007 – 2009 as a high debt initial state that required a period of deleveraging. However, it is reasonable to think that the financial crisis that occurred in the Fall of 2008 represented a qualitatively different phenomenon than simply high levels of household debt. Arguably this was an exogenous credit shock, with subsequent recovery slowed by high levels of household debt and the lingering effects of the credit shock.

We can model this as an initial high level of household debt together with a temporary credit shock. In particular, suppose that initial household assets are as shown in Figure 12, and that the economy experiences a temporary credit shock as analyzed in section 5. Then the equilibrium path reflects both the effects of a high level of initial debt that requires a period of deleveraging, and a temporary credit crunch facing households.

The resulting equilibrium path is shown in Figure 18, with the recovery from the high debt state with no exogenous credit shock shown for comparison. The initial fall in employment is 10.7 pct. points, compared to a fall of 4 in the high debt case without a credit shock, and a fall of 5 in the case of a credit shock that hits an economy already at the stationary distribution of assets. Thus the total initial fall in employment is greater than the sum of its constituent parts, implying an interaction between high initial levels of debt and the credit shock. One way to interpret these results is that greater initial dispersion in
asset holdings increases the sensitivity of the economy to financial shocks. This suggests that higher moments of the asset distribution are important variables for central banks to monitor, and that policies that aim to reduce excessive levels of debt held by households can be welfare-improving. Such policies are an example of macroprudential policies.

This interaction occurs because the high debt initial asset distribution has many more households close to the borrowing constraint than the stationary distribution, as can be seen in Figure 11. Thus when the borrowing constraint tightens substantially following the shock, the initial fall in demand is much greater than with the stationary distribution. This can be illustrated by a decomposition of the natural rate using equation (14), which reveals that 56% of the cumulative fall of the natural rate in the credit shock case is due to deleveraging by constrained households, compared to 23% in the high debt path with no credit shock.

Although the initial fall in employment is much greater with the credit shock than without, the persistence of the fall in employment is much less. The reason is that deleveraging happens much faster in the credit shock case, as can be seen directly from the bottom middle panel of Figure 11. After 20 quarters, excess household debt has been reduced by 76% in the financial shock case, compared with 47% in the no shock case. Likewise, after 40 quarters 90% of deleveraging has been accomplished in the financial shock case, compared with 74% in the no shock case.

The faster pace of deleveraging implies a much faster recovery in employment, as can be seen from the first panel in Figure 11. Although initial employment falls more than twice as far following the financial shock than in its absence, the employment path following the financial shock surpasses the no shock employment path after just 5 quarters, and is nearly fully recovered after 10. This leads to the surprising result that the cumulative loss of employment and therefore output is nearly twice as large in the absence of the credit shock! Cumulative employment loss in the credit shock case comes to 37.8 pct. points of employment, compared with 63.5 points in the no shock case. Thus the total decline in output is less in an economy with an additional adverse demand shock, if that shock speeds up the pace of deleveraging.

This is particularly surprising given that the initial demand shock is so much greater in the credit shock case. Taking the magnitude of the initial demand shock to be the initial fall in employment implied by the flexible price path of the interest rate, the credit

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32 Heathcote and Perri (2014) obtain a similar result by showing how a low level of household wealth can produce greater volatility by allowing low-employment equilibria to become feasible.

33 For an analysis of macroprudential policies in a model with a demand externality from household debt, see Korinek and Simsek (2014). For a general analysis of macroprudential policies in the presence of nominal rigidities, see Farhi and Werning (2013).
shock generates a 6 times greater fall in initial employment than that due to the high debt distribution alone. Yet despite a six-fold greater initial shock, the faster recovery in the financial shock case produces significantly less cumulative employment loss. Part of the reason for this is that the faster employment recovery produces less amplification compared with the no financial shock case: 42% of the initial fall in aggregate consumption is due to amplification in the financial shock case, compared with 70% in the no shock case. This implies an initial demand multiplier of 1.73 instead of 3.38.

7 Expectations and Recovery

Thus far we have assumed that the economy follows the path with the highest initial level of employment subject to the central bank’s no-commitment policy rule. However, there are many other equilibrium paths that are consistent with the equilibrium conditions of the model. Which one the economy follows depends on the expectations of households, which may be influenced by the policy rule adopted by the central bank.

