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# THE ECONOMICS OF TIE-IN SALES

M. L. Burstein\*

A TIE-IN sale or lease is ordinarily defined as one in which the seller of the "tying" good requires that one or more other goods *used with* the tying good also be purchased from him. However, I intend to use the term more broadly and define a tie-in sale as one which simply requires that the purchaser of the tying good purchase his "requirements" of one or more "tied" goods from the seller of the tying good. Since I do not treat tie-ins requiring purchase of *specific quantities* of tied goods, it could be said that this paper is essentially concerned with full-line forcing. An important conclusion of the paper is that complementarity of the tied with the tying good is not essential to the *rationale* of a tie-in sale; all of the major results can be derived on the assumption that the tying and tied goods are independent in demand in the sense that  $\partial x_i / \partial p_j = 0$ . The tying arrangement is seen as a means of extracting the profit inherent in an "all or nothing" selling arrangement and can be analyzed in very general terms.<sup>1</sup> The model is static; the paper shows that tying arrangements *can* be viewed in a context apart from extension of monopoly or exclusion of entry. Of course, it does not follow that tying arrangements cannot be viewed dynamically. On the other hand, it is submitted that there are, for example, many cases of full-line forcing that cannot be explained by any hypothesis thus far advanced.

Let us begin by assuming that all of the factors of production and intermediate products of

the economy are sold in purely competitive markets as are all of the final products  $X_2, X_3, \dots, X_n$  but that  $X_1$  is monopolized.<sup>2</sup> We assume further that the producer of  $X_1$  is not a monopolist and that  $X_1$  is not so important a good that there are significant income effects incident to changes in its price. Finally, we shall assume that goods  $X_2, X_3, \dots, X_n$  are produced under conditions of constant cost, an assumption that becomes significant only for goods with respect to which the actions of the producer of  $X_1$  might cause meaningful changes in equilibrium output.

The story begins with the monopolist charging a single price,  $p_1^*$ , which permits him to maximize profits in the absence of price discrimination or tying arrangements. He is aware, however, that practically all of his customers are paying less for the quantities of  $X_1$  they are using than they would be willing to pay on an all-or-nothing basis, given  $[p_1, p_2, \dots, p_n]^*$ . Now assume that direct price discrimination or lump-sum exaction schemes are foreclosed, and recall that, regardless of his actions, the vector  $[p_2, p_3, \dots, p_n]^*$  will remain unchanged with respect to those not using  $X_1$  (recalling the assumption of constant costs). However, we assume it is possible for him to enforce the following requirement for purchase or lease of  $X_1$ :

Permission to use  $X_1$  requires purchase of all "requirements" of goods  $X_2, X_3, \dots, X_r$  (where  $r$  is less than  $n$ ) from me at prices  $p_2^{**}, p_3^{**}, p_r^{**}$ , the vector of prices  $[p_2, p_3, \dots, p_r]^{**}$ .<sup>3</sup>

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<sup>1</sup> A point on an all-or-nothing demand curve can be found by determining the maximum amount a consumer would pay for  $x$  units of a good if confronted with the choice of purchasing  $x$  units or none at all. It is, of course, true that, if we observe a consumer purchasing  $\bar{x}$  units in an unrestricted market at the going price  $\bar{p}$ , he would generally be willing to pay more than  $\bar{x}\bar{p}$  on an all-or-nothing basis for the  $\bar{x}$  units. The difference is old-fashioned consumer surplus.

<sup>2</sup> In what follows the  $X$  notation indicates the goods, the  $x$ 's rates of purchase, and the  $p$ 's the prices;  $[ ]^*$  indicates a vector of prices prevailing before the tying arrangement is imposed or one remaining unchanged afterwards;  $[ ]^{**}$  indicates a vector of prices prevailing after imposition of a tying arrangement. The number of consumers is indexed by the subscript  $j$ . Thus,  $x_{ij}$  indicates the rate of purchase of the  $i$ th good by the  $j$ th consumer. It is sometimes found convenient to assume that the tying firm is able to charge as many prices for one or more goods as there are consumers. On the other hand, multi-part tariffs with respect to any one consumer are excluded throughout.

