Financial Intermediation and the Supply of Liquidity

Jonathan Kreamer

University of Maryland, College Park

November 11, 2012
Question

- Growing recognition of the importance of the financial sector.
- Relatively neglected: Role of financial system in liquidity provision.
- Two questions:
  1. How do changes in bank net worth affect the supply of liquidity, interest rates, and output?
  2. What is the role of government liabilities in providing liquidity over the business cycle?
Liquidity

- Liquidity here refers to assets that can serve as stores of value.
- Firms hold liquid assets as insurance against losing access to credit markets.
- Credit lines from banks serve as liquidity.
Basic Mechanism

Develop novel channel through which the financial sector can affect the real economy:

1. Liquidity is an input into production.
2. Liquidity may be scarce.
3. Financial intermediaries provide liquidity.
4. Intermediaries’ capacity for liquidity provision depends on their net worth.
5. Therefore changes in intermediary net worth affect the economy through the liquidity channel.
Results

- Lower outside liquidity or bank assets implies lower equilibrium investment, in some cases higher liquidity premium.
- If positive equilibrium liquidity premium, optimal for government to supply some public liquidity.
- For our benchmark specification, optimal supply of public liquidity decreasing in bank assets.
Related Literature

Environment

- Three periods \((t = 0, 1, 2)\).
- Unit measure of three types of agents: Households, entrepreneurs (firms), and intermediaries (banks).
- One non-storable good, used for both investment and consumption.
- All agents have linear utility with no discounting:

\[
    u(c_0, c_1, c_2) = c_0 + c_1 + c_2
\]
Households

- Households have a sufficient endowment of the good in each period that it never constrains their ability to lend.
- Thus supply of loanable funds is perfectly elastic at an expected return of 1.
Outside Liquidity

- Households enter period 0 holding a stock $\bar{l}$ of trees that yield a unit return of the good in period 2.
- Trees sell at a price $q$ in period 0 and can be resold in period 1.
- I refer to $\bar{l}$ as “outside liquidity”, $q$ as the “price of liquidity,” and $q - 1$ as the “liquidity premium.”
Firms

- Firms have initial endowment of $A$ in period 0, no endowment in other periods.
- In period 0, firms invest $I$ in projects that yield $\rho_1 I$ in period 2.
- During period 1, all projects must receive an injection of additional funds or be lost.
- A fraction $p$ of firms must inject $\rho_L I$, while a fraction $1 - p$ must inject $\rho_H I$, where $\rho_H > \rho_L$.
- I refer to $\rho$ as a “liquidity shock.”
Firm Pledgeability

- Firm borrowing is subject to **limited pledgeability** arising from moral hazard.
- Entrepreneurs have specific human capital that allows them to operate their project.
- Between period 1 and 2, every entrepreneur with a functioning project receives an opportunity to employ their specific human capital to generate private benefit $BI$.
- Let $\rho_0 = \rho_1 - B$.
- I call $\rho_0$ firm pledgeability, because in equilibrium, investors can receive no more than $\rho_1I - BI = \rho_0I$. 
I make several assumptions on parameters to restrict attention to the interesting case:

A1: $\rho_L < \rho_0 < \rho_H$
A2: $\rho_1 > 1 + p\rho_L + (1 - p)\rho_H$
A3: $\rho_0 < 1 + p\rho_L + (1 - p)\rho_H$
A4: $p\rho_0 < 1 + p\rho_L$
A5: $\rho_0 > p\rho_L + (1 - p)\rho_H$

A2 implies that it is socially optimal to invest all available funds in the entrepreneur’s projects.

A3 and A4 imply that this is not feasible since external financing is insufficient.
Banks sell credit lines that can be drawn on in period 1 to meet liquidity shocks. Credit lines have a credit maximum $M$.

Banks are also subject to an agency cost: they must receive at least profits $\pi = C \left( \frac{D}{1-p}, K \right)$, where $D$ are funds that banks receive from their borrowers in period 2.

I assume that $C(\cdot)$ is continuously differentiable, with $C_1(\cdot) > 0$ for $D > 0$, $C_{11}(\cdot) > 0$, $C_2(\cdot) < 0$, $C_1(\cdot) \to 0$ as $D \to 0$ and $C_1(\cdot) \to \infty$ as $D \to \infty$. 
Initial Investors

- In period 0, firms raise $I + ql - A$ from households.
- Initial investors are promised $R^I_L$ in the event of a low shock, and $R^I_H$ in the event of a high shock.
- This repayment must satisfy
  \[ pR^I_L + (1 - p)R^I_H \geq I + ql - A \]  \hspace{1cm} (1)
In period 1, firms receive a liquidity shock and choose continuation rules $\lambda_L \in 0, 1$ and $\lambda_H \in 0, 1$.

To continue, firms must obtain $\lambda \rho I$ funds from banks and from households.