7.1 Forward Guidance

The analysis in sections 5 and 6 assumed that the central bank followed the no commitment policy rule after the economy exits the liquidity trap. However, if the central bank can credibly commit to allowing $p > p^*$ after the recovery, then an equilibrium path with higher employment is possible. If the central bank has a loss function that penalizes deviations from the target job-finding rate in both directions, then it would like to commit to producing such a hiring boom. In this case, the central bank commits to allowing excessive hiring after the trap without raising interest rates.

This is a form of forward guidance, analogous to committing to keeping interest rates low despite high inflation in a standard New Keynesian model. Here the cost of this policy is not excessive inflation, but an inefficiently high rate of hiring, which is costly due to the hiring cost $\vartheta$. The mechanism for raising demand is also somewhat different: in the standard New Keynesian model, the expectation of a future consumption boom raises current demand due to consumption smoothing, whereas in this paper an expected hiring boom also raises demand by reducing unemployment risk, which reduces precautionary saving.

Figure 19 shows the dynamics of the economy in response to the credit shock analyzed in section 5. The solid line shows the path of recovery when the central bank lacks commitment (identical to Figure 6), whereas the dashed line shows the recovery path un-
under forward guidance. The forward guidance equilibrium allows a hiring boom with the job-finding rate topping out at 1.55, 15% higher than the central bank’s preferred level. This policy rule was chosen to minimize the loss function

$$L = \int_0^\infty e^{-\rho t} [zn(t) - zn^*]^2 dt$$

where $n^*$ is the steady state level of employment. That is, the central bank seeks to minimize the discounted quadratic output gap.\(^{34}\)

The equilibrium path under forward guidance sees an initial fall of employment of just over 3 percentage points, about 40% smaller than in the no-commitment case. Employment recovers to its steady state level after just 5 quarters (well before the shock begins to dissipate) and then overshoots the steady state, reaching a maximum level of 0.8 percentage points above steady state after 10 quarters. Overall, the average level of employment over the entire period is about equal to steady state employment (0.14 percentage points higher).

The initial fall in the job-finding rate is likewise about half of what it is in the baseline case (a fall of 0.27 instead of 0.56), and the job-finding rate likewise overshoots the

\(^{34}\)A discounted quadratic loss function in the output gap and inflation is common in the New Keynesian literature because this is the linear approximation to the optimal policy rule around the efficient zero-inflation steady state. This rule is not optimal in the present model because of the presence of incomplete markets, but offers a simple benchmark to illustrate the power of forward guidance in this setting.
steady state rate by about 0.20. This higher job-finding rate both mechanically produces
the higher growth path of employment, and creates the demand that enables it by re-
ducing precautionary saving and by relaxing the borrowing constraint relative to the no-
commitment path.

7.2 Power of Forward Guidance

Section 7.1 showed that news about future job-finding rates can be very effective at stim-
ulating current demand. This is in line with recent work showing powerful effects from
forward guidance in New Keynesian models. However, McKay, Nakamura, and Steins-
son (2015) argue that the presence of incomplete markets reduces the power of forward
guidance, since constrained households do not respond to news about future consump-
tion. They show that this is equivalent to discounting the future path of interest rates in
the aggregate Euler equation.

I also find that the behavior of constrained households leads to discounting of future
interest rates in the aggregate Euler equation (see equation (14)). However, I find that
the aggregate Euler equation contains another forward-looking term corresponding to
precautionary saving by unconstrained households. In the presence of endogenous un-
employment risk, a decrease in future interest rates that raises the path of employment
will decrease unemployment risk facing households, lowering precautionary savings and
raising demand. Thus, in the presence of endogenous unemployment risk, the addition of
incomplete markets has an ambiguous effect on the power of forward guidance. Whether
the discounting effect of constrained households or the forward-looking precautionary ef-
effect of unconstrained households dominates depends on the fraction of households that
are constrained, and the strength of precautionary behavior.