<sup>3</sup> It is implicitly assumed that the producer of  $X_1$  can purchase desired quantities of tied goods ( $X_2, X_3, \dots, X_r$ ) in competitive markets and resell them at tied prices ( $p_2^{**}, p_3^{**}, \dots, p_r^{**}$ ) and that the equivalent of transport costs

It remains to examine the criteria employed in establishing the new price vector, including the question whether it is worth his while to tie goods independent in demand with  $X_1$  at the outset (where the sign of  $\partial x_i/\partial p_j$  is used as the criterion for "relatedness": goods are taken to be "independent" when  $\partial x_i/\partial p_j = 0$ , as "complementary" when  $\partial x_i/\partial p_j < 0$ , as "substitutional" when  $\partial x_i/\partial p_j > 0$ ). We do not consider problems of enforcing actual or prospective tying arrangements!

It is easiest to begin by considering the utility position of a consumer of  $X_1$ . This will enable us to develop notions about the limits of exploitation and eventually permit at least heuristic criteria for the determination of the optimal (profit-maximizing) vector  $[p_1, p_2, \dots, p_r]**$ . Once the tying arrangement is imposed with respect to  $X_2, X_3, \dots, X_r$ , the consumer is faced with a dichotomous all-or-nothing choice. He can either (a) accept the tying arrangement and allocate his income optimally among  $n$  goods in accordance with the price vectors  $[p_1, p_2, \dots, p_r]**$ ,  $[p_{r+1}, p_{r+2}, \dots, p_n]^*$  or (b) reject the tying arrangement and allocate his money income optimally among the  $n - 1$  goods  $X_2, X_3, \dots, X_n$  in accordance with the price vector  $[p_2, p_3, \dots, p_n]^*$ . We assume he will accept the first option if the resulting utility position is preferred to that stemming from the second.

A number of intriguing implications are already apparent. First, if the consumer accepts the tie-in, he would not reduce his rate of purchase of  $X_1$  at  $p_1^*$  if the tied goods are independent in demand with the tying good: the marginal cost to him of  $X_1$  is unchanged and  $\partial x_i/\partial p_j$  is assumed to be zero. The monopolist could extract some of his "victim's" consumer surplus from  $X_1$  entirely through his manipulation of the prices of tied goods. Secondly, the weight of theoretical considerations suggests that  $p_1$  will fall after the tie-in if the tying and tied goods are either independent or complementary in demand (the substitutional case is uncertain). Similarly, if the monopolist had previously exercised price discrimination between consumers (without multipart tariffs), the  $p_{1j}$ 's can be expected to fall. Taking the former case (and

can be ignored. Of course, he might produce some or all of the tied products himself. The analysis is seen to have special interest for study of the conglomerate firm.

assuming independence), upon establishment of a tie-in,  $x_1$  (taken as a function of  $p_1$ ) will be less for any given value  $p_1$  (the demand curve with axes labeled  $x_1, p_1$  will have shifted leftward) since a non-discriminating tie-in (a tie-in whose terms are the same for all customers) would result in *complete withdrawal* of some customers. Again, what might be called the "net profitability" of a price decrease tends to be higher than before to the extent that it encourages more customers to "stay," permitting collection of more profits from tied sales. There is at least one force working toward flattening out the demand curve for  $X_1$  after imposition of a tie-in: a given increase in price will now result not only in reductions in the rate of purchase on the part of non-withdrawing customers but also in complete withdrawal of others. On the other hand, to the extent that the demand curves are convex to the origin ( $\partial x_1/\partial p_1$  increasing with  $x_1$ ) a countering force is established. On balance, however, if the marginal cost curve for  $X_1$  is flat or slopes upward, these considerations lead to a strong presumption that  $p_1**$  will be less than  $p_1^*$ .<sup>4</sup> As for the price-discriminating

