(a) Static problem: Can write as

\[ \text{Max } T \frac{w-1}{w} + N \frac{w-1}{w} + \mathbb{I}(EX - T - RN) \]

FOC's: \[ \frac{w-1}{w} T^{-\frac{1}{w}} = 1 \]

\[ \frac{w-1}{w} N^{-\frac{1}{w}} = \lambda R \]

\[ \Rightarrow N = T(R^{-w}) \]

Plug into budget constraint: \[ T + R^{1-w} T = EX \]

\[ T = \frac{EX}{1 + R^{1-w}} \quad N = \frac{EX \cdot R^{-w}}{1 + R^{1-w}} \]

Plug into definitions of \( p, c \):

\[ p \cdot EX \cdot \left[ \frac{w - \left(1 + R^{1-w}\right)^{1-w} + R \left(1 + R^{1-w}\right)^{1-w} \frac{w}{w}}{w} \right] = c \]

\[ p = \left(1 + R^{1-w}\right)^{1-w} \]
Key 8/09 Macro comp p1, page 2

Note that \( \frac{dp}{dR} = \frac{1}{1-w^*} R^{-w} = R^{-w} > 0 \)

(b) State variables: \( At \) (Endogenous)
\( R_t, Y_t, X_t \) (Exogenous)

Controls: \( T_t, N_t \)

Bellman Equation

\[
V_t(At) = \max_{T_t, N_t} \left[ u(T_t, N_t) + \beta E_t V_{t+1}(At+1) \right]
\]

FOC's: \( T_t; \quad C_t \frac{1}{\delta} - T_t \frac{1}{w} = \beta(1+r) E_t V'_{t+1}(At+1) \)

\( N_t; \quad C_t \frac{1}{\delta} - N_t \frac{1}{w} = \beta(1+r) R_t E_t V'_{t+1}(At+1) \)

Envelope: \( V'_t(At) = \beta(1+r) E_t V'_{t+1}(At+1) \)
Combining FOCs for \( T, N; \)
\[
R_t = \left( \frac{T_t}{N_t} \right)^{1/w}
\]
Plugging in (5): \( R_t = \left( \frac{T_t}{X_t} \right)^{1/w} \rightarrow \) increasing in \( T \)
Decreasing in \( X \)

Combining envelope and FOC for \( T_t \rightarrow \) Euler Equation:
\[
C_t^{1 - 1/w} T_t^{-1/w} = \beta (1 + r) E_t \left[ C_{t+1}^{1 - 1/w} T_t^{1/w} \right]
\]

Note if \( w = 0 \), we have
\[
T_t^{-1/w} = \beta (1 + r) E_t (T_t^{-1/w})
\]
And dynamics of \( T \) are independent of \( N \) or \( X \).

(C) Complete Markets Problem:
\[
\max \quad \sum_{t=0}^{\infty} \beta^t \pi_t(z_t) u(C_t(z_t))
\]
\( s.t. \ (2), (5'), \) and (6)
Assuming \( w = 0 \), FOC for \( T_t(z_t) \) simplifies to:
\[
\beta^t \pi_t(z_t) T_t^{1/w} \rightarrow \text{Multiplier on } (6)
\]
Note that since world tradeables output is constant and countries are small, we have
\[ \frac{Q_0(\tau t)}{\beta_t \pi(\tau t)} \]
will be independent of $\tau t$, so that $T^e(\tau t) = \bar{T}$ (constant for each country, although $\bar{T}$ may vary across countries)

So, under complete markets,
\[ C_t = \left( \frac{\bar{T} w^{-1} - X_t}{w} + X_t \frac{w^{-1}}{w} \right)^{\frac{w}{w-1}} \]
\[ P_t = (1 + R)^\frac{1-w}{w-1} = (1 + (\frac{
bar{T} - X_t}{X_t})^{\frac{1-w}{w-1}})^\frac{1}{1-w} \]

Note: Only shocks to $X_t$ affect $C$ or $P$, since under complete markets, tradeables consumption $T$ is fully insured against country-specific tradeables endowment shocks.

Since an increase in $X_t$ increases $C_t$ and reduces $R_t$ and $P_t$, we have perfect negative correlation of $(C, P)$.
Under incomplete markets,

\[ C_t = \left( \frac{w-1}{w} + \frac{w-1}{t_t w} \right)^{\frac{w}{w-1}} \rightarrow \text{Increasing in } T_t, X_t \]

\[ P_t = \left( 1 + \left( \frac{T_t}{X_t} \right)^{-1-w} \right)^{\frac{1}{1-w}} \rightarrow \text{Increasing in } T_t, \text{ Decreasing in } X_t \]

Note: now that positive tradeables endowment shocks will increase \( T_t \), which will increase both \( C_t \) and \( P_t \).

As before, positive non-tradeables endowment shocks will increase \( C_t \) while reducing \( P_t \).

So correlation is no longer perfectly negative.