3. Consider a continuous time model of search and matching in the labor market with the following features:

a) Firms have one worker.
b) An employer must post a vacancy to hire a worker with a per vacancy flow cost of c. Free entry drives the value of posting a vacancy to zero.
c) Workers and employers are matched via a matching function \( m(u,v) \) where \( u \) is unemployment, \( v \) is vacancies and the matching function exhibits constant returns to scale.
d) Only unemployed workers can search for jobs.
e) Workers who are not employed have reservation income equal to \( b \).
f) Employers face aggregate and idiosyncratic shocks with the productivity of a job equal to \( \bar{p} \bar{q} \) where \( p \) is the aggregate component of productivity and \( q \) is the idiosyncratic component of productivity drawn from a bounded distribution with a cumulative distribution function \( F(.) \). Firms enter at the upper bound of the distribution but face a probability \( \pi \) that they will get a new draw from this distribution at each instant of time. The aggregate shock takes on two values with the probability of switching states given by \( \Pi \).
g) Both workers and employers discount the future using the same interest rate \( r \).
h) Wages are determined by Nash Bargaining over the joint surplus with the bargaining power of employers equal to \( (1-\pi) \) and the bargaining power of workers equal to \( \pi \).

Answer the following questions:

i. If there is no aggregate uncertainty \( (p \) is constant), formally characterize the solution to the model in terms of job creation, job destruction, unemployment and vacancies. (Hint: Derive the steady state conditions and show graphically the properties of the steady state solution).

ii. Without formally deriving, describe what happens to job creation, job destruction, unemployment and vacancies in response to aggregate shocks when there is aggregate uncertainty.

iii. Discussion: Can this model account for the following observed empirical patterns? (Hint: If so, describe the mechanism in the model that yields this pattern. If not, discuss why the model cannot yield this pattern. Be as specific as possible in terms of the assumptions and properties of the model that are relevant for whether the model can account for the empirical pattern."

1. Procyclical job finding rate for unemployed workers and a countercyclical job filling rate for firms.
2. The volatility of aggregate job destruction is greater than the volatility of aggregate job creation.
3. The volatility of aggregate hires is greater than the volatility of aggregate separations.
4. The flow of workers who directly transit from one job to another without a spell of unemployment is procyclical.
5. The job filling rate for firms is increasing in the growth rate of the firm in the cross section.

iv. For all of your answers to part "iii" where you indicated that the model cannot account for the observed empirical pattern, discuss which of the assumptions in "a-h" above that likely needs to change to match the pattern in the data.