<sup>4</sup> For  $r = 2$  and marginal costs  $k_1$  and  $k_2$  (increasing marginal costs strengthen the argument), and denoting the tying firm's profit as  $\pi$ , the argument can be put formally. Prior to the tying arrangement (using partial notation for simplicity) we know that

$$\partial \pi / \partial p_1 = x_1^* + p_1^* \partial x_1 / \partial p_1 - k_1 \partial x_1 / \partial p_1 = 0, t = 0$$

and we want to show that the expression is negative if evaluated at  $p_1** = p_1^*$  at  $t = 1$ , that

$$\left. \begin{aligned} \partial \pi / \partial p_1 \Big|_{t=1} &= x_1** + p_1^* \partial x_1 / \partial p_1 \Big|_{t=1} + p_2** \partial x_2 / \partial p_1 \Big|_{t=1} \\ &\quad - k_1 \partial x_1 / \partial p_1 \Big|_{t=1} - k_2 \partial x_2 / \partial p_1 \Big|_{t=1} < 0. \end{aligned} \right.$$

Therefore, taking the *absolute* values of the partials, that

$$\left. \begin{aligned} (x_1^* - x_1**) + p_1^* \left[ \partial x_1 / \partial p_1 \Big|_{t=1} - \partial x_1 / \partial p_1 \Big|_{t=0} \right] \\ + p_2** \partial x_2 / \partial p_1 \Big|_{t=1} &> \\ k_1 \left[ \partial x_1 / \partial p_1 \Big|_{t=1} - \partial x_1 / \partial p_1 \Big|_{t=0} \right] \\ + k_2 \partial x_2 / \partial p_1 \Big|_{t=1} &. \end{aligned} \right.$$

The text shows that  $x_1^*$  exceeds  $x_1**$  where  $X_1$  and  $X_2$  are either independent or complementary and that  $\partial x_2 / \partial p_1$  is negative in these cases. Where  $X_1$  and  $X_2$  are substitutes, these results are uncertain since  $x_1**$  exceeds  $x_1^*$  for those *not withdrawing* and  $\partial x_2 / \partial p_1$  is positive with respect to those "remaining in the game" (but the effect of withdrawals is to exert an opposite influence on  $\partial x_2 / \partial p_1$  — cf. note 1).

case (assuming imperfect discrimination as above), the problem can be put heuristically as follows: we can expect the consumer's gain from additional output to exceed the monopolist's loss (in the absence of a tying arrangement). Consequently, if the tie-in yield (from sales of tied goods) can exceed this "loss," it would be profitable to reduce  $p_1$  (increase output of  $X_1$ ) after the tying arrangement is imposed.<sup>5</sup> Third-

By assumption of monopoly, we have  $p_1^*$  greater than  $k_1$  and we assume that  $p_2^{**}$  exceeds  $k_2$ .

The absolute values of the  $\partial x_1/\partial p_1$  terms are all that remain untreated. However, the text showed that rather artificial assumptions are necessary to find  $\partial x_1/\partial p_1$  absolutely greater at  $t=0$  than at  $t=1$  when evaluated at  $p_1^*$ . It follows that  $\partial \pi/\partial p_1$  can be expected to be negative if evaluated at  $p_1^*$  and will be even more so if marginal cost is increasing (since the absolute value of the right-hand side will then be less) in the case of independent and complementary goods. Finally, we conclude that, if the position of equilibrium is unique, the optimal price of the tying good is less after the tying arrangement than it was before in the case of tied independent and complementary goods.

We see that the conclusion is even stronger in the case of complementary goods than in the case of independence, since, in the former event,  $\partial x_2/\partial p_1$  is negative even for those "remaining in the game," and  $x_1^{**}$  is further reduced by virtue of reduction in the rate of purchase of  $X_1$  on the part of those remaining as a consequence of the increase in  $p_2$ . Thus the American Can Company once tied its cans to its can-closing machinery, renting the machines "below cost." Cf., James W. McKie, "The Decline of Monopoly in the Metal Container Industry," *American Economic Review*, Vol. XLV, *Proceedings*, R. B. Hefebower and G. W. Stocking, eds. (Homewood, Illinois, 1958), 96-104.