Firms promise $R^1_L$ and $R^1_H$ to new investors in each state, which must satisfy

\begin{align*}
\lambda_H \rho_H I & \leq R^1_H + M \tag{2} \\
\lambda_L \rho_L I & \leq R^1_L \tag{3}
\end{align*}
Firms all purchase the same credit line $M$, with promised return $R^B_L$ and $R^B_H$.

Banks receive profits $\pi = pR^B_L + (1 - p)R^B_H$.

Incentive compatibility requires

$$pR^B_L + (1 - p)R^B_H \geq C(M, K)$$

(4)
Firms only have $\lambda \rho_0 I + l$ funds available in period 2 to repay all their creditors.

This pledgeability constraint can be expressed as

$$R^I_L + R^1_L + R^B_L \leq \lambda_L \rho_0 I + l \quad (5)$$

$$R^I_H + R^1_H + R^B_H \leq \lambda_H \rho_0 I + l \quad (6)$$
Firms choose the profit-maximizing contract

\[
\max_{R, \lambda, I, l} \quad p \left( \lambda_L \rho_1 I - R_L^I - R_L^1 - R_L^B + l \right) \\
+ (1 - p) \left( \lambda_H \rho_1 I - R_H^I - R_H^1 - R_H^B + l \right) \\
\text{s.t.} \quad (1) - (6) \\
R \geq 0, \ I \geq 0, \ M \geq 0, \ \pi \geq 0
\]

where \( R = \{ R_L^I, R_H^I, R_L^1, R_H^1, R_L^B, R_H^B \} \).
Proposition 5

Theorem
At the solution, $\lambda_L = 1$, and constraints (1) - (6) hold with equality.

Note: Will restrict attention to case that it is optimal for all firms to meet the high liquidity shock, and all firms choose the same contract with banks.
Leverage ratio

We can combine (1) - (6) to obtain an overall constraint arising from limited pledgeability

$$\rho_0 I + l \geq I + ql - A + [p\rho_L + (1 - p)\rho_H] I + \pi$$

We can write this as a leverage constraint

$$I \leq \frac{A - (q - 1) l - \pi}{\chi_1}$$

where \(\chi_1 = \frac{1 - \rho_0 + p\rho_L + (1 - p)\rho_H}{\chi_1}\).

What are \((q, l, \pi, M)\) in equilibrium?
Equilibrium

- From calculating the optimal contract, we have an arbitrage condition
  \[ q - 1 = C_1(M, K) \]

- From market clearing, we have
  \[ (\bar{l} - l) \cdot (q - 1) = 0 \]

- Calculating the available pledgeable funds in period 1, we have
  \[ \rho_H I = \rho_0 I + l + M \]

- Bank profits satisfy
  \[ \pi = C(M, K) \]
If $q = 1$, then the equations above imply $I = A/\chi_1$, $M = 0$. This will be an equilibrium as long as

$$\bar{l} \geq \frac{\left(\rho_H - \rho_0\right)A}{\chi_1}$$

Otherwise, $q > 1$ and equilibrium $(M, I)$ is the solution to the two equations

\[
I = \frac{A - \bar{l}C_1(M, K) - C(M, K)}{\chi_1}
\]

\[
I = \frac{M + \bar{l}}{\rho_H - \rho_0}
\]
Comparative Statics

Higher available liquidity $K$ or $\bar{l}$ implies higher equilibrium $I$.

\[
\frac{dI}{d\bar{l}} = \frac{\bar{l}C_{11}(\cdot)}{(\rho_H - \rho_0) \left[ \bar{l}C_{11}(\cdot) + C_1(\cdot) \right] + \chi_1} > 0
\]

\[
\frac{dI}{dK} = -\frac{\bar{l}C_{12}(\cdot) + C_2(\cdot)}{(\rho_H - \rho_0) \left[ \bar{l}C_{11}(\cdot) + C_1(\cdot) \right] + \chi_1} > 0
\]
Public Provision of Liquidity

- Government has distortionary tax instrument and perfect credibility.
- Government can issue bonds in period 0 that pay in period 2, pay using taxes in period 2.
- Issuing a bond equivalent to increasing $\bar{l}$. 
Public Provision of Liquidity

- Government issues bonds $x$ with a face value of 1, at a price $q$.
- The government imposes deadweight loss $D(x)$ to raise these funds. Suppose that $D(x)$ is convex is $x$.
- All equilibrium equations are unchanged with $x + \bar{l}$ in place of $l$.
- Increase in government bonds raises $I$ and lowers $q$. 
Optimal Public Liquidity

- Hard to get a nice result without functional forms.
- Assume deadweight loss \( C(x) = \frac{\sigma}{2} x^2 \) and bank agency costs
  \( C(M, K) = \frac{\psi}{2} \frac{M^2}{K} \)
- Then the optimal supply of government bonds is increasing in \( K \).
Conclusions

- Developed novel channel through which changes in bank net worth affects real activity.
- Found role for government provision of public liquidity.
- Under benchmark specification, countercyclical public liquidity provision is optimal.