To illustrate the power of forward guidance in my model, I consider the case of a 50bp
reduction in the interest rate for one quarter, announced at time $t = 0$, that occurs for $t \in
[19, 20]$. Under complete markets, the one-quarter decrease in the interest rate implies
that aggregate consumption declines at a 50bp annual rate for one quarter, which works
out to about an 18bp fall in employment during that quarter. Since by assumption the
economy returns to steady state employment in the long-run, and demand is flat after
$t = 20$, this fall in employment during $t \in [19, 20]$ implies that employment rises by
about 12bp above steady state in $t = 0$ when the policy is announced, and then remains
there until $t = 19$, when it descends to its steady state level over one quarter. This implies

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35See e.g. Eggertsson and Woodford (2003) and Carlstrom, Fuerst, and Paustian (2012).
37Remember that consumption includes nonlabor income, and so is about 1.5 times employment.
Figure 20: Employment following 50bp decrease in $r$ for $t \in [19, 20]$.

a very significant cumulative increase in employment.

We would like to compare the complete markets result to the path of employment generated by our model. However, as discussed in section 3.3, there are many equilibria consistent with this path of interest rates. Since we are interested in a case of forward guidance in which the Fed allows $p > p^*$ for $t < 20$, we cannot use the equilibrium selection rule we used in our baseline experiments $p \leq p^*$. This makes sense, because that selection rule assumed that the Fed was trying to implement $p = p^*$, but was constrained by the zero lower bound. Instead, I assume that the Fed is trying to stimulate job growth by employing forward guidance and so will not force employment below its steady state level at any point along the equilibrium path. This is a potentially dangerous assumption, since it would imply some additional “forward guidance” if the natural path of demand were to fall below the steady state at some point. However, the dynamics of adjustment for this case will not imply this, and so this assumption turns out to be innocuous. Further, among those paths of employment that maintain $n(t) \geq n^{ss}$ throughout, I choose the lowest path of employment — i.e. I assume no additional forward guidance beyond that embodied in the path of the interest rate.

The resulting path of employment is shown in Figure 20, with the complete markets benchmark shown for comparison. Employment initially rises by 52bp, and then steadily falls over the next 19 quarters, finally reaching about 28bp above steady state at $t = 19$. Employment then falls sharply to about 5bp at $t = 20$. It then remains somewhat elevated
Figure 21: Equilibrium following 50bp decrease in $r$ for $t \in [19, 20]$.

and gradually falls to its steady state level. Thus the initial rise in employment is almost 3 times greater than in the complete markets case, and the cumulative employment effect before $t = 20$ is more than 2 times greater than in the complete markets case.

To properly understand these dynamics, we must consider the behavior of other variables. Figure 21 depicts the path of the job-finding rate, total household debt, and the interest rate, which are the three key variables determining the path of demand, and therefore of employment. The higher level of consumption induced by the expectation of future low interest rates causes demand and therefore employment to remain above its steady state level for 20 quarters. This implies a substantial hiring boom to maintain the higher level of employment, as indicated by the path of the job-finding rate shown in the left panel of Figure 21. While constrained households’ consumption is unaffected by the forward guidance, except insofar as the higher employment rate implies fewer (necessarily unemployed) households are constrained, unconstrained households anticipate this hiring boom and respond by reducing precautionary saving. This increased spending raises demand and employment further, amplifying the effect of forward guidance. As time goes on, the end of the hiring boom grows nearer, and so households steadily increase their precautionary saving, though spending remains above the complete markets level to the end because households anticipate the continued employment boom after $t = 20$. During the quarter in which the interest rate is low, consumption falls quickly because of the Euler equation. This implies that employment must also fall quickly, and to achieve this the job-finding rate is sharply lower for one quarter.

After the boom induced by forward guidance, demand (and therefore employment) remain elevated. This occurs because of the changes in the asset distribution that take place during the hiring boom. The variance of the asset distribution is proxied by the total debt held by indebted households, whose path is shown in the middle panel. Total debt falls during the boom, indicating that the asset distribution contracts. This occurs both
because there are fewer unemployed households running up high debts, and because employed households are engaging in less precautionary saving, and so pushing out the top end of the asset distribution less. A narrower asset distribution implies higher demand post-boom, and since under our equilibrium selection assumptions the Fed does not raise interest rates to kill off this boom, the job-finding rate must be elevated to maintain it. As the household asset distribution returns to its steady state variance, this post-forward-guidance boom dissipates, and employment returns to its steady state level.