A tying arrangement can be used to make more effective a pricing scheme that calls for  $X_{r+1}$ , a very strong complement to  $X_1$  (probably with minuscule marginal utility if not used with  $X_1$ ) to be offered below cost to the users of  $X_1$ . (Cf. R. G. D. Allen, *Mathematical Analysis for Economists* (London, 1942), 359-62). In the absence of tying arrangements, this scheme can have the effect of both increasing the firm's profits and increasing the consumer surplus gained from the right to use  $X_1$ . It follows that a given tie-in requirement (offering tied goods above competitive prices) might now have a larger yield. The argument is on all fours with that of note 6 and, of course, suggests that the event (offer of related goods below cost) is more likely if a tying arrangement can also be imposed.

When goods substitutional with  $X_1$  are tied in (with  $[p_2, p_3, \dots, p_r]^{**}$  such that each element is higher than the corresponding element in  $[ ]^*$ ),  $x_1^{**}$  may exceed  $x_1^*$  and  $p_1^{**}$  may exceed  $p_1^*$ . Here an effect of the tie-in is to cause "acceptance" of  $X_1$ .

<sup>5</sup>The loss in profits from  $X_1$  (in the absence of a tying arrangement) is  $(MC - MR)dq = [dC/dq - p(1-1/N)]dq = A$ , where  $N$  is the absolute price elasticity of demand. The value to the consumer in money terms of his gain (recalling that this is a case of simple price discrimination without all-or-nothing conditions) can be approximated by

ly, simple criteria emerge as to which goods to tie in. Obviously, there is no point in choosing goods which the buyers of  $X_1$  were not previously purchasing. Furthermore, own-price elasticities of the tied goods should not be too "high." Little will be gained by the tying firm if the "victims" drastically reduce purchases of the tied goods at the tied prices. Thus, the seller of radio tubes might find it advantageous to tie in a good such as salt, imposing a substantial increase in the price of salt, but not so drastic that Option I becomes inferior to Option II in "too many" cases. The reader can doubtless supply numerous examples according to his fancy, but should keep in mind the problems of enforcement, the author's example not withstanding.<sup>6</sup>

Fourthly, under no circumstances can the monopolist achieve the gain permitted by a simple all-or-nothing imposition with respect to  $X_1$  (imposition of a lump-sum tax as a prerequisite for the right to purchase  $X_1$ , for example); here the perfectly-discriminating monopolist is in a position superior even to that of the tying monopolist who imposes different tying arrangements on different customers. This conclusion follows from standard criteria for welfare effects of poll taxes *vis-à-vis* excise taxes. Let us assume, for example, that the  $j$ th consumer is purchasing  $x_{1j}$  units of  $X_1$  at  $p_1^*$  before the tie-in is imposed and would be willing to pay  $\$A$  in addition to the amount  $\$x_{1j}p_1^*$  on an all-or-nothing basis, given  $[p_2, p_3, \dots, p_n]^*$ .<sup>7</sup> The tying arrangement is, of course, equivalent to a series of excise taxes on the  $r-1$  tied goods together with either a subsidy on  $X_1$  or possibly an excise tax on  $X_1$  in the case of tied substitutes. But, as

$-q \cdot dp$ , but, by definition of an elasticity,  $-q \cdot dp = \frac{p \cdot dq}{N} = B$ . We wish to show that  $B$  exceeds  $A$ , that

- (1)  $p \cdot dq/N > dC - p \cdot dq + p \cdot dq/N$ , that
- (2)  $p \cdot dq > dC$ , that
- (3)  $p > dC/dq$ .

But (3) is obviously true of a monopoly equilibrium position. We know that price exceeds marginal cost in the neighborhood of the initial price-output equilibrium.

<sup>6</sup>It should be stressed that, ideally, tie-ins would be discriminating; different tie-ins would be devised for different purchasers.

<sup>7</sup>Compare J. R. Hicks, *Value and Capital* (2nd ed., Oxford, 1946), 38-41 for the technique of deriving "a perfectly general representation of consumer's surplus independent of any assumption about the marginal utility of money."