Overall, I find that the forward-looking precautionary behavior of unconstrained households is more powerful than discounting by constrained households, so that incomplete markets make forward guidance more powerful. While determining which force predominates in the U.S. economy is ultimately an empirical question that this paper cannot clearly answer, there is some evidence that the effects on demand of endogenous variation in unemployment risk are quite large. For example, Carroll, Slacalek, and Sommer (2012) use find that the rise in the saving rate following 2008 was in large part due to a rise in precautionary saving in response to increased income risk. Moreover, they find that the effect of tightening household credit constraints on the saving rate was much smaller than the precautionary effect. Similarly, Parker and Preston (2005) decompose sources of aggregate consumption growth in the U.S. data, and find that changes in consumption growth due to time-varying precautionary saving are comparable in size to variation due to changes in the interest rate.

In combination with such empirical findings, the results from my model suggest caution in viewing incomplete markets as a means of reducing forward-looking behavior of households in macro models. I show that this result is sensitive to the assumption of limited endogenous unemployment risk. When endogenous unemployment risk is included, I find that the resulting feedback may be sufficient to make aggregate consumption more forward-looking, rather than less. Thus there is no unambiguous relationship between incomplete markets and the responsiveness of aggregate demand to future economic conditions, and if households’ responses to endogenous unemployment risk are large, the effect may plausibly go the other way.

7.3 Pessimistic Expectations Following Credit Shock

The discussion above assumes that the economy always follows the highest path of employment consistent with the central bank’s policy rule. However, there is no reason that this must be true. While the central bank has the means to rule out any path with excessive hiring by threatening to raise the interest rate, it cannot rule out paths with lower
Figure 22: Pessimistic recovery from credit shock (solid) vs. baseline (dashed).

hiring because it is constrained by the lower bound on the interest rate.

Figure 22 shows a pessimistic path of recovery in response to the credit shock analyzed in section 5. This produces an initial fall in employment that is 2 percentage points greater than the highest no-commitment path. The resulting path of employment remains about 1 percentage point the below steady state for a significant period after the credit shock has dissipated.

Simultaneous to the slow recovery in employment is a period of slow hiring, which lowers demand and tightens the borrowing constraint. The job-finding rate remains well below the steady state level for the entire period shown, whereas in the baseline case it returns to the steady state level after 12 quarters and remains close to this level thereafter. The larger initial fall in the job-finding rate is by construction — this is exactly what it means in this model for households to be pessimistic about the rate of recovery, and this is the proximate cause of the larger initial fall in employment. However, a further effect arises from the dynamics of the asset distribution. The lower rate of job-finding raises precautionary savings, while the lower employment rate increases the number of borrowers in the economy. This increases the dispersion of asset holdings relative to the baseline case, raising the total debt held by net borrowers as seen in the bottom middle panel of Figure 22. This greater dispersion of asset holdings weakens demand, further lowering the path of employment.
7.4 Pure Expectational Shock

Given the significant effects of pessimistic expectations following a shock to credit market fundamentals found above, a natural question is what effect a pure expectational shock can produce. Suppose that the economy is at the steady state for $t < 0$, and at time 0 households come to expect a period of slow hiring. This will lead them to reduce their desired spending, decreasing demand and causing $n(0)$ to fall. The central bank cannot rule out this equilibrium because it cannot lower interest rates, and so it is powerless to prevent this equilibrium.

Under our maintained assumption that the economy converges to the steady state in the long-run, there is a unique path corresponding to each level of $n(0) < n^*$, with lower $n(0)$ corresponding to a larger shock to expectations. Figure 23 shows the path corresponding to a 5 percentage point drop in initial employment relative to the steady state. The first thing to note is how persistent the fall in employment is. The time horizon shown in Figure 23 extends to 50 quarters, 12.5 years after the initial shock. By the end of this time, employment has still not fully recovered to its steady state level.

This persistence is driven by the increase in asset dispersion during the recovery. As discussed above, a low job-finding rate increases asset dispersion because it stimulates greater saving from employed households while also increasing the total number of households that are borrowers (i.e. unemployed). This effect is sufficiently great that it offsets the narrowing of the asset distribution due to the tightening of the borrowing con-
straint, at least after the first quarter. This greater asset dispersion generates persistence of low employment by lowering demand. Intuitively, high unemployment worsens household balance sheets, which lowers demand and perpetuates high unemployment. Since the asset distribution is a relatively slow-moving variable (excepting initial deleveraging), this generates substantial persistence.

8 Conclusion

Unemployment risk has significant implications for the dynamics of recovery from a liquidity trap. Unemployment risk both amplifies and increases the persistence of demand shocks. Amplification occurs due to the feedback from slow hiring to weak demand, through precautionary savings and endogenous borrowing constraints. Persistence arises from the dynamics of the asset distribution, since a period of high unemployment raises the burden of debt, reducing demand during the recovery.

One important conclusion of this paper is that the distribution of debt is a critical variable in determining the dynamics of the economy in response to demand shocks. From a positive perspective, I find that persistence of demand shocks is driven in part by the evolution of the asset distribution, and that high initial asset dispersion increases the sensitivity of the economy to shocks. From a normative perspective, these results suggest that central banks should monitor the distribution of debt, and take steps to reduce high household leverage. These results are particularly relevant given recent trends toward increased credit access and greater wealth inequality in the developed world.

This paper also highlights the significant role of expectations in determining the path of recovery. These results suggest that if the central bank can credibly manage expectations about future policy, i.e. engage in forward guidance, it can significantly mitigate negative shocks. More ominously, these results also suggest the possibility of self-fulfilling negative expectations about the pace of recovery. Given the zero lower bound on interest rates, the central bank may find itself powerless to prevent such outcomes. In light of slowing growth and demographic transitions around the world that depress the natural rate of interest, these possibilities are highly relevant to policymakers today.

I leave many questions to future research. This paper does not explicitly model durable goods and housing, although these constitute a large fraction of household credit use. Particularly given the centrality of mortgage debt and house prices in the 2007 – 2009 recession, explicitly considering such forms of debt could be a fruitful avenue of further research. In addition, this paper assumes that prices are fixed, which eliminates the role of inflation in equilibrium dynamics. Allowing for partially flexible prices could generate
interesting interactions between inflation, unemployment risk, and the asset distribution.

Overall, my results suggest that a precautionary saving channel in response to high unemployment may have played a role in the 2007 – 2009 recession and the subsequent recovery. Such a channel could help explain the size of the initial fall in demand, as increased precautionary saving led households to decrease spending in the Fall of 2008. Further, this channel could also explain the slow recovery, as high unemployment increased the burden of debt and slowed the process of deleveraging.
References


A Computational Algorithm

I begin by solving for the steady state equilibrium. For given parameters, this involves finding the job-finding rate $p$ and borrowing constraint $a$ such that the stationary distribution of assets resulting from the household decision rules satisfies $A = 0$.

A.1 Household problem

I solve the household problem for given $(p, a)$ using value function iteration. For given values of $(V, U)$, we can define consumption using (4) and (5). The consumption rules define saving rules by (1). If the implied saving rule violates the borrowing constraint, saving at $a$ is set to zero.

Letting $(V_n, U_n)$ be the value function at step $n$ of value function, we can define the next step of the iteration $(V_{n+1}, U_{n+1})$ by

$$
\rho V_{n+1} = u(c_c) + \frac{dV_{n+1}}{da} \cdot \dot{a} + \frac{V_n - V_{n+1}}{dt} + s (U_{n+1} - V_{n+1})
$$

$$
\rho U_{n+1} = u(c_u) + \frac{dU_{n+1}}{da} \cdot \dot{a} + \frac{U_n - U_{n+1}}{dt} + p (V_{n+1} - U_{n+1})
$$

where $dt$ is the step size. Note that this is equivalent to iterating backward through time, with $dt$ equal to the time step. The system of equations above can be quickly solved for the next step by inverting a single matrix. This process continues until the process converges, which generally takes on the order of 20 iterations.