Hotelling has shown in his classic article, a welfare loss valued by the consumer at  $\$A$  would be commensurate with a yield from him through the excises of approximately  $\$A - \frac{1}{2}(\Sigma \Delta p_i \Delta x_{ij})$ .<sup>8</sup>

Summarizing the argument up to this point, tying arrangements have been regarded as systems of excise taxes which are made effective by forcing the consumer into an all-or-nothing choice. We have seen that there is no necessary connection between the choice of tied goods and complementarity of tying and tied goods. But, of course, there are various practical considerations which lead us to expect that tying arrangements will more often concern goods that are used together: enforcement is likely to be much easier in this case, since the tying firm then has reasonably accurate knowledge of his customers' rates of use of the tied good (gauging this from sales of the tying good) and can detect "cheating"; in the real world it would be very difficult to tie goods not produced by the tying firm, and goods produced jointly are likely to be related in demand. Nevertheless, the "independent" case has definite empirical implication for the selling practices of conglomerate firms and for full-time forcing generally; all of us are aware of cases where dealers are required to take on "weak" lines in order to acquire the franchise for "strong" lines. Another case that comes to mind concerns an electronics manufacturer selling a wide range of goods, most at least distantly related, but having very different market power with respect to the various items. We would predict that a given selection of tied goods might reveal inclusion of a number much less related in demand to the tying good(s) than others excluded but also produced by the tying firm.

<sup>8</sup> Harold Hotelling, "The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates," *Econometrica*, VI (July 1938), 242 ff., reprinted in *Readings in the Economics of Taxation*, R. A. Musgrave and C. S. Shoup, eds. (Homewood, Ill., 1959), 139-67. The reader should note that the argument of the text is heuristic, since a monopolist with the power to impose a lump-sum tax would set  $p_1$  equal to marginal cost (compare note 5). Obviously,  $p_1^* > MC_1$ . But, if we begin with  $p_1 = MC_1$  (together with a lump-sum tax), and if a tying arrangement is wholly or partially substituted for the lump-sum tax, the potential profits of the firm are in all cases reduced. This is because the firm's profits are here equal to the "benefit" — the sum of consumers' and producer's surplus — associated with the production and use of  $X_1$ . Hotelling shows that this "benefit" will be reduced by the substitution of excise for poll taxes when the poll taxes are accompanied by marginal-cost pricing.

It remains to achieve fuller generality of the argument by taking up the case of a multiproduct firm producing a number of goods related in demand and/or production.<sup>9</sup> It has been made abundantly clear that the analysis of tied sales leads directly to the theory of the multiproduct firm; even when goods are initially independent in demand, the effect of a tying arrangement is to create dependence (such terms as  $\partial x_1 / \partial p_2$  become negative through the effects of complete withdrawal of some consumers. We proceed then to blend together the "conventional" analysis of the multiproduct firm with the special considerations already shown to pertain to a tying firm.

This might best be done by directly grafting our analysis onto the well-known treatment of a firm producing  $r$  goods, all related in demand and/or production, and a monopolist with respect to  $r-s$  of these, the other  $s$  goods (together with the remaining  $n-r+s$  goods in the economy) being sold in purely competitive markets (thus implicitly abandoning our assumption that all non-monopolized goods are produced at constant costs). We assume he has hitherto arrived at an optimal set of  $r-s$  prices for the monopolized goods ( $p_{r-s+1}^*, p_{r-s+2}^*, \dots, p_r^*$  are parametric for him). We now assume that the joint monopolist becomes aware of tying possibilities and forces consumers to purchase from him all "requirements" of  $X_{r-s+1}, X_{r-s+2}, \dots, X_r$  if they are to be entitled to buy  $X_1, X_2, \dots, X_{r-s}$ .<sup>10</sup> An heuristic statement of

<sup>9</sup> The problem can be given an explicitly monopolistic-competition cast, although it should be noted that this involves abandonment of the assumption that all nontying goods are sold in purely competitive markets. Assume that the tying firm produces seven goods and has a very strong market position with respect to  $X_1$  and  $X_2$  and a much weaker one for  $X_3, X_4, \dots, X_7$ . He might simply require that all those entitled to use  $X_1$  and  $X_2$  must *not* purchase  $X_3, X_4, \dots, X_{12}$ , goods produced by others and substitutional with one or more of  $X_3, X_4, \dots, X_7$ . This scheme would find the yield being obtained from increased demand for  $X_3, X_4, \dots, X_7$ . It could coincide with an attempt to monopolize the entire product field,  $X_3, X_4, \dots, X_{12}$  but need not. Its success depends upon the correlation of use of  $X_1$  and  $X_2$  with the use of  $X_3, X_4, \dots, X_{12}$ . Obviously, if the latter products are ordinarily *used with* the former, the correlation is established.