A.2 Calculating the stationary distribution

Once we have calculated the decision rules of households, we can quickly solve for the implied stationary distribution of assets. From (28) and (29), the stationary asset distribution satisfies

$$
pm_u - sm_e - \frac{d}{da} (m_e \dot{a}_e) = 0
$$

$$
sm_e - pm_u - \frac{d}{da} (m_u \dot{a}_u) = 0
$$

To calculate the stationary distribution numerically we discretize the state space to a set of points $a_i$, and let $\Delta = a_{i+1} - a_i$ be the distance between adjacent points. Then we

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38 For a detailed description of this algorithm, see Achdou et al. (2013).
39 Convergence of the algorithm requires an “upwind” approximation of the derivative of the value function $dV/da$, as described in Candler (2001).
approximate the derivative as
\[
\frac{d}{da} (m \dot{a}) \approx \frac{1}{\Delta} \left[ \max (m_{i-1} \dot{a}_{i-1}, 0) - \min (m_{i+1} \dot{a}_{i+1}, 0) - |m_i \dot{a}_i| \right]
\]
This corresponds to taking the left difference when \( \dot{a}_i > 0 \), and the right difference when \( \dot{a}_i < 0 \). When \( \dot{a}_i \approx 0 \) so that \( \dot{a}_{i-1} > 0 \) and \( \dot{a}_{i+1} < 0 \), the expression corresponds to computing the first difference in the direction in which \( m > 0 \). This is important because \( m_e \dot{a}_e \) may not be differentiable at a point where \( \dot{a} = 0 \), but the derivative in one direction will exist. Moreover, since we are in a two-state case, we know that \( m = 0 \) will hold in one direction when \( \dot{a} = 0 \). In particular, for employed households \( m_{i+1} = 0 \), and for unemployed households \( m_{i-1} = 0 \).

In addition to the expression above, we also have the requirement that \( \sum_i m_i^e = \frac{p}{p+s} = N \) and \( \sum_i m_i^u = \frac{s}{p+s} = 1 - N \). Note that in this case \( m_i \) is the mass at point \( i \), rather than the pdf, which would equal \( m_i / \Delta \).

Now we write these equations in matrix form as \( Tm = v \), where \( m = (m_e^1, ..., m_e^n, m_u^1, ..., m_u^n) \), and \( v \) a vector with \( N \) in the \( n \)th position, and \( 1 - N \) in the \( 2n \)th position, and otherwise zero. (Here \( n \) is the number of grid points). \( T \) is a matrix with entries in terms of \( p, s, \) and \( \dot{a}_i \), that encodes the rate of transitions above, but with the \( n \)th and \( 2n \)th row all 0s and 1s to encode the summations \( \sum_i m_i^e = \frac{p}{p+s} = N \) and \( \sum_i m_i^u = \frac{s}{p+s} = 1 - N \).

Then we simply invert the matrix \( T \) to find the stationary distribution \( m = T^{-1}v \). This matrix is invertible because \( \dot{a}(a) \) is strictly decreasing, and so equals zero at just one point.

### A.3 Finding a transition path

Once we have the steady state, we can calculate the transition path following a shock as follows. First we guess a path of hiring probabilities \( p(t) \) and initial employment \( n(0) \). Given this path of hiring probabilities, we can iterate the household problem backward from the steady state using one step of the value function iteration algorithm used to solve for the steady state. At every step we must also calculate the borrowing constraint implied by the previously computed value function for unemployed households \( UU \).

Once we have the sequence of decision rules, we can iterate the initial asset distribution and employment forward to compute the path of asset demand. We then search for the path of hiring probabilities that makes asset demand zero at every point in time.

Because changing the hiring probability at any point in time has nonlinear effects on asset demand before and after this time, it is difficult to define a simple rule for updating the path of \( p(t) \). Directly searching for the path of \( p \) that satisfied \( A(t) = 0 \) at every
it is prohibitively computationally expensive. I instead search for values of \( p \) at several points, and interpolate the intermediate values, with more points immediately following the shock to capture the more complicated dynamics in this region. I confirmed that this method produces a very close approximation to \( A = 0 \), and varying the number of gridpoints at the margin does not alter the resulting dynamics.