<sup>10</sup> In general there is no reason why he might not include as tied goods goods that he does not himself produce. If this is done, the additional goods are brought into the model in exactly the same way, as are  $X_{r-s+1}, X_{r-s+2}, \dots, X_r$ .

There is, of course, no reason why *all*  $r-s$  goods should be tying goods nor need all  $s$  goods be tied goods. The text

this model might be as follows: an initial "optimum" is obtained by the monopolist through the price vector  $[\hat{p}_1, \hat{p}_2, \dots, \hat{p}_{r-s}]^*$ , a position leaving the users of the  $r-s$  products with exploitable rent, some of which is extracted by imposing excises on the remaining  $s$  goods (although in general  $[\hat{p}_1, \hat{p}_2, \dots, \hat{p}_{r-s}]^{**}$  differs from  $[\ ]^*$ ). Here the effect of the tie-in is not to change the nature of the complex but to change some of the parameters and functional forms as a result of the all-or-nothing conditions.<sup>11</sup>

A necessary condition for profit maximization is that, where  $\Pi$  is the joint monopolist's profit,

$$\partial\Pi/\partial\hat{p}_1 - \partial\Pi/\partial\hat{p}_2 = \dots = \partial\Pi/\partial\hat{p}_r = 0,$$

makes this assumption in the interest of simplicity. Compare note 14.

<sup>11</sup> A more formal treatment of the tying monopolist's maximization criteria is easily shown. We assume below that  $X_1$  alone serves as the tying good and that goods  $X_1, X_2, \dots, X_{r-1}$  are related in demand but that  $X_r$  is unrelated to the others. Goods  $X_2, X_3, \dots, X_r$ , all of which are available in the free market at marginal cost, are tied (showing the transition from single to multiproduct firm analysis through the tie-in in the previous analysis. Total cost (recalling that the monopolist can buy the tied goods at marginal cost and resell them if he doesn't produce them) is denoted as  $C$  and is a function of all  $r$  goods. The necessary conditions for profit maximization become

$$\begin{aligned} I. \quad & x_1 + \hat{p}_1 \partial x_1 / \partial \hat{p}_1 + \dots \\ & + \hat{p}_r \partial x_r / \partial \hat{p}_1 = (\partial C / \partial x_1) (\partial x_1 / \partial \hat{p}_1) + \dots \\ & + (\partial C / \partial x_r) (\partial x_r / \partial \hat{p}_1) \end{aligned}$$

$$\begin{aligned} R. \quad & x_r + \hat{p}_1 \partial x_1 / \partial \hat{p}_r + \dots \\ & + \hat{p}_r \partial x_r / \partial \hat{p}_r = (\partial C / \partial x_1) (\partial x_1 / \partial \hat{p}_r) + \dots \\ & + (\partial C / \partial x_r) (\partial x_r / \partial \hat{p}_r). \end{aligned}$$

With respect to the  $j$ th consumer, the profits function can be discontinuous. Thus, given a price vector  $[\hat{p}_1, \hat{p}_2, \dots, \hat{p}_{r-1}]^{**}$ , there will be a range of variation for  $\hat{p}_r$  for which  $\partial x_{i,j} / \partial \hat{p}_r = 0$  (where  $i$  runs from 1 to  $r-1$ ). But, once  $\hat{p}_r$  attains a certain value, he might withdraw altogether, driving all the  $x_{i,j}$ 's to zero. On the other hand, when we deal with many consumers, each faced with the same prices but each having a different joint-demand function, the profits function can be assumed continuous and such expressions as  $\partial x_i / \partial \hat{p}_r$  will be negative, reflecting absolute withdrawal of some purchasers. With respect to the  $j$ th consumer only, we know that, if the tying firm is maximizing profits *with respect to him*,  $\partial x_{i,j} / \partial \hat{p}_r = 0$  (where  $i$  runs from 1 to  $r-1$ ), and that equilibrium  $\hat{p}_r$  (in this special sense) is that for which  $\partial \pi_j / \partial \hat{p}_r = 0$ .

Such expressions as  $\partial x_3 / \partial \hat{p}_2$  are now harder to interpret,  $X_3$  and  $X_2$  could be substitutes, but  $\partial x_3 / \partial \hat{p}_2$  could be negative in equilibrium. Taking  $[\hat{p}_1, \hat{p}_2, \dots, \hat{p}_r]^{**}$ , it is possible that for a small increase in  $\hat{p}_2$ , sales of  $X_2$  will fall more as a result of complete withdrawals than they will rise as a result of greater purchases of  $X_2$  by those remaining. The model reduces to that of page 69 when  $r-s=1$  (as here) and all goods in the tying complex are initially independent in demand and production with respect to each of the other  $r-1$  goods.

recalling that the  $\hat{p}$ 's are elements of the vector  $[\hat{p}_1, \hat{p}_2, \dots, \hat{p}_r]^{**}$ . Once again the  $j$ th consumer accepts the tie-in so long as the utility position associated with the choice of  $n$  goods with price vectors  $[\hat{p}_1, \hat{p}_2, \dots, \hat{p}_r]^{**}$ ,  $[\hat{p}_{r+1}, \hat{p}_{r+2}, \dots, \hat{p}_n]^*$  is preferred to that associated with the choice of  $n-r+s$  goods with the price vector  $[\hat{p}_{r-s+1}, \hat{p}_{r-s+2}, \dots, \hat{p}_n]^*$ . In the event that the chosen tying arrangement finds all  $s$  tied goods initially independent in demand and production with respect to the tying good, the tying aspect of the joint monopolist's policy is reduced to the same terms as that of the simple (single-product) monopolist.<sup>12</sup>

Before leaving what might be called the pure theory of tying arrangements, let us note the possibility of a tying arrangement permitting "metering" demand to achieve an approximation to a multi-part tariff.<sup>13</sup> A frequently cited example of such a device is the practice of the International Business Machine Corporation of requiring that lessees of its computing machines purchase the associated punch cards from IBM (we are not interested here in alternative explanations of the IBM Case; if the reader wishes he can take what follows as a mere hypothetical example). Since it is evident that the lessees using the machines more intensively would be precisely those using more cards, we see at once that this class of tying arrangement has real advantages, *ceteris paribus*, over the hypothetical radio-tube and salt tie-in discussed above. After all there is no reason why those whose demand for salt is more intensive should be those willing to pay higher prices for radio tubes; the "metering-device" tie-in opens attractive possibilities for price discrimination. The punch-card example illustrates other aspects of the ideal tied good: the relevant elasticity of substitution is low and the demand for punch cards is correspondingly inelastic; punch cards contribute relatively little to the cost of computing (when the full carrying cost of the machine is considered) so that a given increase

<sup>12</sup> Including the conclusion that no purchaser who accepts the tie-in would reduce his purchases of  $X_1, X_2, \dots, X_{r-s}$  at  $[\hat{p}_1, \hat{p}_2, \dots, \hat{p}_{r-s}]^*$ .

<sup>13</sup> Cf. Ward S. Bowman, Jr., "Tying Arrangements and the Leverage Problem," *Yale Law Journal*, November 1957, at pp. 23-24. This excellent article has been of great benefit to me.

in their price has relatively little effect on the demand for computers despite the fact that punch cards and computers are strong complements.<sup>14</sup> Our argument, however, has been that

<sup>14</sup> Similarly, it makes much more sense to tie seat covers to automobiles than *vice versa*, a further extension of the *caveat* that the ideal tied good is not necessarily complementary with the tying good and that the joint monopolist

if a computer manufacturer is prevented from tying punch cards to computers, he could profitably select salt as second best, at least in the worst of all possible worlds.

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will not in general make all of the  $r$ - $s$  monopolized goods tying goods nor will he make all of the remaining  $s$  goods he produces (in competitive markets) tied goods.